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Inlet and Exit-Header Shapes for Uniform Flow Through a Resistance Parallel to the Main Stream

An analytical and experimental study of flow in headers with a resistance parallel to the turbulent and incompressible main stream has been made. The purpose was to shape the inlet and exit headers, which had a large length-to-height ratio, so that the fluid would pass through the resistance uniformly. Analytical wall shapes and estimated total pressure drop through the headers were compared with experimental results. Good agreement between analysis and experiment was found for the cases compared.

Introduction

THERE has been a great interest in the flow of fluids through resistances parallel to the main fluid stream, Figs. 1 and 2. There are many applications of this type of flow in such equipment as air driers and heat exchangers. The advantage of this flow geometry as compared with the case where the resistance is normal to the main stream is that a larger sized resistance can be used giving a lower total-pressure drop for the same mass flow and frontal area or, if the flow-resistance size is constant, smaller frontal area and less fluid holdup in the system are obtained. The fluid holdup refers to the amount of fluid in the line during steady running conditions.

A disadvantage commonly found in this type of folded-flow system is that the fluid does not pass through the resistance uniformly. In many applications uniform flow through the resistance is desirable primarily because then full use can be made of the active surfaces of the equipment, and usually the total-pressure drop through the system will be less. Several approaches to the problem of obtaining uniform flow exist. One method is that of using turning vanes [1];¹ however, these are difficult to fabricate and install. Because of flow separation along the vanes they are not always successful. Another method might be to vary the resistance along the channel; however, this may be impractical in

¹ Numbers in brackets designate References at end of paper.

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some applications. In this study uniform flow through a porous resistance was obtained by shaping the inlet and exit headers. The fluid was assumed turbulent and incompressible and the

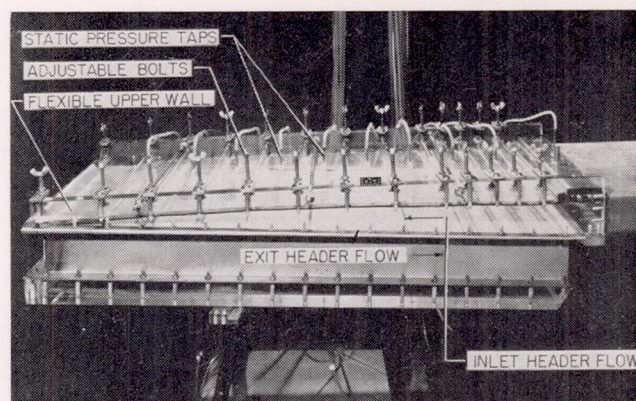


Fig. 1 Test section

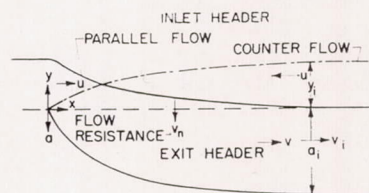


Fig. 2 Analytical model

Nomenclature

a = nondimensional co-ordinate normal to resistance for exit header, a'/a_i

B = integration constant, $(a_i/y_i)^2 - D$

D = integration constant, $(2 - n)/(1 + \beta - n)$

F = friction constant, $\frac{L}{4a_i} \frac{(0.316)}{\left(\frac{v_i 2a_i}{\nu}\right)^{0.25}}$

f = friction factor, $8\tau_w/\rho v_i'^2$

L = length of flow resistance

\mathcal{L} = defined by equation (26)

n = parameter to fix exit-header shape, $0 \leq n < 1$

p = nondimensional static pressure, $p'/(\rho v_i'^2/2)$

u = nondimensional velocity in inlet header, u'/v_i

v = nondimensional velocity in exit header, v'/v_i

v_n' = normal velocity through resistance

x = nondimensional co-ordinate along resistance, x'/L

y = nondimensional co-ordinate normal to resistance for inlet header, y'/a_i

β = term defined in equation (13)

ν = kinematic viscosity

ρ = density

τ_w = shear stress at wall

χ = defined by equation (21)

Subscripts

f = term to be added to account for friction

i = term evaluated at outlet of exit header or entrance of inlet header

0 = cases where friction is neglected

Superscript

' = (prime) dimensional term

length of the header was considered large compared with the height. The case of shaping only the inlet header to achieve uniform flow and allowing the fluid to exhaust directly on leaving the resistance into the atmosphere has been treated previously by Loeffler and Perlmutter [2]. The analysis in this paper for the inlet header makes similar assumptions as those used in [2].

The case where the fluid enters from the atmosphere through a resistance into an exit header which exhausts the fluid parallel to the resistance has been studied by Taylor [3] using both one and two-dimensional analyses while ignoring friction. Good agreement for both analyses with experiments was obtained. In the analysis of the exit header in this paper the one-dimensional analysis and similar assumptions as Taylor's are used. It can be shown that it is impossible to obtain uniform flow from the atmosphere through a resistance into an exit header which exhausts the fluid parallel to the resistance by shaping the exit header. This is because the exit header requires a pressure decrease in the direction of the outlet of the header to force out the fluid. Since the pressure on the upstream side of the resistance is constant, this causes the largest pressure difference, and hence the greatest flow across the resistance to be near the outlet of the exit header.

Cichelli and Boucher [4] studied inlet and exit-header shapes to achieve uniform flow through the resistance. They assumed that the fluid leaving the inlet header carried no forward component of velocity. This did not prove to be true in the present case, as shown by experiment. They also ignored frictional effects. Heyda [5] carried out an analytical solution for shaping inlet and exit headers to obtain uniform flow through the resistance using a two-dimensional approach for the exit header. Similar assumptions to the present paper were made except friction was neglected and the solution carried out for linear shaped outer walls of the exit headers.

In this paper the headers are designed so that the static-pressure difference across the resistance will be constant for any point along the resistance. Since the resistance is constant, this will insure uniform flow through the resistance at every point. This assumes that the streamlines are identical through the resistance. The problem was broken up into two separate parts, the inlet header and the exit header. The two problems are related by the fact that the pressure distribution in the exit header is the same as in the inlet header except for a constant difference across the resistance. The analysis was finally checked by experiment, Fig. 1, and the theoretical wall shapes were compared with experimental wall shapes. Also, estimates for all-over total-pressure drops were compared with experimental over-all total-pressure drops.

Analysis

The analytical model is shown in Fig. 2. For the exit header for fairly large resistance, the fluid enters with no x -component of velocity. For very thin resistances with large open areas this will not be true. The one-dimensional momentum balance for the exit header is

$$-a'dp' - 2\tau_w dx' = \rho d(a'v'^2) \quad (1)$$

where terms in equation (1) are defined in Fig. 2 or in the nomenclature. Stress τ_w can be defined from the Blasius law using a hydraulic diameter based on infinite parallel plates in the definition of the Reynolds number

$$f = \frac{8\tau_w}{\rho v'^2} = \frac{0.316}{\left(\frac{v'2a'}{\nu}\right)^{1/4}} \quad (2)$$

Actually, since there is an addition or removal of fluids and the wall cross section is changing, the Blasius relationship cannot be

expected to hold exactly for this case. However, the frictional effects are only minor compared to the change in momentum and the inclusion of the friction is useful to indicate the trend and the magnitude of its effects. It will be shown later that the fluid in the inlet header enters the resistance with almost its full forward velocity. This would cause the shear at that resistance to be very small, hence the effect of friction should be smaller than indicated. Nondimensionalizing as follows:

$$v = \frac{v'}{v_i}, \quad a = \frac{a'}{a_i}, \quad p = \frac{p'}{(\rho v_i^2/2)}, \quad \text{and } x = \frac{x'}{L}$$

Letting $F = (f_i/4)(L/a_i)$ where

$$f_i = \frac{0.316}{\left(\frac{v_i 2a_i}{\nu}\right)^{1/4}}$$

equation (2) becomes

$$-\frac{a}{2} dp - \frac{Fv^2}{(va)^{1/4}} dx = d(av^2) \quad (3)$$

Since uniform flow through the resistance is desired, from continuity it is required that

$$va = x \quad (4)$$

Substituting equation (4) into (3) yields

$$-\frac{a}{2} dp - \frac{Fx^{7/4}}{a^2} dx = d\left(\frac{x^2}{a}\right) \quad (5)$$

At this point the pressure can be given as a function of x and equation (5) can be solved for the exit-header wall shape. Also, the exit-header wall shape can be given and then it is possible to solve for the pressure. In choosing the first case care must be taken to insure that the pressure decreases in the direction in which the fluid is to flow. This is because the fluid entering the exit header has no velocity component in the direction of the outlet. In order for the fluid to flow in the direction of the outlet the only force available would be the pressure gradient. The pressure distribution can be chosen as follows:

$$p - p_i = \frac{(2-n)}{(1-n)} [1 - x^{2(1-n)}] \quad (6)$$

where n is arbitrary. To keep the pressure finite at $x = 0$, n must be less than 1. Differentiating equation (6) gives

$$\frac{dp}{dx} = 2(n-2)x^{1-2n} \quad (7)$$

Substituting equation (7) into equation (5) yields

$$\frac{da}{dx} - \frac{2a}{x} = -\frac{a^2(2-n)}{x^{1+2n}} + \frac{F}{x^{1/4}} \quad (8)$$

Exit Header With No Friction. If friction F were neglected in equation (8), we would obtain the solution for a

$$a_0 = x^n \quad (9)$$

where the subscript 0 refers to the cases where friction is neglected.

To prevent a_0 from becoming infinite at $x = 0$, n must be greater than or equal to zero. Therefore n is restricted between the limits $0 \leq n < 1$, where the upper limit is from the previous paragraph. By letting n be a given positive value between these limits, a variety of exit-header shapes can be obtained, Fig. 3.

Exit Header With Friction. It seems reasonable to assume that the effect of friction will only change the wall shape slightly; a can then be written as

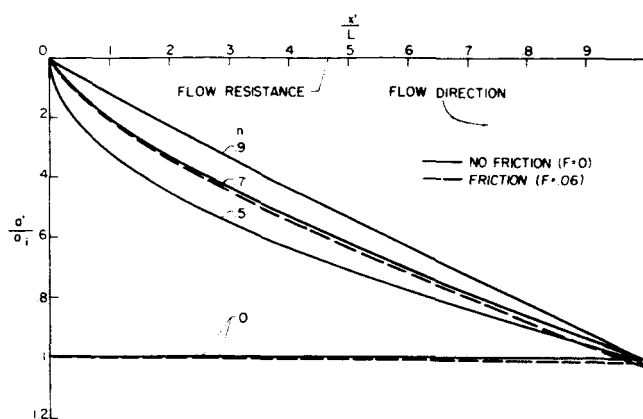


Fig. 3 Exit-header shapes for various values of n , for cases of friction and no friction

$$a = a_0 + a_f \quad (10)$$

where $a_0 \gg a_f$. We can consider a_f to be the change in wall shape due to friction. Substitution of equation (10) into equation (8), and discarding the squares of a_f as negligible, results in

$$\frac{da_f}{dx} + \frac{a_f}{x} (4 - 3n) = \frac{F}{x^{1/4}} \quad (11)$$

This can be solved to give

$$a_f = \frac{Fx^{3/4}}{\left(\frac{19}{4} - 3n\right)} \quad (12)$$

Adding equation (12) to equation (9), the wall shape of the exit header can be found for the case including friction. These results are plotted in Fig. 3 as dashed lines for a given value of F . For different values of F the resulting a_f can be found by the ratio $a_{f,2} = (F_2/F_1)a_{f,1}$. The friction causes the wall to be farther from the resistance than in the case for no friction, because, to maintain the given pressure distribution, the drop in pressure due to the effect of friction must be compensated for by slowing down the fluid by widening the channel. This causes an increase in static pressure to compensate for the frictional pressure loss.

Inlet Header. The fluid entering the inlet header can be in the same direction (parallel flow) or in the opposite direction (counterflow) to the fluid in the exit header, Fig. 2. A momentum balance for either case in nondimensional terms is

$$-\frac{y}{2} dp - F \frac{u|u|}{|uy|^{1/4}} dx - d(yu^2) + (1 - \beta)ud(yu) = 0 \quad (13)$$

where $y = y'/a_i$, $u = u'/v_i$ and the other terms are made nondimensional as before. In the case of parallel flow, u is positive, while for the case of counterflow u is negative. The term $(1 - \beta)$ in equation (13) can be considered the fraction of the x -component of velocity of the main-stream fluid contained in the fluid particles passing into the resistance. The value of β will be considered a constant of the resistance and will be discussed more fully later on.

Counterflow Inlet Header With No Friction. For the case where the inlet-header velocity is in the opposite direction to the exit-header velocity (counterflow), assuming uniform flow through the resistance, from continuity it is found that

$$yu = -x \quad (14)$$

The pressure gradient found from equation (7) for the exit header must be the same as for the inlet header because the static-pres-

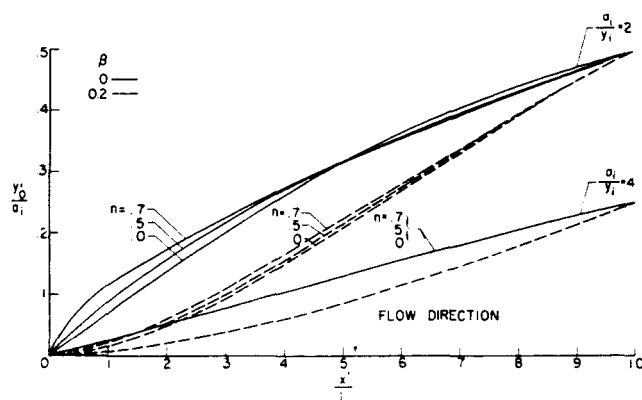


Fig. 4 Wall shapes for inlet header in counterflow, neglecting friction

sure difference between the two headers at each x must be a constant. Using equations (7) and (14), equation (13) gives

$$\frac{dy}{dx} = (1 + \beta) \frac{y}{x} - \frac{F}{x^{1/4}} - \frac{(2 - n)}{x^{1+2n}} y^3 \quad (15)$$

The analytical solution for this equation is not obvious. It can be solved numerically for the inlet-header shape. If friction is neglected ($F = 0$), equation (15) can be solved to yield

$$\frac{1}{y_0^2} = \frac{D}{x^{2n}} + \frac{B}{x^{2(\beta+1)}} \quad (16)$$

where

$$D = \frac{2 - n}{1 + \beta - n} \quad \text{and} \quad B = \left(\frac{a_i}{y_i}\right)^2 - D$$

The value of a_i/y_i has a fixed lower limit for the condition of uniform flow through the resistance, otherwise y_0^2 becomes negative. As long as

$$\left(\frac{a_i}{y_i}\right)^2 \geq \left(\frac{2 - n}{1 + \beta - n}\right) \quad (16a)$$

y_0^2 will always be positive. y_0 is plotted against x in Fig. 4. It can be seen that as a_i/y_i gets large the problem reduces to that of the fluid exhausting into a large reservoir as given in Ref. [2]. This condition is approached for the case of $a_i/y_i = 4$. For this case the inlet-header shape does not seem to depend on the exit-header shape n .

Counterflow Inlet Header With Friction. Since the effect of friction would change the wall shape only slightly, y can be written as

$$y = y_0 + y_f \quad (17)$$

where $y_0 \gg y_f$. Substituting into equation (15) and neglecting squares of y_f yield

$$\frac{dy_f}{dx} = -\frac{(2 - n)3y_0^2 y_f}{x^{1+2n}} + (1 + \beta) \frac{y_f}{x} - \frac{F}{x^{1/4}} \quad (18)$$

Using y_0 from equation (16) in equation (18), y_f can be integrated as follows:

$$\begin{aligned} \frac{y_f}{F} \frac{(Dx^{2(1+\beta-n)} + B)^{3/2}}{x^{1+\beta}} &= \frac{1}{3n - 2\beta - \frac{11}{4}} \\ &\left[x^{-\frac{1}{4}-\beta} (Dx^{2(1+\beta-n)} + B)^{3/2} + 3B^{3/2}(1 + \beta - n) \right. \\ &\quad \left. \int x^{-\frac{5}{4}-\beta} \left(1 + \frac{D}{B} x^{2(1+\beta-n)}\right)^{1/2} dx \right] + c \end{aligned} \quad (19)$$

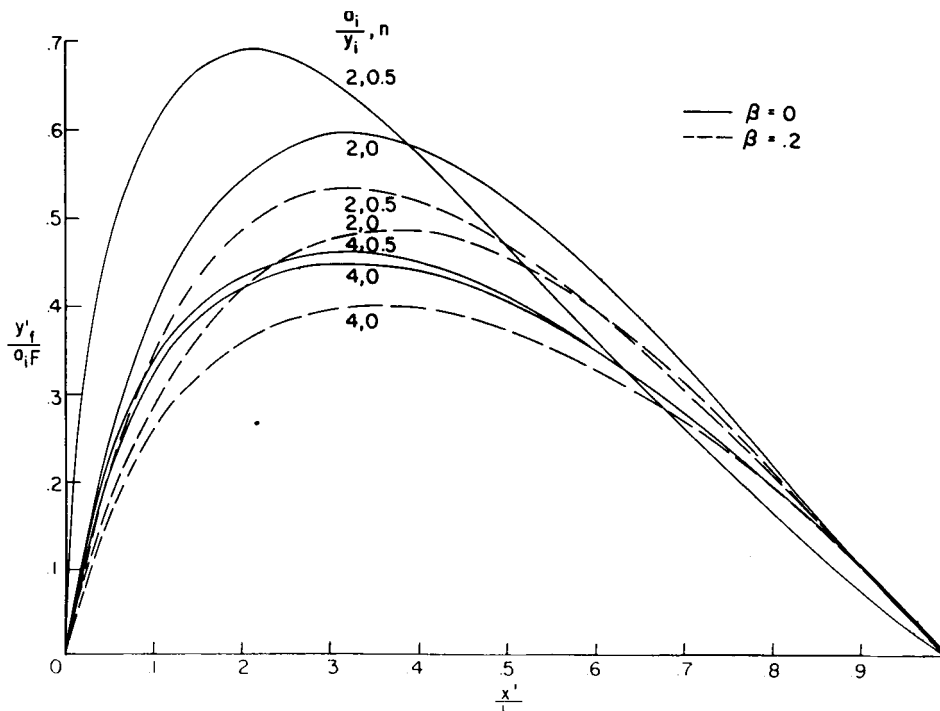


Fig. 5 Correction to inlet-header shapes owing to friction for the counter flow case

By expansion of the integrand into a series the integral can be solved to yield

$$\frac{y'_f}{F}(Dx^{2(1+\beta-n)} + B)^{1/2} \left(3n - 2\beta - \frac{11}{4} \right) = \chi - \chi_x = 1x^{1+\beta} \quad (20)$$

where

$$\chi = x^{1/4}(Dx^{2(1+\beta-n)} + B)^{1/2} + 3B^{1/2}(1 + \beta - n) \left[-\frac{x^{3/4}}{\beta + \frac{1}{4}} + \frac{D}{2B} \frac{x^{\frac{11}{4} + 2(\beta-n)}}{\left(\frac{7}{4} + \beta - 2n\right)} - \frac{1}{2 \cdot 4} \left(\frac{D}{B}\right)^2 \frac{x^{\frac{19}{4} + 4(\beta-n)}}{\left(\frac{15}{4} + 3\beta - 4n\right)} + \frac{3}{2 \cdot 4 \cdot 6} \left(\frac{D}{B}\right)^3 \frac{x^{\frac{27}{4} + 6(\beta-n)}}{\frac{23}{4} + 5\beta - 6n} - \frac{3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \left(\frac{D}{B}\right)^4 \frac{x^{\frac{35}{4} + 8(\beta-n)}}{\frac{31}{4} + 7\beta - 8n} + \dots \right] \quad (21)$$

In Fig. 5 the values of y'_f/F are plotted. These values must be added to y_0 to obtain the wall shape when the effects of friction are included.

Parallel-Flow Inlet Header With No Friction. For the case where the inlet-header velocity is in the same direction as the exit-header velocity, the momentum balance is given by equation (13). From continuity, since flow through the resistance is constant,

$$u = \frac{1-x}{y} \quad (22)$$

Substituting equations (7) and (22) into equation (13) gives

$$\frac{1}{y^3} \frac{dy}{dx} + \frac{(1+\beta)}{(1-x)y^2} + \frac{(2-n)}{(1-x)^2} x^{1-2n} = \frac{F}{y^3(1-x)^{1/4}} \quad (23)$$

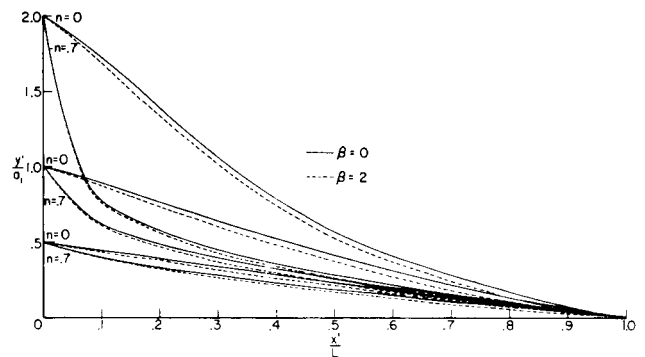


Fig. 6 Inlet-header wall shape for parallel flow neglecting friction for values of $y_i/a_i = 0.5, 1, \text{ and } 2$

After neglecting friction ($F = 0$), this can be integrated to

$$\frac{1}{y_0^2}(1-x)^{2(1+\beta)} = 2(2-n) \int x^{1-2n}(1-x)^{2\beta} dx + c \quad (24)$$

Expanding the integrand in a series yields

$$y_0^2 = \frac{(1-x)^{2(1+\beta)}}{\mathcal{L} + \left(\frac{a_i}{y_i}\right)^2} \quad (25)$$

where

$$\mathcal{L} = 2(2-n)x^{2(1-n)} \left[\frac{1}{2-2n} - \frac{2\beta x}{3-2n} + \frac{2\beta(2\beta-1)x^2}{2!(4-2n)} - \frac{2\beta(2\beta-1)(2\beta-2)}{3!(5-2n)} x^3 + \dots + \frac{(-1)^K(2\beta)!x^K}{(2\beta-K)!K!(2-2n+K)} \right] \quad (26)$$

These results are shown in Fig. 6.

Parallel-Flow Inlet Header With Friction. Breaking y up as in equa-

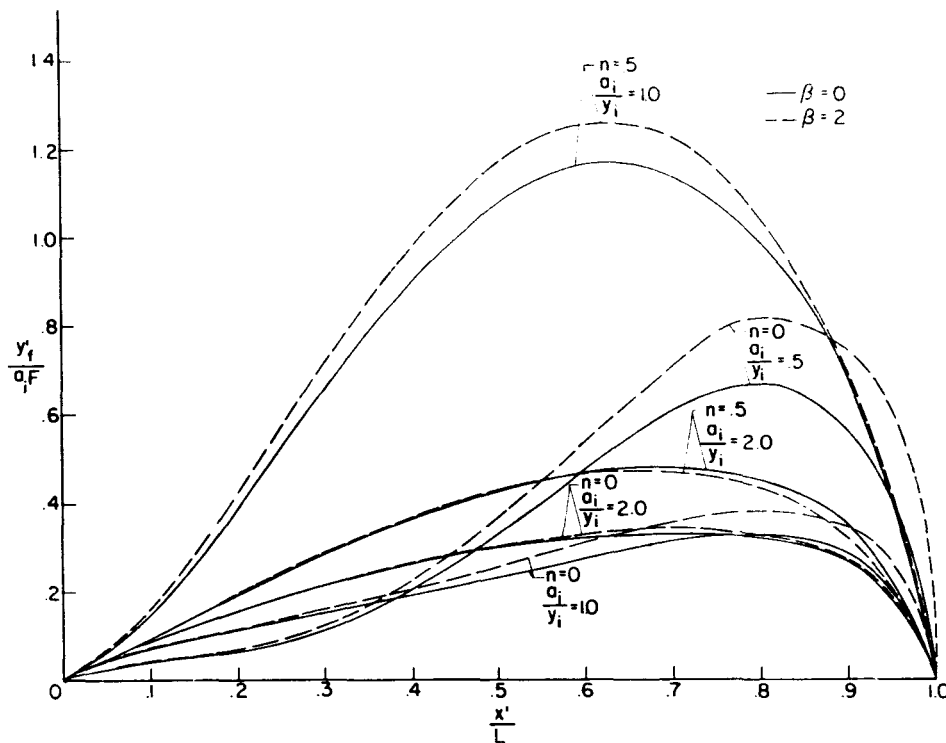


Fig. 7 Correction to inlet-header shapes owing to friction for parallel-flow case

tion (17), substituting into equation (23), and ignoring terms with y_f^2 yield

$$\frac{dy_f}{dx} + y_f \left[\frac{(1 + \beta)}{(1 - x)} + 3y_0^2 \frac{(2 - n)}{(1 - x)^2} x^{1-2n} \right] = \frac{F}{(1 - x)^{1/4}} \quad (27)$$

This can be integrated as follows:

$$y_f \exp \left[\frac{3y_0^2(2 - n)(x^{1-2n})/(1 - x)^2}{F(1 - x)^{1+\beta}} \right] = \int_{x=0}^x \frac{\exp[3y_0^2((2 - n)(x^{1-2n})/(1 - x)^2)]}{(1 - x)^{5/4 + \beta}} dx \quad (28)$$

The solution can be found by numerical integration of the integral. The results from equation (28) are plotted in Fig. 7.

Experiment

An experimental study was undertaken to check the results of the analysis and study over-all pressure drops. A view of the test section is shown in Fig. 1. The flow enters through the upper inlet header. The upper wall of the inlet header, which was flexible, was made of 1/8-in-thick Plexiglas. It could be shaped by means of bolts which were spaced along the sides. The entering height of 2 in. and the width of 16 in. were designed to approximate a two-dimensional model. The flow left the entrance header through the lower surface, which was a porous plate 32 in. long. One or more layers of cotton cloth could be placed on top of the porous plate to increase the flow resistance. The exit header consisted of a wall at a fixed distance away from the resistance. This corresponded to an n -value of zero in the analysis. This distance could be made either 1 or 4 in., corresponding to an a_i/y_i , respectively, of 0.5 and 2. The fluid exhausted into the room at the same end that the fluid had entered (counterflow). Instrumentation on the test section consisted of pressure taps located

along the center line of the upper and lower walls. The static pressure measured at the wall was shown to be the same value as at the resistance below by static-pressure profile measurements in [2]. Measurements of total-pressure drop through the porous plates and the cloths when the fluid flow is normal to the resistance are given in [2] also. This pressure drop was found to follow the relationship

$$5.2\Delta H = \frac{K\rho}{2g} v_n'^m \quad (29)$$

where ΔH is pressure drop across the resistance in inches of water and v_n' is the velocity normal through the resistance in feet per second. The values of K and m were found to be 17 and 1.9 for a porous plate only; 470 and 1.8 for a porous plate and one cloth; and 435 and 1.94 for a porous plate and two cloths. Dry air at 125 psig was available as the operating fluid. This air was filtered and passed through a standard ASME orifice run to measure the mass-flow rate. In determining the upper wall shape which would yield a uniform flow through the resistance, the mass flow was first set at some desired rate. Next the bolts, Fig. 1, along the sides of the flexible wall were adjusted until a constant pressure difference across the resistance was obtained at all points down the channel. If the static-pressure difference at a point was too high, the fluid in the upper header was accelerated by lowering the wall. This caused the static pressure to drop until the desired pressure difference was obtained. Measurements of the wall shape were then made by means of a depth gage.

The experimentally obtained uniform wall shapes are compared with the theoretical wall shapes in Fig. 8. The points seem to be close to $\beta = 0$, except near the entrance where the value of β is higher. The porous plate without the cloth seems to stay close to $\beta = 0$, even in the upstream region. The reason for this behavior of β will be discussed later.

Fig. 9 shows the static-pressure difference across the resistance along the channel. For the shaped wall the static-pressure difference has been made constant along the channel by shaping the wall. The pressure difference was found to remain constant along

the channel over the flow range studied. This flow range can be found from the F -range in Fig. 8. Numerical calculation will show that changes over a large range of velocity will have a very small effect on the wall shape. As shown in Fig. 9, shaping the wall did not give constant pressure difference across the resistance down the channel when a_i/y_i was changed from 2 to 0.5. It was impossible to adjust the wall to achieve uniform flow for this case. This agrees with the analysis for the counterflow case, which claims that there is a minimum value of a_i/y_i that is above 0.5 but below 2, equation (16a).

To study the variation from uniform flow for some different simple wall shape, the upper wall was made to increase linearly, ($y' = y_i x'/L$).

Since a linear wall shape gives smaller channel heights than found from the analysis as the flow travels downstream, the static pressure drops faster in this case than for the shaped wall. This is the reason that the pressure difference across the resistance shown

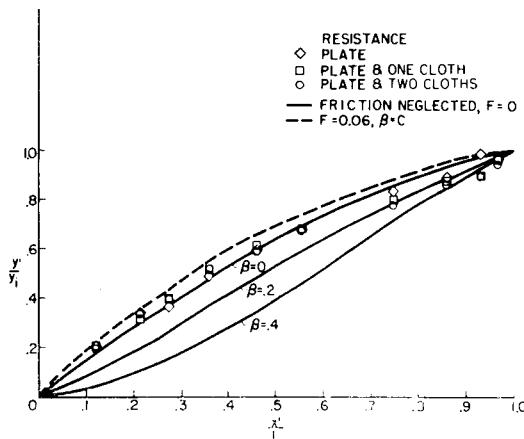


Fig. 8 Comparison of experimental wall shape with theoretical shapes for counterflow case, $a_i/y_i = 2$

in Fig. 9 becomes smaller down the channel. Thus most of the normal flow passes through the resistance near the entrance of the inlet header for this case. For an inlet header wall a constant distance away from the resistance ($y' = y_i$), the opposite effect occurs. The pressure rises downstream so that the pressure difference across the channel increases as the fluid travels downstream. For this case most of the fluid passes through the resistance in the downstream portion of the resistance. For the parallel flow case a similar deduction can be made. If, instead of the shaped wall, the height is made to rise linearly ($y' = y_i[1 - (x'/L)]$), then the wall will be higher than the theoretical shape for uniform flow, and the pressure in the inlet header will rise as the fluid travels downstream. This causes the largest pressure drop across the resistance and therefore also the largest normal velocity to occur close to the outlet of the exit header.

The ratio of normal velocity through the screen at $x' = 0$ over the normal velocity at $x' = L$, ($v_{n, x=0}/v_{n, x=L}$), is approximately equal to the square root of the ratio of the pressure difference across the resistance at $x' = 0$ over the pressure difference at $x' = L$. This ratio was found to be only slightly dependent on flow rate for the straight-wall case. However, the ratio is strongly dependent on the flow resistance. The higher the resistance, the closer the ratio of $v_{n, x=0}/v_{n, x=L}$ approaches 1. Experimentally for counterflow with the upper wall straight $y' = y_i(x'/L)$, it was found that for the porous plate $v_{n, x=0}/v_{n, x=L} = 0.6$. For the porous plate plus a cloth $v_{n, x=0}/v_{n, x=L} = 0.75$. For the porous plate plus two cloths $v_{n, x=0}/v_{n, x=L} = 0.8$.

Total-Pressure Drop Through Test Section. A complete discussion on the causes of over-all pressure drops in this type of flow is given in [6]. Plotted in Figs. 10, 11, and 12 are the total-pressure drops versus flow rates for the different resistances. Shown as solid lines are the theoretical total-pressure losses of Kuchemann and Weber [7]. Their maximum estimate I assumes that the total head loss represents the head loss through the resistance based on the normal velocity v_n' as given by equation (29) plus the velocity head drop $(\rho/2)(u_i'^2 - v_n'^2)$, in turning into the screen

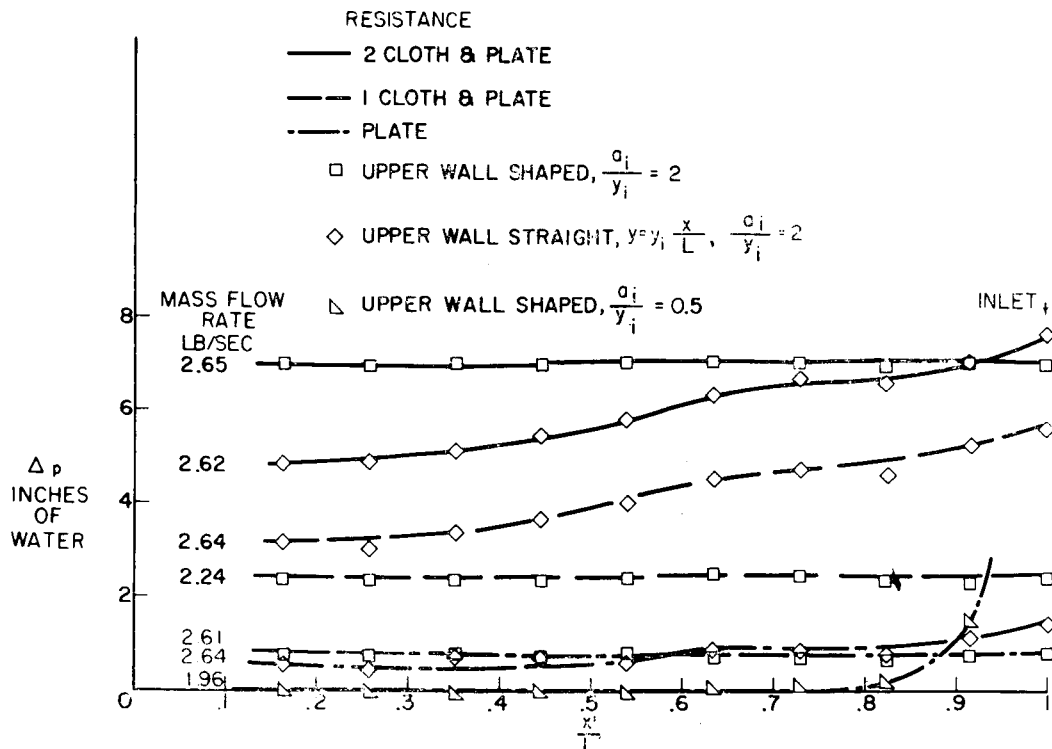


Fig. 9 Pressure drop across resistance along channel

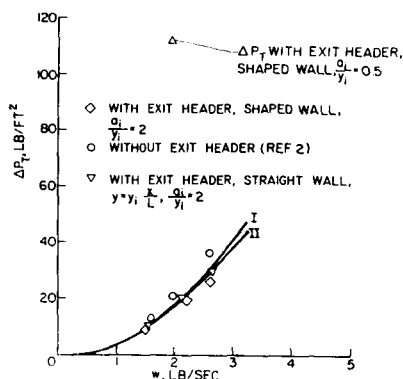


Fig. 10 Total-pressure drop through system for porous plate

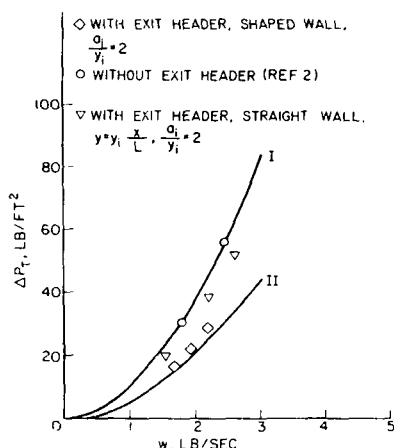


Fig. 11 Total-pressure drop through system for porous plate plus one cloth

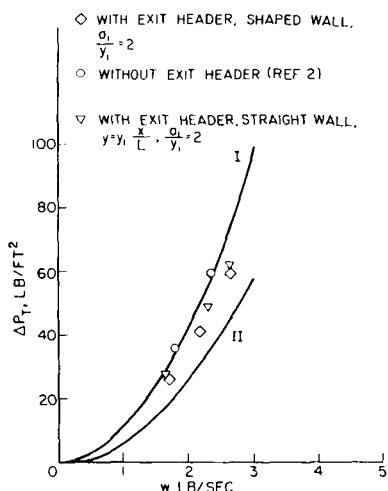


Fig. 12 Total-pressure drop through system for porous plate plus two cloths

which is not considered recovered. The reacceleration in the exit header for this case is assumed to be achieved without losses. Their minimum estimate II represents either the first or second term of estimate I, whichever is larger. Notice that the total-pressure drop for the straight upper wall is higher than for the shaped wall. This can be explained as follows:

For counterflow the total-pressure drop through the system can be considered equal to the pressure drop across the resistance at

$x'/L = 1$ plus the difference in velocity head between the incoming and outgoing velocity at this same point. Since for the straight-wall case, $y' = y_i(x'/L)$, the pressure difference across the resistance at $x'/L = 1$ is a maximum (as discussed earlier), which is higher than for the shaped wall for the same mass-flow rate, a larger over-all pressure drop would be expected. A similar result was found in [2]. It was concluded previously that, if the upper wall were straight so that $y' = y_i$ the minimum static-pressure drop across the resistance would be at $x'/L = 1$. This would imply that for the same mass-flow rate the total-pressure drop through the system will be smaller for this case than for the shaped wall. Similar reasoning can be used in the parallel-flow case.

Also shown in Fig. 10 is the total-pressure drop for the shaped wall where $a_i/y_i = 0.5$. Here the total-pressure drop is much larger than for the other cases, since all the flow passed through only a small section of the resistance and thus gave a very high pressure drop through the resistance.

Notice also that the case for uniform flow without an exit header from [2] has a higher total-pressure drop than for the case with the exit header. This is because the velocity head out of the exit header is not recovered for the case with no exit header.

Discussion of β . The problem of predicting the value of β is difficult. One of the most complete discussions of this parameter appears in a paper by Soucek and Zelnick [8], and in the discussion at the end of that paper by McNown. They came to the conclusion that β was dependent on the surface characteristics of the resistance and on the ratio of the exiting velocity to the main-stream velocity. McNown felt that β could never be greater than 0.5. Cichelli and Boucher [4] in their analysis assumed a value of β of 1. Heyda [5] assumes a β of 0. $1 - \beta$ in equation (13) can be considered as the fraction of the forward velocity of the main-stream fluid contained in the fluid leaving the inlet header. β would be 0 if the exiting fluid had an x -component of velocity equal to that of the main stream. β would be 1 if the fluid particle turned and left the inlet header with no forward component of velocity. On the basis of smoke streamline studies made by Comtois [1], it was found that the fluid enters the screen with very little turning. This would be expected, since no pressure gradient was found normal to the resistance. Thus there would be no force to turn the particle until it enters the resistance. This means that the particle leaves the inlet header with its full forward momentum. The experimental results obtained here show a β greater than zero near the entrance of the inlet header which drops to 0 farther downstream. The reason for this is that initially the boundary layer is being removed. The average velocity across the boundary layer is less than the average velocity across the header. Therefore, although the boundary-layer fluid leaves with its full forward component of velocity this is less than that of the mean velocity across the channel. However, as this boundary layer is removed, β approaches zero.

Conclusions

A study was made of turbulent incompressible flow through resistance parallel to the stream direction for the purpose of obtaining uniform flow through the resistance. By shaping the inlet and exit headers, which had a large length-to-height ratio, that was found to be possible. Theoretical and experimental wall shapes for uniform flow are given and show good agreement. These wall shapes were found to stay constant over a large velocity range. Experimental results of the effect of using some simple wall shapes different from those calculated theoretically are also shown.

Measurements of over-all total-pressure drops were taken and compared with theoretical estimates with good agreement. It was shown that for certain shapes giving nonuniform flow it was

possible to obtain lower over-all pressure drops than for the case of uniform flow through the resistance.

Acknowledgment

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DISCUSSION

J. F. Heyda² and C. D. Fulton²

We are in substantial agreement with the author but offer the following remarks.

There is no reason to believe in the existence of the quantity beta, which implies in Equation (13) that if any fluid becomes retarded its energy will be added to the remaining fluid. This is impossible in either potential or viscous flow.

Beta may have originated in the notion that the fluid would start turning while still in the inlet header because the turn must be complete immediately afterward and in subsonic flow nothing should happen abruptly. In potential flow, even if there were such a turning, there could be no transmission of energy from one stream tube to another, and therefore beta could not exist. But more than that, there is no such turning. A map of curvilinear squares—a Laplacian field—constructed in the inlet header proves this. The porous resistance dictates only the position at which each streamline will be swallowed and not the angle at which it will approach. The angle is, in fact, equal to that of a line drawn from the far end of the resistance to the header wall opposite the point of swallowing. Thus the flow strikes the resistance with full force and delivers its momentum to it after leaving the inlet header.

If the surface of the resistance contains large holes, vanes, or other configurations, the potential flow pattern will show repetitive stagnations and regions of increased velocity. But the average velocity just outside the holes or vanes will be the same as in the inlet header, and so will the average pressure. Thus, whether the microscopic or the macroscopic view be taken, the results will be the same in the inlet header in potential flow, and beta will not exist.

Nor can beta exist because of frictional effects. If viscous shear retards the adjacent fluid in the header before it is swal-

lowed, the momentum extracted from this fluid will again appear as a forward force in the solid material of the resistance. The total force on the material will be the same with or without viscous shear, since the material is bound to receive all the momentum one way or another. The momentum extracted by shear will not appear as a forward push on the main stream of the header.

In the turbulent flow, viscosity can also create eddies of retarded fluid which will attempt to propagate into the main stream at an acute angle. In some cases, that angle will be smaller than the angle at which the flow is leaning into the resistance, and all the eddies will be swallowed. In other cases, a few eddies will propagate and will thereby reduce the pressure in the header. They can be accounted for by adding a small increment to the value of F . What Equation (13) actually states, instead, is that if friction underlies the existence of beta, then friction will produce a rise in pressure, which is absurd.

The notation used by the author in defining dimensional and nondimensional variables is not quite consistent in the Nomenclature. This produces difficulties later. Thus, to be consistent, a_i should be replaced by a_i' , and a_i would not need to be used since its value would then be unity. Similar remarks apply to y_i and y_i' .

In the exit header, two types of flow may be conceived. One is mixed or one-dimensional and can be generated if the holes in the plate squirt out very energetic jets or if some other strong stirring mechanism is at work. This corresponds to Equation (3).

The other type of flow is stratified and two-dimensional. It can occur if the flow leaves the resistance in a relatively quiescent state. Each stream tube retains its identity and moves in a curved, accelerating path toward the outlet. The velocity profile is steeply tilted. We have analyzed this flow using a one-dimensional pressure field, which is justifiable both analytically and experimentally. The result is the following equation, here written in dimensional form but omitting the author's prime notation.

$$a(x) = v_n \sqrt{\rho/2} \int_0^x \frac{dt}{\sqrt{p(t) - p(x)}}$$

The integral computes the header width by finding the vertical widths of stream tubes originating at positions t when they reach position x . Standing at one position x , the variable t is run from 0 to x and the integral computed. Then x is changed and the process repeated. This is a Volterra improper integral. As t approaches x , the vanishing of the denominator reflects the fact that each stream tube starts out as a second-degree parabola that takes up a great deal of space. This voluminousness is partly compensated for by the narrowness of the high-speed tubes near the header wall. The equation can be converted into Abel's integral equation. It can also be inverted approximately for p by the methods of Reference [5]. It can be elaborated for compressibility and variable flow rate through the resistance. It can be solved in closed form for certain simple functions.

The results of the equations for mixed and stratified flows in the exit header are not greatly different although the equations appear very different and the mathematics are more difficult in the improper integral. For a given header shape, the form of the pressure curve is the same and the pressure gradient is merely somewhat steeper in the stratified flow. For this reason, one may elect to use the mixed flow equation corrected by a multiplier to handle stratified flow, or any flow between the two. Many experimental results agree closely with stratified flow and show the associated velocity profile. The following table shows how the two flows compare.

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		End-to-end pressure fall in velocity heads at mean outlet velocity	
Exit header shape	Shape of pressure curve $p(0) - p(x)$	Mixed flow	Stratified flow
$a \sim x$ Triangle	$C - \log 1/x$ Logarithmic*	4.60*	7.23*
$a \sim \sqrt{x}$ Parabola	x^2 Linear	3	4
$a = C$ Rectangle	x^2 Square law	2	2.47

* In the triangular header the pressure rises to an infinite peak at $x = 0$. The constant C is indefinitely large. The tabulated pressure falls are for the latter 90 per cent of the header length, for this case only.

The shapes of the inlet and exit headers can be related directly by eliminating the pressure. For a given exit header, there is a multiplicity of inlet headers nested one inside another. The largest possible inlet header in counterflow is the one producing stagnation at its closed end. It requires the least possible inlet energy and may be called the economic shape. Omitting friction, that shape is given by the following equations when mixed flow occurs in the exit header. One equation is the inversion of the other. The choice of which one to use depends upon which header is given at the beginning. These equations may be regarded as either dimensional or nondimensional.

$$\left[\frac{x}{y(x)} \right]^2 = \left[\frac{x}{a(x)} \right]^2 + 2 \int_0^x \left[\frac{t}{a(t)} \right]^2 \frac{dt}{t}$$

$$\left[\frac{x}{a(x)} \right]^2 = \left[\frac{x}{y(x)} \right]^2 - \frac{2}{x^2} \int_0^x \left[\frac{t}{y(t)} \right]^2 t dt$$

When stratified flow occurs in the exit header, the corresponding equation is the following, which is not easy to invert. Therefore one must start with $y(x)$. There is a multiplicity of shapes $y(x)$ that give the same $a(x)$. Economic shapes are those having appreciable width at the closed end—i.e., an infinite slope there.

$$a(x) = \int_0^x \frac{dt}{\sqrt{\left[\frac{x}{y(x)} \right]^2 - \left[\frac{t}{y(t)} \right]^2}}$$

The results of the foregoing three equations are shown in the following table:

Exit header shape	Economic inlet header shape	Economic inlet header width	
		Mixed exit flow	Stratified exit flow
Triangle	Impossible		
Parabola	Parabola	0.577	0.5
Rectangle	Rectangle	0.707	0.636

J. F. Thorpe³

The author has investigated a very interesting problem and presents information which is valuable to the fluid mechanics engineer. One particularly useful aspect of this investigation is that it defines a class of flow problems to which the one-dimensional approximation can be applied with acceptable results.

The author has assumed that the fluid enters the exit header with no x -component of velocity. This assumption is valid for the high resistance of the porous cloths which he used. As a result of this assumption he finds that it is impossible to maintain a constant pressure in the exit header. It was also found that the momentum factor β , which is applied to the fluid leaving the inlet header, was essentially zero or slightly positive for the resistances tested.

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The discussor has performed experiments⁴ on flow configurations similar to those investigated by the author. However, the transverse resistances were long slots which permitted the fluid entering the exit header to do so with the full (inlet header) x -component of velocity. In fact, as far as the inlet header is concerned, it was found that the factor β was slightly negative. These conditions represent the opposite case to the author's and it is interesting to see what conclusions can be drawn using the author's method of analysis.

Assuming that the fluid enters the exit header with an x -component of velocity u' , the nondimensional momentum equation (for parallel flow) corresponding to the author's equation (3) becomes with his symbols,

$$-\frac{a}{2} dp - \frac{Fv^2}{(va)^{1/4}} dx + u d(av) = d(av^2) \quad (3)'$$

The continuity equations for parallel flow and uniform transverse discharge become

$$va = x \quad (4)'$$

$$uy = 1 - x \quad (22)'$$

Substituting the continuity equations into equation (3)',

$$-\frac{a}{2} dp - \frac{Fx^{1/4}}{a^2} dx + \frac{1-x}{y} dx = d\left(\frac{x^2}{a}\right) \quad (5)'$$

Now if friction is neglected ($F = 0$), it can be seen that equation (5)' admits of a solution in the case $n = 1$. That is if

$$a = a_0 = x^n = x; \quad (n = 1), \quad (9)'$$

Then

$$-\frac{x}{2} dp + \frac{1-x}{y} dx = dx$$

All that is necessary is to take

$$y = y_0 = 1 - x \quad (30)$$

Then

$$dp = 0; \quad u = v = 1 \quad (31)$$

Thus it is possible to keep the static pressure constant in the exit header by shaping both the inlet and exit headers with straight walls.

If the solution outlined above is to be entirely consistent then equation (13) must reduce to $dp = 0$. Otherwise the assumed uniform transverse flow will not occur. With $F = \beta = 0$ it is seen that equation (13) is satisfied with $dp = 0$ (recall $u = 1$).

The discussor has measured the pressure distributions⁴ for the flow configuration defined by equations (9)' and (30). It was found that the pressure distributions in the inlet and exit headers

⁴ J. F. Thorpe, "A Parallel Duct Flow Problem," Doctoral Dissertation, University of Pittsburgh, 1960.

were constants. From this result, it was concluded (as did the present author) that the effect of friction has little to do with the wall shape for uniform transverse discharge.

In the experiments performed by the discussers, the transverse openings were slots in a vicinity where the local fluid velocity was greater than the average velocity in the inlet header. From the measured velocity profiles the factor β was estimated to be about -0.035 . In calculating the axial pressures it was found that best results were obtained if this negative value was actually used in the calculations. Thus, it would appear that the present method of handling this momentum factor is valid.

Author's Closure

The discussers' comments were very worth while and have added a great deal to our understanding of the present problem. Some points have been raised which should be enlarged upon. A β not equal to zero does not always imply that some of the energy of the leaving fluid is added to the remaining fluid in the inlet header. If there is some nonuniform entering velocity profile in the inlet header, the mean velocity near the porous wall will not be equal to the mean velocity across the channel and when this fluid near the wall is removed with all its forward momentum, β will not be zero.

The discussers' conclusions on the shapes with the lowest total pressure drops were very interesting. For the author's case this economical shape can be arrived at as follows. The total pressure drop through the system would be in nondimensional form

$$p_{\text{total}} = p_{\text{inlet}} - p_{\text{outlet}} + \left(\frac{u_i}{v_i}\right)^2 - 1 \quad (32)$$

For the counterflow case $p_{\text{inlet}} - p_{\text{outlet}}$ will differ by some constant of the screen resistances R . By using continuity (32) results in

$$p_{\text{total}} = R + \left(\frac{a_i}{y_i}\right)^2 - 1 \quad (33)$$

It is desired to minimize a_i/y_i so as to reduce p_{total} .

Rewriting (16) as follows for $\beta = 0$

$$\frac{1}{y_0^2} = \frac{1}{x^2} \left[\left(\frac{2-n}{1-n} \right) \left(x^{2(1-n)} - 1 \right) + \left(\frac{a_i}{y_i} \right)^2 \right] \quad (34)$$

Since $1/y_0^2$ can never be negative the term in the bracket [] must be positive for all x . Since it is most likely to be negative at $x = 0$ the result for the bracket yields

$$\left(\frac{a_i}{y_i} \right)^2 \geq \left(\frac{2-n}{1-n} \right) \quad (35)$$

The smallest possible value of a_i/y_i then will occur at an n of 0. Then $y_i/a_i = 0.707$ and both the inlet header and exit header will be rectangles as found by the discussers. The total pressure drop will then be

$$p_{\text{total}} = R + 1 \quad (36)$$

Since from (33) the smallest value of a_i/y_i possible is desired for the most economical shape this would occur in mixed flow with rectangular headers, as shown in the discussion.

For the case of parallel flow from (32) and (6)

$$p_{\text{total}} = R + \frac{2-n}{1-n} + \left(\frac{a_i}{y_i} \right)^2 - 1 \quad (37)$$

This will be a minimum for the rectangular exit header ($n = 0$) then

$$p_{\text{total}} = R + 1 + \left(\frac{a_i}{y_i} \right)^2 \quad (38)$$

The most economical shape would be the largest inlet and smallest outlet feasible. However, only in the limit ($a_i/y_i \rightarrow 0$) will it be as economical as the counterflow case (36).

Finally there is some question whether β actually equals -0.035 for the slotted resistance case. This small effect could be possibly due to friction in the header.