MITIGATION OF THE IMPACT OF SENSING NOISE ON THE PRECISE FORMATION FLYING CONTROL PROBLEM
Final Report

NASA Grant NCC5-724

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INTRODUCTION

This is the final report of the NASA Grant NCC5-724 that was awarded from a proposal submitted in response to NRA-03-GSFC/AETD-01.

OBJECTIVES

The specific objectives of this proposed research were:

- Further investigation into the impact of CDGPS sensing errors for high Earth orbit missions.
- Identify augmentation approaches of the CDGPS that will improve the relative state for low and high Earth orbit missions.
- Integration of the navigation and control concepts into the GSFC Formation Flying Testbed

In addition this was a cooperative effort with Dr. Jonathan How at MIT. Dr. Alfriend was to spend two weeks working with Dr. How and his students. The travel for these two weeks was paid by the Texas Engineering Experiment Station (TEES) as cost sharing.

SUMMARY

Dr. Alfriend spent the week of March 1-5 at MIT working with Dr. How and the week of April 26-29 at NASA Goddard Space Flight Center with Dr. How's students integrating the navigation and control concepts on the GSFC Formation Flying Testbed.

In addition, Dr. Alfriend developed an approximate analytical solution to the linearized relative motion navigation problem that has provided insights into the achievable accuracy. This solution identifies the critical factors for reducing the relative motion navigation accuracy. This research will be submitted for publication and/or presentation at a conference in the near future.

In previous research [1] sponsored by AFOSR Dr. Alfriend developed a state transition matrix (STM) that is valid for all eccentricities and includes 1st order absolute and differential J_2 effects. In this contract he been working with the MIT group to integrate this STM into their control methods and implement it on the Testbed. This work is on-going and not yet complete.

RELATIVE NAVIGATION DETAILS

The planar Hill's equations and solution are

$$\ddot{x} - 2n\dot{y} - 3n^2 x = 0$$
$$\ddot{y} + 2n\dot{x} = 0 \tag{1}$$

$$x = 2\left(2x_0 + \dot{y}_0 / n\right) - \left(3x_0 + 2\dot{y}_0 / n\right)\cos nt + \left(\dot{x}_0 / n\right)\sin nt$$
$$y = \left(y_0 - 2\dot{x}_0 / n\right) - 3\left(2x_0 + \dot{y}_0 / n\right)nt + 2\left(3x_0 + 2\dot{y}_0 / n\right)\sin nt + 2\left(\dot{x}_0 / n\right)\cos nt \tag{2}$$

The secular term in the y equation is just the differential semi-major axis. Therefore, for two satellites to stay close together it is necessary that

$$\delta a = 2\left(2x_0 + \dot{y}_0 / n\right) \tag{3}$$

The standard deviation is

$$\sigma_{\delta a} = 2\left[4\sigma_x^2 + \left(4/n\right)\sigma_x \sigma_{\dot{y}} \rho_{x\dot{y}} + \left(1/n^2 \sigma_{\dot{y}}^2\right)\right]^{1/2}$$
$$\sigma_{\delta a} = 2\left[\left(2\sigma_x - \sigma_{\dot{y}}/n\right)^2 + \left(4/n\right)\sigma_x \sigma_{\dot{y}}\left(1 + \rho_{x\dot{y}}\right)\right]^{1/2} \tag{4}$$

where $\rho_{x\dot{y}}$ is the correlation coefficient. It is obvious from eq. (4) that the semi-major axis error is minimized if $\rho_{x\dot{y}} = -1$. In response to this observation Carpenter and Alfriend [2] concluded that a good Kalman filter for the relative navigation would have $\rho_{x\dot{y}} \approx -1$. However, the results obtained by How [3] with differential carrier phase GPS (DCGPS) have resulted in $0 < \rho_{x\dot{y}} < -0.2$. The purpose of this research was to explain this difference between what was expected and what was observed.

The coupling between the radial displacement and in-track speed is a result of the equations of motion being derived in a rotating coordinate system. The period of the rotation is the orbit period. Thus, for a filter to provide the expected correlation coefficient of (-1) a significant portion of a period is needed. However, measurements are being obtained much faster, in this case every 30 seconds. Basically the filter is essentially seeing a double integrator and is being overwhelmed by the measurements. We now address this problem. The equations of motion are

$$
\begin{aligned}
&\ddot{x} - 2n\dot{y} - 3n^2 x = v_x \\
&\ddot{y} + 2n\dot{x} = v_y \\
&v = N(0,\sigma_v), E(vv^T) = Q_c \\
&\mathbf{z} = H\mathbf{x} + \mathbf{w} \\
&\mathbf{w} = N(0,\sigma_w), E(ww^T) = R_c \\
&\mathbf{x} = (x,\dot{x},y,\dot{y})^T \\
&H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\end{aligned}
\tag{5}
$$

Let Δt be time between measurements and make the time transformation

$$
\tau = \frac{t}{\Delta t}, (\ )' = \frac{d}{d\tau}, \frac{d}{dt} = \frac{1}{\Delta t}\frac{d}{d\tau} \tag{6}
$$

Letting $\varepsilon = n\Delta t$ the equations of motion become

$$
\begin{aligned}
x'' &= 2\varepsilon y' + 3\varepsilon^2 x + (\Delta t)^2 v_x \\
y'' &= -2\varepsilon x' + (\Delta t)^2 v_y
\end{aligned}
\tag{7}
$$

The Riccati equation is

$$
AP + PA^T + BQB^T = PH^T R^{-1} HP \tag{8}
$$

This is a 4th order nonlinear matrix equation. In addition, the system is discrete, not continuous. An exact solution of the discrete version of eq. (8) is probably not obtainable. To obtain an approximate solution two things are done. First, we assume that the solution of the steady state continuous equation is approximately equal to the discrete solution of the discrete equation and, secondly since $\varepsilon << 1$ we expand each of the matrices in a series in powers of ε. This results in the 0th order equation being two decoupled double integrators that can be solved. Then the 1st order equation is linear and it can be solved. This approach results in

$$
\begin{aligned}
\sigma_x &= 2^{1/4}\sigma_{vd}^{1/4}\sigma_{wd}^{3/4}\Delta t^{1/2} \\
\sigma_{\dot{y}} &= 2^{1/4}\sigma_{vd}^{3/4}\sigma_{wd}^{1/4}\Delta t^{1/2} \\
\rho_{x\dot{y}} &= -n\left(\frac{\sigma_{wd}}{\sigma_{vd}}\right)^{1/2} \\
\sigma_{\delta a} &= \sigma_{\delta a} = \frac{2}{n}\sigma_{\dot{y}} = \frac{2^{5/4}}{n}\sigma_{vd}^{3/4}\sigma_{wd}^{1/4}\Delta t^{1/2}
\end{aligned}
\tag{9}
$$

What is very interesting is that to the 1st approximation the error in the semi-major axis is proportional to the error in the in-track velocity. The drift rate per orbit is $d = 3\pi\delta a$. Figures 1 and 2 show the semi-major axis standard deviation and drift rate standard deviation as a function of the process noise, σ_{vd}, and the discrete measurement noise, σ_{wd}. Figure 3 shows the correlation coefficient as a function of the position measurement error (not measurement noise). For the process noise of $\sigma_{vd} = 10^{-5} m/s^2$, which is the value used by MIT, we see that $\rho_{xy} > -0.1$ which is in the neighborhood of the value obtained by MIT. To reduce the correlation coefficient it is necessary to reduce the process noise.

These results should aid in designing a filter and providing insight into the dynamics of the relative navigation.

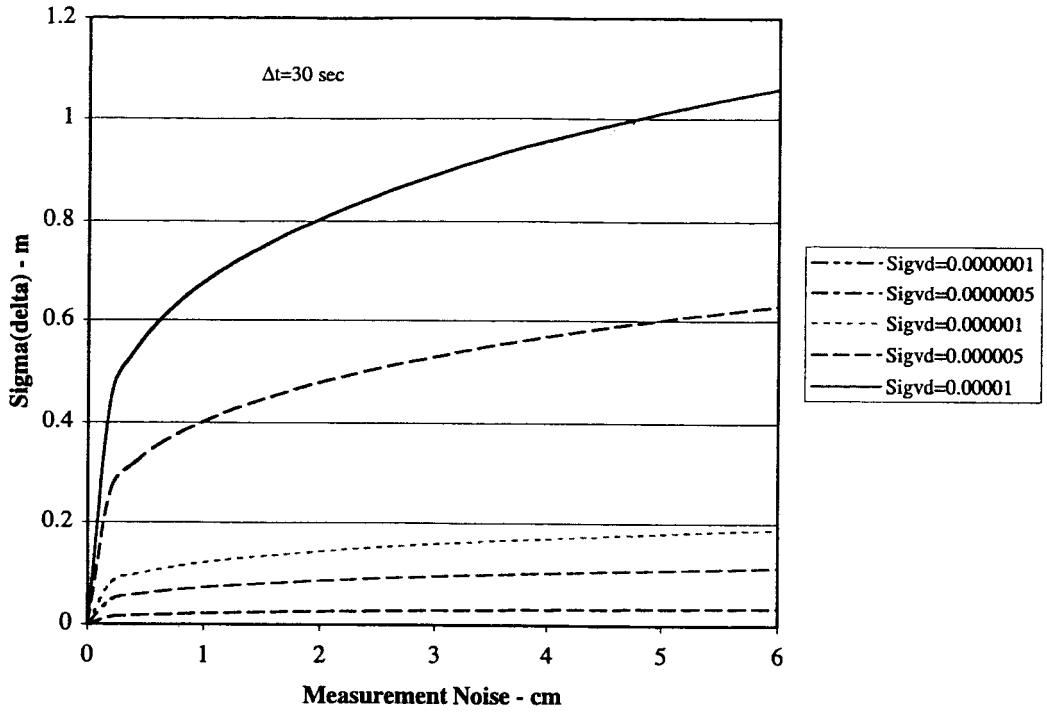

Figure 1 Semi-major Axis Standard Deviation

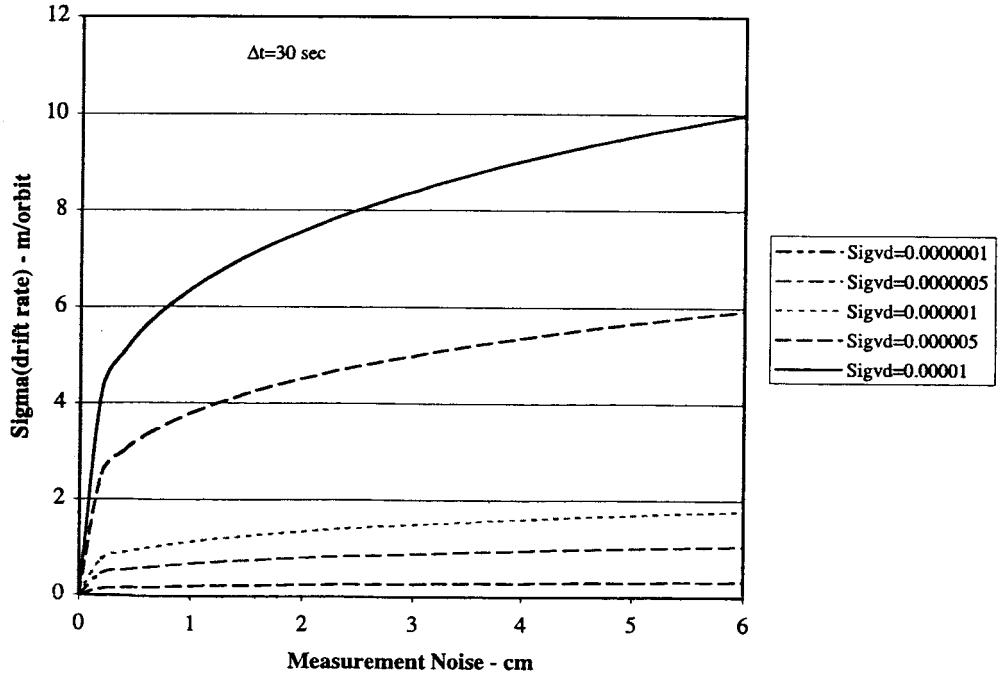

Figure 2 Drift Rate Standard Deviation

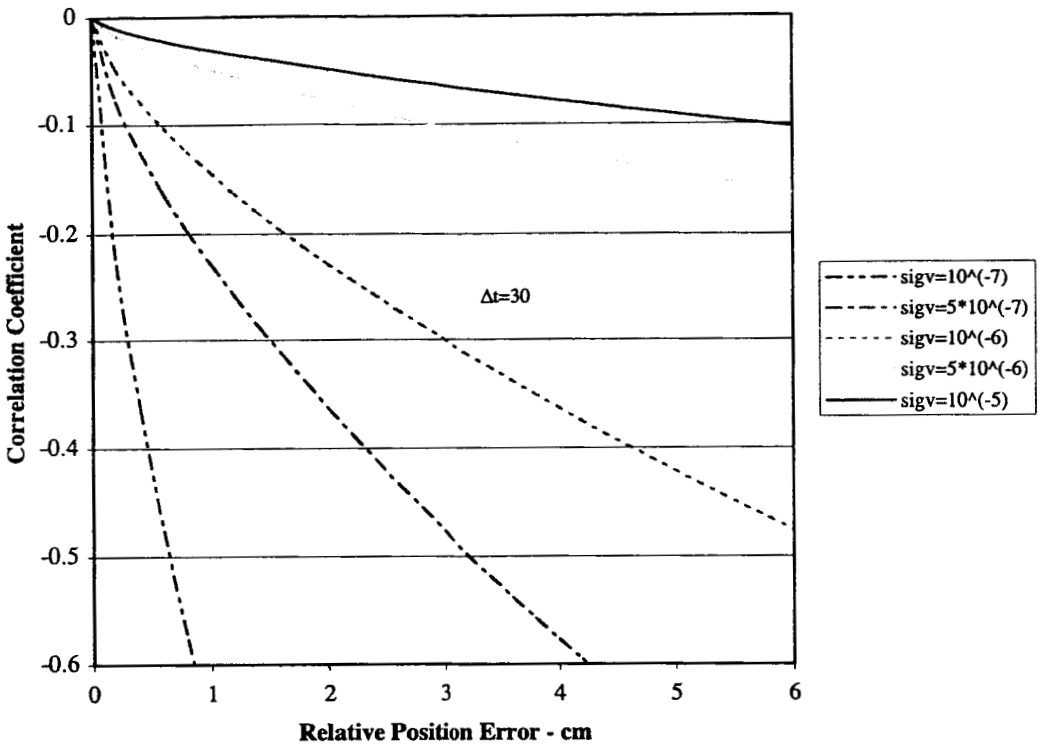

Figure 3 Correlation Coefficient

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