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TECHNICAL TRANSLATION

F-54

CONCERNING A METHOD FOR THE MEASUREMENT OF VERY HIGH TEMPERATURES IN NEARLY TRANSPARENT ARC COLUMNS

By Rudolf Wilhelm Larenz

Translation of "Über ein Verfahren zur Messung sehr hoher Temperaturen in nahezu durchlässigen Bogensäulen."
Zeitschrift für Physik, vol. 129, 1951.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON December 1960

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CONCERNING A METHOD FOR THE MEASUREMENT OF VERY HIGH TEMPERATURES IN NEARLY TRANSPARENT ARC COLUMNS*

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By modifying astrophysical methods and enlarging on an idea developed by Hormann, a method has been developed which permits measurements up to extremely high temperatures in arc columns [1].

§1. INTRODUCTION

In the report presented here a spectrophotometric method is presented which permits the measurement of temperature distributions in arc columns up to very high temperatures. In this particular method, an earlier method reported by Hörmann [2] is being enlarged and made useful for measurement of high temperatures by combining it with astrophysical methods found by the Saha method by Fowler and Milne [3]. The assumptions of Fowler and Milne must be adjusted for this purpose to the physical conditions prevailing in an arc column.

In contrast to a method recently developed by H. Bartels [4] for the measurement of high temperatures, which method assumes strong self reversal of the lines used for measurement, it is here assumed that the absorption in the arc column is only very small in the spectral region in which measurements are carried out. The application of this method is

*"Über ein Verfahren zur Messung sehr hoher Temperaturen in nahezu durchlässigen Bogensäulen." Zeitschrift für Physik, vol. 129, 1951, pp. 327-342. (Presented at the meeting of the German Physical Society in the British Zone in Münster on April 15, 1950.)

further limited by the possibility that in the arc column, temperatures can be reached where the degree of ionization reaches the magnitude of 1. In section 2, the general basis for the method is given. In section 3, the formal bases are developed, and also the differences are treated which arise when the calculation of Fowler and Milne are applied to a different physical situation. In section 4 a theory of errors is developed, as far as this can be possible in a general way.

§2. BASES OF THE METHOD

We assume local temperature equilibrium in the arc column. "Local temperature equilibrium" is to be understood as meaning that the distribution of the energy among individual particles is given by a Boltzmann distribution. Therefore, it is not necessary that the radiation density also reflect this temperature. The radiation density will always be very much smaller under conditions required in the application of this method. Whether the fiction of such a local temperature equilibrium has been sufficiently approached in the arc column can be determined to a certain degree by controls which will be further discussed below.

We further assume axial symmetry of the column and observe the emission of rays in a plane perpendicular to that of the column axis. The column points in this plane are related to a coordinate system whose origin is placed at the column axis, the x-axis is oriented in the direction of observation, while the y-axis is perpendicular to that. Further, $\underline{r}$ is the radial distance of a point from the column axis and r_0 is the column radius. The observed rays parallel to the x-axis are uniquely defined by the distance $\underline{y}$ from this column axis.

For the measurement of the temperature distribution, either a line emission or emission in a continuum can be used. The important assumption in either case is that the absorption in the column is only very small in the frequencies which are used for measurement. Possibilities on how to control this assumption adequately will be discussed later.

We indicate the principle of this method first in the simplest possible

case, by limiting ourselves to the measurement of a line. It is assumed that the line emission is not disturbed by underlying continua. Then I(y) is the total radiation density of the line emitted by a ray with axial distance y. The emission coefficient for this line is represented in its radial course by $\epsilon(r)$. If the absorption in the column can be neglected this yields

$$I(y) = 2 \int_0^{\sqrt{r_0^2 - y^2}} \varepsilon\left(\sqrt{x^2 + y^2}\right) dx. \tag{1}$$

If I(y) is measured over the whole cross-section of the column, then $\epsilon(r)$ results from the solution of Abel's integral equation (1). If, on the other hand, ϵ can be calculated as a function of temperature T and pressure P, then using

$$\varepsilon(r) = \varepsilon[T, P] \tag{2}$$

the radial temperature distribution in the column can be determined at a given pressure. The experimental application of this theoretically simple method is made more difficult by the fact that under certain conditions, great radiation densities must be measured absolutely. Furthermore, the calculation of $\epsilon(T,P,)$ will generally be limited by the fact that the transition probabilities which enter $\epsilon(T,P)$ as factors are known only insufficiently. Both difficulties immediately disappear when for a given radius $\tilde{r}$ the temperature $\tilde{T}$ is known. Then equation (2) can be replaced by

$$\varepsilon^*(r) = \frac{\varepsilon(r)}{\varepsilon(\tilde{r})} = \frac{\varepsilon[T, P]}{\varepsilon[\tilde{T}, P]} = \varepsilon^*[T, P]. \tag{3}$$

The function $\epsilon^*(r)$ can be determined by suitable normalizing if the relative course of the function $\epsilon(r)$ is known. A function $\epsilon'(r)$ which represents the relative course of $\epsilon(r)$ correctly is obtained from the solution of the equation

$$\frac{I(y)}{I(0)} = 2 \int_0^{\sqrt{r_0^2 - y^2}} \varepsilon'\left(\sqrt{x^2 + y^2}\right) dx \tag{4a}$$

with

$$\varepsilon'(r) = \frac{\varepsilon(r)}{I(0)}. \tag{4b}$$

Here it is only necessary to measure the function I(y)/I(o) which is generally known in astrophysical usage as center-boundary variation. Furthermore, in the function $\epsilon^*(T,P)$ the transition probability is eliminated. $\epsilon^*(T,P)$ can generally be calculated with sufficient accuracy. In this way Hörmann determined the temperature decrease in a carbon arc by substituting for $\tilde{r} = 0$ a value $\tilde{T}$ which was determined from other measurements [5].

As long as the present method is dependent on the use of other temperature measurements, this method cannot lead to an independent temperature measurement and is therefore not applicable for measurements in ranges in which temperatures have not been previously measured. An independent temperature measurement based on the derivations above can only be made if the standard temperature $\tilde{T}(\tilde{r})$ can be derived from equation (3). By analogy to the thought process of Fowler and Milne, it will now be shown that such is possible, if the temperatures which can be achieved in the particular light source are sufficiently high. In this connection, the temperature function of the emission coefficient at constant pressure will be considered in general terms. The carrier of the line to be measured is the ℓth degree ionized atom. Then the number of these atoms per unit volume is n_ℓ, whereby n_{ℓ_0} represents the ground state, and $n_{\ell a}$, the excited state of the line whose transition probability is designated as A. Then

$$\varepsilon = \frac{A}{4\pi} h\nu \frac{n_{\ell a}}{n_{\ell 0}} \cdot n_{\ell 0}.$$

The fraction $n_{\ell a}/n_{\ell 0}$ increases monotonically with increasing temperature, and approaches, at sufficiently high temperature, a constant limiting value. The behavior of $n_{\ell 0}$ is particularly simple when dealing with the emission of an arc line ($\ell = 0$). Then $n_{\ell 0}$ decreases monotonically with increasing temperature and approaches zero at sufficiently high temperature. The causes for this decrease are decrease in density, decrease in ground states due to excitation, fewer emission carriers because of ionization, and displacement of atoms and ions by released electrons. If the emission carrier is an ion, then $n_{\ell 0}$ will first increase with rising temperature, and after

passing through a maximum will fall monotonically to zero, because the same causes for the decrease are first overwhelmed by an increase in newly formed atoms. In any case, these conditions lead to the fact that ϵ passes through a maximum at a specific temperature. At this same temperature, every other function, which differs from ϵ by a constant factor, also has a maximum. Therefore, the temperature at which this maximum occurs can be calculated. We define this as temperature norm $\tilde{T}$ so that according to the normalization shown in (3), $\epsilon^*(T,P)$ becomes equal to 1 in the maximum.

The maximum of the function $\epsilon^*(T,P)$ must also appear as a maximum in the radial function of the emission coefficient $\epsilon(r)$ and that of the measured relative function $\epsilon'(r)$. If $\tilde{r}$ is the radius, at which such a maximum occurs, then at a distance from the axis $\tilde{r}$, the temperature $\tilde{T}$, obviously occurs and therefore, as required above, a certain radius corresponds to a temperature norm as determined from the measured function $\epsilon'(r)$ and the calculated function $\epsilon^*(T,P)$. One obtains $\epsilon^*(r)$ by normalizing the relative function $\epsilon'(r)$, which was obtained from the center-boundary variation, in such a way that it becomes equal to unity at its maximum, and then, according to the simple graphic method based on the relationship

$$\varepsilon^*(r) = \varepsilon^*[T, P]$$

the radial temperature variation $T(r)$ can be determined.

The determination of the center-boundary variation of the radiation density for the measurement of temperature requires two tasks: first, the function $\epsilon^*(r)$ must be determined by solving Abel's integral equation. Secondly, $\epsilon^*(T,P)$ must be calculated. The first task is of a purely formal nature. The systematic solution has already been treated in detail by Hörmann and can be passed over here. In the appendix of the next report a method of solution will be explained which, although somewhat less systematic, has been very useful if very many curves must be evaluated. The second task, the calculation of $\epsilon(T,P)$ needs a somewhat more detailed treatment. Before this is done the principles of the method must be explained in more general terms.

It appears at first that the measurement of the radial temperature

distribution is only then possible when the axis temperature actually exceeds the value $\tilde{T}$. It is immediately apparent that this limits its application much too severely. Because, if we have a standard light source where for a specified emission line the critical temperature is exceeded, then in all light sources which emit the same line with the same function $\epsilon(T,P)$ and at the same pressure, temperature measurements can also be made even if $\tilde{T}$ has not been reached. We only have to be concerned that the radiation densities measured from the light sources can be related to those measured from the standard light source. Then the solution of equation (4a) yields in every case immediately the function

$$\varepsilon'(r) = \frac{\varepsilon(r)}{I(0)}.$$

as defined in (4b). For clarity the units which refer to measurements of the standard light source are designated by $\underline{n}$. Then the required function is

$$\varepsilon^*(r) = \frac{\varepsilon(r)}{\varepsilon_n(0)} \cdot \frac{\varepsilon_n(0)}{\varepsilon_n(\tilde{r})} = \frac{\varepsilon'(r)}{\varepsilon'_n(0)} \cdot \frac{I(0)}{I_n(0)} \cdot \varepsilon^*_n(0),$$

which has to be substituted in equation (3) to determine the temperature.

In order to explain this principle on which the temperature measurement is based, we first dealt with the measurement of a line. However, there are no changes in principle when dealing with the center-boundary variation of the radiation density in a spectral region of continuing emission. Only in this case, radiation density and emission coefficient must be related to the unit of the frequency scale. Furthermore, the dependence of the emission coefficient in the continuum on T and P must be ascertainable. The conditions are generally complicated by the fact that several continua of different emission mechanisms contribute to the emission. The measurement of a single line only yields the relative course of the function $\epsilon(T,P)$, but not its absolute value. This will also be true in the presence of several continua if the relative dependence of the particular continua on temperature is the same. Generally, however, the different mechanisms of emission cause differences in their relative temperature dependence, which permits determination of the absolute value for the different emission contributors, or at least determination of their relationship in $\epsilon_\nu^*(T,P)$. Here the calcu-

F-54

lation of $\epsilon_\nu^*(T,P)$ presents somewhat greater difficulties for the theoretical understanding of the emission.

Previously, at the start of this paragraph, it was pointed out that for the utilization of this method two general requirements are necessary. It is assumed, first, that the fiction of local temperature equilibrium is sufficiently approached, and secondly, that the self-absorption of emission frequencies used for measurements is very small. Two remarks will follow dealing with the possibility of how to control the fulfilment of those requirements.

The result could demonstrate that the first requirement had not been fulfilled if measurements of lines with different excitation potentials, or of lines of different ionization levels, and measurements of significantly different continuum frequencies, lead to significantly different temperature distributions. It is thus possible to prove experimentally that the first requirement has been fulfilled.

The fulfilment of the second requirement also can be tested. Such possibilities are only indicated here. Since the absorption within a line varies greatly with frequencies and its effect on the total radiation density cannot simply be overlooked, measurements of lines will be based on the absorption in the line center. We, therefore, base the control of line measurements, as well as measurements on continuous emission, on the radiation density which can be related to the units of the frequency scale. Then, in every case, the control consists of an estimation of the optical layer thickness on the observed ray for the particular frequency. The basis for a systematic estimation of a layer thickness can be found in a theory of emission from an inhomogeneous layer developed by H. Bartels [6].

The equations on which this estimate is based are briefly stated here. Assume that

$$\frac{\varepsilon_\nu}{\kappa} = J_\nu$$

is the yield (κ = absorption coefficient). It is further assumed that $d\tau = \kappa dx$ the optically measured path element, and that $\overline{J}^{\tau}$ is the average value of the yield measured over the optical standard. Then the relationship

$$\frac{I_\nu}{\overline{J}^T} = Y(\tau_0, p)$$

is a function of the optical layer thickness τ_0 and of a parameter p, which may vary between 0 and 1, and depends on the relative distribution of the yield over the optical standard. The function $Y(\tau_0, p)$ was calculated by Bartels. For thin layer thicknesses ($\tau_0 < 1$) is a sufficient approximation

$$Y(\tau_0, p) = \tau_0 - \frac{\tau_0^2}{2} + \frac{\tau_0^3}{24}(3+p) + \cdots,$$

showing that Y is barely dependent on p. In order to estimate the layer thickness, it is necessary to measure absolutely the radiation density and to estimate the average value $\overline{J}^T$. In many cases the second requirement is secured even if only the order of magnitude of the layer thickness is estimated. This requires only a rough measurement of I_ν and an estimate of $\overline{J}^T$ based on temperature data resulting from the apparent ionization levels.

§3. CONCERNING THE CALCULATION $\epsilon(T,P)$

The calculation of ϵ is dealt with here only in so far as it concerns its determination in a general way. It is sufficient if we assume a column which burns in a univalent atomic gas. The extension to a gas mixture introduces nothing new and may be accomplished without difficulties [7]. For the same reason, we will only deal with the measurement of a line when formulating the method of calculation.

It is assumed that n_i ($i = 0, 1, 2, \ldots \ell \ldots$) is the number adjusted for unit value U_{ji} the ionization potential, and u_i the sum of the states of the atoms ionized to the ith degree, P the gas pressure and P_e the electron pressure. The carrier of the line emission is the atom ionized to the ℓth degree, the excitation voltage of the line is U_a, the weight of the initial g_a. The ionization levels which have to be considered in the calculation have the upper limit $i = z$. For the calculation of ϵ the following is available:

a) the equation for

$$\varepsilon = \frac{A}{4\pi} h \nu \frac{g_a}{u_l} \cdot n_l \cdot e^{-\frac{e U_a}{k T}}, \tag{5}$$

b) the Saha equation:

$$\frac{n_{i+1}}{n_i} P_e = \frac{(2\pi m)^{\frac{3}{2}}}{h^3} \cdot 2 \cdot \frac{u_{i+1}}{u_i} (k T)^{\frac{5}{2}} \cdot e^{-\frac{e U_i}{k T}} = F_i(T), \tag{6}$$
$$i = 0,\ 1,\ 2 \ldots l \ldots (z-1).$$

c) the gas equation:

$$P = [n_0 + 2 n_1 + 3 n_2 + \cdots + (z+1) n_z] k T, \tag{7a}$$

$$P_e = [n_1 + 2 n_2 + \cdots + z n_z] k T. \tag{7b}$$

If the equation for continuous emission were to be formulated, then equation (5) is to be substituted by an equation for ϵ_ν, in which in addition to the known functions of T, n_i-values also appear. In any case, the problem is reduced to the determination of n_i from the equations (6), (7a), and (7b). From (6) the following results:

for i = 0

$$n_0 = \frac{P - P_e}{k T} \cdot \frac{1}{1 + \sum_{i=1}^{z} \prod_{m=0}^{i-1} \frac{F_m}{P_e}}, \tag{8a}$$

for i = 1, 2 ... l ... z

$$n_i = \frac{P - P_e}{k T} \cdot \frac{\prod_{m=0}^{i-1} \frac{F_m}{P_e}}{1 + \sum_{i=1}^{z} \prod_{m=0}^{i-1} \frac{F_m}{P_e}}. \tag{8b}$$

Substituting in (7b) then yields in the usual manner the equation from which P_e as a function of P and T can be determined:

$$P_e\left\{1+\sum_{i=1}^{r}(i+1)\prod_{m=0}^{i-1}\frac{F_i}{P_e}\right\}=P\cdot\sum_{i=1}^{r}i\prod_{m=0}^{i-1}\frac{F_i}{P_e}. \tag{9}$$

With the solution $P_e(T,P)$ of equation (9) one obtains from (8) the functions $n_i(T,P)$, which—substituted in equation (5) for ϵ or the respective expression for ϵ_ν—results in the functions $\epsilon(T,P)$ or $\epsilon_\nu(T,P)$. From these, after dividing by the maximum values $\epsilon(\tilde{T},P)$ or $\epsilon_\nu(\tilde{T},P)$, the functions $\epsilon^*(T,P)$ or $\epsilon_\nu^*(T,P)$ are determined, which are used as the basis for the evaluation.

In general, in solving equation (9) one is limited to graphic methods or suitable iteration methods. Only in the P – T regions, in which only 2 ionization levels appear in noticeable concentrations, can (9) be reduced to a quadratic equation with sufficient approximation. Particularly in cases where, in addition to the neutral atom only a simple charged ion occurs, F-54

$$P_e=F_0\left\{\sqrt{1+\frac{P}{F_0}}-1\right\}. \tag{10}$$

substituting in (8a) or (8b) yields in this case

$$n_0=\frac{P}{kT}\left\{1-2\frac{F_0}{P}\left[\sqrt{1+\frac{P}{F_0}}-1\right]\right\}, \tag{11a}$$

$$n_1=\frac{P}{kT}\cdot\frac{F_0}{P}\left[\sqrt{1+\frac{P}{F_0}}-1\right]. \tag{11b}$$

The temperature region in which temperature measurements can be carried out according to this method for lines of certain kinds of atoms or ions, is determined by the parameters which define $\tilde{T}$. Thus we ask: what relationship exists between $\tilde{T}$—and at the same time the respective measurement region—and the excitation voltage of the line, the weights, the ionization potential of the emission carriers, or the pressure? A sufficiently complete view of these relationships can be obtained, if for simplification of the calculations it is assumed that no more than two ionization levels occur at the same time in noticeable concentration, and that the sum of the states can be replaced by the weights of the respective ground states [8]. In this frame of reference we deal with two cases: in

the first case we deal with temperature measurements of an arc line, and in the second case we assume the atom ionized to the ℓth degree as an emission carrier.

In the first case, according to (5)

$$\varepsilon = \frac{A}{4\pi} h\nu \frac{g_a}{g_0} \cdot n_0 \cdot e^{-\frac{eU_a}{kT}}$$

It is possible to substitute for n_0

$$n_0 = \frac{1-c}{1+c} \cdot \frac{P}{kT} \tag{12}$$

with the ion concentration c [9]. The Saha equation then becomes

$$\frac{c^2}{1-c^2} \cdot P = F = \frac{(2\pi m)^{\frac{3}{2}}}{h^3} 2 \frac{g_1}{g_0} \cdot (kT)^{\frac{5}{2}} e^{-\frac{eU_j}{kT}}. \tag{13}$$

if ε is differentiated with respect to kT —differentiations with respect to kT will be designated by a period—and if the differential quotient is set equal to zero, we obtain the equation

$$\left[\frac{1-c}{1+c}\right]^{\cdot} - \frac{1}{kT}\frac{1-c}{1+c} + \frac{1-c}{1+c}\frac{eU_a}{(kT)^2} = 0, \tag{14}$$

from which $\tilde{T}$ must be determined. However,

$$\left[\frac{1-c}{1+c}\right]^{\cdot} = -\frac{2}{(1+c)^2} \cdot \dot{c}$$

and according to (13)

$$\left.\begin{aligned} \dot{c} &= \frac{(1-c^2)^2}{2c} \cdot \frac{\dot{F}}{P} = \frac{(1-c^2)^2}{2c} \frac{F}{P} \frac{1}{kT}\left[\frac{eU_j}{kT} + \frac{5}{2}\right] \\ &= \frac{c}{2}(1-c^2)\frac{1}{kT}\left[\frac{eU_j}{kT} + \frac{5}{2}\right], \end{aligned}\right\} \tag{15a}$$

and also

$$\left[\frac{1-c}{1+c}\right]^{\cdot} = -\frac{1-c}{1+c}\cdot c\left[\frac{eU_j}{kT}+\frac{5}{2}\right]. \tag{15b}$$

If we set the ion concentration $c(\tilde{T}) = \tilde{c}$, then according to (15b) we get from (14)

$$\tilde{c} = \frac{\frac{eU_a}{k\tilde{T}} - 1}{\frac{eU_j}{k\tilde{T}} + \frac{5}{2}} \tag{16}$$

and after combination with (13) as the final equation for $\tilde{T}$

$$\frac{1}{\frac{(2\pi m)^{\frac{3}{2}}}{h^3}\cdot e^{\frac{3}{2}}}\cdot\frac{1}{2}\cdot\frac{g_0}{g_1}\frac{P}{U_j^{\frac{3}{2}}}\left(\frac{eU_j}{k\tilde{T}}\right)^{\frac{3}{2}}\cdot e^{+\frac{eU_j}{k\tilde{T}}} = \left[\frac{\frac{eU_j}{k\tilde{T}}+\frac{5}{2}}{\frac{eU_a}{k\tilde{T}}-1}\right]^2 - 1. \tag{17}$$

For the solution of this transcendental equation and its dependency on P and U_j, a graphic representation is simplest. We set $eU_j/k\tilde{T} = \xi$ and $U_a/U_j = \alpha$, further we substitute for the constant factor on the left-hand side a measurement factor which permits us to measure P in atmospheres and set

$$\frac{g_0}{g_1}\cdot\frac{P_{[\mathrm{Atm.}]}}{U_{j\,[\mathrm{Volt}]}^{\frac{3}{2}}} = K, \tag{18}$$

so that the graphic representation is based on the equation

$$1.03\cdot 10^{-4}K\cdot\xi^{\frac{3}{2}}e^{+\xi} = \left[\frac{\xi+\frac{5}{2}}{\alpha\xi-1}\right]^2 - 1 \tag{19}$$

In Figure 1, the logarithms of the left and right hand side are found in terms of different values for the parameters K and α. The point of intersection corresponding to a given pair of α, K yields on the abscissa the solution $\xi = eU_j/k\tilde{T}$, which corresponds to this pair of parameters. The variation of K is sufficiently limited by

F-54

$$10^{-7} < K < 1$$

if one considers that the application of this method at rising pressures is limited by the increasing absorption. α cannot reach the value 1, as long as a line is measured, but it will not reach a value below 0.4. From the diagram one obtains at first formally: ξ varies in the opposite direction, with α when K is held constant and with K if α is held constant, but not to the same degree. In particular, a change in K by an order of magnitude yields only a variation in ξ of 20 to 25%. From (18) follows that for constant excitation potential changes in pressure or ionization potential cause only small changes in $\xi = eU_j/k\tilde{T}$. This means on the one hand that temperature $\tilde{T}$ increases only very little with pressure at fixed ionization

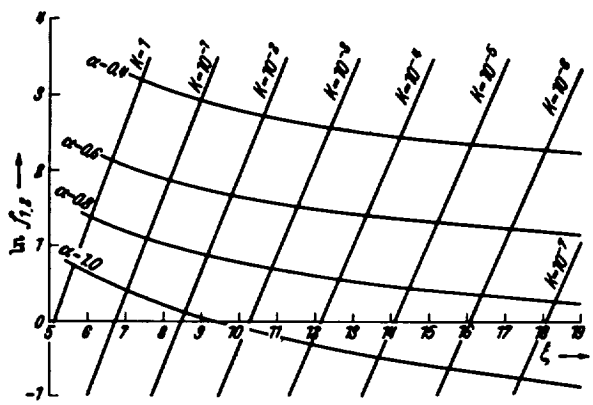

Fig. 1. For the solution of the transcendental equation (17).
$f_1 = 1.03 \cdot 10^{-4} K\, \xi^{\frac{5}{2}} \epsilon^{\xi}, \quad f_2 = \left[\frac{\xi + \frac{5}{2}}{a\xi - 1}\right]^2 - 1.$

potential, and on the other hand, with fixed pressure $\tilde{T}$ increases to an almost equal measure with the ionization potential, because of the way ξ approaches a constant value. From the diagram one can also get certain values for $\tilde{T}$: at an ionization potential of 10 volts, a pressure of 1 atm. with $g_0/g_1 = 1$ and $\alpha = 0.8$, ξ becomes equal to 10, or $\tilde{T} \sim 12{,}000°K$. For the lines of the Balmer series at the same pressure the average $\tilde{T} \sim 16{,}000°K$.

If, as is assumed in the second case above, the emission carrier is the fth degree ionized atom, then

$$\varepsilon = \frac{A}{4\pi} h\nu \frac{g_a}{g_l} \cdot n_l \cdot e^{-\frac{eU_a}{kT}}$$

with

$$n_l = \frac{1 - c_{l+1}}{(l+1) + c_{l+1}} \cdot \frac{P}{kT} \tag{20}$$

and the Saha equation

$$\frac{c_{l+1}}{1 - c_{l+1}} \cdot \frac{l + c_{l+1}}{(l+1) + c_{l+1}} \cdot P = F_l. \tag{21}$$

is valid. In analogous manner as above, for $\tilde{c}_{l+1}$, the concentration of the $(l+1)^{th}$ degree ionized atom results at temperature $\tilde{T}$,

$$\tilde{c}_{l+1} \cdot \frac{l+2}{l+1} \cdot \frac{l + \tilde{c}_{l+1}}{l + 2\tilde{c}_{l+1}} = \frac{\frac{eU_a}{k\tilde{T}} - 1}{\frac{eU_j}{k\tilde{T}} + \frac{5}{2}}. \tag{22}$$

By combination with (21) there results in parallel to (17)

$$\left.\begin{aligned} &\frac{1}{\frac{(2\pi m)^{\frac{3}{2}}}{h^3} e^{\frac{3}{2}}} \cdot \frac{1}{2} \cdot \frac{g_l}{g_{l+1}} \cdot \frac{P}{U_{j_l}^{\frac{3}{2}}} \cdot \left[\frac{eU_{j_l}}{k\tilde{T}}\right]^{\frac{3}{2}} \cdot e^{+\frac{eU_{j_l}}{k\tilde{T}}} \\ &= \left\{\frac{\frac{eU_{j_l}}{k\tilde{T}} + \frac{5}{2}}{\frac{eU_a}{k\tilde{T}} - 1} \cdot \frac{l+2}{l+1} \cdot \frac{l + \tilde{c}_{l+1}}{l + 2\tilde{c}_{l+1}} - 1\right\} \frac{(l+1) + \tilde{c}_{l+1}}{+ \tilde{c}_{l+1}}. \end{aligned}\right\} \tag{23}$$

Although here, in contrast to (17), $\tilde{T}$ still appears implicitly—in $\tilde{c}$—equation (23) is sufficient to show, that with constant U_{j_l} the solution for ξ does not change significantly from the solution in the first case. The left-hand side of (23) completely corresponds to the left-hand side of (17), so that nothing changes in the graphic representation of the corresponding curves. The right-hand side is somewhat modified: instead of the quadratic expression

$$\left[\frac{\frac{eU_j}{k\tilde{T}} + \frac{5}{2}}{\frac{eU_a}{k\tilde{T}} - 1}\right]^2$$

the fraction itself appears. In addition, it contains a newly appearing factor as well as the braces on the right-hand side. Particularly, when dealing with higher ionization levels, both of these factors have values within narrow limits and only little larger than 1. Furthermore, they change with $\tilde{T}$ in the same sense as the quotient

$$\frac{\frac{eU_j}{k\tilde{T}} + \frac{5}{2}}{\frac{eU_a}{k\tilde{T}} - 1}.$$

The right-hand side of equation (23) decreases monotonically with increasing ξ just as does the right-hand side of equation (17). The curves in the diagram corresponding to a given value of α, as can be seen from (23), are displaced toward lower values compared to the corresponding curves in (17). One can see, however, that the solution of ξ at constant K values is changed to only insignificantly smaller values of ξ. Somewhat stronger—but in the opposite direction—is the effect of the higher ionization potential on the decrease in K values. In general, the $\tilde{T}$ values for the transition from neutral atoms to the successive ionization levels will behave as the respective ionization potentials.

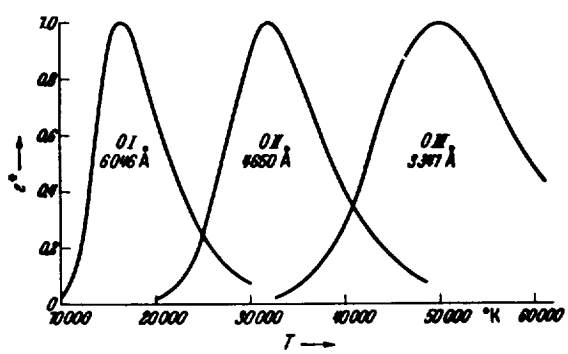

Fig. 2. $\epsilon^*(T,P)$ functions for oxygen arc and spark lines (P = 1 atm.).

For example, for $\epsilon^*(T,P)$ functions, curves were chosen which were used for measurements of the Gerdien arc. The Gerdien arc burns in an atomic gas mixture with hydrogen and oxygen appearing in the ratio 2 : 1. Fig. 2 shows the $\epsilon^*(T,P)$ functions for each of one line of O I, O II and O III. The fact that these curves refer to components of a mixture is here

of no concern. They show that measurements of lines of a given emission carrier are only possible within a limited temperature region. This region is limited on both sides by the lowest values to which $\epsilon^*(r)$ can be measured. However, the regions corresponding to the individual ionization levels generally lie so close to each other that generally measurement can proceed to the lines of the next higher ionization level without a significant hole. It is shown in Fig. 2 that the curves embrace in their entirety a region from 10,000 to 60,000°K.

As already pointed out in the introduction, the method developed here for temperature measurements is built on the same basis as the methods which Fowler and Milne used for the ordering of stellar classes in a temperature scale. The development of the basis for calculation as given in these paragraphs makes the differences apparent which are caused by the significantly different type of problem encountered. When measuring the temperature changes in an arc column, the total pressure is given as a constant parameter, and the electron pressure must be calculated using equation (9) as function of pressure and temperature. In the determination of stellar classes the relationships are different. Instead of a statement on the total pressure, Fowler and Milne use a statement regarding the electron pressure which is determined from the conditions prevailing in the stellar atmosphere. This makes the solution of equation (9) unnecessary.

The observations in these paragraphs primarily dealt with temperature measurements of lines. In the temperature variation of continual emission, the portion representing the end continua of the neutral atoms leads to a decided maximum. Further maxima can appear due to limiting continua of newly appearing types of ions. However, if they do appear in the temperature variation of the basic continua, they will be only little different, and therefore less suitable for measurement. For the first maximum, the equations in these paragraphs can be used.

§4. REGARDING CAUSES OF ERROR IN THE EVALUATION

Errors arise in this manner of evaluation if the parameters used in the evaluation are not substituted with their proper values. The position of the

maxima which determine the norm temperature $\tilde{T}$, and the shape of the curves which is determined from measurements of other temperatures, are dependent on pressure, ionization potentials, excitation potentials, and weights. Excitation potentials and weights are sufficiently known so that they do not cause errors. On the other hand, as was shown first by Rompe and Schulz, the ionization potential of plasma can decrease to such an extent that it can only be roughly estimated in many cases. Furthermore, the determination of the pressure may contain considerable uncertainty under several experimental conditions. It is, therefore, necessary to determine to what extent these uncertainties will affect the result.

For temperature $\tilde{T}$, errors appear in the approximation of (17) or (23) if the small variability of the right-hand side is neglected because of the much larger variability of the left-hand side. We denote with U_j, $\tilde{T}$ and P the true value, and with $\Delta\tilde{T}$ the deviation from $\tilde{T}$ due to the errors ΔU_j or ΔP. This results in the measurements of arc as well as spark lines in

$$\frac{\Delta\tilde{T}}{\tilde{T}} = +\frac{\Delta U_j}{U_j}\frac{\xi}{\xi+\frac{1}{2}}, \tag{24a}$$

$$\frac{\Delta\tilde{T}}{\tilde{T}} = +\frac{\Delta P}{P}\frac{1}{\xi+\frac{1}{2}}. \tag{24b}$$

However, when ionization tension and pressure are faulty, not only the maximum, but the entire curve $\epsilon^*(T,P)$ moves. Then if the error is to be pursued over the entire temperature scale, the calculation has to be built on a broad basis. At the same time the approximation formula (24) can be defined. The errors occur due to the fact that the ϵ^* value, which was measured for the radius r, because of the deformation of the curve receives a temperature $T + \Delta T$ instead of the correct temperature T. Therefore, the relative error $\Delta T/T$ corresponds to the temperature change which is obtained if one determines $\epsilon^*(T,P,U_j)$ at constant ϵ^* by varying first the ionization potential U_j by ΔU_j at constant pressure and then the pressure P by ΔP at constant ionization potential.

If the error in measuring an arc line is calculated in this manner, the result is

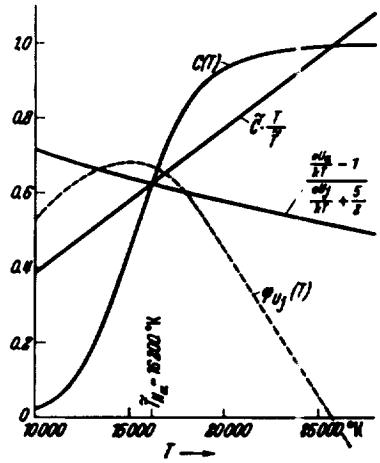

Fig. 3. For the interpretation of the function $\phi U_j(T)$.

$$\frac{\Delta T}{T} = +\frac{\Delta U_j}{U_j}\cdot\frac{\frac{eU_j}{kT}}{\frac{eU_j}{kT}+\frac{5}{2}}\cdot\frac{c-\frac{\tilde{c}T}{\tilde{T}}}{c-\frac{\frac{eU_a}{kT}-1}{\frac{eU_j}{kT}+\frac{5}{2}}} = \frac{\Delta U_j}{U_j}\cdot\frac{\frac{eU_j}{kT}}{\frac{eU_j}{kT}+\frac{5}{2}}\cdot\varphi_{U_j}(T), \tag{25a}$$

$$\frac{\Delta T}{T} = +\frac{\Delta P}{P}\cdot\frac{1}{\frac{eU_j}{kT}+\frac{5}{2}}\cdot\frac{c-\tilde{c}}{c-\frac{\frac{eU_a}{kT}-1}{\frac{eU_j}{kT}+\frac{5}{2}}} = \frac{\Delta P}{P}\cdot\frac{1}{\frac{eU_j}{kT}+\frac{5}{2}}\cdot\varphi_P(T). \tag{25b}$$

Both functions ϕT can be clearly interpreted. This is shown for $\phi U_j(T)$. For this purpose

$$c, \quad \tilde{c}\frac{T}{\tilde{T}} \qquad \text{and} \qquad \frac{\frac{eU_a}{kT}-1}{\frac{eU_j}{kT}+\frac{5}{2}}$$

is superimposed over T in Fig. 3. The example is based on a measurement of H_α. From the general characteristics of the curves it can be seen that $\phi U_j(T)$ changes only a small amount in the vicinity of $\tilde{T}$, and that it is always positive and less than 1. For significantly lower temperatures, $\phi U_j(T)$ gets smaller. The same is true for higher temperatures. Here $\phi U_j(T)$ can even pass through zero. But in that case the concentrations of

F-54

the neutral atoms become so small that it probably almost reaches the limit of the temperature region which can be measured. It seems at first that the interpretation of the function $\phi U_j(T)$ up to this temperature limit is only meaningful for hydrogen since with other atoms the appearance of doubly charged ions negates the assumption for the calculation. However, the results shown in the example in Fig. 3 can be applied also to higher temperatures of other atoms, since c within the framework of the calculation actually only measures as $1 - c$ the disappearance of the neutral atoms, and c, therefore, with the appearance of the next higher ionization levels is equally effective as concentrations of all ions. Completely analogous considerations can be made for $\phi p(T)$. We can therefore conclude generally, that for the whole temperature range to be considered for measurement in arc lines, the following is valid:

$$\frac{|\Delta T|}{T} < \frac{\Delta U_j}{U_j} \cdot \frac{\frac{eU_j}{kT}}{\frac{eU_j}{kT} + \frac{5}{2}}, \tag{26a}$$

$$\frac{|\Delta T|}{T} < \frac{\Delta P}{P} \cdot \frac{1}{\frac{eU_j}{kT} + \frac{5}{2}}. \tag{26b}$$

In a completely analogous way, both equations also can be proven for measurements in spark lines.

Hannover, Physical Institute of the Technical College,
December 1950

FOOTNOTES

[1] In the next report, measurements of the Gerdien arc up to 34,000°K will be given.

[2] H. Hörmann, Z. Physik, 97, p. 539, (1935).

[3] R. H. Fowler, Statistische Mechanik, Leipzig, 1931. Refer also A. Unsöld, Physik der Sternatmosphären (Physics of Stellar Atmospheres), Berlin, (1938). Contains extensive literature references.

[4] H. Bartels, Z. Physik, 127, p. 243, (1950); 128, p. 546, (1950).

[5] H. Hörmann, Z. Physik, 97, p. 539, (1935).

[6] H. Bartels, Z. Physik, 125, p. 597, (1949); 126, p. 108, (1949).

[7] In view of the high temperatures, only atomic gases can be considered as components of the mixtures.

[8] If one calculates $\epsilon(T,P)$ for application to specific quantitative measurements, then these simplifications are not always valid, and must be explained at least from case to case.

[9] Since we are dealing here only with one kind of ion, the subscripts for c and F are unnecessary. Generally, the term $c_l = \frac{n_l}{\sum_{i=0}^{z} n_i}$ refers to the concentration of atoms ionized to the lth degree.

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