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GIMBAL GEOMETRY AND ATTITUDE SENSING OF THE ST-124 STABILIZED PLATFORM

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SUMMARY

The ST-124 platform is a four gimbal platform. Each gimbal resolves two orthogonal vectors about the third orthogonal axis; therefore, resolvers are fixed to the gimbal pivot points to furnish attitude signal for vehicle control. The resolvers are connected in a chain to provide the coordinate transformation computations. The chain is unique because two separate carrier frequencies are utilized in a single chain to provide yaw, roll, and pitch signals that would otherwise require two complete resolver chains. The two frequencies are demodulated in the guidance computer since the phasing of the signals is significant.

INTRODUCTION

To steer a vehicle, such as the Saturn, in space requires:

1. A knowledge of the existing location of the vehicle
2. A knowledge of the location of the target or destination
3. A calculation of the proper action to take the vehicle from its present location to the destination
4. A system for converting the calculated action in paragraph 3 into vehicle dynamics.

While this is an over-simplification of the problem, it will suffice for this discussion.

Paragraph 1, describing the existing location of the vehicle, consists of translation in X, Y, and Z, and rotation about X, Y, and Z. This is the function of the platform.

Paragraph 2 is the mission.

Paragraph 3 is the function of the navigation computer.

Paragraph 4 is a function of the control computer and actuators.

A block diagram of this system is shown in FIGURE 1.

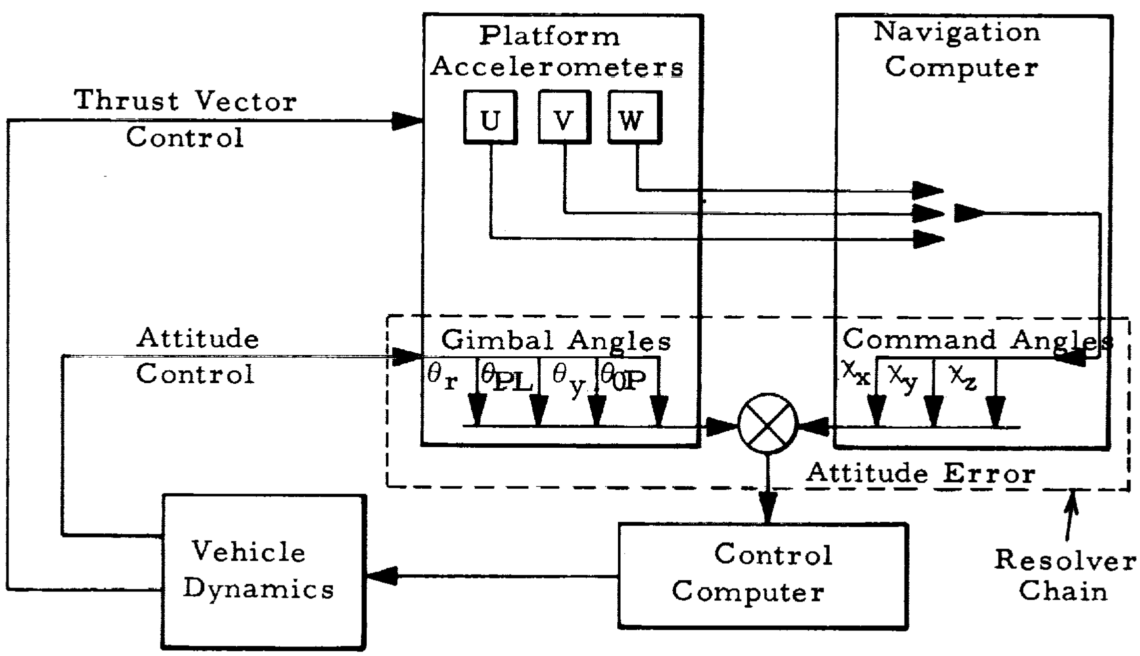


FIGURE 1

This block diagram does not show such devices as sun seekers, rate gyros, etc. Note that the loop is closed twice; through the gimbal angles and through the accelerometers.

This paper will deal with that part of the block diagram shown in the dotted lines, labeled "resolver chain."

DEFINITION OF SYMBOLS

Symbol	Definition
$\overline{X}_m$	Unit vector through Fin I
$\overline{Y}_m$	Unit vector along the longitudinal axis
$\overline{Z}_m$	Unit vector equal to $\overline{X}_m \times \overline{Y}_m$
M	Coordinate system containing $\overline{X}_m$, $\overline{Y}_m$, and $\overline{Z}_m$
$\overline{X}_c$	Unit vector in the direction of the program position of Fin I
$\overline{Y}_c$	Unit vector in the direction of the program position of the longitudinal axis
$\overline{Z}_c$	Unit vector equal to $\overline{X}_c \times \overline{Y}_c$
C	Coordinate system containing $\overline{X}_c$, $\overline{Y}_c$, and $\overline{Z}_c$
$\overline{U}$	Unit vector in inertial space; normally down range
$\overline{V}$	Unit vector in inertial space ; normally up at liftoff
$\overline{W}$	Unit vector in inertial space equal to $\overline{U} \times \overline{V}$
P	Coordinate system containing $\overline{U}$, $\overline{V}$, and $\overline{W}$
ϕ yaw	Attitude error in $\overline{X}_m$ or yaw
ϕ roll	Attitude error in $\overline{Y}_m$ or roll
ϕ pitch	Attitude error in $\overline{Z}_m$ or pitch
R	Directed line. A rotation about R will reduce all attitude errors to zero
α	The amount of rotation about R to reduce all attitude errors to zero

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
θ_r	Roll gimbal angle
θ_{PL}	Pitch limited gimbal angle
θ_y	Yaw gimbal angle
θ_{OP}	Outer pitch gimbal angle
χ_z, χ_y, χ_x	Euler angle program
a	Angle between $\overline{X}_c$ and $\overline{X}_m$
b	Angle between $\overline{X}_c$ and $\overline{Y}_m$
c	Angle between $\overline{X}_c$ and $\overline{Z}_m$
d	Angle between $\overline{Y}_c$ and $\overline{X}_m$
e	Angle between $\overline{Y}_c$ and $\overline{Y}_m$
f	Angle between $\overline{Y}_c$ and $\overline{Z}_m$
g	Angle between $\overline{Z}_c$ and $\overline{X}_m$
h	Angle between $\overline{Z}_c$ and $\overline{Y}_m$
k	Angle between $\overline{Z}_c$ and $\overline{Z}_m$
G	A matrix describing the relation between C and M
Ψ	Angle between $(\overline{Y}_m \times \overline{Y}_c)$ and $\overline{X}_c$
β	Rotation about $\overline{X}_c$
Γ	Rotation about $\overline{Z}_c$

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
Δ	Rotation about $\overline{Y}_c$
h_A	Value of h after rotation β
h_B	Value of h after rotation β and Γ
h_G	Value of h after rotation β, Γ, and Δ
$\overline{\omega}_\alpha$	$= \frac{d\alpha}{dt}$
$\overline{\omega}_x$	$\overline{X}_m$ component of $\overline{\omega}_\alpha$
$\overline{\omega}_y$	$\overline{Y}_m$ component of $\overline{\omega}_\alpha$
$\overline{\omega}_z$	$\overline{Z}_m$ component of $\overline{\omega}_\alpha$
A, B, E	Resolver inputs
C, D, F	Resolver outputs
θ_{PL_o}	Zero value of θ_{PL}

DISCUSSION

A resolver chain is used to perform the function of the block diagram enclosed in the dotted line in FIGURE 1. This function is to compare the program position of the vehicle (as described by the navigation computer) to the actual position of the vehicle (as described by the platform gimbal angles) and to supply the control computer with the information that it needs to turn the vehicle into the position described by the navigation computer. A review of the kinematics involved is presented to give a better understanding of how this is accomplished.

Define a set of cartesian coordinates made up of unit vectors $\overline{X}_m$, $\overline{Y}_m$, and $\overline{Z}_m$ where $\overline{X}_m$ is perpendicular to the longitudinal axis of the vehicle and passes through Fin 1, $\overline{Y}_m$ is along the longitudinal axis of the vehicle, and $\overline{Z}_m = \overline{X}_m \times \overline{Y}_m$ is the vector or cross product of $\overline{X}_m$ and $\overline{Y}_m$. Rotation about $\overline{X}_m$ is yaw, about $\overline{Y}_m$ is roll, and about $\overline{Z}_m$ is pitch. This is the "M" coordinate system (FIGURE 2).

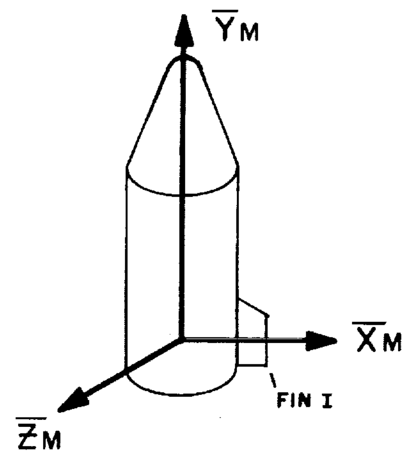


FIGURE 2

Define a second set of cartesian coordinates made up of unit vectors $\overline{X}_c$, $\overline{Y}_c$, and $\overline{Z}_c$. These are the axes of the command position of the vehicle described by the navigation computer. Again $\overline{X}_c$ is through Fin 1, $\overline{Y}_c$ is along the longitudinal axis and $\overline{Z}_c = \overline{X}_c \times \overline{Y}_c$. This is the "C" coordinate system.

Define a third set of cartesian coordinates made up of unit vectors $\overline{U}$, $\overline{V}$, and $\overline{W}$, which are the coordinates of the platform, or inertial space. The orientation of these vectors to an observer on earth is established by the platform erection system prior to liftoff. A likely orientation at some time (t), prior to liftoff is $\overline{U}$ coincident with $\overline{X}_m$, $\overline{V}$ coincident with $\overline{Y}_m$, and $\overline{W}$ coincident with $\overline{Z}_m$. This is the "P" coordinate system.

"P" is the invariant inertial reference to which all motions are referred after liftoff. The origin of M, C, and P are coincident.

The resolver chain is to provide signals to the control computer to turn M into C.

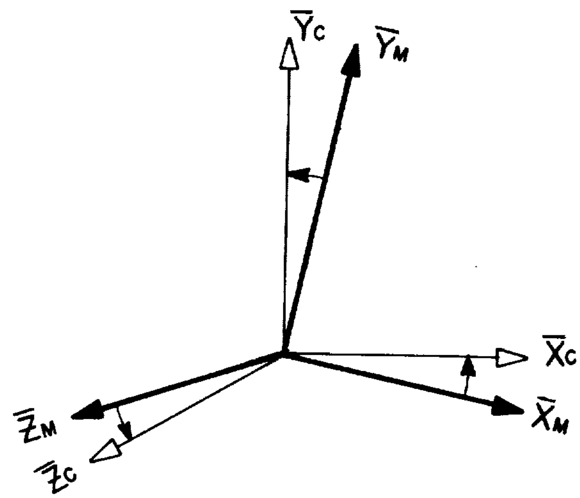


FIGURE 3

From FIGURE 3, it can be seen that rotation about X_m will affect the relation of $\overline{Y}_m$ to $\overline{Y}_c$, and $\overline{Z}_m$ to $\overline{Z}_c$, but will not affect the relation of $\overline{X}_m$ to $\overline{X}_c$. This is true for each axis in turn. Therefore, to turn $\overline{X}_m$ into $\overline{X}_c$, a motion about $\overline{Y}_m$ and $\overline{Z}_m$ is required.

If errors exist (M not coincident with C) in all axes simultaneously, the resolver chain should provide the control computer information so

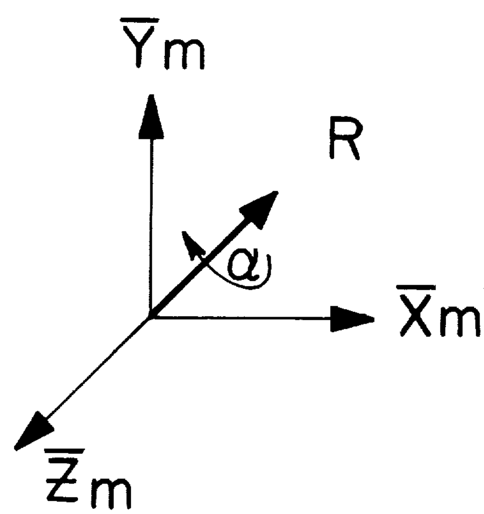


FIGURE 4

that one motion will reduce all three errors to zero. This information is the location of some axis of rotation R, and some amount of rotation α (angle) that will simultaneously turn $\overline{X}_m$ into $\overline{X}_c$, $\overline{Y}_m$ into $\overline{Y}_c$, and $\overline{Z}_m$ into $\overline{Z}_c$ (FIGURE 4). First thoughts would indicate that the rotation could be distributed between the axes and M in the same ratio as R is distributed, by the direction cosines between $\overline{X}_m$ and R, $\overline{Y}_m$ and R, and $\overline{Z}_m$ and R giving:

$$
\begin{aligned}
\phi \text{ yaw} &= \alpha \cos \angle \overline{X}_m R \\
\phi \text{ roll} &= \alpha \cos \angle \overline{Y}_m R \\
\phi \text{ pitch} &= \alpha \cos \angle \overline{Z}_m R
\end{aligned}
\tag{1}
$$

where ϕ yaw, ϕ roll, and ϕ pitch represent the error in each axis.

However, this is not true because angles cannot be treated as vectors, but it is a very good approximation for small errors between M and C.

This is the same approximation as saying:

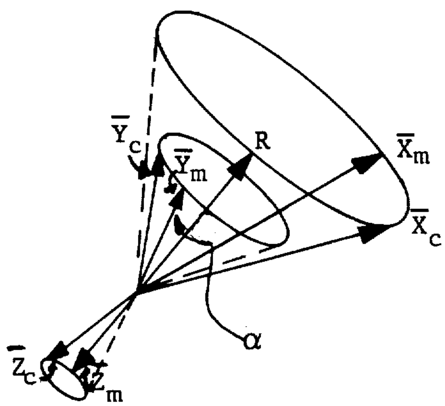
$$
\begin{aligned}
\phi \text{ yaw} &= \overline{X}_m \text{ comp } (\overline{Y}_m \times \overline{Y}_c + \overline{Z}_m \times \overline{Z}_c) \\
\phi \text{ roll} &= \overline{Y}_m \text{ comp } (\overline{X}_m \times \overline{X}_c + \overline{Z}_m \times \overline{Z}_c) \\
\phi \text{ pitch} &= \overline{Z}_m \text{ comp } (\overline{X}_m \times \overline{X}_c + \overline{Y}_m \times \overline{Y}_c)
\end{aligned}
$$

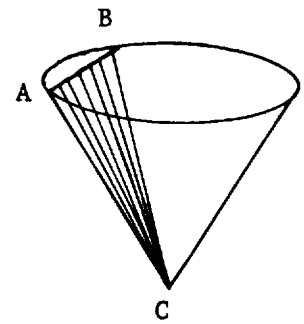
where $\alpha = (\overline{X}_m \times \overline{X}_c + \overline{Y}_m \times \overline{Y}_c + \overline{Z}_m \times \overline{Z}_c)$ in the R direction.

The component may be expressed as a dot or scalar product between the vector and component direction:

$$
\begin{aligned}
\phi \text{ yaw} &= \overline{X}_m \cdot (\overline{Y}_m \times \overline{Y}_c + \overline{Z}_m \times \overline{Z}_c) \\
\phi \text{ roll} &= \overline{Y}_m \cdot (\overline{X}_m \times \overline{X}_c + \overline{Z}_m \times \overline{Z}_c) \\
\phi \text{ pitch} &= \overline{Z}_m \cdot (\overline{X}_m \times \overline{X}_c + \overline{Y}_m \times \overline{Y}_c).
\end{aligned}
\tag{2}
$$

This may be shown to be untrue by noting in FIGURE 5 that $\overline{X}_m$ moves into $\overline{X}_c$, not in a plane but on the surface of a cone. $\overline{X}_m \cdot (\overline{Y}_m \times \overline{Y}_c + \overline{Z}_m \times \overline{Z}_c)$ states that the motion is in the $\overline{X}_m$ $\overline{X}_c$ plane.





Plane ABC approximates conic section ABC.

FIGURE 5

Again, if α is small, a plane approximates the conic section. If α is large, this approximation cannot be used.

The information needed for either the approximate or exact calculation is a description of M and C in a common coordinate system. The M-coordinate system is chosen, since the vehicle control actuators are fixed in this system. C, then, must be expressed in M. To derive the instrumentation needed to express C in M, it is necessary to be aware of the kinematics involved.

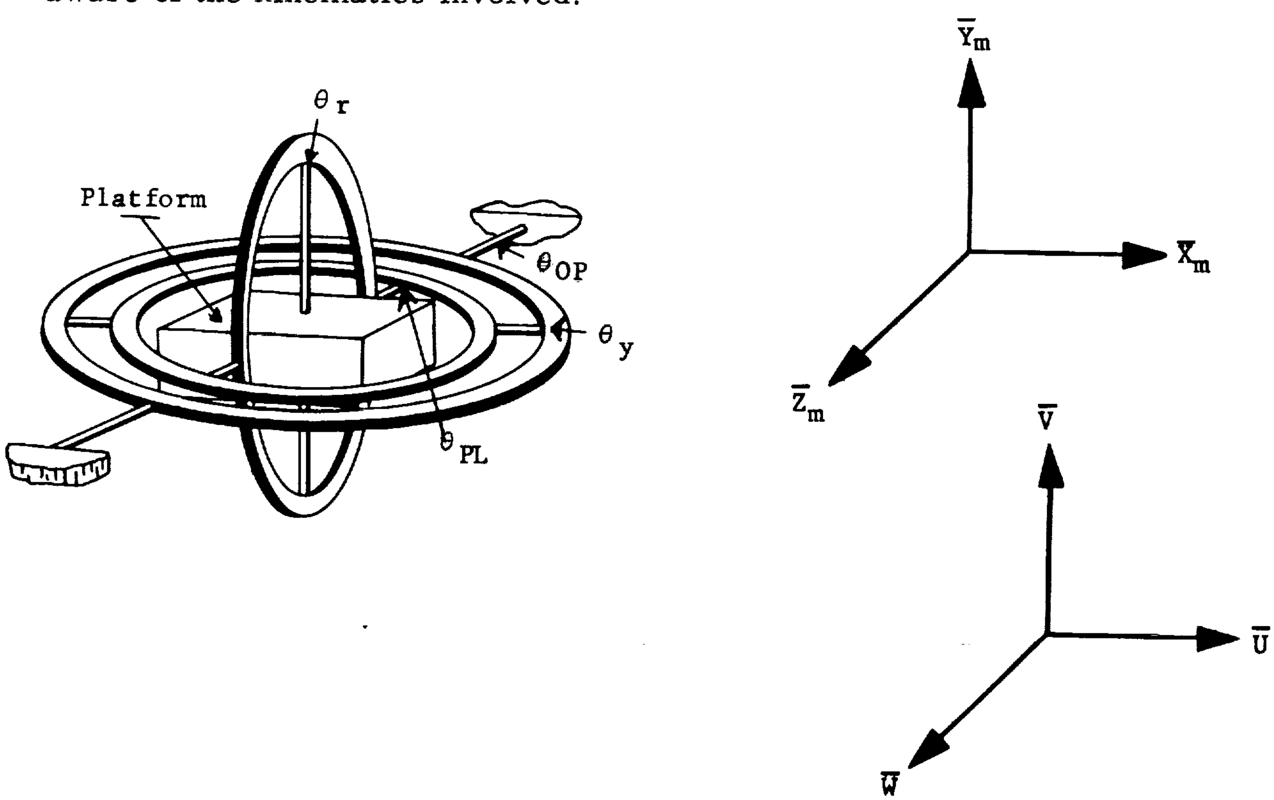


FIGURE 6

The ST-124 is constructed with the gimbals arranged as shown in FIGURE 6.

Starting from the inside, the gimbal angles are θ roll, θ pitch limited, θ yaw, and θ outer pitch. Prior to liftoff with $\overline{U} = \overline{X}_m$, $\overline{V} = \overline{Y}_m$, and $\overline{W} = \overline{Z}_m$, angle θ roll is along $\overline{V}$ and $\overline{Y}_m$; θ pitch limited and θ outer pitch are along $\overline{W}$ and $\overline{Z}_m$; and θ yaw is along $\overline{U}$ and $\overline{X}_m$.

The angles θ roll,, θ pitch, limited θ yaw, and θ outer pitch describe the relation that exists between P and M. From matrix algebra this may be expressed as:

$$\begin{vmatrix} U \\ V \\ W \end{vmatrix} = \begin{vmatrix} \cos\theta_r & 0 & \sin\theta_r \\ 0 & 1 & 0 \\ -\sin\theta_r & 0 & \cos\theta_r \end{vmatrix} \begin{vmatrix} \cos\theta_{PL} & -\sin\theta_{PL} & 0 \\ \sin\theta_{PL} & \cos\theta_{PL} & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_y & -\sin\theta_y \\ 0 & \sin\theta_y & \cos\theta_y \end{vmatrix} \begin{vmatrix} \cos\theta_{OP} & -\sin\theta_{OP} & 0 \\ \sin\theta_{OP} & \cos\theta_{OP} & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \overline{X}_m \\ \overline{Y}_m \\ \overline{Z}_m \end{vmatrix}$$

which may be written as:

$$|P| = |\theta_r|\ |\theta_{PL}|\ |\theta_y|\ |\theta_{OP}|\ |M|. \qquad (3)$$

These matrices give the P coordinates in M.

The Euler angle which is the output of the navigation computer (χ_z, χ_x, and χ_y) may be thought of as imaginary gimbal angles of an imaginary gimbal system connecting the command position vehicle to the platform as shown in FIGURE 7.

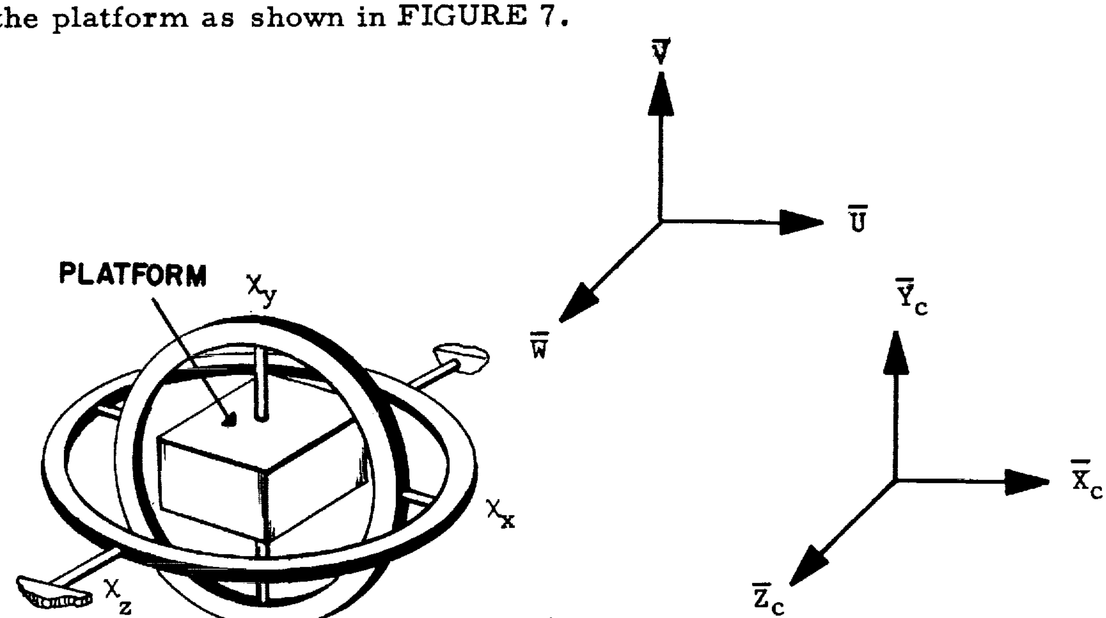


FIGURE 7

When the program is zero, χ_z is in the $\overline{Z}_c$ or $\overline{W}$ direction, χ_x is in the $\overline{U}$ or $\overline{X}_c$ direction, and χ_y is in the $\overline{V}$ or $\overline{Y}_c$ direction. The reason for describing the command position in this manner is covered later.

To express the C coordinates in terms of the P coordinates, from matrix algebra:

$$\begin{vmatrix} \overline{X}_c \\ \overline{Y}_c \\ \overline{Z}_c \end{vmatrix} = \begin{vmatrix} \cos\chi_z & -\sin\chi_z & 0 \\ \sin\chi_z & \cos\chi_z & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\chi_x & -\sin\chi_x \\ 0 & \sin\chi_y & \cos\chi_x \end{vmatrix} \begin{vmatrix} \cos\chi_y & 0 & \sin\chi_y \\ 0 & 1 & 0 \\ -\sin\chi_y & 0 & \sin\chi_y \end{vmatrix} \begin{vmatrix} \overline{U} \\ \overline{V} \\ \overline{W} \end{vmatrix}$$

which may be written:

$$|C| = |\chi_z|\,|\chi_x|\,|\chi_y|\,|P|. \tag{4}$$

Combining equations 3 and 4:

$$|C| = |\chi_z|\,|\chi_x|\,|\chi_y|\,|\theta_r|\,|\theta_{PL}|\,|\theta_y|\,|\theta_{OP}|\,|M|, \tag{5}$$

which gives the C coordinates in M. Expanding equation 5 yields,

$$|C| = \begin{vmatrix} \cos a & \cos b & -\cos c \\ -\cos d & \cos e & \cos f \\ \cos g & -\cos h & \cos k \end{vmatrix} |M| \tag{6}$$

where, from analytic geometry, cos a, cos b, cos c, cos d, cos e, cos f, cos g, cos h, and cos k are the direction cosines between the six vectors $\overline{X}_m$, $\overline{Y}_m$, $\overline{Z}_m$, $\overline{X}_c$, $\overline{Y}_c$, and $\overline{Z}_c$; and a, b, c, d, e, f, g, h, and k are the direction angles between $\overline{X}_m$, $\overline{Y}_m$, $\overline{Z}_m$, $\overline{X}_c$, $\overline{Y}_c$, and $\overline{Z}_c$. Equation 6 may be written as:

$$|C| = |G|\,|M|. \tag{7}$$

Matrix $|G|$ then contains all the information needed for an approximate or exact kinematic solution to the motion needed to make the errors become zero.

Since the nine direction cosines in matrix $|G|$ are known, the approximate and exact solution may be formulated. From equation 2, the approximate solution is:

$$\phi \text{ yaw} = \bar{X}_m \cdot (\bar{Y}_m \times \bar{Y}_c + \bar{Z}_m \times \bar{Z}_c)$$

$$\phi \text{ roll} = \bar{Y}_m \cdot (\bar{X}_m \times \bar{X}_c + \bar{Z}_m \times \bar{Z}_c)$$

$$\phi \text{ pitch} = \bar{Z}_m \cdot (\bar{X}_m \times \bar{X}_c + \bar{Y}_m \times \bar{Y}_c).$$

In deriving $\bar{Y}_m \times \bar{Y}_c$ in terms of direction cosines, consider the unit sphere,a portion of which is shown in FIGURE 8.

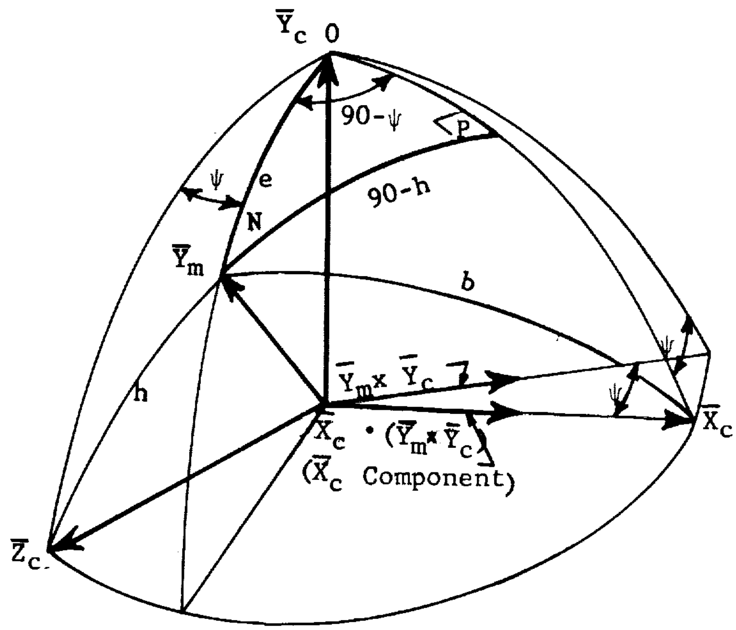


FIGURE 8

The angles in FIGURE 8 correspond to the angles in G . The only angle shown in Figure 8 that is not in $|G|$ is angle Ψ. Angle Ψ is defined as the angle between $(\bar{Y}_m \times \bar{Y}_c)$ and $\bar{X}_c$.

$\bar{Y}_m \times \bar{Y}_c$ has a magnitude $|\sin e|$ in a direction perpendicular to the $\bar{Y}_m \bar{Y}_c$ plane, or perpendicular to e. $\bar{Y}_m \times \bar{Y}_c$ is also perpendicular to $\bar{Y}_c$, and hence lies in the $\bar{X}_c \bar{Z}_c$ plane. (The negative of $\bar{Y}_m \times \bar{Y}_c$ is shown in FIGURE 8 for clarity.)

The magnitude of the component of $\bar{Y}_c \times \bar{Y}_m$ along $\bar{X}_c$ (a scalar) is $(\sin e \cos \Psi)$, which is expressed in equation 8.

$$\bar{X}_c \cdot (\bar{Y}_m \times \bar{Y}_c) = \sin e \cos \Psi. \tag{8}$$

Using spherical trigonometry on the right spherical triangle NOP gives:

$$\sin e = \frac{\sin (90^\circ - h)}{\sin (90^\circ - \Psi)}$$

$$\sin e = \frac{\cos h}{\cos \Psi} \tag{9}$$

$$\cos h = \sin e \cos \Psi. \tag{10}$$

Then, from equations 8 and 10:

$$\overline{X}_c \cdot (\overline{Y}_m \times \overline{Y}_c) = \cos h. \tag{11}$$

In a like manner, it may be shown that

$$\overline{Z}_c \cdot (\overline{Y}_m \times \overline{Y}_c) = \cos b. \tag{12}$$

Taking each vector of M in turn ,

$$\begin{aligned} \phi \text{ yaw} &= \cos h + \cos f \\ \phi \text{ roll} &= \cos c + \cos g \\ \phi \text{ pitch} &= \cos b + \cos d \end{aligned} \tag{13}$$

If ϕ yaw, ϕ roll, and ϕ pitch are small (α small), then:

$$\begin{aligned} \cos h &\simeq \cos f \\ \cos c &\simeq \cos g \\ \cos b &\simeq \cos d \end{aligned} \tag{14}$$

which may be demonstrated as shown in FIGURES 9A and 9B.

First consider a rotation about $\overline{X}_c$ an amount β. Then $h = 90^\circ + \beta$ and $f = 90^\circ - \beta$, and $(-\cos h) = (+\cos f) = (+\sin \beta)$.

If $\overline{X}_m$ is displaced from $\overline{X}_c$ an amount Γ parallel to the $\overline{X}_c\overline{Y}_c$ plane and an amount Δ parallel to the $\overline{X}_c\overline{Z}_c$ plane to give a general displacement, the Γ motion does not effect the value of h ($h_A = h_B$) and the Δ motion does not effect the value of f. The Δ motion will have a very

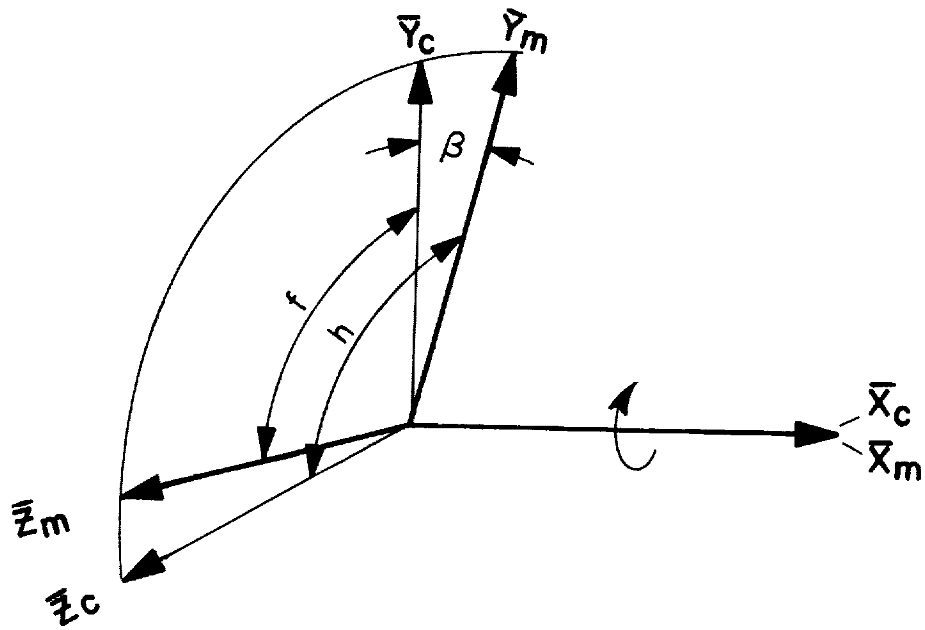


FIGURE 9A

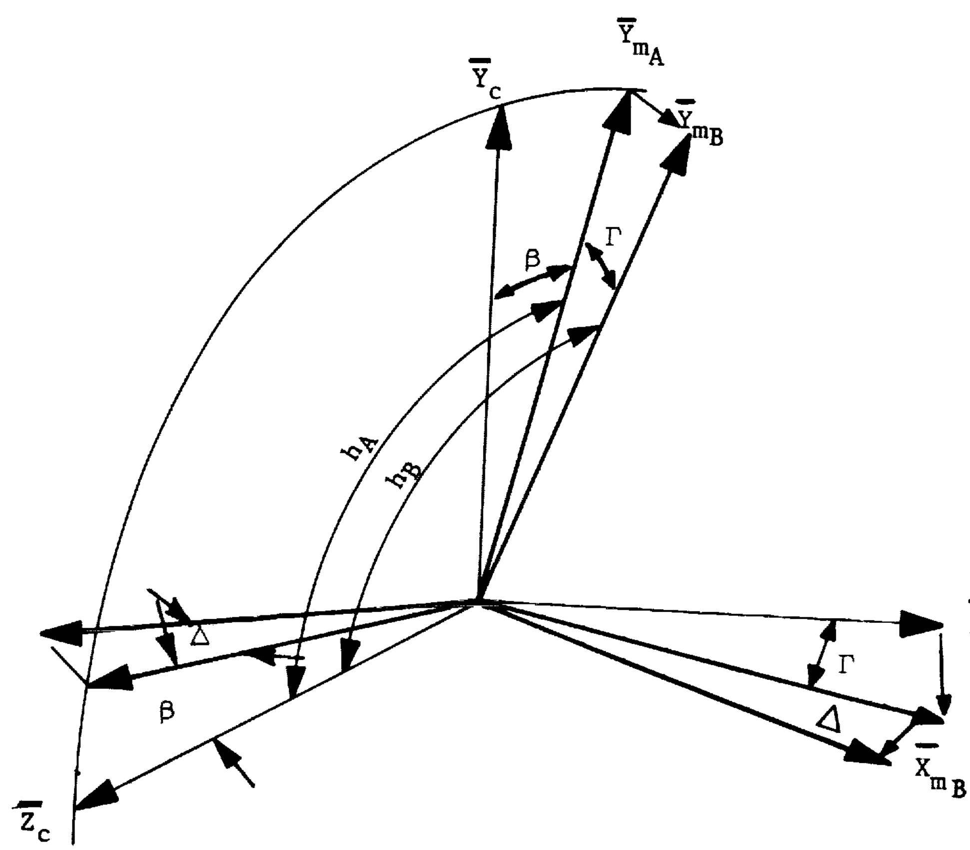


FIGURE 9B

small effect upon h. The exact value of this effect is difficult to show but may be approximated by $h_C \simeq h(1 + \sin\Gamma \sin\Delta)$. The Δ motion primarily represents rotation of the $\bar{Y}_m$ vector about its self with very little displacement of the vector from its initial position. Similarly, Γ has only a small effect upon f.

If the errors are small (α small), then:

$$\begin{aligned} \phi \text{ yaw} &= 2 \cos f \\ \phi \text{ roll} &= 2 \cos c \\ \phi \text{ pitch} &= 2 \cos b. \end{aligned} \tag{15}$$

For an approximate solution, only three of the nine terms of $|G|$ are needed. If the error between M and C is large (α large), then an angle cannot be treated as a vector. For an exact solution, both the angle and the rate must be given. If α is differentiated and $\bar{\omega}_\alpha$ is obtained, $\bar{\omega}_\alpha$ will lie along R. Then $\bar{\omega}_\alpha$ may be broken into components according to direction cosines, because $\bar{\omega}_\alpha$ is a vector. For a large α, equation 14 is not valid. The rates then are:

$$\begin{aligned} \bar{\omega}_x &= \bar{\omega}_\alpha (\cos h + \cos f) \\ \bar{\omega}_y &= \bar{\omega}_\alpha (\cos c + \cos g) \\ \bar{\omega}_z &= \bar{\omega}_\alpha (\cos b + \cos d). \end{aligned} \tag{16}$$

$\bar{\omega}_\alpha$ may be derived from α in accordance with the dynamics of the problem. This is not considered here. From quaternions or Eulers parameters, it may be shown that:

$$\cos^2 \alpha/2 = \frac{1 + \cos a + \cos e + \cos k}{4}$$

or

$$\alpha = 2 \sqrt{\cos^{-1} \frac{1 + \cos a + \cos e + \cos k}{4}} \; . \tag{17}$$

Here all nine terms of $|G|$ are needed.

Resolvers are used to instrument these systems. The resolver inputs and outputs are shown in FIGURE 10.

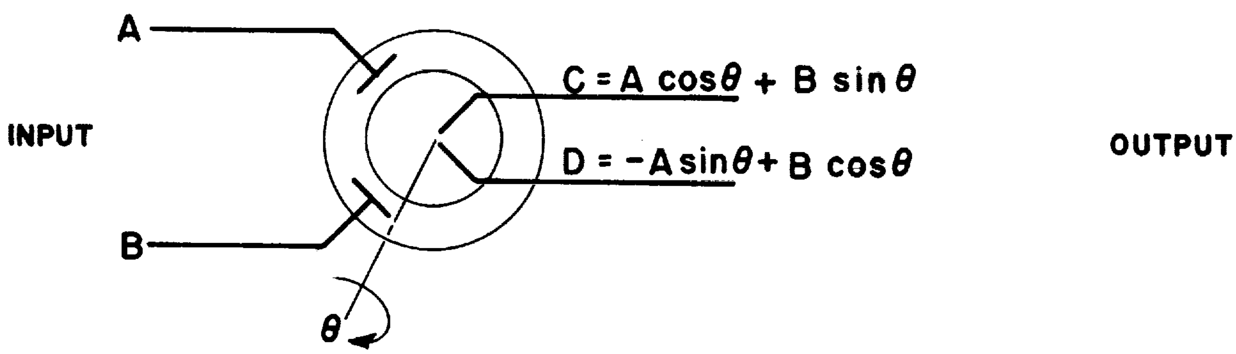


FIGURE 10

The circuit shown in FIGURE 11 results from the addition of a third lead to the resolver shown in FIGURE 10.

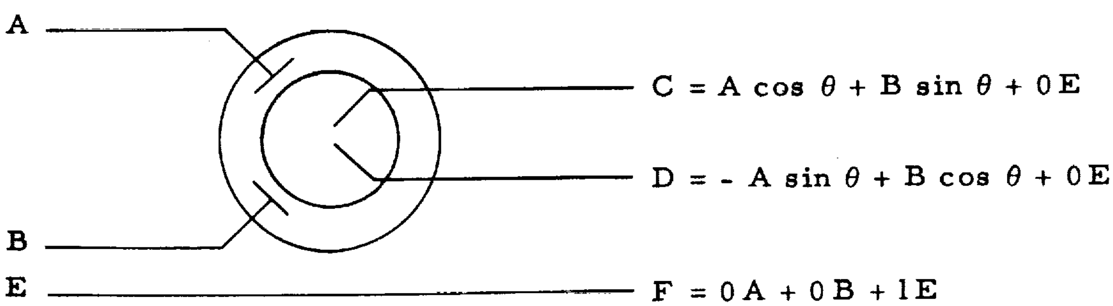


FIGURE 11

The matrix for the output of the resolver in FIGURE 11 is:

$$\begin{vmatrix} C \\ D \\ F \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A \\ B \\ E \end{vmatrix} . \qquad (19)$$

The rows may be interchanged and the columns may be interchanged by the electrical hookup. Hence, with a resolver, all the matrices in equations 3 and 4 can be generated, and the output of a chain of resolvers can be made equal to $|G|$. The hookup of such a chain is shown in FIGURE 12.

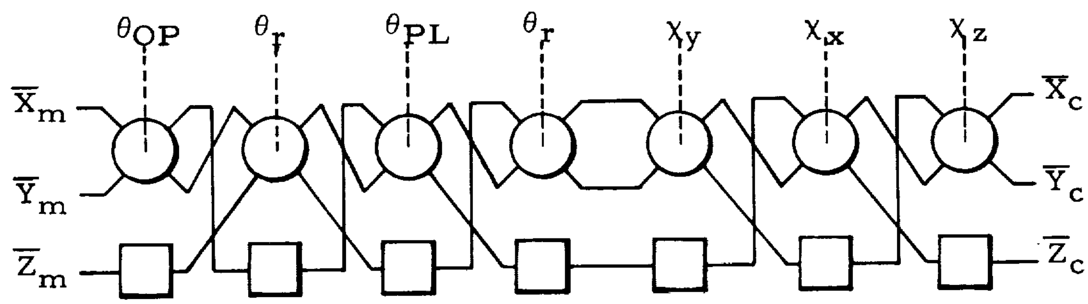


FIGURE 12

However, note that if M were expressed in terms of C that:

$$|M| = \begin{vmatrix} \cos a & \cos d & -\cos g \\ -\cos b & \cos e & \cos h \\ \cos c & -\cos f & \cos k \end{vmatrix} \; |C|. \qquad (20)$$

This matrix contains exactly the same information as $|G|$. The chain for this matrix is:

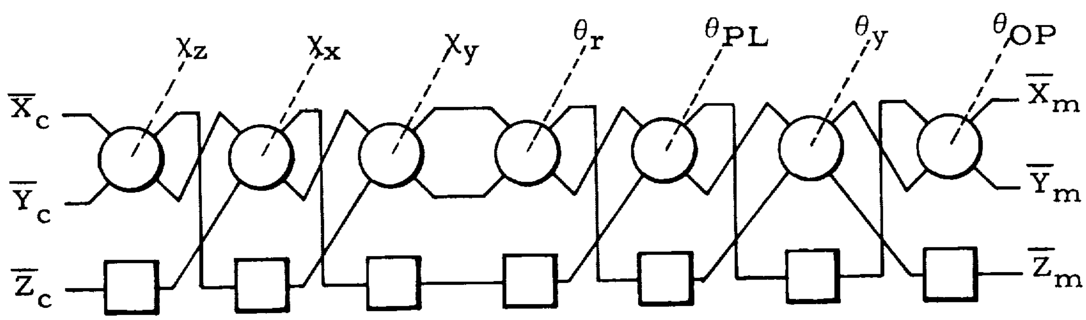


FIGURE 13

The chain in FIGURE 13 is easier to instrument than the chain in FIGURE 12 because the input to the resolver chain is located in the navigation computer near the exciting oscillator. The higher power levels are kept confined to a small area. Otherwise, the choice is immaterial. The chain in FIGURE 13 is used in the ST-124.

Certain changes in impedance level, voltage gradient and power occur as the signal passes through the resolver. To have the signal by passing the resolver compatible with the signal through the resolver, impedance networks are placed in the signal path of the by-pass signal. These are shown in the boxes in FIGURES 12 and 13.

On the output, it is necessary to separate the signal due to X, due to Y and due to Z (inputs). There are several ways to do this; for example, make $\overline{X}_m$ one frequency, $\overline{Y}_m$ a second frequency, and $\overline{Z}_m$ a third frequency. Also, three chains could be used, exciting only X_m in one, only $\overline{Y}_m$ in the second, and only $\overline{Z}_m$ in the third. Other methods are time sharing or sampling, and so forth.

Note that for the approximate solution, only two inputs are needed to generate the three terms in the matrix.

In normal flight, the attitude error is not expected to exceed 5^o The approximate solution is accurate to better than 1/2 percent for attitude errors of 10^o, and this error approachs zero as angle α goes to zero. The zero or null of the error signal is exact.

For the ST-124, the approximate solution is instrumented using two frequencies and one chain. The oscillators for driving the ST-124 resolver chain:

1. Have a frequency stability of such a value that the filter characteristics on the output do not change over the range of $f \pm \Delta f$, and the resolver characteristics are very nearly constant for $f \pm \Delta f$.

2. Deliver the maximum power to the load, yet keep all resolvers well below magnetic saturation.

3. Have a ratio of fundamental frequency to harmonics so there is no apparent null shift or other resolver characteristic change.

4. The output impedance should be low to make the output signal independent of any ΔZ impedance change of the resolver chain because of shaft rotation or other disturbance.

The resolvers to be used on the ST-124 are being developed by a vendor; and, until the design is finished and parameters are given, it is impractical to put exact numerical values on the source design. However, experience in resolvers indicates the order of magnitude of the source parameters.

Some of these orders of magnitude are:

1. Frequency stability at 1600 cps and 1900 cps is ± 10 cps.

2. Power output is 5 watts per oscillator; saturation is a function of NI maximum and, as both primary windings are excited, the maximum instantaneous $NI = N_1I_1$ maximum + N_2I_2 maximum. (Note: This means the power per oscillator is 1/4 the power that could be used if there were only one oscillator.) The first resolver is the only one where saturation is a consideration because the resolvers are connected for less than maximum power transfer.

3. The ratio of the fundamental frequency to harmonics should be approximately 20 to 1.

4. The source output voltage is anticipated to be 26 volts with a gradient on the output of the chain of 0.25 volt/degree.

The reason for expressing the program in the angles shown in FIGURE 7 can be demonstrated. If the commands are given in yaw, pitch, and roll, the attitude of the vehicle is not defined.

Consider the case at some time, t, after liftoff when a command of 90° roll, 90° yaw, and 90° pitch has been given. In what order or what steps these commands have been given is not known. FIGURE 14 shows the final positions of the vehicle after a 90° movement in yaw, pitch, and roll (a), and pitch, roll, and yaw (b). Clearly, these are not in the same final position.

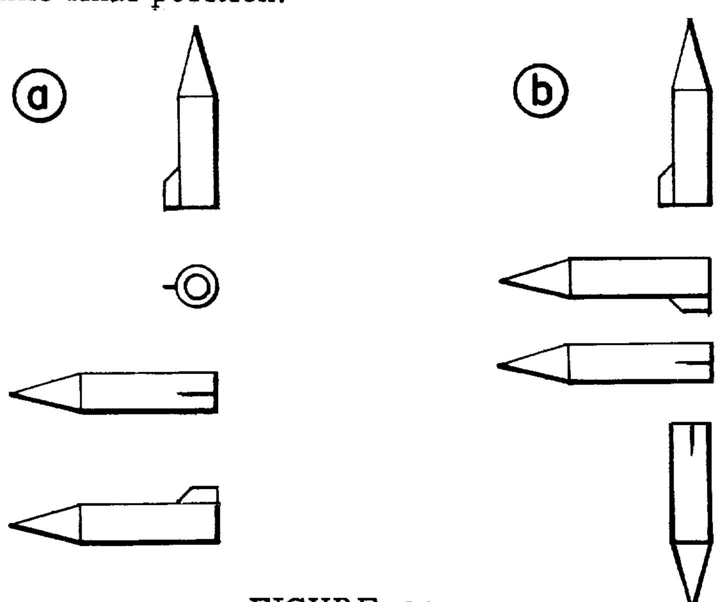


FIGURE 14

The imaginary gimbals shown in FIGURE 7 have three gimbals for complete freedom in all axes; the mechanical system requires four gimbals to reduce the torque applied to the platform because of vehicle dynamics. This effect is called gimbal lock and is illustrated in FIGURE 15.

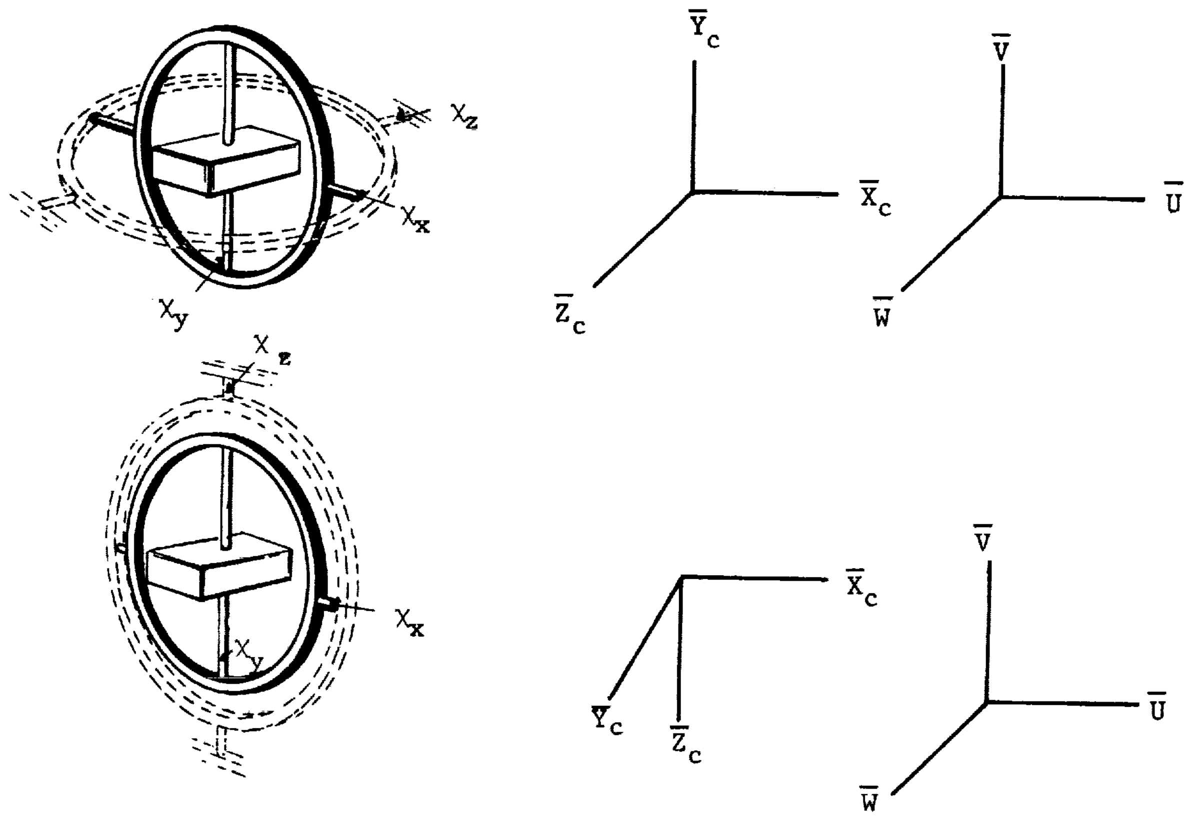


FIGURE 15

After a 90° yaw motion, the imaginary gimbal system has no freedom in the $\bar{Y}_c$ or $\bar{W}$ direction. Any motion about $\bar{Y}_c$ then will exert a torque on the platform in the $\bar{W}$ direction.

Note that χ_x and χ_y are attached to the same gimbal ring, hence χ_x is always perpendicular to χ_y. If an additional gimbal could be inserted perpendicular to both χ_x and χ_y, there would always be freedom in all axes. This design is used on the platform. The angle

θ_{PL} is always kept in its zero position (θ_{PL_o}) so that θ_{PL}, θ_r, and θ_y are mutually perpendicular. Then, in the steady state, referring to FIGURE 6 and 7:

$$\begin{aligned} \chi_z &= \theta_{OP} \\ \chi_x &= \theta_y \\ \chi_y &= \theta_r \\ 0 &= \theta_{PL} . \end{aligned} \tag{21}$$

Here, any perturbation of the vehicle about the platform is absorbed by θ_r, θ_{PL}, and θ_y. Any command or program motion is absorbed by θ_r, θ_y, and θ_{OP}. In the vehicle, the perturbations are small, allowing a restricted range on θ_{PL}. This is $\pm 20^{\circ}$ in the ST-124.

A control system to hold $\theta_{PL} = \theta_{PL_o}$ (steady state) is needed. This control system is involved with the platform stabilization loops. The platform angles θ_r, θ_{PL}, and θ_y are driven by gyro stabilizer servo loops. θ_{OP} is driven by a servo loop to maintain $\theta_{PL} = \theta_{PL_o}$. One way to drive θ_{OP} is with a null position pick-off located on θ_{PL}. A block diagram of such a loop is shown in FIGURE 16.

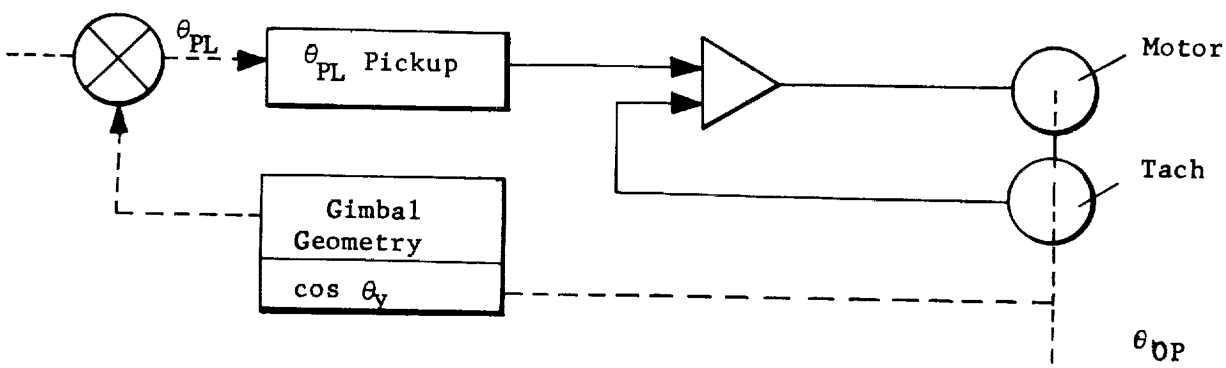


FIGURE 16

The motion in θ_{PL} because of the motion in θ_{OP} is:

$$\theta_{PL} = \theta_{OP} \cos \theta_y .$$

To compensate for the cos θ_y term, a secant θ_y is added as shown in FIGURE 17

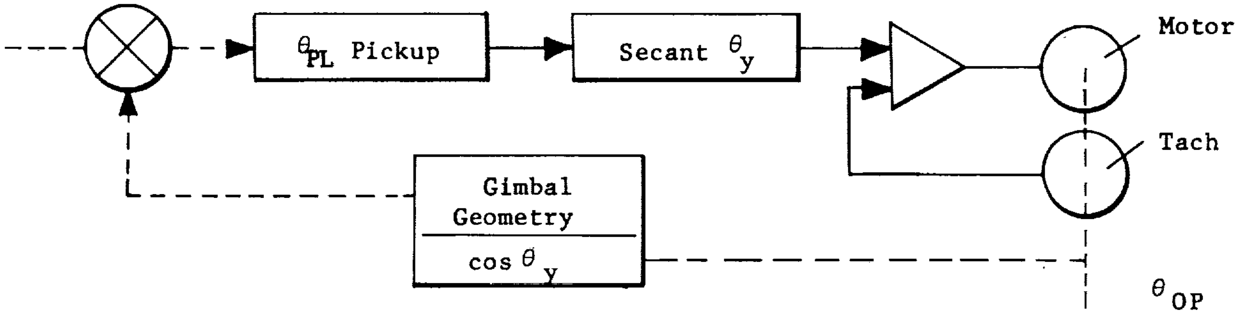


FIGURE 17

The difficulty here is to instrument a secant function. To avoid the secant function instrumentation, another loop for driving θ_{OP} is shown in FIGURE 18.

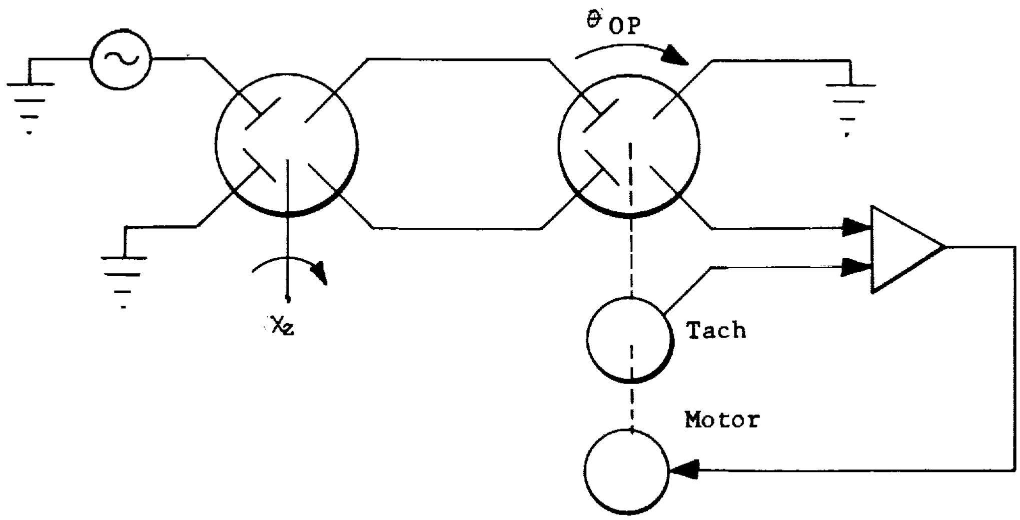


FIGURE 18

The loop will drive θ_{OP} until it is equal to χ_z. The premise for this control loop is that if $\theta_{OP} = \chi_z$ and all errors are zero (steady state) then:

$$\theta_r = \chi_y$$

$$\theta_y = \chi_x$$

$$\theta_{OP} = \chi_z$$

$$\theta_{PL} = 0.$$

FIGURE 6 and 7 show that this is one condition that can exist. To show that it is a unique condition, consider:

If $\theta_{PL} = \theta_{PL_o}$, then $\theta_r = \chi_y$, $\theta_y = \chi_x$, and $\theta_{OP} = \chi_z$.

If $\theta_{PL} \neq \theta_{PL_o}$ and the error is zero, the motion about θ_{PL} must be absorbed by θ_r and θ_y. (θ_{OP} is held equal to χ_z.)

However, θ_r and θ_y are both perpendicular to θ_{PL}, and hence cannot absorb any motion about θ_{PL}.

Therefore, θ_{PL} must equal θ_{PL_o}.

If the attitude error is greater than 20^o (in θ_{PL}), the gimbal will hit the stops and tumble the platform. On a mission where the vehicle attitude error could be greater than 20^o, either another system to hold $\theta_{PL} = \theta_{PL_o}$ must be used, or another system used for a backup when θ_{PL} is greater than 15^o.

CONCLUSION

A unique system has been provided in the platform resolver chain for coordinate transformation in the normal flight mode of operation for attitude control, alignment, and various stabilization and mechanization functions in the platform. Another method would be to have velocity control only, which would give control over translational but not rotational motion. Encoders could be placed on the platform gimbal shafts, with all calculations made in a digital computer.

The problem could be approached from a dynamic rather than a kinematic point of view. This would combine the functions of the resolver chain and the control computer into one system.

Since the physical systems are less than ideal, the approximate solution is comparable to the other vehicle system. It behaves as the exact solution if the approximation of equation 14 is not used.

NASA-Langley, 1962

NASA TN D-1118
National Aeronautics and Space Administration.
GIMBAL GEOMETRY AND ATTITUDE SENSING OF THE ST-124 STABILIZED PLATFORM. Richard L. Moore and Herman E. Thomason. May 1962. 23p. OTS price, $0.75.
(NASA TECHNICAL NOTE D-1118)

The results of an extensive study for obtaining attitude signals from a four-gimbal platform are summarized. The basic requirements were to furnish roll, yaw, and pitch attitude signals within an accuracy of 6 minutes of arc with a minimum number of resolvers. The system coordinates are defined, and by establishing the direction cosines in matrix algebra, the problem solution is mathematically formulated. Supplementary information pertaining to redundant gimbal control is also provided.

I. Moore, Richard L.
II. Thomason, Herman E.
III. NASA TN D-1118

(Initial NASA distribution: 29, Navigation and navigation equipment; 50, Stability and control.)

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