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HIGH-EFFICIENCY MODIFIED FRESNEL REFLECTORS FOR SOLAR-ENERGY CONCENTRATION

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SUMMARY

A study has been made of the Fresnel reflector and two variations of this reflector for use as solar-energy collectors. One variation is the conical Fresnel reflector in which the serrations are located on the inner surface of a cone. It is shown that this reflector can have a collection efficiency of 1.00 for any rim angle, if the proper cone angle is selected. Equations are developed for the design of the second variation which consists of a reflector plane that is not perpendicular to the incoming light rays. Segments of this reflector can be used to form a pyramidal collector which combines the desired flatness of the Fresnel reflector with the high efficiency of the conical Fresnel reflector. This collector can have an efficiency which is very close to 1.00 when a sufficient number of reflective sides are used.

INTRODUCTION

The efficient use of solar energy as a source of electrical power in space requires that the energy be concentrated for many methods of conversion. For example, a boiler-turbine-alternator system with mercury as a working fluid requires temperatures around 1,300° F, which can be obtained only by concentrating the sun's energy.

Solar-energy power systems will be desirable in the power range up to 30 kilowatts. (See ref. 1.) Collector diameters might have to be as large as 50 to 100 feet for systems which would use a single collector. Present or proposed booster rockets will be unable to accommodate such large collectors, unless the collectors are folded. Since the collector must be folded, there would appear to be an advantage in using a collector which is flat or composed of flat panels so that a compact package would result from folding. The Fresnel reflector, which is essentially flat, has been proposed for a solar collector in reference 2, where it is shown that the collector folds easily into a relatively compact package. The Fresnel reflector, however, has one disadvantage. Certain areas of the reflector produce a shadowing effect and shield the focal point from the reflected rays, thus reducing the collection efficiency.

The purpose of this report is to investigate modifications of the Fresnel reflector which will reduce the loss in efficiency due to shadowing without sacrificing the favorable feature of the Fresnel reflector, its flatness. With these objectives in mind, a development is made of the geometry of a Fresnel reflector whose plane is not perpendicular to the incoming solar rays. A proposed configuration would consist of several panels of Fresnel reflectors arranged as the sides of a pyramid. As an intermediate step in the development of the reflector whose plane is not perpendicular to the incoming solar rays, the conical Fresnel reflector, which does not have flat panels, is investigated because it can be designed to have no loss in efficiency due to shadowing.

SYMBOLS

A_p	projected area of collector
A_s	surface area of collector
f	focal length of collector
h	height of serration on Fresnel reflector
R	reflector radius, measured perpendicular to focal axis
R_1	upper limit of reflector radius for local efficiency of 1.0
r	radius to any point on collector measured perpendicular to focal axis
s	projected length, in radial direction, of portion of reflector serration, from which reflected rays do not reach focal point
X,Y,Z	coordinate axes of tilted Fresnel reflector
x	coordinate along X-axis
Y_1	axis in plane of tilted Fresnel reflector
y_1	coordinate along Y_1-axis
$(y_1/f)_0$	smaller nondimensional y_1 ordinate of serration at $x/f = 0$ (identifies serration curves of fig. 5)

α — angle whose tangent is slope of loci of serrations in X-Y_1 plane of tilted Fresnel reflector, $\tan^{-1} \dfrac{dy_1}{dx}$

β — local serration angle with respect to X-Y_1 plane of tilted Fresnel reflector (see fig. 3)

η — reflector efficiency, ratio of energy reaching focal plane to energy specularly reflected

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η_θ — local efficiency at any point on reflector

θ — angle between focal axis and any reflected ray

θ_R — rim angle, angle between focal axis and a ray reflected from rim of reflector, or from rim at Y_1-axis of a panel for a pyramidal collector

ϕ — complement of one-half the cone apex angle (see fig. 2); see figure 3 for pyramidal collector

DEVELOPMENT OF COLLECTOR MIRRORS

Basic Fresnel Reflector

The Fresnel reflector has been analyzed very thoroughly and the results are reported in reference 3. However, a brief analysis of this reflector is included here for completeness. A sketch of the reflector and a detail of the shadow area are shown in figure 1. The reflecting surfaces of the serrations are shown as straight lines in the detail, but in an ideal Fresnel reflector cross section these lines would be sections of parabolas, all having a common focus. Straight-line elements would probably be used in the actual construction of a reflector because of the ease of fabrication. It is seen from the figure that any rays impinging on the length of the serration marked s will not be reflected to the focal point, but will be reflected from the side of the serration back into space. Thus, the local efficiency for any infinitely small serration is

$$\eta_\theta = \frac{(r_2 - r_1) - s}{r_2 - r_1} \tag{1}$$

or

$$\eta_\theta = \frac{h \cot \frac{\theta}{2} - h \sin \theta}{h \cot \frac{\theta}{2}} \tag{2}$$

which reduces to

$$\eta_\theta = \cos \theta \tag{3}$$

The efficiency of the reflector then becomes

$$\eta = \frac{2\pi \int_0^R (\cos \theta) r \, dr}{2\pi \int_0^R r \, dr} \tag{4}$$

Substitution and integration gives

$$\eta = \frac{2 \cos \theta_R}{1 + \cos \theta_R} \tag{5}$$

Conical Fresnel Reflector

In order to reduce or eliminate the useless area shown in figure 1, the conical Fresnel reflector is investigated as a possible solution to the problem. Of course, this reflector does not have flat panels, but there may be some applications for which this reflector may be suitable. Figure 2 shows a planform and two details of the reflector. Detail A shows that one area of the reflector can have no shadowing effect and thus has an efficiency of 1.0. However, when the angle θ is greater than 2ϕ, shadow areas (see fig. 2, detail B) similar to those on the basic Fresnel reflector result. Angle θ is the angle between the focal axis and a line from the focus to any point on the reflector, and ϕ is the complement of one-half the cone apex angle. Therefore, when $\theta > 2\phi$, the efficiency at any point on that section of the reflector is

$$\eta_\theta = \frac{\left(r_2 - r_1\right) - s}{r_2 - r_1} \tag{6}$$

or

$$\eta_\theta = \frac{\dfrac{h}{\tan\frac{\theta}{2} - \tan\phi} - h\sin\theta}{\dfrac{h}{\tan\frac{\theta}{2} - \tan\phi}} \tag{7}$$

which reduces to

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$$\eta_\theta = \cos\theta + \sin\theta\tan\phi \tag{8}$$

For the general case where the rim angle θ_R is greater than 2ϕ, the efficiency of the whole reflector becomes

$$\eta = \frac{2\pi\displaystyle\int_0^{R_1} r\,dr + 2\pi\displaystyle\int_{R_1}^{R} \eta_\theta r\,dr}{2\pi\displaystyle\int_0^{R} r\,dr} \tag{9}$$

Substitution gives the following:

$$\eta = \frac{R_1^2}{R^2} + \frac{\displaystyle\int_{\theta=2\phi}^{\theta_R} \frac{(\cos\theta + \sin\theta\tan\phi)\sec^2\theta\tan\theta\,d\theta}{(1 + \tan\theta\tan\phi)^3}}{\frac{1}{2}R^2} \tag{10}$$

Graphical methods were used in making calculations from equation (10).

Tilted Fresnel Reflector

The tilted Fresnel reflector is proposed as a means of minimizing the loss in efficiency due to shadowing. Flat panels of a modified Fresnel reflector are combined into a solar-energy collector shaped like a pyramid. The planform view of the serrations of the basic Fresnel reflector is a series of concentric circles. (See fig. 1.) However, for the tilted panels no simple relation exists, and the shape of the serrations in the planform view must be determined. A sketch of one reflector panel with the geometry necessary for development of the loci of the serrations and the angles of the serrations β with respect to the reflector plane is shown in figure 3. In order to obtain the loci and the angles, however, several construction lines were necessary. At the point (x, y_1), a line normal to the reflecting surface was drawn to intersect the focal axis (Z-axis). From this intersection a line was drawn normal to the X-Y_1 plane, and the third side of the triangle was drawn in the X-Y_1 plane back to point (x, y_1). Thus figure 3 shows that the angles α, from which the loci can be determined, and β can be readily obtained. From figure 3 it can also be seen that $\tan\alpha = \frac{dy_1}{dx}$.

Thus, the differential equation which gives the loci of the serrations is

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$$\frac{dy_1}{dx} = \frac{x}{\left(\sqrt{x^2 + y_1^2\cos^2\phi}\,\cot\frac{\theta}{2} + y_1\sin\phi\right)\sin\phi - y_1} \tag{11}$$

Substitution for θ gives the following equation, from which the loci were calculated on an electronic computing machine by means of a numerical solution for one value of apex angle:

$$\frac{dy_1}{dx} = \frac{x}{\left[\sqrt{\left(f - y_1\sin\phi\right)^2 + x^2 + y_1^2\cos^2\phi} + f\right]\sin\phi - y_1} \tag{12}$$

The loci of the equation are plotted in figure 4 to indicate the pattern of the serrations in the planform view (X-Y_1 plane) for an apex angle of 135°.

The local serration angle β, with respect to the X-Y_1 plane, is shown in figure 3 for one point on the reflector. The equation for calculation of these angles is

$$\beta = \cos^{-1}\frac{\left[\left(\sqrt{x^2 + y_1^2\cos^2\phi}\,\cot\frac{\theta}{2} + y_1\sin\phi\right)\cos\phi\right]\cos\frac{\theta}{2}}{\sqrt{x^2 + y_1^2\cos^2\phi}\,\cot\frac{\theta}{2}} \tag{13}$$

Substitution for θ gives

$$\beta = \cos^{-1}\left\{\cos\phi\sqrt{\frac{1}{2}\left[1 + \frac{f - y_1\sin\phi}{\sqrt{\left(f - y_1\sin\phi\right)^2 + x^2 + y_1^2\cos^2\phi}}\right]}\left[1 + \frac{y_1\sin\phi}{\sqrt{\left(f - y_1\sin\phi\right)^2 + x^2 + y_1^2\cos^2\phi} + f - y_1\sin\phi}\right]\right\} \tag{14}$$

The values of β for the loci shown in figure 4 have been calculated and are presented in figure 5. Note that in contrast to the basic Fresnel reflector, for which the reflecting-surface angle does not vary for a given serration, this reflector has serrations for which the angle varies widely.

The efficiency of the tilted Fresnel reflector has also been determined for one apex angle. An examination of figure 6 shows that a plane containing the focal axis and the point (x,y_1) produces a cross section identical to a radial cross section taken through a conical Fresnel reflector as shown in figure 2.

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Equation (8) for the local efficiency of a conical Fresnel reflector then becomes the following equation for the tilted reflector:

$$\eta_\theta = \cos\theta + \frac{y_1 \sin\phi}{\sqrt{x^2 + (y_1 \cos\phi)^2}} \sin\theta \tag{15}$$

Substitution for θ gives

$$\eta_\theta = \frac{f}{\sqrt{f^2 - 2y_1 f \sin\phi + x^2 + y_1^2}} \tag{16}$$

The efficiency at various points on the reflector has been calculated, and contours of equal efficiency are shown in figure 4. The total efficiency has also been calculated by graphical methods.

RESULTS AND DISCUSSION

The efficiencies of the various reflectors as a function of rim angle have been calculated and are presented in figure 7 for the basic and conical Fresnel reflectors, and in figure 8 for the tilted Fresnel reflector. The paraboloid of revolution which has an efficiency of 1.00 at all rim angles is used as a basis for comparison with the Fresnel reflectors. In contrast to the paraboloid efficiency, the efficiency of the basic Fresnel reflector varies from 1.00 at a rim angle of 0° to 0 at a rim angle of 90°. In order to compare reflector efficiencies realistically, factors other than rim angle must be taken into consideration. One important factor is the ability of the reflector to concentrate the reflected energy in the focal plane. High concentrations are necessary in order to obtain high heat-receiver temperatures. Analyses

have been made in references 3 and 4 to determine the optimum rim angle for highest concentration of solar rays obtainable with paraboloidal reflectors that have geometric inaccuracies. Slightly different methods of analysis were used in each reference so that a rim angle of 53° was obtained in reference 3 and a rim angle of 60° was obtained in reference 4. The optimum rim angle for the basic Fresnel reflector is smaller than that for the paraboloid because of the loss in efficiency with increased rim angle that is characteristic of the basic Fresnel reflector. This optimum rim angle is in the range of 40° to 47°. The basic Fresnel reflector has an efficiency of only 0.86 at a rim angle of 40°. (See fig. 7.) However, figure 7 indicates that if a conical Fresnel reflector L
with a cone apex angle of about 140° is selected, the efficiency would be 1
raised to about 1.00 for the optimum rim angle of 40°. This reflector 9
may be made up of conic surfaces, thus avoiding construction of the double- 3
curvature surface of the paraboloid. 8

Local efficiency has been calculated for a tilted Fresnel reflector panel with an apex angle of 135°. The contours of equal efficiency are shown in figure 4. Unlike the conical Fresnel reflector, much of the area of the tilted Fresnel reflector is affected by shadowing. However, if a practical collector is constructed, in the form of a pyramid, as shown in figure 9, the segments used are mostly in the shadow-free area for rim angles up to about 45°, which corresponds to $y_1/f = 0.765$. This is shown in figure 4, where the boundary for a four-sided pyramidal collector is indicated on the reflector planform. The variation of total efficiency with rim angle has been computed for collectors with 3, 4, 8, 12, and an infinite number of equal reflective sides. Curves for all but the 8- and 12-sided reflectors are shown in figure 8. The curves for these two reflectors are very close to that for the reflector with an infinite number of sides so the data are not shown. These collectors were designed to have a rim angle of 45° which would give an efficiency of 1.00 for a conical Fresnel reflector with an apex angle of 135°. Note that for the four-sided pyramid, the efficiency is about 0.985 at the design rim angle, whereas the three-sided pyramid has a slightly lower efficiency. The curve for the collector with an infinite number of sides also corresponds to the data for a conical Fresnel reflector. This curve represents the maximum efficiency that can be obtained from a tilted Fresnel collector with an apex angle of 135°.

One obvious requirement for collectors of solar power in space is that they be lightweight. However, the desired optical accuracy will determine to a large extent how light a structure is practical. For purposes of analysis, the weight of a collector may be replaced by the surface area so that the relative merits of the collectors may be assessed. Shown in figure 10 for the paraboloidal, basic Fresnel, and conical Fresnel collectors are their efficiencies, weighted by the ratio of projected area to surface area. It should be noted here that the

curve for the conical Fresnel collector of figure 10 is an optimum curve for this type of collector. That is, for a given rim angle, a cone angle must be determined in order to obtain the collector with the highest weighted efficiency. The variation of the product of efficiency and area ratio with cone apex angle is given for several values of rim angle in figure 11. The maximum values of weighted efficiency from this figure were used to determine the curve for the conical Fresnel collector of figure 10. In the case of the Fresnel collectors the surface area has been assumed to be that of a flat plate for the basic Fresnel and of a cone for the conical Fresnel. This assumption is made on the premise
L1938 that most of the weight of a collector is concentrated in the supporting structure rather than in the thin serrated reflecting surface skin. Both the paraboloid and the conical Fresnel collectors, which can have an efficiency of 1.00 for all rim angles, show a decrease in weighted efficiency at the higher rim angles, as seen in figure 10. The conical Fresnel collector is still more efficient than the basic Fresnel collector for all rim angles. The weighted efficiency of the conical Fresnel collector has been presented because it is the upper limit that may be obtained from a pyramidal collector by using the tilted Fresnel reflector segments.

CONCLUDING REMARKS

A study has been made of the Fresnel reflector and two variations of this reflector for use as solar energy collectors. One variation is the conical Fresnel reflector in which the reflective serrations are located on the inner surface of a cone. It is shown that this reflector can have a collection efficiency of 1.00 for any rim angle if the proper cone angle is selected. The second variation consists of tilting the plane of the reflector so that the incoming rays are no longer perpendicular to the reflector plane. Segments of this reflector can be used to form a pyramidal collector which combines the desired flatness of the Fresnel reflector with the high efficiency of the conical Fresnel reflector. It is shown that this type of collector can have an efficiency which is very close to 1.00 if a sufficient number of reflective sides are used to form the collector.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 5, 1962.

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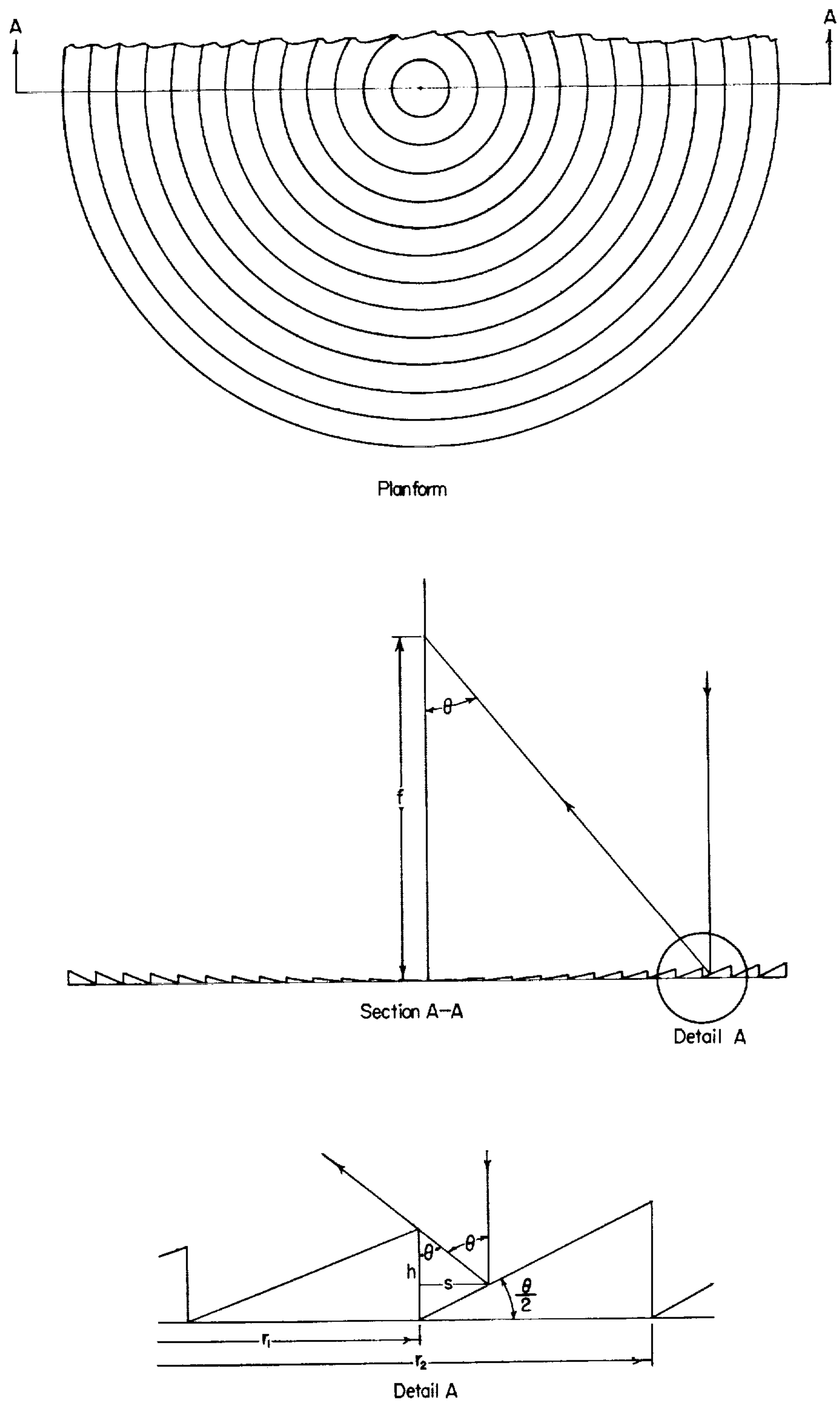

Figure 1.- Planform of basic Fresnel reflector and detail of shadow area.

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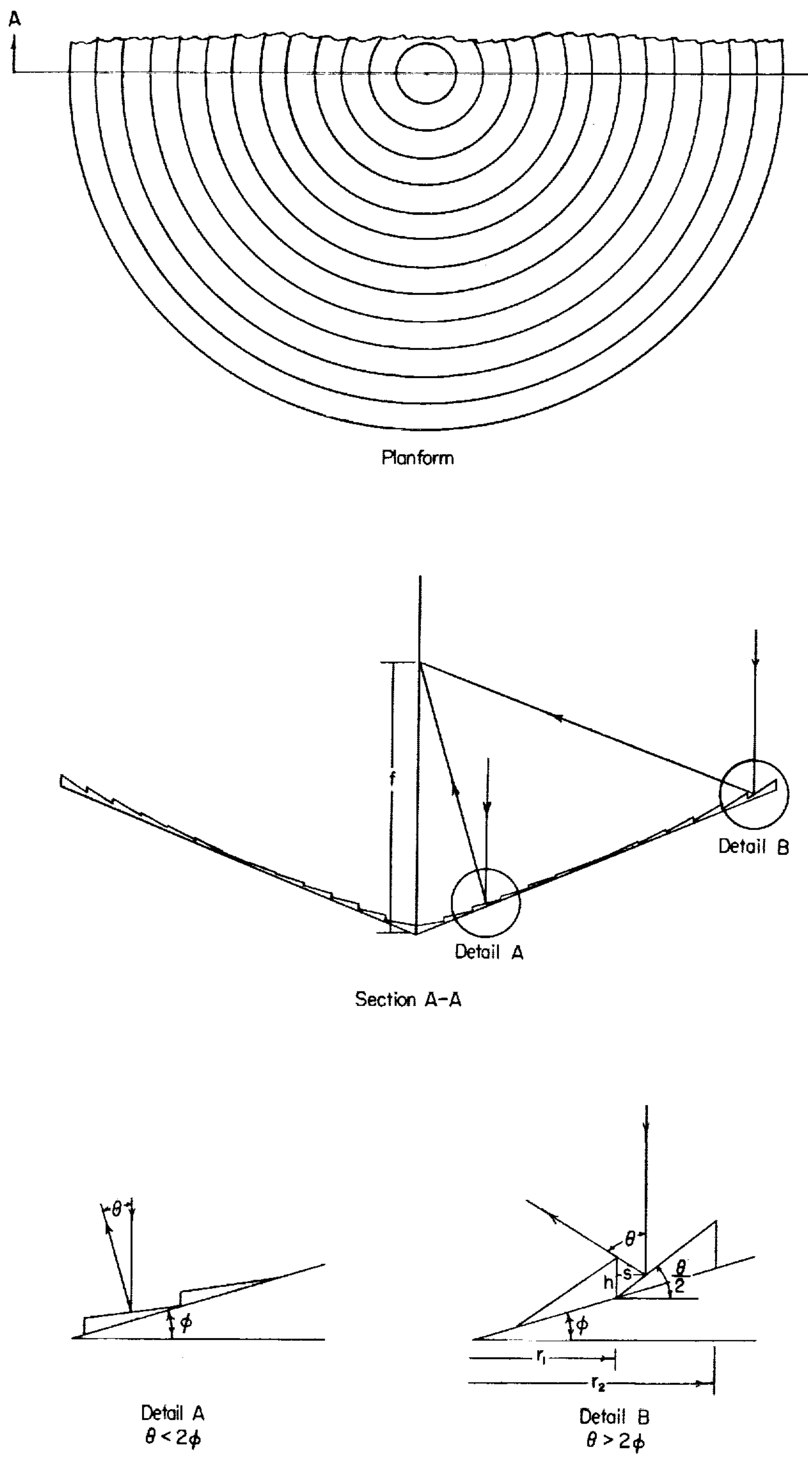

Figure 2.- Conical Fresnel reflector and reflection characteristics of different areas of mirror.

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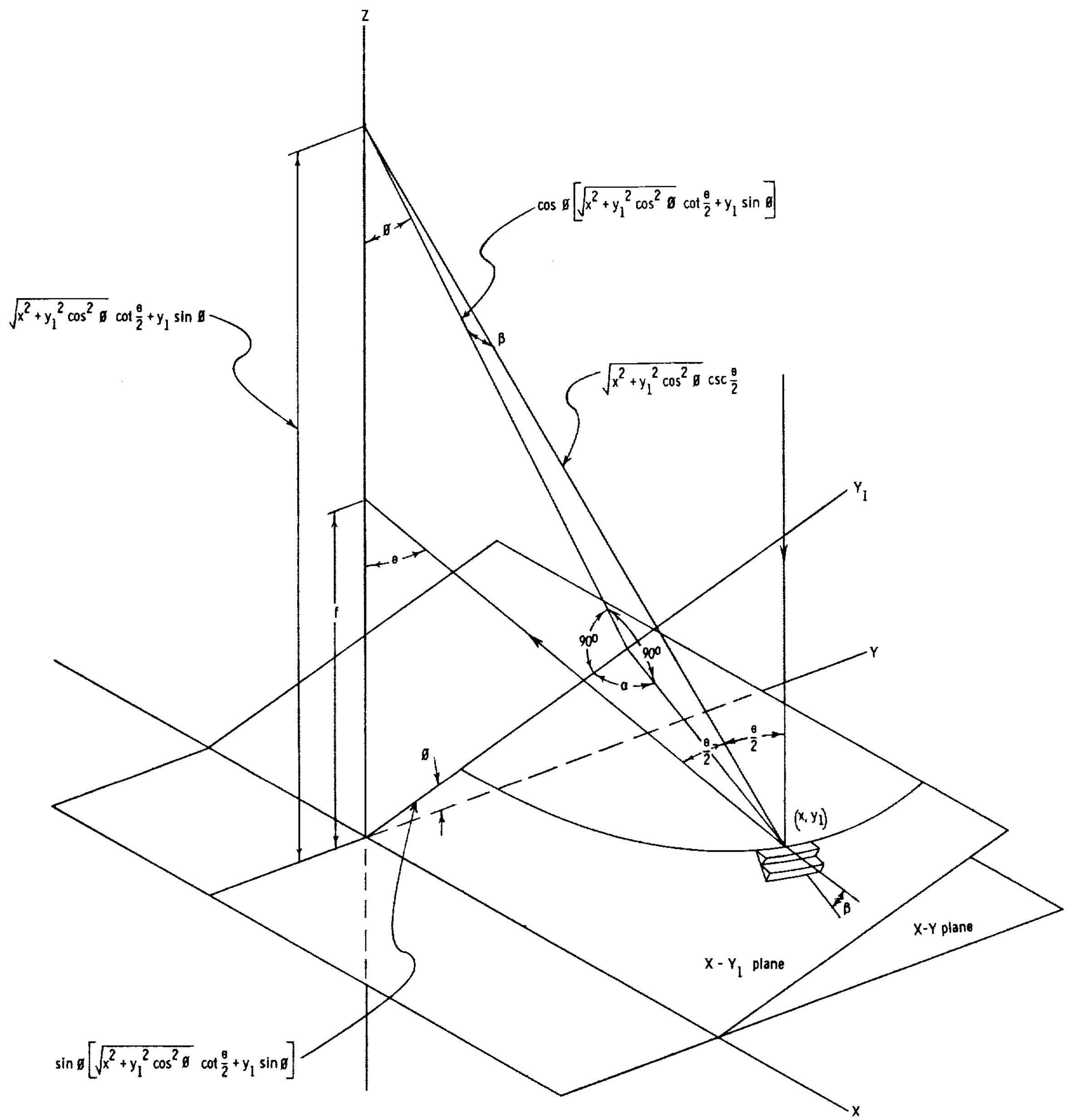

Figure 3.- Sketch of tilted Fresnel reflector with geometric details.

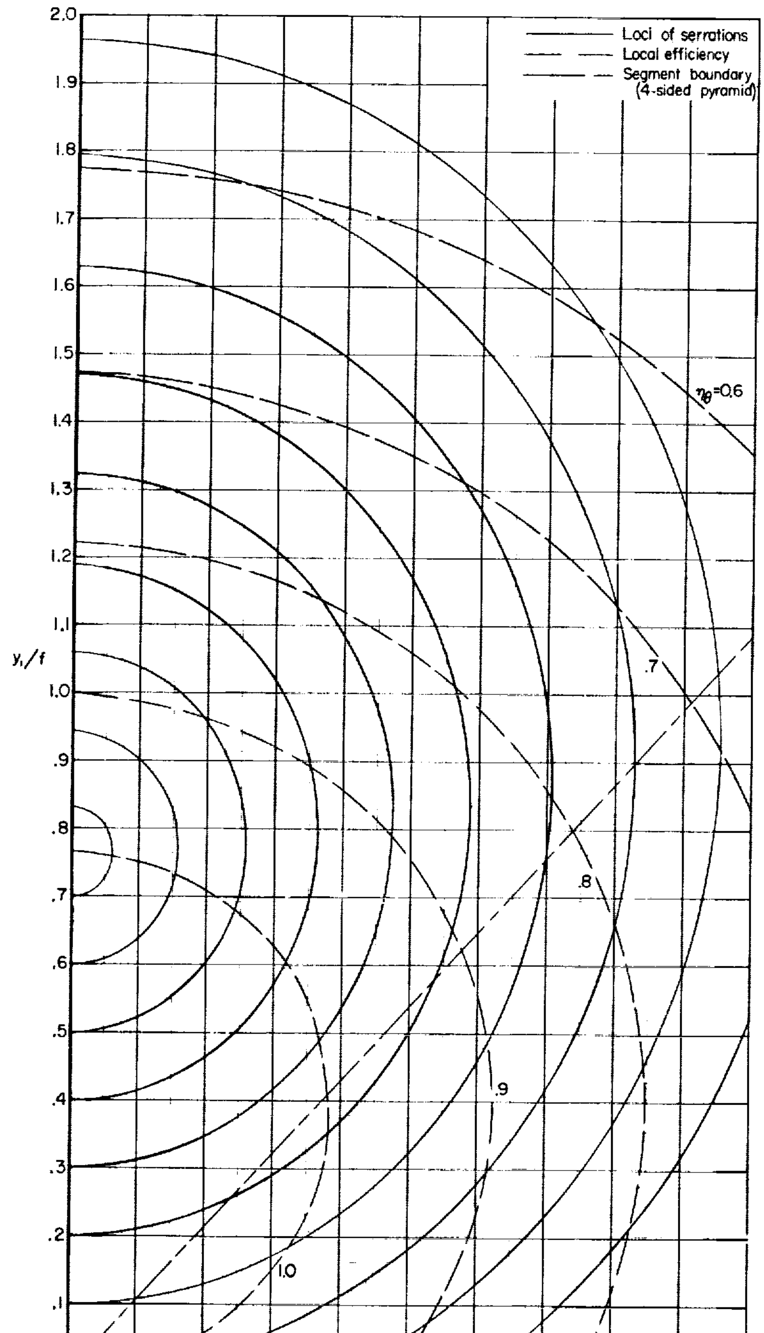

Figure 4.- Planform view of tilted Fresnel reflector showing loci of typical serrations and contours of local efficiency. Apex angle = 135°.

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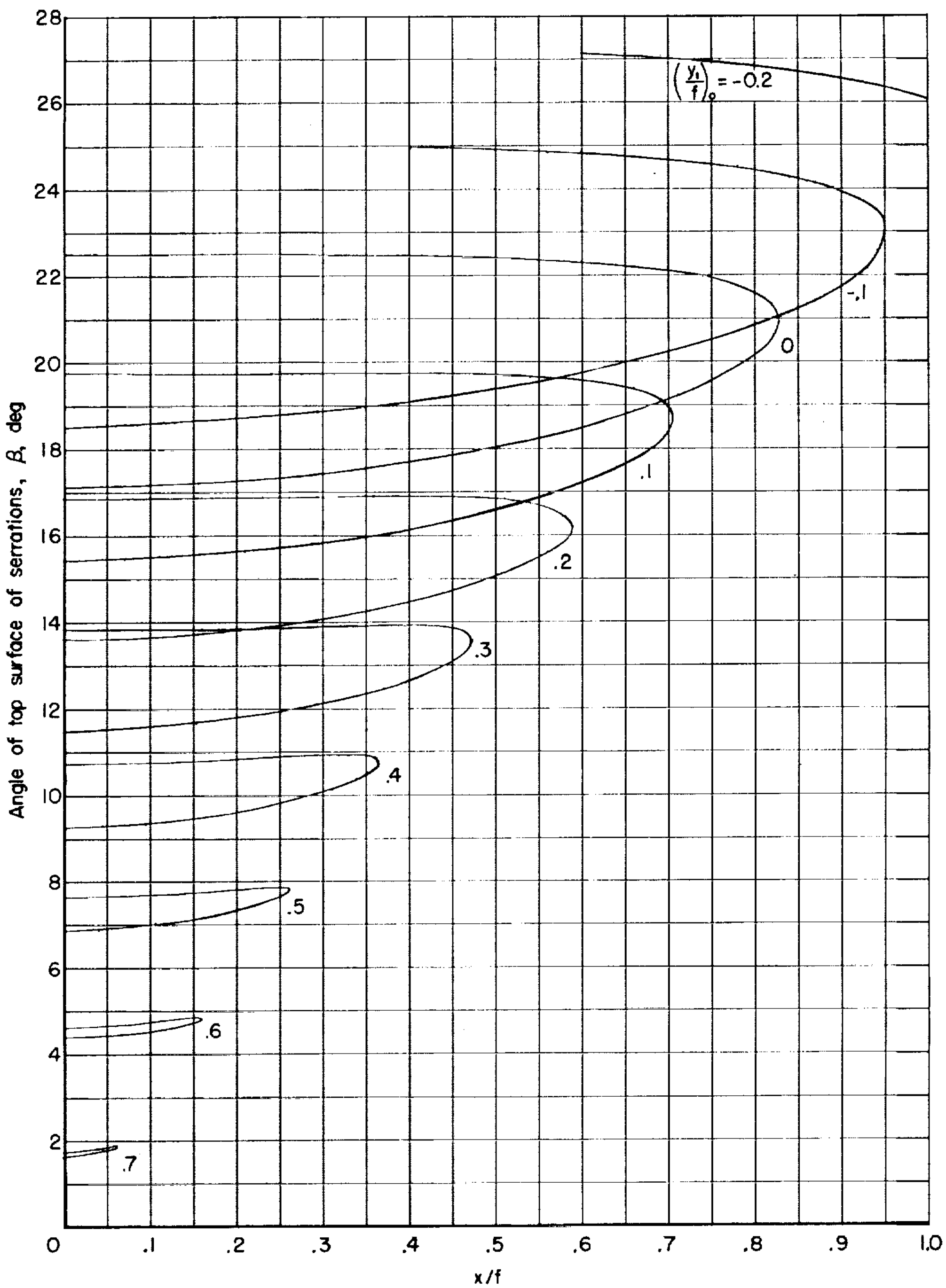

Figure 5.- Angles of top surfaces of serrations shown on planform in figure 4 for tilted Fresnel reflector of 135° apex angle. (Data on upper arms of these curves correspond to points on lower arms of curves of fig. 4.)

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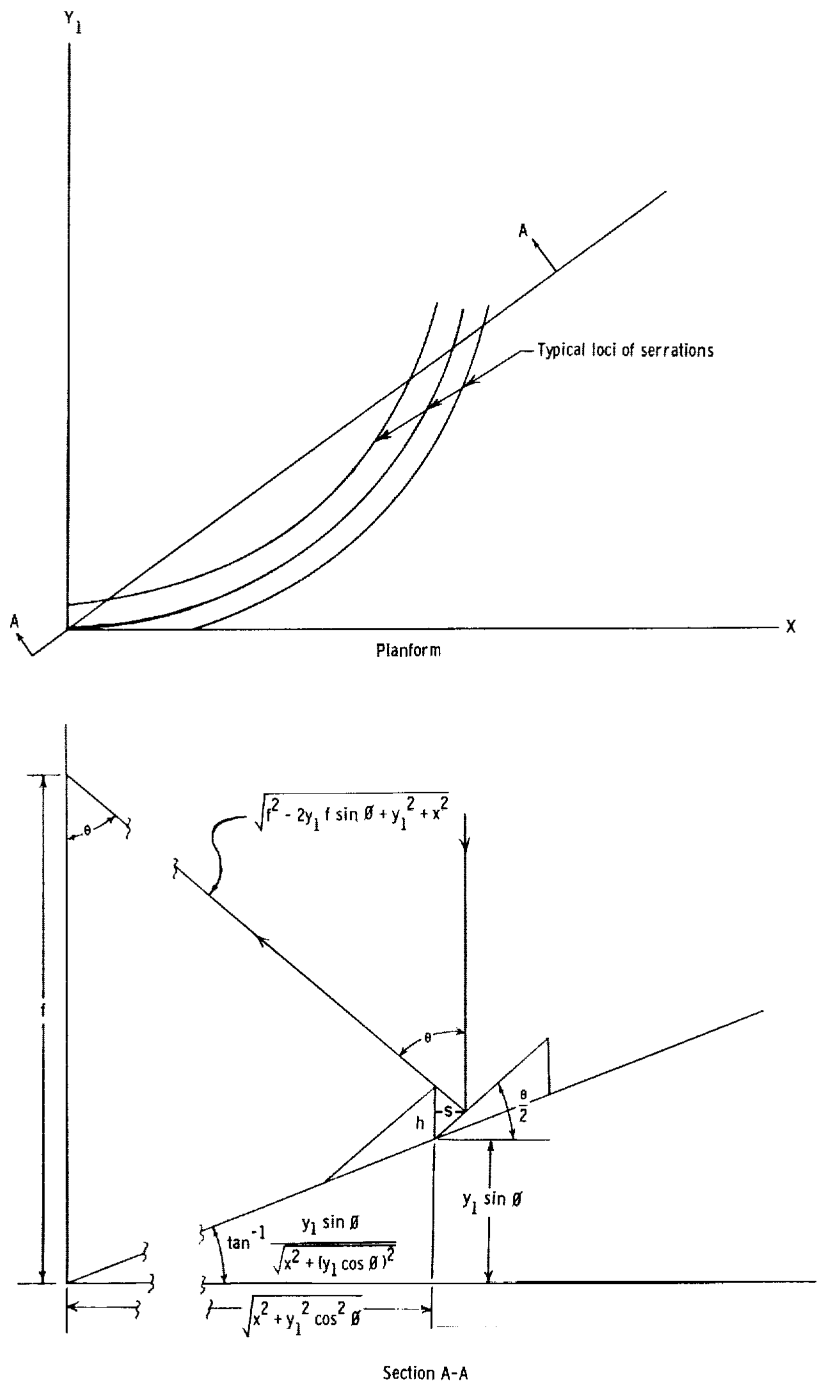

Figure 6.- Sketch of tilted Fresnel reflector and shadow area.

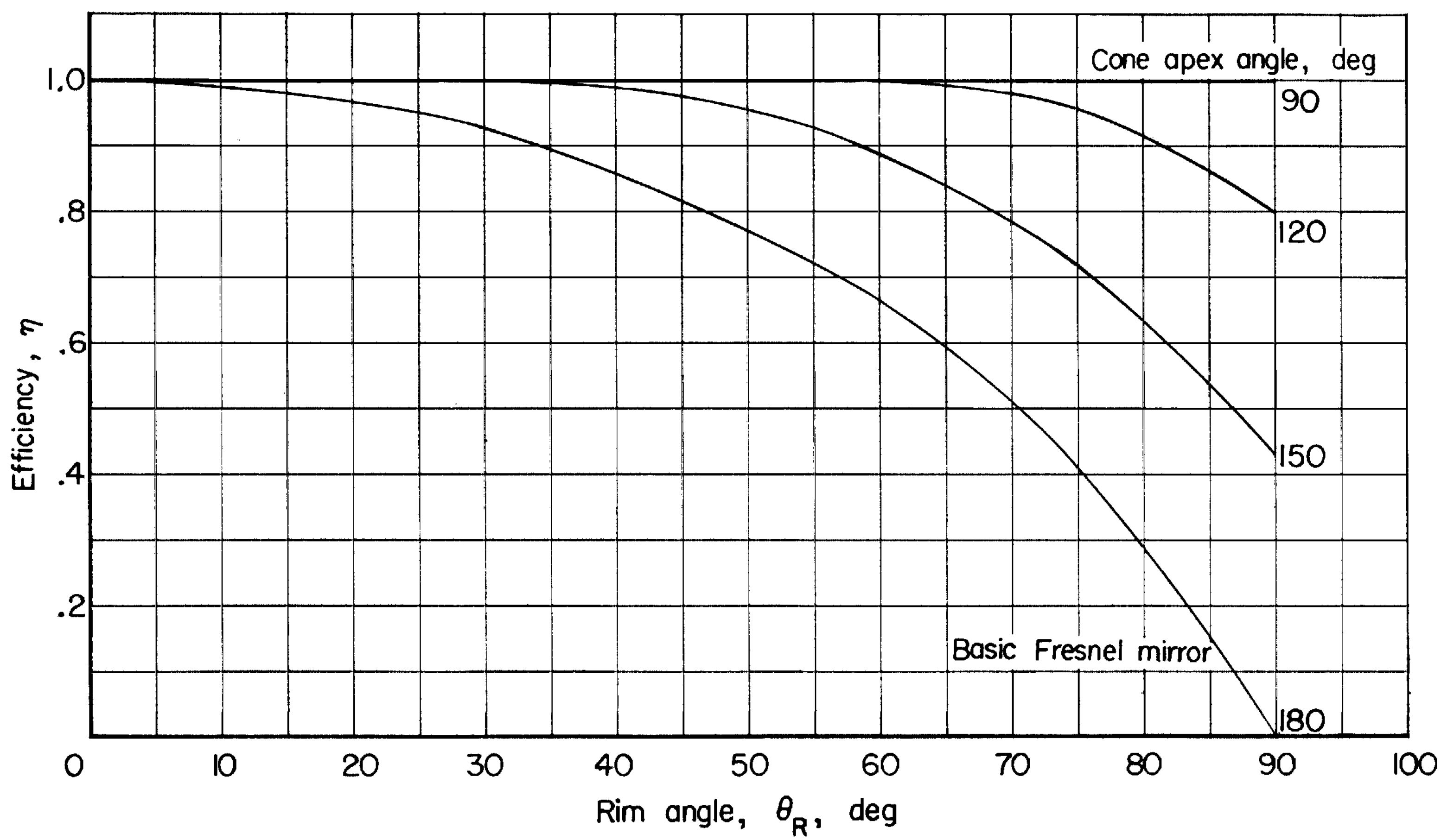

Figure 7.- Efficiency as a function of rim angle for the basic and conical Fresnel collectors with various cone apex angles.

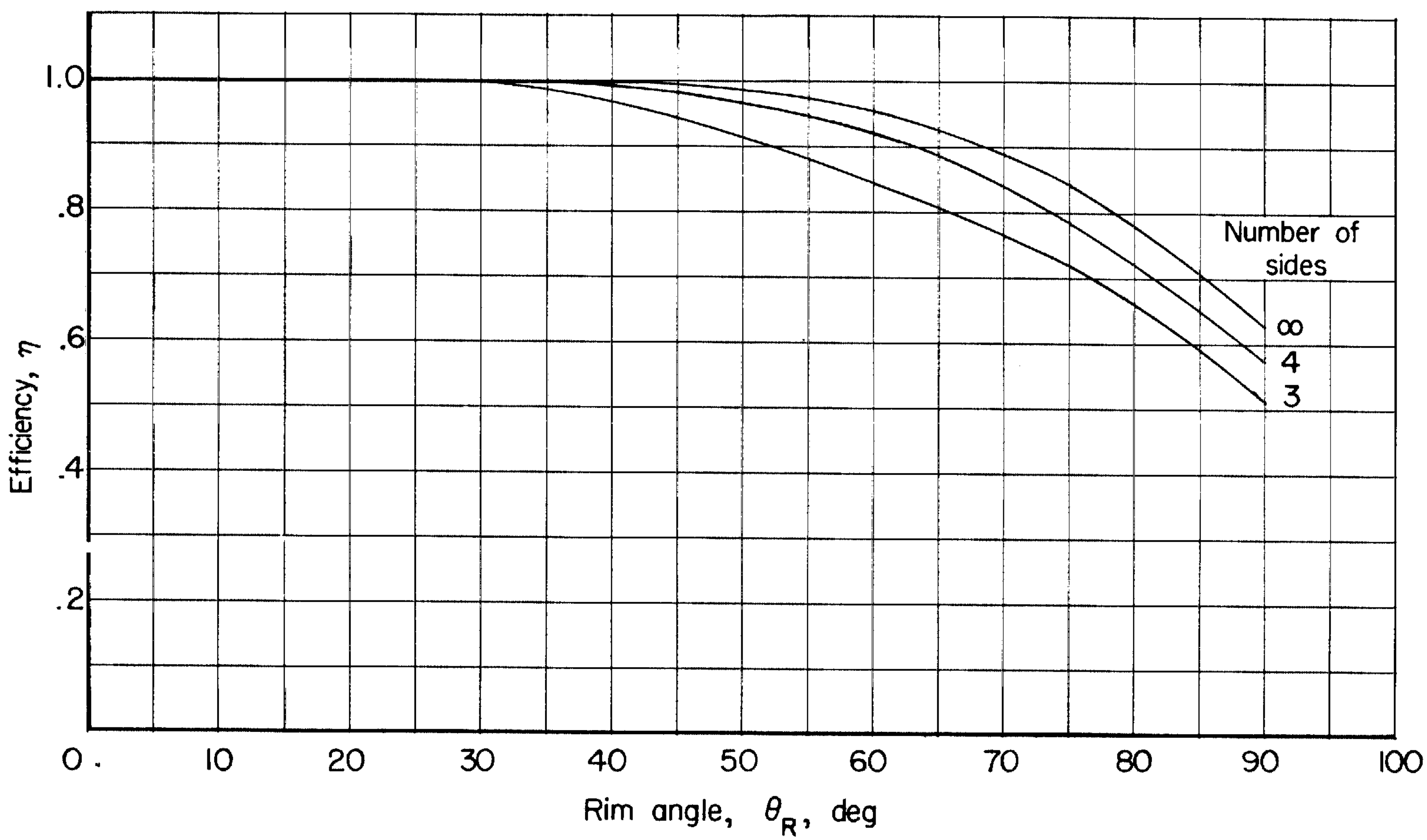

Figure 8.- Efficiency as a function of rim angle for pyramidal collectors employing panels of the tilted Fresnel reflector and having various numbers of sides. Apex angle = 135°; design rim angle = 45°.

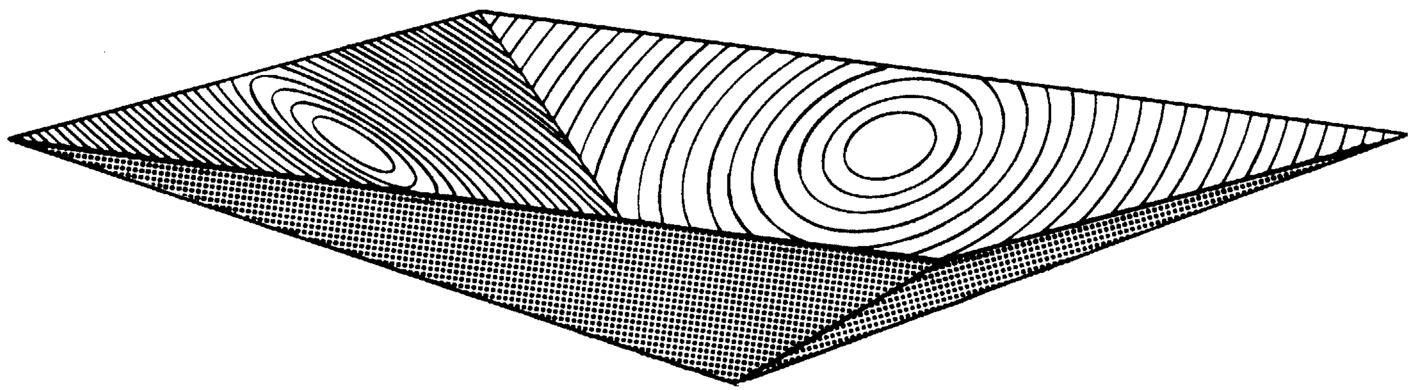

Figure 9.- Sketch of pyramidal collector made of panels of tilted Fresnel reflectors.

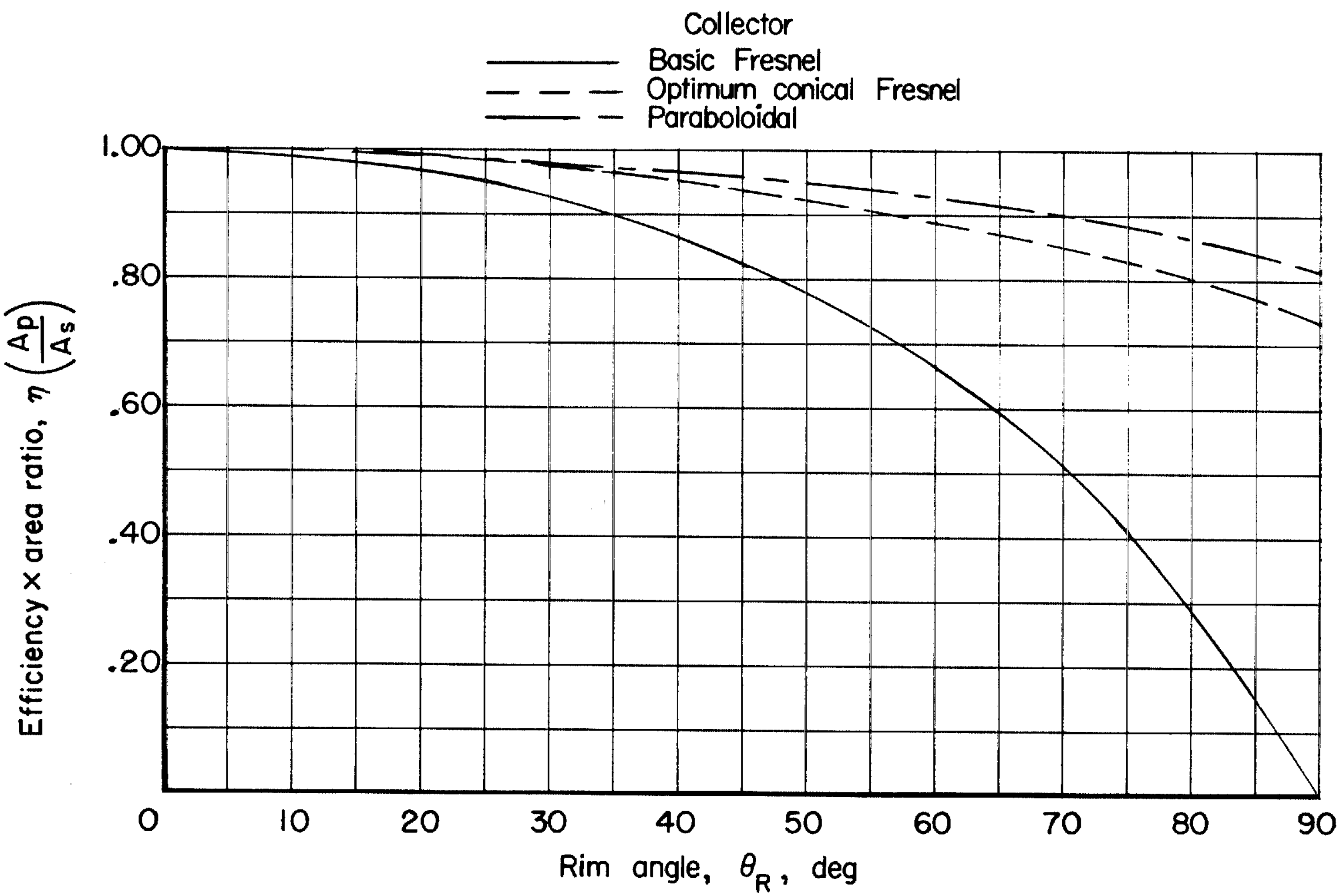

Figure 10.- Effect of rim angle on weighted efficiency of three different types of solar-energy collectors.

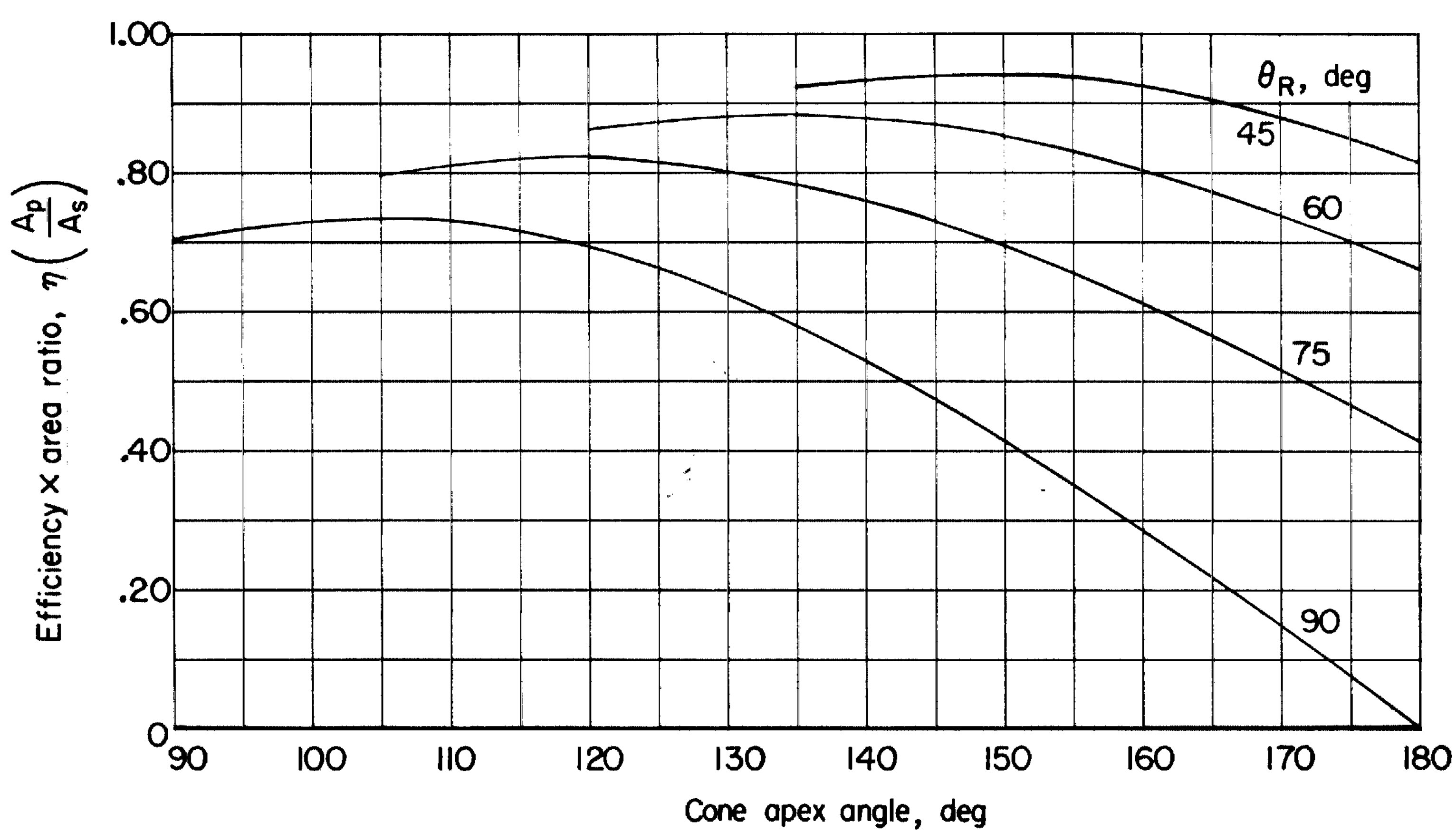

Figure 11.- Variation of product of efficiency and area ratio with cone apex angle for conical Fresnel collectors of various rim angles.

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