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# A Computer Program for the Computation of Running Gear Temperatures Using Green's Function

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# A COMPUTER PROGRAM FOR THE COMPUTATION OF RUNNING GEAR TEMPERATURES USING GREEN'S FUNCTION

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## ABSTRACT

A new technique has been developed to study two dimensional heat transfer problems in gears. This technique consists of transforming the heat equation into a line integral equation with the use of Green's theorem. The equation is then expressed in terms of eigenfunctions that satisfy the Helmholtz equation, and their corresponding eigenvalues for an arbitrarily shaped region of interest. The eigenfunction are obtained by solving an integral equation. Once the eigenfunctions are found, the temperature is expanded in terms of the eigenfunctions with unknown time dependent coefficients that can be solved by using Runge-Kutta methods. The time integration is extremely efficient. Therefore, any changes in the time dependent coefficients or source terms in the boundary conditions do not impose a great computational burden on the user. The method is demonstrated by applying it to a sample gear tooth. Temperature histories at representative surface locations are given.

## INTRODUCTION

The cooling of gears is an important problem that has been studied for a number of years. An early model of oil cooling is given in DeWinter and Block (1972). El-Bayoumy et al. (1989) expands on the model of DeWinter and Block (1972) mainly by noting the importance of the Coriolis force on oil cooling of gears. El-Bayoumy et al. (1989) also develops a finite element model of the gear tooth. We now believe that the finite element method has speed and accuracy limitations and have abandoned this approach in favor of the Green's function method described herein.

In the next section, a new dynamic, accurate and efficient solution method for two dimensional heat transfer problems in gears is described. The solution method consists of transforming the heat equation into a line integral equation with the use of Green's functions with unknown

time dependent coefficients. The transformed integral equation is used to obtain the dynamic equations for the time dependent coefficients for each eigenfunction. In order to obtain the eigenvalues and corresponding eigenfunctions, the Helmholtz equation for the eigenfunctions is transformed into a line integral equation by the use of the two-dimensional free space Green's function. The integral equation is discretized into a set of homogenous simultaneous equations. The discretized version of the eigenfunction can be obtained by solving a set of homogenous simultaneous equations. The obtained dynamic equations can be integrated extremely efficiently. Therefore, any changes in the boundary conditions do not impose a great computational burden. In the third section, the computational results and a discussion is presented.

## FORMULATION

In this section, an accurate and efficient solution method for solving a time dependent two dimensional heat problems in gears is developed. The temperature field is expanded in terms of the eigenfunctions with unknown time dependent dynamic coefficients. The dynamic equation for the time dependent coefficients of each eigenfunction is obtained by the use of an integral equation.

Consider a gear tooth geometry shown in Fig. 1. The transient heating of the gear tooth is described by

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \quad (1)$$

where  $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$  and  $\alpha$  is the thermal diffusivity. The boundary condition for the left side and top of the gear is given by

$$K \frac{\partial T}{\partial n} = h(\xi) (T(\xi) - T_c) \quad (2)$$

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where the equation represents the heat output to cooling oil,  $\bar{\xi}$  is the vector coordinate on the boundary,  $K$  is the thermal conductivity,  $n$  is the outward pointing normal unit vector,  $h$  is the heat transfer coefficient and  $T_c$  is the temperature of the cooling oil. The boundary condition for loaded side (meshing surface) of the gear (right side in Fig. 1) determines the heat conduction into gear body and is given by

$$K \frac{\partial T}{\partial n} = F_0(\bar{\xi}) \quad (3)$$

where  $F_0$  is the heat flux into the gear and the remainder of the boundary is described by the vanishing normal derivative of the temperature,  $\partial T/\partial n = 0$ .

### Dynamical Equations for Transient Heat Flow

Rather than solve the physical equations directly, as in a finite element method or finite difference method, we develop here a Green's function method that reduces the two-dimensional problem to a one-dimensional line integral over the gear tooth boundary. The line integral equation yields the dynamical equations. This procedure yields a computational advantage over the previous approaches where there is no reduction in the problem dimensionally and the time and spatial integrations are performed simultaneously.

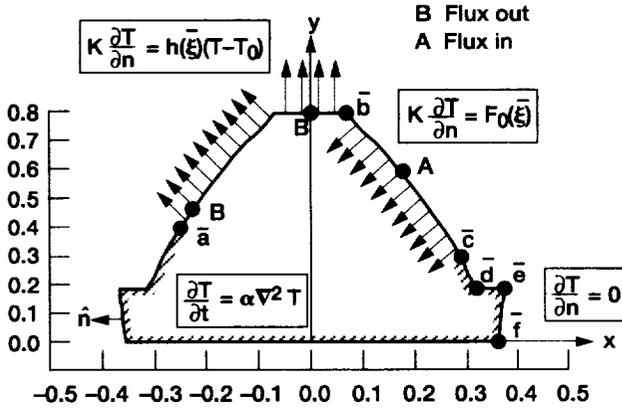


Figure 1.—Simulated gear tooth geometry and all the boundary conditions.

We assume, the following eigenfunctions,  $\psi_n$ , and eigenvalues,  $k_n$  are known and the eigenfunctions satisfy the Helmholtz equation,

$$(\nabla^2 + k_n^2)\psi_n = 0 \quad (4)$$

inside the gear tooth with vanishing normal derivative,  $\partial\psi_n/\partial n = 0$ . (The method for obtaining the eigenvalues and eigenfunctions is given in the following section.) It can be shown that these eigenfunctions are orthogonal. The eigenfunctions will be normalized to unity by  $\iint \psi_n^2 dx dy = 1$ .

The heat equation is transformed into a line integral equation by using Green's theorem, (Wyld, 1972)

$$\iint (T\nabla^2\psi_n - \psi_n\nabla^2T) ds = \int \left( T \frac{\partial\psi_n}{\partial n} - \psi_n \frac{\partial T}{\partial n} \right) dl \quad (5)$$

where  $ds$  is the surface element and  $dl$  is the line element. Substituting Eqs. (1) and (4) into the left hand side of Eq. (5) yields

$$k_n^2 \iint T\psi_n ds + \frac{1}{\alpha} \iint \psi_n \frac{\partial T}{\partial n} ds = \int_{\bar{a}}^{\bar{c}} \psi_n \frac{\partial T}{\partial n} dl \quad (6)$$

where the vectors  $\bar{a}$  and  $\bar{c}$  in the integration limits are shown in Fig. 1 (a driven gear). Note that a considerable simplification has been accomplished for the line integral on the right hand side of Eq. (5). The only contribution comes from the boundary conditions described by the non-vanishing normal derivatives, Eqs. (2) and (3). Substitution of Eqs. (2) and (3) and expansion of the temperature field in terms of the eigenfunctions as

$$T(\bar{x}, t) = \sum_n \eta_n(t) \psi_n(\bar{x}) \quad (7)$$

yield a dynamic equations for the time dependent coefficients,  $\eta_n$ ,

$$\dot{\eta}_n + \alpha k_n^2 \eta_n = \frac{\alpha}{K} \left[ \sum_m A_{nm} \eta_m + B_n + C_n \right] \quad (8)$$

where

$$A_{nm} = \int_{\bar{a}}^{\bar{b}} h(\bar{\xi}) \psi_m(\bar{\xi}) \psi_n(\bar{\xi}) dl \quad (9a)$$

$$B_n = -T_c \int_{\bar{a}}^{\bar{b}} h(\bar{\xi}) \psi_n(\bar{\xi}) dl \quad (9b)$$

$$C_n = \int_{\bar{b}}^{\bar{c}} F_0(\bar{\xi}) \psi_n(\bar{\xi}) dl \quad (9c)$$

The line integrals are easily calculated as the boundary conditions are changed. Therefore, the calculation burden for changing the boundary conditions is minimal. These coupled first order equations in Eq. (8) are integrated efficiently with Runge-Kutta method.

### Eigenvalues and eigenfunctions

The method for obtaining the eigenvalues and eigenfunctions used to effect the simplification of the previous section is given below. These functions are two dimensional analogs of the trigonometric functions used in Fourier methods.

The eigenvalues,  $k_n$  and the corresponding eigenfunctions,  $\psi_n$  are obtained through the use of two dimensional free space Green's function (Koshigoe and Tubis 1989),

$$G(\bar{x}|\bar{x}') = \frac{1}{4} H_0^{(1)}(k_n |\bar{x} - \bar{x}'|) \quad (10)$$

where  $H_0^{(1)}$  is the Hankel function of the first kind and the Green's function satisfies the inhomogeneous Helmholtz equation,

$$(\nabla^2 + k_n^2)G(\bar{x}|\bar{x}') = -\delta(\bar{x} - \bar{x}') \quad (11)$$

where  $\delta(\bar{x})$  is the delta function.

Again utilizing Green's theorem, one obtains

$$\begin{aligned} & \iint (G(\nabla'^2 + k_n^2)\psi_n - \psi_n(\nabla'^2 + k_n^2)G) ds' \\ &= \int \left( G(\bar{x}|\bar{\xi}') \frac{\partial \psi_n}{\partial n'} - \psi_n \frac{\partial G(\bar{x}|\bar{\xi}')}{\partial n'} \right) dl' \end{aligned} \quad (12)$$

With the use of Eqs. (4) and (11) and vanishing normal derivative boundary condition for the eigenfunctions the integral equation is simplified

$$\psi_n(\bar{x}) + \int \psi_n(\bar{\xi}') \frac{\partial G(\bar{x}|\bar{\xi}')}{\partial n'} dl' = 0 \quad (13)$$

This integral equation yields the value of  $\psi_n$  at any location inside the gear tooth when the values of  $\psi_n$  on the gear tooth boundary are known. Now let  $x$  approach a point,  $\bar{\xi}$  on the gear tooth boundary, then Eq. (13) becomes

$$\beta_{\bar{\xi}} \psi_n(\bar{\xi}) + P \int \psi_n(\bar{\xi}') \frac{\partial G(\bar{\xi}|\bar{\xi}')}{\partial n'} dl' = 0 \quad (14)$$

where  $\beta_{\bar{\xi}}$  is the contribution from the singularity in the integrand (Burton and Miller 1971) and is given by:

$$\beta_{\bar{\xi}} = \text{inside angle at the point } \bar{\xi} / 2\pi \quad (15)$$

and  $P$  designates the Cauchy principal value integral. The eigenfunction,  $\psi_n$  is discretized in Eq. (14) and yields a set of simultaneous equations. The eigenvalues,  $k_n$  are determined by setting the determinant of the simultaneous equations to zero. Once the eigenvalues are determined, the corresponding eigenfunctions can be obtained through the simultaneous equations. The formulation developed in this section can now be applied to the gear tooth geometry (shown in Fig. 1) and the calculation result discussed.

### SAMPLE CALCULATION

The technique developed in the previous section is applied to the gear tooth geometry shown in Fig. 1. Various coordinates are labeled in Fig. 1

in order to specify the key features of the gear tooth geometry. The coordinates of these points are given in table 1. The physical constants used for the calculation are: the gear thermal diffusivity,  $\alpha = 0.452 \text{ ft}^2/\text{hr}$ ; the thermal conductivity,  $K = 25 \text{ Btu/hr/ft/F}$ ; the oil temperature,  $T_c = 200 \text{ F}$ ; the heat transfer coefficient,  $h = 0.34 \text{ Btu/sec/ft}^2/\text{F}$  follows from DeWinter and Block (1972) and El-Baypoumy et al. (1989).

The heat flux,  $F_0$ , along the boundary from the location  $\bar{\xi} = \bar{b}$  to  $\bar{c}$  (shown in Fig. 1), is given as a function of the distance measured from the point  $\bar{b}$  (shown in Fig. 2). This is the heat generated in mesh for a 1-inch wide gear, with a pitch radius of 6 inches, rotating a 10,000 rpm and transmitting 500 hp. We are interested in gear steady-state temperatures that take hundreds of seconds to reach. Hence, the detailed temperature changes that occur as the gear goes in and out of mesh cannot be resolved. Thus, both the heating and the cooling have been averaged over a complete revolution cycle.

Table 1.—Coordinate of Points in Figure 1

Figure 1 location	x, inches	y, inches
$\bar{a}$	-0.197	0.541
$\bar{b}$	.073	.795
$\bar{c}$	.274	.337
$\bar{d}$	.318	.184
$\bar{e}$	.371	.184
$\bar{f}$	.362	.000

The temperature calculation was performed using 21 eigenmodes. (This was found to provide adequate convergence.) The results presented herein were generated on a 486-PC with a total running time, including the costly eigenfunction generation, of less than one hour.

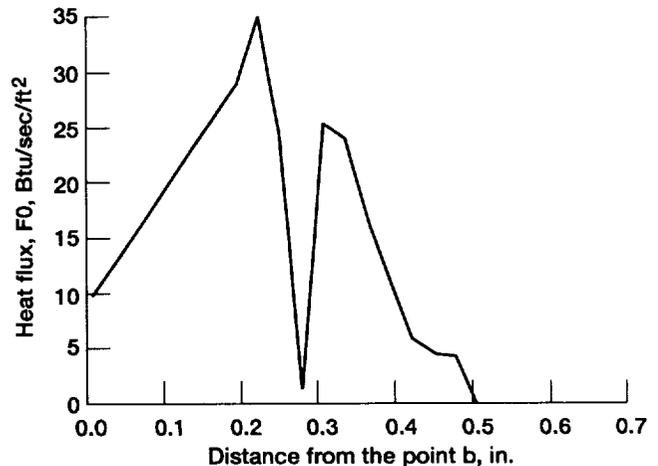


Figure 2.—Time-averaged heat input to gear.

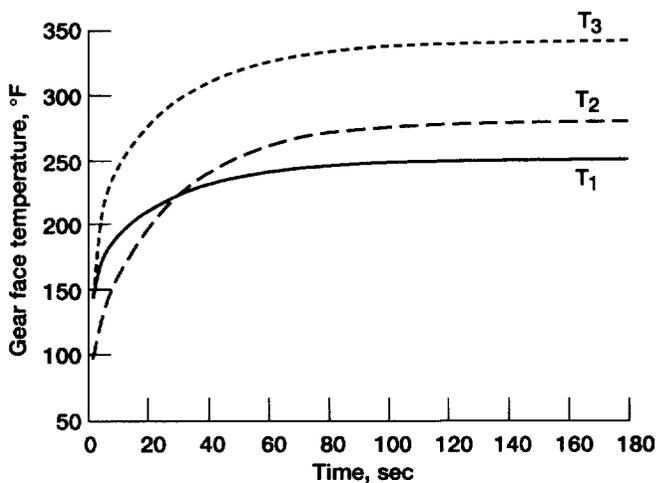


Figure 3.—Gear surface temperature at selected locations.

The eigenvalues and the corresponding eigenfunctions are obtained through the use of discretized version of Eq. (14). The discretization is carried out with 40 line elements and on each element, the second order approximation is used to represent the geometry as well as the eigenfunctions. The temperature calculations at three different locations on the gear tooth boundary,  $(0.0, 0.397)$  represented by the solid line,  $(-0.228, 0.066)$  by the dashed line, and  $(0.174, 0.194)$  by the dotted line are shown in Fig. 3 as functions of time. The curve labeled T<sub>1</sub> is centered on the top of the gear at location  $(0.0, 0.795)$ . T<sub>2</sub> on the cooled side of the gear in the region cooled by the oil at  $(-0.228, 0.464)$ . T<sub>3</sub>, the highest curve, is on the side heated during mesh, slightly above the pitch point at  $(0.174, 0.592)$ . Because both the heating and cooling functions have been averaged over a complete gear revolution, these temperatures can best be interpreted as out-of-mesh temperatures.

At the present time the calculation method is hard wired for a single, but representative, problem. Furthermore, we have not yet included the triangular portion of the gear extending to the axis of rotation. Generalization of the method is planned now that its application to a specific problem has been demonstrated. Appendix A "Thermal Analysis of Spur Gears" provides the geometric formulation input.

## CONCLUSION

A new technique has been developed to study two-dimensional heating of gears. The computational advantage of this technique over

previous approaches using the finite element method or the finite difference method results from two features: First, the problem is reduced from two dimensions to one. Second, the time and spatial integrations are separated. Therefore, when compared with other methods, this new technique can provide substantial improvements (one order of magnitude) in computational speed. However, the benefit of the dimensional reduction is manifested not only in the computation speed but also in the ease of problem set-up since one is required to deal with the boundary not the entire two-dimensional gear geometry. The other benefit of this technique, based on the separation of time and spatial integration, is accuracy. This technique takes the full advantage of the spectral method that has exponential solution convergence. This should be compared to finite element or difference methods where only algebraic convergence is possible.

Since the new technique is extremely efficient, any changes in the time dependent coefficients or source terms in the boundary conditions do not impose a great computational burden on the user. This result is very important when performing accurate Scoring Analysis in gears. This allows the bulk (or blank) temperature to be accurately known. Furthermore, the gear out-of-mesh temperature is not a constant along the tooth profile at steady state running conditions, as is often assumed by gear engineers. The method is also more adaptable for use in small, lubricated, concentrated contacts, such as gears, since high resolution can be obtained without using large numbers of elements.

Currently, we are planning to extend this technique to a multiblock application that includes the remainder of the gear sector and that further optimizes the computation accuracy and speed.

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The polar/vectorial angle  $\theta$ , in Fig. 1 may be calculated from:

$$\theta = (x^2 + y^2 - R_b^2)^{1/2} \frac{1}{R_b} = \frac{(R^2 - R_b^2)^{1/2}}{R_b}$$

$$3. \quad -\tan^{-1} \left[ \frac{(R^2 - R_b^2)^{1/2}}{R_b} \right] = \tan \phi - \phi = \text{inv} \phi$$

where:  $R = \frac{N}{2P}$  &  $R_o = \frac{N+2}{2P}$  &  $R_b = R \cos \phi$

so that the pressure angle  $\phi$  in Fig. 1 is:

$$4. \quad \theta = \tan^{-1} \frac{(R^2 + R_b^2)^{1/2}}{R_b} = \cos^{-1} \frac{R_b}{R} \text{ and the roll angle is } \epsilon = \theta + \phi =$$

$\tan \phi$ . The important radii of curvature equations, some at critical locations, are shown below in equation set 5, and see Figs. 1 and 2:

$$5. \quad \rho = R \sin \phi = \epsilon R_b \text{ and for the mating gear } \rho_m = R_m \sin \phi = \epsilon_m R_{bm}$$

The lowest point of contact for the mesh is  $\rho_c$  calculated from:

$$\rho_c = C \sin \phi - (R_o^2 + R_b^2)^{1/2}$$

and

$$\rho_{cm} = (R_{om}^2 + R_{bm}^2)^{1/2}$$

$$R_c = \left[ \left( C \sin \phi - \sqrt{R_o^2 + R_b^2} \right)^2 + R_b^2 \right]^{1/2}$$

where  $C \sin \phi = L_a$  the line of action, and

$$R_{cm} = \left[ \left( C \sin \phi - \sqrt{R_{om}^2 + R_{bm}^2} \right)^2 + R_{bm}^2 \right]^{1/2}$$

is the lowest point on the mating gear. The radii of curvature at the lowest and highest points of single tooth contact may be calculated from:

$$\rho_l = (R_o^2 + R_b^2)^{1/2} \quad R_l = (\rho_l^2 + R_b^2)^{1/2}$$

$$\rho_h = C \sin \phi - (R_{om}^2 - R_{bm}^2)^{1/2} + p_b$$

$$R_h = (\rho_h^2 + R_b^2)^{1/2}$$

where the base circle pitch is  $p_b = \frac{2\pi}{N} \cos \phi$  and the circular pitch

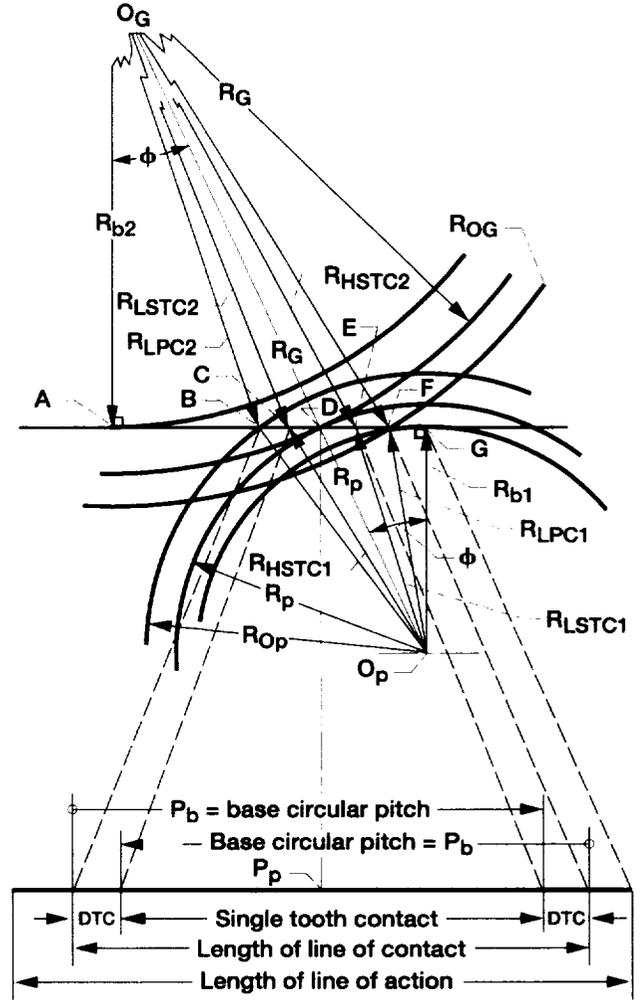


Figure 2.—Gear mesh line of action geometry.

is  $p = \frac{\pi D}{N}$ . The distance of roll/slide  $S$  along the involute curve may be calculated by integrating over the roll angle  $\epsilon$ , from (vogle, Ref. 17), see Fig. 1:

$$6. \quad S = \int_{\epsilon_1}^{\epsilon_2} R_b \epsilon d\epsilon = \frac{R_b}{2} \epsilon^2 \Big|_{\epsilon_1}^{\epsilon_2} = \frac{R_b}{2} (\epsilon_2^2 - \epsilon_1^2)$$

and since, as can be noted from Fig. 1,  $\rho = R_b \epsilon = R_b \tan \phi$  and  $\epsilon = \frac{\rho}{R_b}$

thus  $S_{1-2} = \frac{\rho_2^2 - \rho_1^2}{2R_b}$  over any arbitrary portion of the profile with sub-

scripts 1 and 2 and  $S_{c-o} = \frac{\rho_o^2 - \rho_c^2}{2R_b}$  over the whole profile from the first

point of mesh contact on the profile at  $S_c$  to the last point at outside diameter  $S_o$ . For example: from  $R_c$  to  $R_l$  to  $R$  (at pitch point) to  $R_h$  to  $R_o$  at the outside radius. The arc length along the base circle  $\Sigma$  can be

calculated from (where subscripts 1 and 2 are at arbitrary locations on the tooth profile) as shown.

$$7. \sum_{1-2} = R_b(\epsilon_2 - \epsilon_1) = R_b(\tan\phi_2 - \tan\phi_1) \text{ so that } \sum_{1-2} = R_b \left( \frac{\rho_2}{R_b} - \frac{\rho_1}{R_b} \right) = \rho_2 - \rho_1 \text{ and thus the ratio between the distance } S \text{ and the distance } \Sigma \text{ becomes } k = \frac{\rho_2 + \rho_1}{2R_b} \text{ a quantity useful in thermal calculations.}$$

The transition time, as a function of radii of curvature and its rolling velocity between any two points along the tooth, can be expressed in equation set 8 as:

$$8. \quad V = \frac{d\rho}{dt} = \frac{d\epsilon}{dt} R_b = R_b \omega \frac{\text{rad}}{\text{sec}}$$

where:

$$\frac{d\epsilon}{dt} = \omega, \text{ a constant and since } \rho = \epsilon R_b \text{ then } d\epsilon = \omega dt \text{ and } \frac{\omega}{\epsilon} dt = \frac{d\rho}{\rho}, \text{ so}$$

$$\text{that } \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho} = \ln \frac{\rho_2}{\rho_1} = \frac{\omega}{\epsilon_2 - \epsilon_1} (t_2 - t_1) = \frac{\omega(t_2 - t_1)}{R_b(\rho_2 - \rho_1)} \text{ and } \Delta t_{1,2} =$$

$$(t_2 - t_1) = \frac{\rho_2 - \rho_1}{R_b \omega} \ln \frac{\rho_2}{\rho_1} \text{ is the time it takes for the gear to rotate from } \epsilon_1 \text{ to } \epsilon_2. \text{ The critical dimensions along the line of action are shown in figure 2 and described below in equation 9:}$$

$$9. \quad Z = \sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C \sin \phi \text{ is the length of the "line of contact } b - f" \text{ as a subset of the "line of action } a - g."$$

Now we can calculate the width of the Hertzian band of mutual contact at the mesh point from equation 10: (Timoshenko Ref.18)

$$10. \quad B = \left( \frac{16 W_n (K_1 + K_2) \rho_1 \rho_2}{F(\rho_1 + \rho_2)} \right)^{1/2} \text{ where: } K_1 = \frac{1 - \nu_1^2}{\pi E_1} \text{ and}$$

$$K_2 = \frac{1 - \nu_2^2}{\pi E_2} \text{ and } K_1 + K_2 = \frac{2}{\pi} \left( \frac{1 - \nu^2}{E} \right) \text{ and since: } \rho_1 + \rho_2 = L_a = C \sin \phi$$

and  $W_n = W_t / \cos \phi = W_t \sec \phi$ , see Fig. 3, and  $E_1 = E_2 = E$

$$B = 3.19 \sqrt{\frac{W_t (1 - \nu^2)}{F \cos \phi C \sin \phi E}} (\rho_1 \rho_2)^{1/2}$$

The rolling velocity for the gear  $V_1$  and it's mating gear  $V_2$  are calculated from equation set 11:

$$11. \quad V_1 = \frac{n_1 \pi \rho_1}{360} \text{ ft/sec and } V_2 = \frac{n_2 \pi \rho_2}{360} \text{ ft/sec, so that the sliding velocity } V_s = V_1 - V_2 = \frac{\pi}{360} (n_1 \rho_1 - n_2 \rho_2) \text{ ft/sec where: } V_1 = V \text{ \& } V_2 = V_m.$$

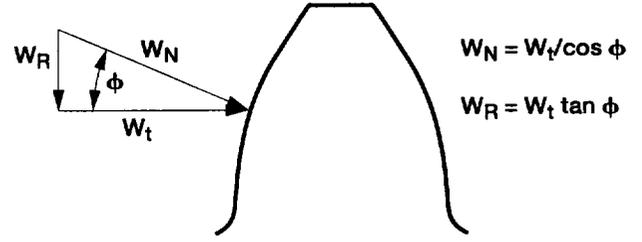


Figure 3.—Gear tooth profile showing normal load  $W_N$ .

Therefore the rolling velocity anywhere along the profile can be calculated from:  $V_1 = \frac{\pi \rho_1}{114.59} \text{ ft/sec}$  and  $V_m = \frac{n_m \rho_m}{114.59} \text{ ft/sec}$  for the mat-

ing gear so that the sliding velocity is calculated from:  $V_{is} = \frac{n \rho_i + n_m \rho_{im}}{114.59}$

ft/sec where  $n$  = speed of gear and  $n_m$  = the speed of the mating gear. Another value needed to calculate the coefficient of friction is the total

velocity from:  $V_t = \frac{n \rho_i + n_m \rho_{im}}{114.59} \text{ ft/sec.}$

We can now calculate the coefficient of friction as:

$$12. \quad f = 0.0127 \log_{10} \left( \frac{3.17 \times 10^6 W_t \sec \phi}{F \mu_{cp} V_s V_t} \right) \text{ where } F = \text{tooth face width,}$$

$\mu_{cp}$  = viscosity in centipoise and  $W_t$  = tangential tooth load in lbs. Thus we find ourselves in a position to calculate the instantaneous heat flux  $q(\rho_i)$  as a function of the radius of curvature at the instantaneous position "i" along the line of contact per equation Set 13.

$$13. \quad q(\rho_i) = \frac{W_t \sec(\phi) f (n \rho_i - n_m \rho_{im})}{170 330} \text{ BTU/min}$$

This equation can also be written in a form more useful using the pinion speed only:

$$q(\rho_i) = \frac{W_t \sec(\phi) n_p f}{170 330} (\rho_{pi} - m_g \rho_{gi}) \text{ BTU/min, where } m_g = \frac{N_g}{N_p} \text{ the}$$

gear ratio. At times it is more convenient to calculate the hear flux from the radius vector at the instantaneous contact points where:

$$qR_i = \frac{W_t \sec(\phi) n_p f}{170 330} \left\{ C \sec \phi (1 - m_g) - \left( R_{gi}^2 \Big|_{R_{gLPC}}^{R_{go}} - R_{gb}^2 \right)^{1/2} + m_g \left( R_{pi}^2 \Big|_{R_{pLPC}}^{R_{po}} - R_{pb}^2 \right)^{1/2} \right\} \text{ BTU/min}$$

## CLOSURE

This appendix develops the geometry analysis for the input for the computer solution of the thermal analysis of spur gears using Green's function to solve for gear blank surface temperatures. This

geometry analysis provides the gear tooth geometry input, the gear tooth sliding distance parameters, the rolling and sliding velocity inputs and the equations for the frictional heating developed during the gear tooth meshing, as a function of the location on the gear tooth.

Using these inputs the program can then determine the transient and steady state temperatures of the gear teeth.

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## NOMENCLATURE FOR APPENDIX A

B	width of the band of mutual contact at the mesh point	$R_o$	outside radius of the gear and end of the involute curve and $R_{om}$ for mating gear
C	center distance for mating pair	$R_l$ & $R_h$	radius vector to lowest and highest points of single tooth contact
E	Young's modulus of elasticity	V	velocity along the curve (involute)
F	face width	$V_s$	the sliding velocity at an instantaneous point $v_1 - v_2$ in the mesh
f	coefficient of friction	$V_t$	the total velocity at an instantaneous point in the mesh $v_1 + v_2$
$M_g$	gear ratio $N_g/N_p$	$V_l$	rolling velocity of the gear and its mating gear
N	number of teeth in gear and $N_m$ number of teeth in the mating gear	$W_t$ & $W_n$	tangential and normal (perpendicular) load, respectively
n	pinion speed, rpm, $n_m$ for mating gear and $n_p$ for pinion	x & y	Cartesian coordinates of the involute curve from its origin at the base circle
P	diametral pitch of the teeth	Z	length of the line of contact as a subset on the line of action
$P_b$	circular pitch of gear and its mating gear	$\Delta t_{1,2}$	the time it takes to rotate from $\epsilon_1$ to $\epsilon_2$
$qR_t$	heat flux calculated using the instantaneous radius vector	$\Delta N$	virtual number of teeth expansion or reduction for long and short addendums
$q(\rho_1)$	heat flux due to sliding friction at instantaneous point of contact using the radius of curvature as a parameter	$\epsilon$	roll angle on tooth
R	radius vector to the pitch point and $R_m$ for the mate at the contact point	$\theta$	involute polar angle = $\text{inv}\phi = \tan\phi - \phi = \epsilon - \phi$ (rad)
$R_b$	base radius of the involute curve (its origin) and $R_{bm}$ for mating gear	$\mu_{cp}$	oil viscosity, in cp
$R_c$	radius vector to lowest point of contact from center of gear and $R_{cm}$ for mating gear	$\rho$	radius of curvature from the base circle and $\rho_m$ for the mating gear at contact point
$R_{gb}$ & $R_{pb}$	base radius of gear and pinion respectively	$\rho_c$	radius of curvature at lowest or initial point of contact from base circle and $\rho_{cm}$ for mating gear at contact point
$R_{gi}$ & $R_{pi}$	instantaneous radius of gear and pinion respectively	$\rho_l$ & $\rho_h$	radius of curvature at lowest and highest points of single tooth contact
$R_{go}$ & $R_{po}$	outside radius of gear and pinion respectively	$\phi$	pressure angle of mesh
$R_{LPC2}$ & $R_{LPC1}$	lowest point of contact for gear and pinion respectively		

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