

Introduction: Cosmic collisions between terrestrial planets resemble somewhat the life cycle of the phoenix: worlds collide, are consumed in flame, and after the debris has cleared, shiny new worlds emerge aglow with possibilities. And glow they do, for they are molten. How brightly they glow, and for how long, is determined by their atmospheres and their moons.

A reasonable initial condition on Earth after the Moon-forming impact is that it begins as a hot global magma ocean^{1,2}. We therefore begin our study with the mantle as a liquid ocean with a surface temperature on the order of 3000-4000 K at a time some 100-1000 years after the impact, by which point we can hope that early transients have settled down.

A second initial condition is the presence of a substantial atmosphere. Earth today has about 270 bars of H₂O and 50 bars of CO₂ near the surface, the first mostly in oceans and the latter mostly in carbonate rocks. Similarly large reservoirs are held in the mantle. Thus it is reasonable to start with 100-1000 bars of H₂O and CO₂ in the atmosphere. The water vapor and carbon dioxide would be supplemented by smaller amounts of CO, H₂, N₂, various sulfur-containing gases, and by a suite of geochemical volatiles evaporated from the magma^{3,4}.

A third initial condition is the existence of the Moon. We start the Moon with its current mass at the relevant Roche limit. This gives Earth its shortest first day and maximizes the role of tidal heating.

A fourth initial condition is the angular momentum of the Earth-Moon system. In canonical models this is held constant, so that with the Moon at the Roche limit, the Earth starts with a 5 hour day. Recent Moon-forming models need loss of angular momentum from the Earth-Moon system^{5,6}. These start Earth with a ~3 hour day and thus imply more tidal heating.

It is well known that an atmosphere's thermal blanketing effect prevents a magma ocean from cooling rapidly. A string of increasingly sophisticated models^{7-11,1,2,12,13} have addressed consequences of thermal blanketing by steam atmospheres over Earth's cooling magma oceans. All of these models presume H₂O-CO₂ atmospheres, and most ignore the Moon.

A different set of studies has addressed the composition of the atmosphere over a magma ocean^{14-16,3,4}. Shortly after the impact the atmosphere over the lava also contains volatiles such as sulfur, sodium, and chlorine. The geochemically enriched atmosphere is markedly more opaque at magma temperatures than

H₂O and CO₂ working alone, and thus the planet cools more slowly and spends more time with a lava surface than previous models would predict.

For Earth we use a ruthlessly simplified model. We characterize the Earth by its surface temperature T_s and an interior temperature T_i . In effect T_i is a potential temperature, the temperature that every parcel in an adiabatic mantle would have were it brought to the surface. We also assume that the mantle is uniform, in the sense that it has the same viscosity at all depths and that it freezes everywhere at the same time. This assumption is roughly equivalent to assuming that the adiabat and the melting curve are parallel (a bad assumption near the surface when the surface is cold, but perhaps not so bad when the mantle has not yet fully congealed). The uniform mantle vastly simplifies the discussion of topics like parameterized convection that can depend on a global viscosity.

We describe silicate freezing by three temperatures: a liquidus T_{liq} hotter than which the mantle is fully molten; a solidus T_{sol} colder than which the mantle is fully solid; and a critical temperature T_{crit} in between at which the rheology of the material changes from that of a solid with melt percolating through it to a liquid carrying suspended solids^{11,17}. The critical temperature T_{crit} corresponds roughly to what is commonly meant by melting. We take $T_{liq}=1800$ K and $T_{sol}=1400$ K. These are arbitrary values; our results are insensitive to them. For simplicity we assume that the melt fraction is linear in T_i between T_{liq} and T_{sol} .

We follow previous workers by using parameterized convection to link the cooling rate to the temperature and heat generation inside the Earth^{17,13}. In parameterized convection, heat flow is related to the size and viscosity of the interior and the temperature gradient across the surface boundary layer through the Rayleigh number.

Viscosity is of central importance to convection and tidal heating. We follow^{11,17} by treating the temperature dependence of viscosity in two regimes divided by T_{crit} . Viscosity is a strong function of temperature, moreover it undergoes a phase transition at $T=T_{crit}$ where it jumps by more than ten orders of magnitude.

Cooling takes place in two regimes, the first a fast-cooling regime in which the geothermal heat flow is comparable to or bigger than the globally-averaged absorbed sunlight, the second a slow-cooling regime in which the radiative cooling is in approximate balance with absorbed sunlight and the heat flow is small. The

transition between regimes, when it comes, is abrupt.

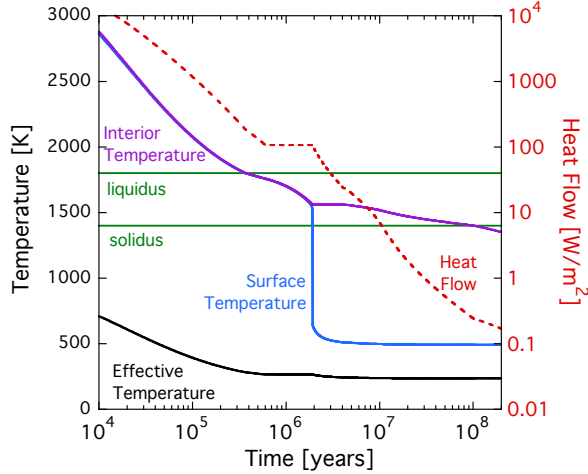


Figure 1. Thermal evolution of Earth after a canonical Moon-forming impact. Results are shown for a 100 bar atmosphere. Internal temperature T_i (mantle), surface temperature T_s , and effective radiating temperature T_{eff} are shown against the left hand axis. The liquidus and solidus are also indicated. Geothermal heat flow (dashed curve) is shown against the right hand axis. The heat flow plateaus at $\sim 100 \text{ W/m}^2$ while the cooling Earth passes through the runaway greenhouse, a phase that lasts from 0.5-1.7 Myr. The runaway greenhouse phase ends abruptly when the monotonically cooling mantle becomes too viscous to sustain the high heat flow, at which point the surface quickly cools to the equilibrium temperature under ~ 100 bars of CO_2 .

Tidal heating by the nearby Moon is a major term in the early Earth's energy budget. Here we assume that tidal dissipation in the magma ocean can be described by viscosity. This is a reasonable approximation in the congealing mantle because the mantle's viscosity would be big enough to make ordinary viscous dissipation a big term. Where viscous dissipation dominates, Q is a computed quantity. But at very early times when the mantle is very hot and effectively inviscid, or at modern times when the mantle is solid, ordinary viscous dissipation is predicted to be negligible and some other source of dissipation will likely be more important.

The Moon is entwined with Earth by a negative feedback between thermal blanketing and tidal heating that comes from the temperature-dependent viscosity of the magma ocean. Because of this feedback, the rate that the Moon's orbit evolves is limited by the modest radiative cooling rate of Earth's atmosphere, which in effect tethers the Moon to the Earth. Consequently the

Moon's orbit evolves orders of magnitude more slowly than in conventional models. Slow orbital evolution promotes capture by orbital resonances that may have been important in the Earth-Moon system^{18,5}.

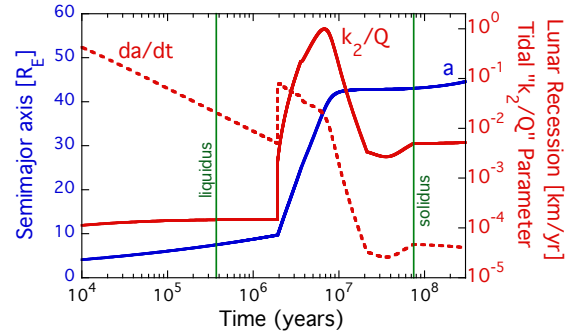


Figure 2. Two parameters (red) that are thought to play key roles in resonance capture and lunar orbital evolution. The velocity da/dt that the Moon recedes from Earth is important to assessing resonance capture. For conventional fixed Q , da/dt is estimated to be 10-100 km/yr when the Moon encounters the evection resonance. At these speeds resonance capture is very unlikely¹⁸. The atmosphere reduces da/dt to 0.1 km/yr or less, for which resonance capture is nearly certain¹⁸. The parameter k_2/Q figures prominently in published work, where it is treated as constant⁵. The distance to the Moon is shown for context. The particular case shown here assumes a canonical Moon-forming impact and a 100 bar atmosphere. The model does not yet include tidal dissipation in the Moon.

- References. [1] Zahnle K et al (2006) Space Sci. Rev. 129, 35-78. [2] Elkins-Tanton, L. (2008) E.P.S.L. 271, 181-191. [3] Schaefer, L. et al (2012) Ap. J. 755, article id. 41. [4] Fegley, B., Schaefer, L. (2013) in Treatise on Geochemistry. 13. eprint:arXiv:1210.0270. [5] Cuk, M., Stewart, S. (2012) Science 338, 1047-1051. [6] Canup R.M. (2012) Science 338, 1052-1055. [7] Matsui, T., Abe, Y. (1986) Nature 319, 303-305. [8] Abe, Y., Matsui, T. J. (1988) Atm. Sci. 45, 3081-3101. [9] Zahnle, K.J. et al (1988) Icarus 74, 62-97. [10] Abe, Y. (1993) Lithos 30, 223-235. [11] Abe, Y. (1997) Phys. Earth Planet. Int. 100, 27-39. [12] Hamano, K. et al (2013) Nature 497, 607-610. [13] Lebrun, T. et al (2013) EGU2013-4653. [14] Miller-Ricci, E. et al (2009) Ap. J. 704, 770-780 (2009). [15] Schaefer, L., Fegley, B. (2010) Icarus 208, 438-448. [16] Lupu et al Ap.J. (2014). [17] Solomatov, V.S., (2009) In Treatise on Geophysics Vol. 9. 91-120. [18] Touma, J., Wisdom, J. (1998) A.J. 115, 1653-1663.