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Expanded Equations for Torque and Force on a Cylindrical Permanent Magnet Core in a Large-Gap Magnetic Suspension System

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Abstract

The expanded equations for torque and force on a cylindrical permanent magnet core in a large-gap magnetic suspension system are presented. The core is assumed to be uniformly magnetized, and equations are developed for two orientations of the magnetization vector. One orientation is parallel to the axis of symmetry, and the other is perpendicular to this axis. Fields and gradients produced by suspension system electromagnets are assumed to be calculated at a point in inertial space which coincides with the origin of the core axis system in its initial alignment. Fields at a given point in the core are defined by expanding the fields produced at the origin as a Taylor series. The assumption is made that the fields can be adequately defined by expansion up to second-order terms. Examination of the expanded equations for the case where the magnetization vector is perpendicular to the axis of symmetry reveals that some of the second-order gradient terms provide a method of generating torque about the axis of magnetization and therefore provide the ability to produce six-degree-of-freedom control.

Introduction

This paper develops the expanded equations for torque and force on a cylindrical permanent magnet core in a large-gap magnetic suspension system. The core is assumed to be uniformly magnetized, and equations are developed for two orientations of the magnetization vector. One orientation is parallel to the axis of symmetry, and the other is perpendicular to this axis. Fields and gradients produced by suspension system electromagnets are assumed to be calculated at a point in inertial space which coincides with the origin of the core axis system in its initial alignment with a reference inertial axis system. Fields at a given point in the core are defined by expanding the fields produced at the origin as a Taylor series. The assumption is made that the fields can be adequately described by expansion up to second-order terms. The expansion of the fields and gradients is presented in appendix A.

The equations for torques and forces on a magnetic core that are produced by a large-gap magnetic suspension system have been presented and discussed in a number of papers. For example, see references 1 through 6. The torques on the core are usually approximated as a function of the external or applied fields at the centroid of the core, and the forces on the core are usually approximated as a function of the gradients of the applied fields at the centroid. It is generally assumed that terms that are a function of second-order or higher gradients of the applied fields at the centroid can be neglected. In practical applications that involve large-gap magnetic suspension systems, these assumptions have proven to be valid. For an axisymmetric core, such as a cylinder, it can be shown that if the direction of magnetization is along the axis of symmetry, then the torque about that axis, produced by the applied fields and gradients of the applied fields, is always zero (ref. 3). Various methods of overcoming this constraint, which include shaping the core and using nonuniform three-dimensional magnetization, are discussed in references 4 and 7. However, examination of the expanded equations for a cylindrical core reveals that for the case of uniform magnetization perpendicular to the axis of symmetry, some of the second-order gradient terms provide a method of generating torque about the axis of magnetization and therefore the ability to produce six-degree-of-freedom control (ref. 8). For completeness, all gradient terms for expansion of the fields up to second order are presented.

Finally, instead of developing the torque and force equations from a set of governing equations that are a function of core volume, core magnetization vector, and suspension system fields and gradients, appendix B presents a development that begins at a more fundamental level in an attempt to provide better insight into the origin of these equations than is commonly available in the literature.

Symbols

A	area, m^2
a	radius of core, m
$\mathbf{B}$	magnetic flux density vector, T
$\tilde{\mathbf{B}}$	expanded magnetic flux density vector, T
$[\partial \mathbf{B}]$	matrix of field gradients, T/m
$[\partial \tilde{\mathbf{B}}]$	matrix of expanded field gradients, T/m
$\mathbf{F}$	total force vector on core, N
$\delta\mathbf{F}$	force vector on incremental volume of core, N
$\mathbf{I}$	coil current vector, A
l	length of core, m
$\mathbf{m}$	magnetic moment vector, A-m^2

$\delta\mathbf{m}$	magnetic moment vector of dipole with incremental volume, A-m^2
$\mathbf{M}$	magnetization vector, A/m
Q_m	pole strength, A-m
$\mathbf{r}$	position vector, m
$\mathbf{T}$	total torque vector on core, N-m
$\delta\mathbf{T}$	torque vector on incremental volume of core, N-m
$\delta\tilde{\mathbf{T}}$	torque on incremental volume of core about core origin, N-m
$\mathbf{T}_m$	inertial coordinate to suspended-element coordinate vector-transformation matrix
U	potential energy
v	core volume, m^3
W	work
δv	incremental volume, m^3
x, y, z	coordinates in orthogonal axis system, m
θ	Euler orientation for 3, 2, 1 rotation sequence, rad
∇	gradient operator

Subscripts:

x, y, z	components along x-, y-, z-axes, respectively
ij	partial derivative of i component in j-direction
$(ij)k$	partial derivative of ij partial derivative in k-direction

Matrix notations:

$[\ ]^T$	transpose of matrix
$\lfloor\ \rfloor$	row vector

A bar over a symbol indicates that it is referenced to suspended-element coordinates.

Magnetic Torques and Forces

The torques and forces on a cylindrical permanent magnet core are developed in this section by integrating the equations for torques and forces on an incremental volume of the core with magnetic moment $\mathbf{M}\delta v$ over the core volume. These equations are developed in appendix B. Figure 1 shows the cylindrical core and the core coordinate system. The core coordinate system consists of a set of orthogonal $\bar{x}, \bar{y}, \bar{z}$ body-fixed axes that are initially aligned with a set of orthogonal x-, y-, z-axes fixed in inertial space. In order to define the fields and gradients

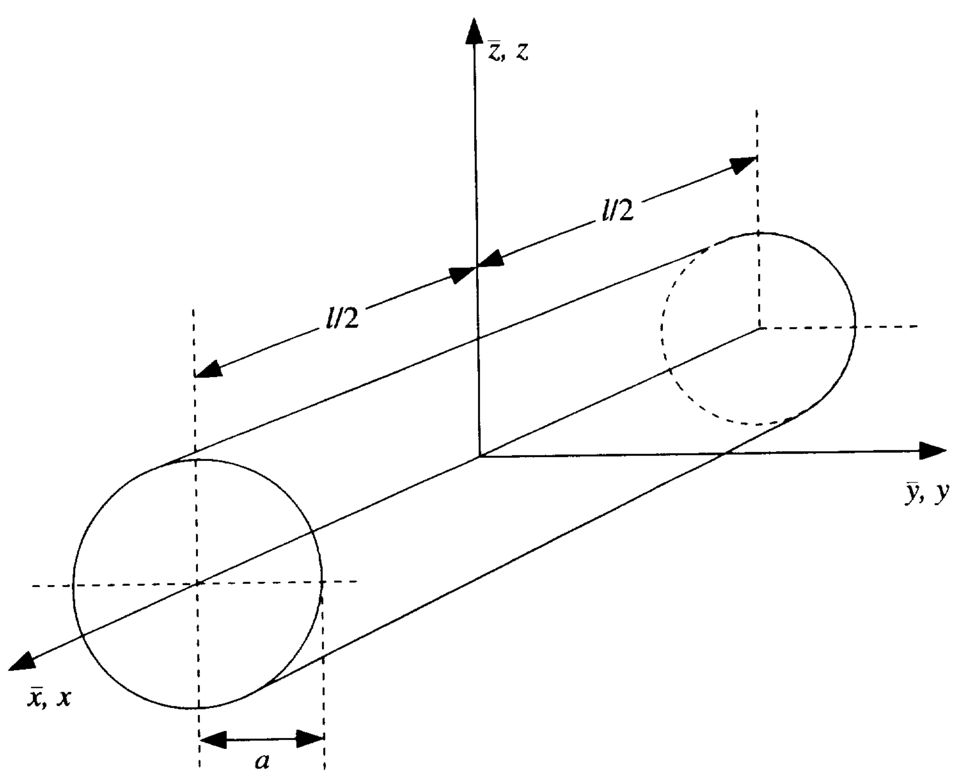

Figure 1. Core coordinate system.

at any point in the core, the fields and gradients at the origin of the core axis system are expanded as a Taylor series. It is assumed that the fields can be adequately described by expansion up to second-order terms. The expanded fields and gradients in both inertial and core coordinates are presented in appendix A. For simplicity in developing the equations in this section, relative motion between the core and the reference inertial coordinate system is assumed to be zero. This assumption removes the requirement to transform between the inertial and core coordinate systems and eliminates a significant number of components which are small relative to the fundamental terms in the equations when small-angle assumptions are used. In particular, the transformation of second-order gradient terms from inertial to core coordinates is very complicated, as illustrated by equation (A14). The torque on an incremental volume of the core, about the core origin, can be written as

$$\delta\bar{\tilde{\bar{\mathbf{T}}}} = \delta\bar{\mathbf{T}} + (\bar{\mathbf{r}} \times \delta\bar{\mathbf{F}}) \tag{1}$$

where $\delta\bar{\mathbf{T}}$ and $\delta\bar{\mathbf{F}}$ are the torque and force on the incremental volume due to the field at that location, and

$\bar{\mathbf{r}} = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$ is the position vector of the incremental volume (fig. 2). The total torque on the core can be written as

$$\bar{\mathbf{T}} = \int_v \delta\bar{\tilde{\bar{\mathbf{T}}}} = \int_v [\delta\bar{\mathbf{T}} + (\bar{\mathbf{r}} \times \delta\bar{\mathbf{F}})] \tag{2}$$

where the integration is over the core. Substituting equations (B20) and (B21) results in

$$\bar{\mathbf{T}} = \int_v \{(\bar{\mathbf{M}} \times \tilde{\mathbf{B}}) + [\bar{\mathbf{r}} \times (\bar{\mathbf{M}} \bullet \nabla)\tilde{\mathbf{B}}]\}\, dv \tag{3}$$

The total force on the core can be written as

$$\bar{\mathbf{F}} = \int_v (\bar{\mathbf{M}} \bullet \nabla)\tilde{\mathbf{B}}\, dv \tag{4}$$

The term $(\bar{\mathbf{M}} \bullet \nabla)\tilde{\mathbf{B}}$ can be written as (ref. 5)

$$(\bar{\mathbf{M}} \bullet \nabla)\tilde{\mathbf{B}} = [\partial\tilde{\mathbf{B}}]\bar{\mathbf{M}} \tag{5}$$

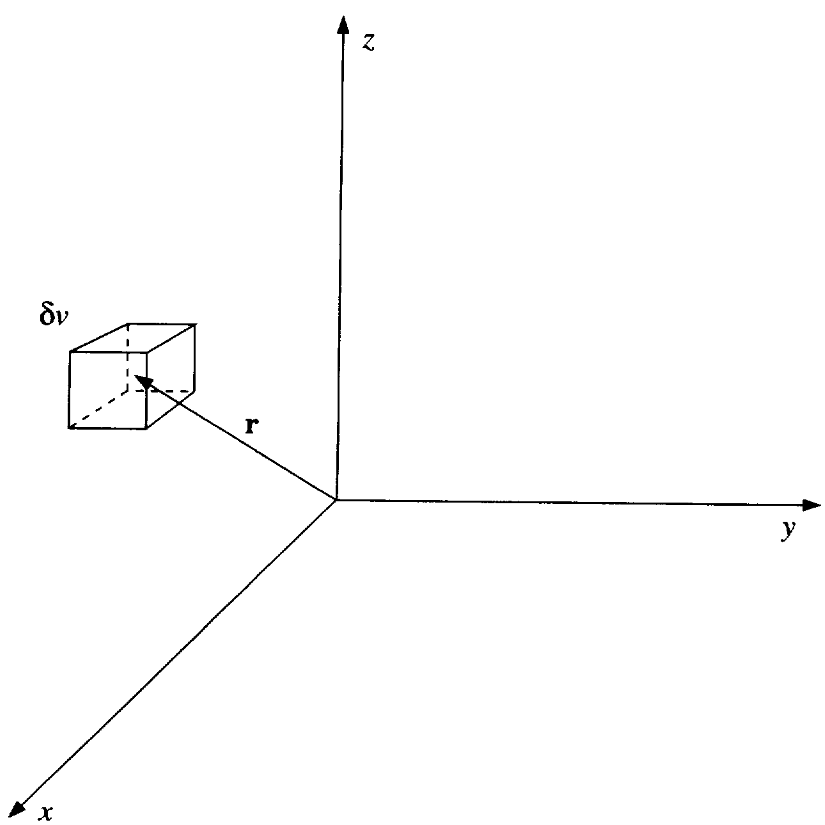

Figure 2. Incremental core volume.

where

$$[\partial\tilde{\mathbf{B}}] = \begin{bmatrix} \tilde{B}_{xx} & \tilde{B}_{xy} & \tilde{B}_{xz} \\ \tilde{B}_{yx} & \tilde{B}_{yy} & \tilde{B}_{yz} \\ \tilde{B}_{zx} & \tilde{B}_{zy} & \tilde{B}_{zz} \end{bmatrix} \qquad (6)$$

and

$$\overline{\mathbf{M}} = \begin{bmatrix} M_{\bar{x}} \\ M_{\bar{y}} \\ M_{\bar{z}} \end{bmatrix} \qquad (7)$$

In equation (6) the notation $f_{ij} = \partial f_i/\partial j$ is used.

Magnetization Along Axis of Symmetry

For orientation of the magnetization vector along the axis of symmetry (x-axis) of the permanent magnet core, the only nonzero term in $\overline{\mathbf{M}}$ is $M_{\bar{x}}$. Expanding equation (5) results in

$$[\partial\tilde{\mathbf{B}}]\overline{\mathbf{M}} = M_{\bar{x}} \begin{bmatrix} \tilde{B}_{xx} \\ \tilde{B}_{xy} \\ \tilde{B}_{xz} \end{bmatrix} \qquad (8)$$

Substituting equation (8) into the second part of equation (3) and expanding results in

$$\bar{\mathbf{r}} \times (\overline{\mathbf{M}} \bullet \nabla)\tilde{\mathbf{B}} = M_{\bar{x}} \begin{bmatrix} (-\tilde{B}_{xy}\bar{z} + \tilde{B}_{xz}\bar{y}) \\ (\tilde{B}_{xx}\bar{z} - \tilde{B}_{xz}\bar{x}) \\ (-\tilde{B}_{xx}\bar{y} + \tilde{B}_{xy}\bar{x}) \end{bmatrix} \qquad (9)$$

Expanding the first part of equation (3) results in

$$\overline{\mathbf{M}} \times \tilde{\mathbf{B}} = M_{\bar{x}} \begin{bmatrix} 0 \\ -\tilde{B}_z \\ \tilde{B}_y \end{bmatrix} \qquad (10)$$

The components of equation (3) become

$$T_{\bar{x}} = 0 + M_{\bar{x}} \int_v (\tilde{B}_{xz}\bar{y} - \tilde{B}_{xy}\bar{z})\, dv \qquad (11)$$

$$T_{\bar{y}} = -M_{\bar{x}} \int_v \tilde{B}_z\, dv + M_{\bar{x}} \int_v (\tilde{B}_{xx}\bar{z} - \tilde{B}_{xz}\bar{x})\, dv \qquad (12)$$

$$T_{\bar{z}} = M_{\bar{x}} \int_v \tilde{B}_y\, dv + M_{\bar{x}} \int_v (\tilde{B}_{xy}\bar{x} - \tilde{B}_{xx}\bar{y})\, dv \qquad (13)$$

Evaluating $\int_v \tilde{B}_{xz}\bar{y}\, dv$ first

$$\int_v \tilde{B}_{xz}\bar{y}\, dv = B_{xz} \int_v \bar{y}\, dv + B_{(xx)z} \int_v \overline{xy}\, dv + B_{(xy)z} \int_v \bar{y}^2\, dv + B_{(xz)z} \int_v \overline{zy}\, dv \qquad (14)$$

where $\tilde{B}_{xz}$ has been expanded by using a Taylor series, as detailed in appendix A, and the notation $f_{(ij)k} = \partial(\partial f_i/\partial j)/\partial k$ has been used. All the integrals involving first-order terms in equation (14) are zero. Evaluating $\int_v \bar{y}^2\, dv$ yields

$$\int_v \bar{y}^2 dv = \int_{-l/2}^{l/2} \int_{-a}^{a} \int_{-y_1(\bar{z})}^{y_2(\bar{z})} \bar{y}^2\, d\bar{y}\, d\bar{z}\, d\bar{x} = (a^4/4)\pi l \qquad (15)$$

where $y_1(\bar{z}) = y_2(\bar{z}) = \sqrt{a^2 - \bar{z}^2}$, a is the radius of the permanent magnet core, and l is the length (fig. 1). Since the area of the face of the permanent magnet core is $A = \pi a^2$ and the volume is $v = Al$, equation (15) reduces to

$$(a^4/4)\pi l = (a^2/4)v \qquad (16)$$

Substituting in equation (14) results in

$$\int_v \tilde{B}_{xz}\bar{y}\, dv = (a^2/4)vB_{(xy)z} \qquad (17)$$

Evaluating $\int_v \tilde{B}_{xy}\bar{z}\, dv$ in a similar manner results in

$$\int_v \tilde{B}_{xy}\bar{z}\, dv = (a^2/4)vB_{(xy)z} \qquad (18)$$

which is equal to equation (17). Therefore the torques about the $\bar{x}$-axis due to second-order gradients cancel out and $T_{\tilde{x}} = 0$ as expected. Going next to equation (12),

$$\int_v \tilde{B}_z\, dv = v\Big[B_z + (l^2/24)B_{(zx)x} + (a^2/8)B_{(zy)y} + (a^2/8)B_{(zz)z}\Big] \tag{19}$$

and

$$\int_v (\tilde{B}_{xx}\tilde{z} - \tilde{B}_{xz}\tilde{x})\, dv = v[(a^2/4) - (l^2/12)]B_{(xx)z} \tag{20}$$

Substituting into equation (12) results in

$$T_{\tilde{y}} = -vM_{\tilde{x}}\Big[B_z + (l^2/24)B_{(zx)x} + (a^2/8)B_{(zy)y} + (a^2/8)B_{(zz)z}\Big] + vM_{\tilde{x}}\Big[(a^2/4) - (l^2/12)\Big]B_{(xx)z} \tag{21}$$

Continuing to equation (13),

$$\int_v \tilde{B}_y\, dv = v\Big[B_y + (l^2/24)B_{(yx)x} + (a^2/8)B_{(yy)y} + (a^2/8)B_{(yz)z}\Big] \tag{22}$$

and

$$\int_v (\tilde{B}_{xy}\tilde{x} - \tilde{B}_{xx}\tilde{y})\, dv = v[(l^2/12) - (a^2/4)]B_{(xx)y} \tag{23}$$

Substituting into equation (13) results in

$$T_{\tilde{z}} = vM_{\tilde{x}}\Big[B_y + (l^2/24)B_{(yx)x} + (a^2/8)B_{(yy)y} + (a^2/8)B_{(yz)z}\Big] + vM_{\tilde{x}}[(l^2/12) - (a^2/4)]B_{(xx)y} \tag{24}$$

Finally, noting that $B_{(ij)k} = B_{(ik)j} = B_{(jk)i} \ldots$ and collecting terms, the components of torque become

$$T_{\tilde{x}} = 0 \tag{25}$$

$$T_{\tilde{y}} = -vM_{\tilde{x}}B_z - vM_{\tilde{x}}[(a^2/4) - (l^2/8)]B_{(xx)z} - vM_{\tilde{x}}(a^2/8)(B_{(yy)z} + B_{(zz)z}) \tag{26}$$

$$T_{\tilde{z}} = vM_{\tilde{x}}B_y + vM_{\tilde{x}}[(l^2/8) - (a^2/4)]B_{(xx)y} - vM_{\tilde{x}}(a^2/8)(B_{(yy)y} + B_{(yz)z}) \tag{27}$$

The force on the core, equation (4), can be evaluated in a similar manner. The components of equation (4) are

$$F_{\tilde{x}} = M_{\tilde{x}}\int_v \tilde{B}_{xx}\, dv \tag{28}$$

$$F_{\tilde{y}} = M_{\tilde{x}}\int_v \tilde{B}_{xy}\, dv \tag{29}$$

$$F_{\tilde{z}} = M_{\tilde{x}}\int_v \tilde{B}_{xz}\, dv \tag{30}$$

Expanding the integral of equation (28) results in

$$\int_v \tilde{B}_{xx}\, dv = \int_v (B_{xx} + B_{(xx)y}\tilde{y} + B_{(xx)z}\tilde{z} + B_{(xx)x}\tilde{x}\, dv \tag{31}$$

Since the integrals containing first-order terms in $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are zero, equation (31) reduces to

$$F_{\tilde{x}} = M_{\tilde{x}}\int_v B_{xx}\, dv = vM_{\tilde{x}}B_{xx} \tag{32}$$

Evaluating equations (29) and (30) results in

$$F_{\tilde{y}} = vM_{\tilde{x}}B_{xy} \tag{33}$$

and

$$F_{\tilde{z}} = vM_{\tilde{x}}B_{xz} \tag{34}$$

Magnetization Perpendicular to Axis of Symmetry

For orientation of the magnetization vector perpendicular to the axis of symmetry (x-axis), the only nonzero term in $\bar{\mathbf{M}}$ is $\mathbf{M}_{\bar{z}}$. Equation (5) becomes

$$[\partial\tilde{\mathbf{B}}]\bar{\mathbf{M}} = \mathrm{M}_{\bar{z}}\begin{bmatrix}\tilde{\mathrm{B}}_{xz}\\ \tilde{\mathrm{B}}_{yz}\\ \tilde{\mathrm{B}}_{zz}\end{bmatrix} \tag{35}$$

and the second part of equation (3) becomes

$$\bar{\mathbf{r}}\times(\bar{\mathbf{M}}\bullet\nabla)\tilde{\mathbf{B}} = \mathrm{M}_{\bar{z}}\begin{bmatrix}(-\tilde{\mathrm{B}}_{yz}\bar{z}+\tilde{\mathrm{B}}_{zz}\bar{y})\\ (\tilde{\mathrm{B}}_{xz}\bar{z}-\tilde{\mathrm{B}}_{zz}\bar{x})\\ (-\tilde{\mathrm{B}}_{xz}\bar{y}+\tilde{\mathrm{B}}_{yz}\bar{x})\end{bmatrix} \tag{36}$$

The first part of equation (3) becomes

$$\bar{\mathbf{M}}\times\tilde{\mathbf{B}} = \mathrm{M}_{\bar{z}}\begin{bmatrix}-\tilde{\mathrm{B}}_{y}\\ \tilde{\mathrm{B}}_{x}\\ 0\end{bmatrix} \tag{37}$$

Substituting (36) and (37) into (3) results in

$$\mathrm{T}_{\bar{x}} = -\mathrm{M}_{\bar{z}}\int_v \tilde{\mathrm{B}}_y\,dv + \mathrm{M}_{\bar{z}}\int_v (\tilde{\mathrm{B}}_{zz}\bar{y} - \tilde{\mathrm{B}}_{yz}\bar{z})\,dv \tag{38}$$

$$\mathrm{T}_{\bar{y}} = \mathrm{M}_{\bar{z}}\int_v \tilde{\mathrm{B}}_x\,dv + \mathrm{M}_{\bar{z}}\int_v (\tilde{\mathrm{B}}_{xz}\bar{z} - \tilde{\mathrm{B}}_{zz}\bar{x})\,dv \tag{39}$$

$$\mathrm{T}_{\bar{z}} = 0 + \mathrm{M}_{\bar{z}}\int_v (\tilde{\mathrm{B}}_{yz}\bar{x} - \tilde{\mathrm{B}}_{xz}\bar{y})\,dv \tag{40}$$

Evaluating the integrals as before and collecting terms results in

$$\begin{aligned}\mathrm{T}_{\bar{x}} = &-v\mathrm{M}_{\bar{z}}\mathrm{B}_y - v\mathrm{M}_{\bar{z}}(l^2/24)\mathrm{B}_{(xx)y}\\ &- v\mathrm{M}_{\bar{z}}(a^2/8)(\mathrm{B}_{(yy)y} + \mathrm{B}_{(yz)z})\end{aligned} \tag{41}$$

$$\begin{aligned}\mathrm{T}_{\bar{y}} = &\, v\mathrm{M}_{\bar{z}}\mathrm{B}_x + v\mathrm{M}_{\bar{z}}[(3a^2/8) - (l^2/12)]\mathrm{B}_{(xz)z}\\ &+ v\mathrm{M}_{\bar{z}}(l^2/24)\mathrm{B}_{(xx)x} + v\mathrm{M}_{\bar{z}}(a^2/8)\mathrm{B}_{(xy)y}\end{aligned} \tag{42}$$

$$\mathrm{T}_{\bar{z}} = v\mathrm{M}_{\bar{z}}[(l^2/12) - (a^2/4)]\mathrm{B}_{(xy)z} \tag{43}$$

The components of the force (eq. (4)) using equation (35) becomes

$$\mathrm{F}_{\bar{x}} = \mathrm{M}_{\bar{z}}\int_v \tilde{\mathrm{B}}_{xz}\,dv = v\mathrm{M}_{\bar{z}}\mathrm{B}_{xz} \tag{44}$$

$$\mathrm{F}_{\bar{y}} = \mathrm{M}_{\bar{z}}\int_v \tilde{\mathrm{B}}_{yz}\,dv = v\mathrm{M}_{\bar{z}}\mathrm{B}_{yz} \tag{45}$$

$$\mathrm{F}_{\bar{z}} = \mathrm{M}_{\bar{z}}\int_v \tilde{\mathrm{B}}_{zz}\,dv = v\mathrm{M}_{\bar{z}}\mathrm{B}_{zz} \tag{46}$$

Discussion of Results

Examination of equations (25) to (27), (32) to (34), (41) to (43), and (44) to (46) reveals that, for expansion of the applied fields up to second-order terms, no coupling exists between force and torque components. As stated earlier, it is generally assumed that the higher order torque terms, which are functions of second order gradients, can be neglected. However, equation (43) indicates that for magnetization perpendicular to the axis of symmetry, torque about the axis of magnetization can be generated by controlling a higher order term directly, thus allowing the core to be controlled in six degrees of freedom. For a cylindrical permanent magnet core magnetized along the axis of symmetry, from equation (25), only five-degree-of-freedom control is possible.

Concluding Remarks

This paper has developed the expanded equations for torque and force on a cylindrical permanent magnet core in a large-gap magnetic suspension system. The core was assumed to be uniformly magnetized, and equations were developed for two orientations of the magnetization vector. One orientation was parallel to the axis of symmetry of the core and the other was perpendicular to this axis. It is generally assumed that terms that are a function of second-order or higher gradients of the applied fields can be neglected. In practical applications involving large-gap magnetic suspension systems, these assumptions have proven to be valid. However, in the case where the magnetization vector is perpendicular to the axis of symmetry of the core, the expanded equations indicate that torque about the magnetization vector can be produced by controlling a second-order gradient directly. This case allows the core to be controlled in six degrees of freedom whereas a cylindrical permanent magnet core magnetized along its axis of symmetry can be controlled only in five degrees of freedom.

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Appendix A

Expansion of Fields and Gradients About the Nominal Operating Point of a Cylindrical Permanent Magnet Core

In appendix A the fields and gradients produced by the suspension system electromagnets are expanded by using a Taylor series about the initial suspension point of the permanent magnet core. The assumption is made that the fields can be adequately described by expansion up to second-order terms. Figure 1 shows the cylindrical core and core coordinate system. The core coordinate system consists of a set of orthogonal $\bar{x}, \bar{y}, \bar{z}$ body-fixed axes that define the motion of the core with respect to an orthogonal x, y, z system fixed in inertial space. The core coordinate system is initially aligned with the x, y, z system. The transformation from inertial coordinates to core coordinates is given by

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = [\mathbf{T}_m] \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{A1}$$

where $[\mathbf{T}_m]$ is the orthogonal transformation matrix for a 3, 2, 1 (z, y, x) Euler rotation sequence and is defined as

$$[\mathbf{T}_m] = \begin{bmatrix} c\theta_z c\theta_y & s\theta_z c\theta_y & -s\theta_y \\ (c\theta_z s\theta_y s\theta_x - s\theta_z c\theta_x) & (s\theta_z s\theta_y s\theta_x + c\theta_z c\theta_x) & c\theta_y s\theta_x \\ (c\theta_z s\theta_y c\theta_x + s\theta_z s\theta_x) & (s\theta_z s\theta_y c\theta_x - c\theta_z s\theta_x) & c\theta_y c\theta_x \end{bmatrix} \tag{A2}$$

where sin has been shortened to s, cos has been shortened to c, and θ_z, θ_y, and θ_x are angles of rotation about the z-, y-, and x-axes, respectively. The field **B** and gradients of **B** produced by the suspension system electromagnets, which are fixed in the inertial frame, are calculated at the origin of the x, y, z system.

Expanding **B** about the origin of the x, y, z system as a Taylor series, up to second order, results in

$$\tilde{\mathbf{B}} = \mathbf{B} + (\mathbf{r} \bullet \nabla)\mathbf{B} + (1/2)(\mathbf{r} \bullet \nabla)^2\mathbf{B} \tag{A3}$$

where $\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and ∇ is the gradient operator. Using compact notation, each element of $\tilde{\mathbf{B}}$ in equation (A3) can be written as

$$\tilde{B}_i = B_i + \frac{\partial B_i}{\partial \mathbf{r}}\mathbf{r} + (1/2)\mathbf{r}^T \frac{\partial^2 B_i}{\partial \mathbf{r}^2}\mathbf{r} \tag{A4}$$

where

$$\frac{\partial B_i}{\partial \mathbf{r}} = \left\lfloor \frac{\partial B_i}{\partial x} \; \frac{\partial B_i}{\partial y} \; \frac{\partial B_i}{\partial z} \right\rfloor \tag{A5}$$

and

$$\frac{\partial^2 B_i}{\partial \mathbf{r}^2} = \begin{bmatrix} \frac{\partial(\partial B_i/\partial x)}{\partial x} & \frac{\partial(\partial B_i/\partial x)}{\partial y} & \frac{\partial(\partial B_i/\partial x)}{\partial z} \\ \frac{\partial(\partial B_i/\partial y)}{\partial x} & \frac{\partial(\partial B_i/\partial y)}{\partial y} & \frac{\partial(\partial B_i/\partial y)}{\partial z} \\ \frac{\partial(\partial B_i/\partial z)}{\partial x} & \frac{\partial(\partial B_i/\partial z)}{\partial y} & \frac{\partial(\partial B_i/\partial z)}{\partial z} \end{bmatrix} \tag{A6}$$

Using the notation $f_{ij} = \partial f_i/\partial j$ and $f_{(ij)k} = \partial(\partial f_i/\partial j)/\partial k$, equations (A5) and (A6) can be written as

$$\frac{\partial B_i}{\partial \mathbf{r}} = \left\lfloor B_{ix} \; B_{iy} \; B_{iz} \right\rfloor \tag{A7}$$

and

$$\frac{\partial^2 B_i}{\partial \mathbf{r}^2} = \begin{bmatrix} B_{(ix)x} & B_{(ix)y} & B_{(ix)z} \\ B_{(iy)x} & B_{(iy)y} & B_{(iy)z} \\ B_{(iz)x} & B_{(iz)y} & B_{(iz)z} \end{bmatrix} \tag{A8}$$

The first-order gradients of $\tilde{\mathbf{B}}$ can be written as

$$\tilde{B}_{ij} = B_{ij} + \frac{\partial(\partial B_i/\partial j)}{\partial \mathbf{r}}\mathbf{r} \tag{A9}$$

where

$$\frac{\partial(\partial B_i/\partial j)}{\partial \mathbf{r}} = \left\lfloor B_{(ij)x} \; B_{(ij)y} \; B_{(ij)z} \right\rfloor \tag{A10}$$

The expanded fields can be expressed in core coordinates as

$$\bar{\bar{\mathbf{B}}} = \bar{\mathbf{B}} + (\bar{\mathbf{r}} \bullet \bar{\nabla})\bar{\mathbf{B}} + (1/2)(\bar{\mathbf{r}} \bullet \bar{\nabla})^2\bar{\mathbf{B}} \qquad \text{(A11)}$$

where $\bar{\mathbf{r}} = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$ is the displacement in core coordinates, $\bar{\mathbf{B}} = [\mathbf{T}_m]\mathbf{B}$, and $\bar{\nabla} = [\mathbf{T}_m]\nabla$. Since, from equation (A1),

$$\mathbf{r} = [\mathbf{T}_m]^T\bar{\mathbf{r}} \qquad \text{(A12)}$$

each element of $\bar{\bar{\mathbf{B}}}$ can be expanded in inertial coordinates by substituting equation (A12) into equation (A4),

$$\tilde{\mathbf{B}}_i = \mathrm{B}_i + \frac{\partial \mathrm{B}_i}{\partial \mathbf{r}}[\mathbf{T}_m]^T\bar{\mathbf{r}} + (1/2)\bar{\mathbf{r}}^T[\mathbf{T}_m]\frac{\partial^2 \mathrm{B}_i}{\partial \mathbf{r}^2}[\mathbf{T}_m]^T\bar{\mathbf{r}} \qquad \text{(A13)}$$

Transforming back into core coordinates,

$$\bar{\bar{\mathbf{B}}} = [\mathbf{T}_m]\begin{bmatrix} \left[\mathrm{B}_x + \frac{\partial \mathrm{B}_x}{\partial \mathbf{r}}[\mathbf{T}_m]^T\bar{\mathbf{r}} + (1/2)\bar{\mathbf{r}}^T[\mathbf{T}_m]\frac{\partial^2 \mathrm{B}_x}{\partial \mathbf{r}^2}[\mathbf{T}_m]^T\bar{\mathbf{r}}\right] \\ \left[\mathrm{B}_y + \frac{\partial \mathrm{B}_y}{\partial \mathbf{r}}[\mathbf{T}_m]^T\bar{\mathbf{r}} + (1/2)\bar{\mathbf{r}}^T[\mathbf{T}_m]\frac{\partial^2 \mathrm{B}_y}{\partial \mathbf{r}^2}[\mathbf{T}_m]^T\bar{\mathbf{r}}\right] \\ \left[\mathrm{B}_z + \frac{\partial \mathrm{B}_z}{\partial \mathbf{r}}[\mathbf{T}_m]^T\bar{\mathbf{r}} + (1/2)\bar{\mathbf{r}}^T[\mathbf{T}_m]\frac{\partial^2 \mathrm{B}_z}{\partial \mathbf{r}^2}[\mathbf{T}_m]^T\bar{\mathbf{r}}\right] \end{bmatrix} \qquad \text{(A14)}$$

The expansion of equation (A14) can be simplified by using small-angle assumptions (ref. 6). Under small-angle assumptions, $\cos\theta = 1$, $\sin\theta = \theta$, and products of angles are neglected. The transformation matrix $[\mathbf{T}_m]$ then becomes

$$[\mathbf{T}_m] = \begin{bmatrix} 1 & \theta_z & -\theta_y \\ -\theta_z & 1 & \theta_x \\ \theta_y & -\theta_x & 1 \end{bmatrix} \qquad \text{(A15)}$$

Appendix B

Torques and Forces on a Magnetic Dipole With Incremental Volume

The torques and forces on a magnetic dipole in a steady magnetic field are identical to those on an infinitesimal current loop with the same magnetic moment (ref. 9). Therefore, the equations for torque and force on an infinitesimal current loop will be developed first by using the fundamental relationship for the force on a current-carrying-conductor element in a uniform, steady magnetic field. For a discussion of magnetic dipoles and infinitesimal current loops, see references 9 and 10.

Infinitesimal Current Loop

Consider a plane loop of conductor with steady current I located in the external, uniform, steady magnetic field $\mathbf{B}$ (fig. B1). In this region $\nabla \times \mathbf{B} = \nabla \bullet \mathbf{B} = 0$. The force on an element $d\mathbf{l}$ of the conductor is given by the fundamental relationship (obtained from the Lorentz force law)

$$d\mathbf{F} = \mathrm{I}\, d\mathbf{l} \times \mathbf{B} \tag{B1}$$

where $d\mathbf{F}$ is a vector indicating magnitude and direction of force on the conductor element; I is the scalar magnitude of the current in the conductor element; $d\mathbf{l}$ is a vector whose magnitude equals the length of the conductor element and whose direction is in the positive direction of the current; and $\mathbf{B}$ is a vector indicating magnitude and direction of the flux density of the external field component. The torque on the loop can be written as

$$\mathbf{T} = \mathrm{I}\oint[\mathbf{r} \times (d\mathbf{l} \times \mathbf{B})] \tag{B2}$$

where $\mathbf{r}$ is the position vector of $d\mathbf{l}$ and the integration is around the loop. By using the identity

$$\mathbf{r} \times (d\mathbf{l} \times \mathbf{B}) = d\mathbf{l}(\mathbf{r} \bullet \mathbf{B}) - \mathbf{B}(\mathbf{r} \bullet d\mathbf{l}) \tag{B3}$$

equation (B2) can be written as

$$\mathbf{T} = \mathrm{I}\left[\oint(\mathbf{r} \bullet \mathbf{B})d\mathbf{l} - \mathbf{B}\oint \mathbf{r} \bullet d\mathbf{l}\right] \tag{B4}$$

Using Stokes's theorem and a related result (ref. 9, p. 289), the line integrals in equation (B4) can be transformed into surface integrals resulting in

$$\mathbf{T} = \mathrm{I}\left\{\int_s [d\mathbf{A} \times \nabla(\mathbf{r} \bullet \mathbf{B})] - \mathbf{B}\int_s (\nabla \times \mathbf{r}) \bullet d\mathbf{A}\right\} \tag{B5}$$

where $d\mathbf{A}$ is a vector whose magnitude is a differential area and whose direction is normal to the plane of the current loop in the sense of the right-hand rule relative to the direction of current flow, ∇ is the gradient operator, and the integrals are over the surface that is defined by the conductor loop. Since $\nabla \times \mathbf{r}$ is zero and $\nabla(\mathbf{r} \bullet \mathbf{B}) = \mathbf{B}$ for constant $\mathbf{B}$, equation (B5) simplifies to

$$\mathbf{T} = \mathrm{I}\int_s (d\mathbf{A} \times \mathbf{B}) \tag{B6}$$

Taking the integral results in

$$\mathbf{T} = \mathrm{I}\mathbf{A} \times \mathbf{B} \tag{B7}$$

An infinitesimal current loop can be defined by letting $\mathbf{A}$ go toward zero and I go toward infinity, keeping the product $\mathrm{I}\mathbf{A}$ finite. For an infinitesimal current loop, the requirement that $\mathbf{B}$ be uniform no longer exists. The product $\mathrm{I}\mathbf{A}$ is called the magnetic moment of the loop and is designated by the letter $\mathbf{m}$. Therefore equation (B7) becomes

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \tag{B8}$$

The torque $\mathbf{T}$ acts on the infinitesimal current loop in a direction to align the magnetic moment $\mathbf{m}$ with the external field $\mathbf{B}$. If $\mathbf{m}$ and $\mathbf{B}$ are misaligned by the angle θ, the magnitude of the torque is

$$\mathbf{T} = \mathbf{mB}\sin\theta \tag{B9}$$

To increase θ by the amount $d\theta$, work dW must be done against the torque $\mathbf{T}$ resulting in an increase in potential energy dU:

$$dU = dW = \mathbf{T}\,d\theta = m\mathbf{B}\sin\theta\, d\theta \tag{B10}$$

The potential energy of an infinitesimal current loop in an external magnetic field can then be obtained by integrating equation (B10):

$$U = -\mathbf{mB}\cos\theta = -\mathbf{m} \bullet \mathbf{B} \tag{B11}$$

where the constant of integration is chosen to be zero when $\mathbf{m}$ is perpendicular to $\mathbf{B}$. The force on the infinitesimal current loop can be obtained from equation (B11). If an external force $\mathbf{F}$ displaces the infinitesimal current loop by the infinitesimal distance dr, then the work done dW will be equal to a decrease in potential energy, $-dU$:

$$dW = \mathbf{F} \bullet d\mathbf{r} = -dU = -\nabla U \bullet d\mathbf{r} \tag{B12}$$

Therefore

$$\mathbf{F} = -\nabla U = \nabla(\mathbf{m} \bullet \mathbf{B}) \tag{B13}$$

The right-hand side of equation (B13) can be expanded as

$$\nabla(\mathbf{m} \bullet \mathbf{B}) = \mathbf{m} \times (\nabla \times \mathbf{B}) + (\mathbf{m} \bullet \nabla)\mathbf{B} + \mathbf{B} \times (\nabla \times \mathbf{m}) + (\mathbf{B} \bullet \nabla)\mathbf{m} \qquad \text{(B14)}$$

Since $(\nabla \bullet \mathbf{B})$, $(\nabla \times \mathbf{B})$, and $(\nabla \times \mathbf{m})$ are zero, equation (B13) can be written in the form

$$\mathbf{F} = (\mathbf{m} \bullet \nabla)\mathbf{B} \qquad \text{(B15)}$$

This form is generally used in the development of the equations for large-gap magnetic suspension systems (refs. 1 through 6).

Magnetic Dipole With Incremental Volume

The magnetic moment of a permanent magnet dipole with north and south poles separated by length l and with pole strength Q_m is defined as

$$\mathbf{m} = Q_m l \qquad \text{(B16)}$$

The magnetic moment $\mathbf{m}$ is a vector pointing from the south pole to the north pole. In the case of an actual magnet, Q_m and l may be indefinite but $\mathbf{m}$ can be determined and is sufficient to specify the fields of the magnet at a large distance from it. At large distances, a magnetic dipole with magnetic moment $Q_m l$ can be treated the same as an infinitesimal current loop with magnetic moment IA and is identical in effect if $Q_m l = \mathbf{IA}$. Therefore, in a steady magnetic field $\mathbf{B}$ the equations for torques and forces on a magnetic dipole with magnetic moment $\mathbf{m}$ are the same as equations (B8) and (B15).

In theory, it can be assumed that a permanent magnet of a given volume v consists of a large number of uniformly distributed permanent magnet dipoles with incremental volumes δv which are oriented in the same direction. The magnetic moment $\delta\mathbf{m}$ of a given dipole with incremental volume δv can be conveniently described by a quantity called the magnetization $\mathbf{M}$, which is defined as the magnetic moment per unit volume. That is,

$$\mathbf{M} = \delta\mathbf{m}/\delta v \qquad \text{(B17)}$$

The total magnetic moment $\mathbf{m}$ for a given permanent magnet can then be written as

$$\mathbf{m} = \int_v \mathbf{M}\, dv \qquad \text{(B18)}$$

where the integration is over the volume of the permanent magnet. Magnetization is also a vector and has the same direction as $\mathbf{m}$. If the permanent magnet is uniformly magnetized, that is, $\mathbf{M}$ is constant over the volume of the permanent magnet, then

$$\mathbf{m} = \mathbf{M}v \qquad \text{(B19)}$$

For a discussion of magnetic dipoles and magnetization, see reference 10.

The torques and forces on an incremental volume of permanent magnet material, in terms of the magnetization $\mathbf{M}$, can then be written as

$$\delta\mathbf{T} = (\mathbf{M} \times \mathbf{B})\delta v \qquad \text{(B20)}$$

and

$$\delta\mathbf{F} = (\mathbf{M} \bullet \nabla)\mathbf{B}\delta v \qquad \text{(B21)}$$

from equations (B8), (B15), and (B17).

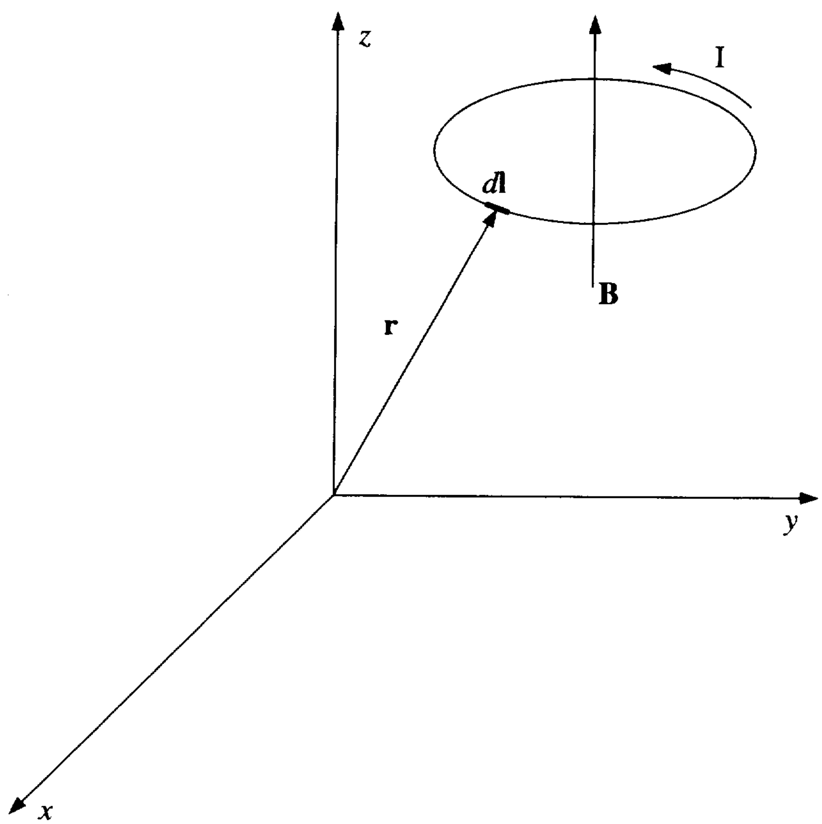

Figure B1. Plane loop of conductor with steady current I in uniform steady magnetic field **B**.

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The expanded equations for torque and force on a cylindrical permanent magnet core in a large-gap magnetic suspension system are presented. The core is assumed to be uniformly magnetized, and equations are developed for two orientations of the magnetization vector. One orientation is parallel to the axis of symmetry, and the other is perpendicular to this axis. Fields and gradients produced by suspension system electromagnets are assumed to be calculated at a point in inertial space which coincides with the origin of the core axis system in its initial alignment. Fields at a given point in the core are defined by expanding the fields produced at the origin as a Taylor series. The assumption is made that the fields can be adequately defined by expansion up to second-order terms. Examination of the expanded equations for the case where the magnetization vector is perpendicular to the axis of symmetry reveals that some of the second-order gradient terms provide a method of generating torque about the axis of magnetization and therefore provide the ability to produce six-degree-of-freedom control.

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