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28

Neutrino Emission Processes, Stellar Evolution
and Supernova. Part II

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Abstract

15603

When the internal temperature of a star exceeds 10^9 °K the evolution is very rapid. The time scale is around 3000 years. Energy dissipated by emission of neutrinos is several orders of magnitude higher than that by optical radiation. Neutrinos have exceedingly long mean free path ($\sim 10^{20}$ gram/cm²) and they will escape as soon as they are produced. The star will contract to release gravitational energy to make up the energy lost to neutrinos and the internal temperature will rise to comply with the virial theorem. It is possible in this fast evolutionary phase, to neglect optical radiations all together as compared with the neutrino radiation. A set of structural equations is thus obtained. The envelope is replaced by boundary conditions. A simple model is constructed to illustrate our approach.

I General Discussions and Outlines

In Part I of this paper we have discussed the evolution of late type stars in general and pointed out the importance of neutrino emission processes in old stars. In this paper we shall investigate the development of a very late star with internal temperature exceeding $10^9 \text{ }^\circ\text{K}$ (1 BK) into a supernova. The general importance of neutrino emission processes in various stages of stellar evolution will appear in another paper which will be published elsewhere.

When the internal temperature T exceeds 1 BK neutrino emission from stars is exceedingly important. The neutrino luminosity (L_ν) is greater than $10^6 L_\odot$ in energy output while optical radiation is less than $10^5 L_\odot$ in general. Neutrino has exceedingly long mean free path ($\sim 10^{20} \text{ g/cm}^2$) in contrast to electromagnetic radiation whose mean free path is around 1 g/cm^2 . Therefore, once created inside stars, the neutrino will escape. The neutrino luminosity is the same as its production rate. Under such circumstances, stars will have quite different structural characteristics from those in which electromagnetic radiation is dominant. By virtue of the Virial Theorem (see Section III), the rate of evolution of such stars is governed by the neutrino

emission rate only. The composition of the star at around 1 BK is mainly Si^{28} . Very little energy is generated when Si^{28} undergoes nuclear transformations to form Fe^{56} at $1 \sim 3$ BK. The supply of nuclear energy is very small. Thus, energy can be supplied only by a gravitational contraction of the star. The structure is mainly determined by two factors: the gravitational contraction and neutrino emission. When optical radiation is neglected as compared to neutrino radiation, the structural equations become sufficiently simplified. Although they still must be solved by numerical methods in general, a simplified model admits analytical solution.

To summarize our approach, we assume the following for pre-supernova stars:

- (1) $T > 10^9 \text{K}$
- (2) Nuclear energy sources are limited, and to the first approximation, they are neglected.
- (3) $L_\nu \gg L_\gamma$ (L_γ is the electromagnetic luminosity of the star).
- (4) The envelope has no feed back effect on the core, i.e. the rapid evolution of the core may change the envelope but the evolution of the envelope has

no effect on the core.

- (5) We consider only the type of supernova caused by the iron-helium transition mechanism as proposed by Burbidge et al.²

Each of the above assumptions will be discussed separately later.

The scheme of this paper is as follows: We first present outlines of the physical properties of matter under the conditions described above. They include: the neutrino emission processes (Sect. II), the virial Theorem (Sect. III), matter in statistical equilibrium (Sect. IV), the hydrostability of a star (Sect. V). In Section VI, we discuss an approach to the structure of pre-supernova stars according to the five assumptions above and derive the differential equations for such stars. In Sect. VII we shall simplify and modify some of the results of Section II through Section V, and construct an analytically soluble model to illustrate our results in Section VI. We do not propose that this model represents a physical pre-supernova star. It has some resemblance to the actual star, but such resemblance should not be too strongly stressed.

A list of symbols used through this paper is provided below:

A	mass number of a nucleus
BK	$10^9 \text{ }^\circ\text{K}$ (one billion degrees)
E	the total internal energy of a star (in ergs)
E_F	Fer surface energy
E_{gr}	the total gavitational energy
E_n	nuclear energy released from $T=0$ to the temperature T
E_r	the energy of an excited state of a nucleus from its ground state
ϵ	specific internal energy of a gas
ϵ_{gr}	the specific gravitational energy
$\frac{d\epsilon_{gr}}{dt}$	the rate of generation of gravitational energy
\dot{E}	rate of ^{net} energy release
G	gravitational constant ($=6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ gm}^{-2}$)
γ	The adiabatic exponent. (The heat capacity ratio C_p/C_v)
I_r	Spin of an excited state of a nucleus
L_γ	electromagnetic luminosity (total electromagnetic energy flux of a star) (in ergs-sec^{-1})
L_ν	neutrino energy flux from a star (in L_\odot units or ergs sec^{-1})
L_\odot	solar luminosity ($=3.78 \times 10^{33} \text{ ergs-sec}$)
L_r	Shell luminosity, the energy flux that flows through a shell of radius r
λ	thermal conductivity

M	mass inside a shell of radius r
m	mass of the core of a star
M_n	mass of neutrons
M_p	mass of protons
M_\odot	solar mass ($=1.985 \times 10^{33}$ g)
μ	chemical potential (ergs)
n	polytropic index
$n(A, Z)$	number density of the nucleus (A, Z)
n_n	number density of neutrons
n_p	number density for protons
η	the absolute value of the ratio of the gravitational energy to the internal energy of a star
ω	statistical weight
P	Pressure
P_d	extremely degenerate fermi gas pressure in which the temperature dependence is neglected
P_F	Fermi gas pressure
P_r	radiation pressure
P_{rel}	pressure of a relativistic but non degenerate gas
$Q(A, Z)$	binding energy of a nucleus (measured from the state of A free nucleons)
ρ	density (in g cm ⁻³)

f_0	initial value
R	gas constant ($=8.32 \times 10^7$ ergs mole ⁻¹ deg ⁻¹ c)
S	specific entropy
T	temperature (in °K)
T_9	T in units of BK
T_0	initial values of T
\mathcal{E}	total energy of a star (the sum of the gravitational energy and the internal energy)
U	energy density (in ergs cm ⁻³)
V	volume (cm ³)
v	Specific volume (volume occupied by a unit of mass, $= \frac{1}{\rho}$ (in cm ³ g ⁻¹))
Z	atomic number

II. Neutrino Process

Since part I of this paper was written, a number of neutrino processes have been investigated. The relative importance of these processes have been summarized in Ref.

(4). Here we shall concern ourselves with the annihilation process of neutrino production:^{3,4,5}



which have been shown to be most important in the temperature regime 10^9 °K - 10^{10} °K. A detailed calculation of (1) has been performed⁴.

In (1) the electron positron pair is created in equilibrium with thermal radiation. Fig. 1 and Fig. 2 reproduce the result of Ref.(5). The rate of energy loss - $\frac{dU}{dt}$ (in ergs/cm³-sec) is approximately given as⁴

$$- \frac{dU}{dt} = 5 \times 10^{18} T_9^3 \exp\left(-\frac{11.9}{T_9}\right) \quad T_9 \ll 6 \quad (2)$$

$$- \frac{dU}{dt} = 4 \times 10^{15} T_9^9 \quad T_9 \gg 6 \quad (3)$$

where $T_9 = \frac{T}{10^9}$ °K, T is the temperature and U is the energy density. The effect of degeneracy on (2) and (3) is complicated (Fig. 2). When $T \sim 10^9$ °K, $\rho \sim 10^7$ g/c.c. degeneracy may not be very important⁵ (also see Fig. 2).

III. The Virial Theorem

The virial theorem is an integral theorem that relates the total gravitational energy of a star to its total internal energy. It does not describe the details of the structure of a star, but by its simplest application one quickly gains an insight to the nature of stellar evolution.

The virial theorem may be derived from the equations of hydrostatic equilibrium:

$$-\frac{dP}{dr} = \rho \frac{GM}{r^2} \quad (4)$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad (5)$$

where P is the pressure, r the radius vector, M the mass inside the sphere of radius r , and G is the gravitational constant. Multiplying (4) by $V = \frac{4\pi}{3} r^3$, one obtains

$$-VdP = \frac{1}{3} \rho \frac{GM}{r} 4\pi r^2 dr = \frac{1}{3} \frac{GM}{r} dM \quad (6)$$

Integrating over the star and partially integrating $\int VdP$, one obtains:

$$\int \frac{GM}{r} dM = 3 \int P dV \quad (7)$$

where $E_{gr} \equiv -\int \frac{GM}{r} dM$ is the total gravitational energy of the star. In general $\int P dV$ is not expressible in terms of $\int \epsilon \rho dV$, the total internal energy of the star. In the case of a

polytropic gas sphere $\int PdV$ is related to the internal energy in a simple way. We have:

$$P = \rho R T \quad \epsilon = \frac{3}{2} R T \quad (8)$$

$$\int P dV = \int \rho R T dV = \frac{2}{3} \int \epsilon \rho dV \quad \text{Perfect gas} \\ -E_{gr} = 2 \int \epsilon \rho dV \quad (9)$$

$$\int P dV = \frac{1}{n} \int \epsilon \rho dV \quad \text{Polytropic gas of index } n. \\ -E_{gr} = \frac{3}{n} \int \epsilon \rho dV \quad (10)$$

In general,

$$-E_{gr} = \eta \int \epsilon \rho dV, \quad \text{where } 1 \leq \eta \leq 2 \quad (11)$$

The case $\eta = 1$ makes an unstable situation since the total energy of the star (gravitational plus thermal) will not be changed for an arbitrary expansion or contraction of the star. This case is of particular importance in the treatment of stellar stability and will be considered later.

In order to calculate the evolution track of a star one must know the equation of state $P = P(\rho, T)$. When one uses that for a non-degenerate gas one over-estimates the temperature dependence of P . When one uses that for an extremely degenerate gas one underestimates the temperature dependence of P by a large margin. In Table I we list the exact value of pressure P_F for a Fermi gas as compared with that for a perfect

gas with relativistic correction P_{rel} and that for an extremely degenerate gas (no temperature dependence) P_d . In fact, for $\frac{\mu}{kT}$ ($\approx \frac{E_F}{kT}$ where E_F is the Fermi energy) as large as 6 the gas is still essentially perfect. This corresponds to a density of roughly 10^6 g/cm^3 at $T = 10^9 \text{ }^\circ\text{K}$, and 10^7 at $T = 3 \times 10^9 \text{ }^\circ\text{K}$, 10^8 at $T = 5 \times 10^9 \text{ }^\circ\text{K}$. Now let us examine whether the condition for a perfect gas holds for a star around $1.3M_\odot$.

Schwarzschild et al. have calculated a model for red giants of mass around $1.3M_\odot$.⁶ Before helium burning starts the central density is around 10^6 g/cm^3 , and the central temperature around $10^8 \text{ }^\circ\text{K}$. The gas is in a very serious state of degeneracy. The helium burning starts quickly and, because the gas is highly degenerate, temperature rise could go very far without upsetting the equilibrium status of the star. Because of this rapid temperature rise, helium burning is accelerated. This is known as the helium flash⁷. Finally the temperature becomes so high that the state of degeneracy is no longer maintained. The core expands to readjust itself by adiabatic cooling. When the core is finally settled, the central density is around 10^4 g/cm^3 and the temperature is around $1.5 \times 10^8 \text{ }^\circ\text{K}$. The core is no longer degenerate.

The law of evolution for a perfect gas sphere is

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0} \right)^3 \quad (12)$$

where ρ_0 and T_0 are the initial values for ρ and T .^k Is it possible that the gas will remain nearly perfect all the time? To answer this question in detail would require a complete knowledge of the evolution of stars beyond the red giant stage which we do not have. Let us assume the validity of (12) and proceed to compute ρ and T for the later stages of evolution. If the resulting ρ and T fall short of degeneracy we may justify ourselves for the condition of being nearly perfect. Even when the resulting ρ and T fall close to degeneracy, the "carbon flash" (which is expected to occur at around 7×10^8 °K) may decrease the central density to a nearly non-degenerate state, just as the helium flash does. Cameron⁸ argued that carbon flash may explode the star, giving rise to supernova phenomenon. We shall assume that this is not so. At present we assume that the star may survive the carbon flash stage.

In Table II we list the central density and temperature as computed from Eq. (12) and the relevant neutrino luminosity, relaxation time, specific energy content at each stage of evolution. It is seen that at $T = 10^9$ °K the neutrino luminosity L_ν

* For derivation, see, for example, Reference (1).

is already greater than $10^6 L_{\odot}$. The star is still quite stable. No stars have been observed with an optical luminosity $> 10^5 L_{\odot}$ without showing a rapidly expanding envelope characteristic of a nova or supernova eruption. Even in the case of nova explosion such high optical luminosity ($\sim 10^6 L_{\odot}$) could not be maintained for more than a few months. It is very remarkable that L_{ν} may be as high as $10^{13} L_{\odot}$ without giving rise to any appreciable instability (at least in the core region). Thus, when $T \geq 10^9$ °K the optical energy radiated is very small compared with the neutrino energy radiated.

When $T \geq 10^9$ °K very little nuclear energy is generated when Si^{28} undergoes further nuclear processes to approach the equilibrium state of elements^{2,15} (which will be briefly discussed in the next section). Anyway, the equilibrium state of elements will be reached at $T \sim 2 \times 10^9$ °K. The core of the star is essentially devoid of all energy sources except the gravitational source. The neutrino process will continue to dissipate energy. Will the star cool down? Only when the central density already approaches that for a white dwarf of approximately the same mass at $T = 0$, the star will be able to cool down without further change. Otherwise the virial theorem (11) must be observed at all times, assuming hydrostatic

equilibrium. Therefore, the star will contract. The central temperature will rise (as a result of the increase of $\int \epsilon \rho dV$). The amount of energy that is dissipated into neutrinos will be

$$\delta [-E_{gr} - \int \epsilon \rho dV] = \delta [(\eta - 1) \int \epsilon \rho dV].$$

Thus the central density and temperature of a star will rise indefinitely until something stops it. At present we shall only investigate the iron-helium phase change theory that has been suggested by Burbidge et al.² and applied to supernova problems by Fowler and Hoyle⁹.

IV. The Property of Elements in Statistical Equilibrium

The equilibrium among elements is the same as a chemical equilibrium process. At a given temperature, the relative abundance of an element is determined by its binding energy and the number of statistical states available. For details of this process the original article of Burbidge et al.² is recommended. Here we shall only quote their result.

Under condition of statistical equilibrium the number density $n(A,Z)$ of the nucleus (A,Z) is given by*

$$\begin{aligned} \log n(A,Z) = & \log w(A,Z) + 33.77 + \frac{3}{2} \log(A T_9) \\ & + \frac{5.04}{T_9} Q(A,Z) + A(\log n_n - 34.07 - \frac{3}{2} \log T_9) \\ & + Z\theta \end{aligned} \quad (13)$$

$$\sum_{A,Z} M(A,Z) n(A,Z) + M_p n_p + M_n n_n = \rho \quad (14)$$

where

$$\begin{aligned} w(A,Z) &= \sum_r (2I_r + 1) \exp[-E_r/R T] \\ Q(A,Z) &= c^2 [(A-Z)M_n + ZM_p - M(A,Z)] \\ \theta &= \log(M_p/M_n) \end{aligned}$$

where I_r is the spin, E_r the energy of the excited state measured above the ground level, $Q(A,Z)$ is the binding energy

* Unless otherwise stated, all logarithms have base 10. For base e, we shall use the notation \ln

of the ground level of the nucleus (A, Z) , M_n , M_p , $M(A, Z)$ are the masses of the free neutron, free proton, and nucleus (A, Z) respectively. $\theta \approx 0$ for $\rho \sim 10^7$ and $T \sim 3 \times 10^9$ °K. The justification for neglecting θ has been given in Reference (5).

In the temperature range $3 \times 10^9 - 10^{10}$ °K the abundance of all elements except He^4 and Fe^{56} may be neglected. Here we shall not consider the case of $T \gtrsim 10^{10}$ °K for which n_n and n_p are comparable to $n(\text{He}^4)$ and $n(\text{Fe}^{56})$. The inclusion of such terms at present would only complicate our analysis. Now we wish only to obtain an insight into the problem.

Thus, after putting into (13) and (14) all relevant constants for He^4 and Fe^{56} , we have

$$\begin{aligned} \log n(4,2) - \frac{1}{14} \log n(56,26) \\ = 32.08 + 1.39 \log T_9 - \frac{34.62}{T_9} \end{aligned} \quad (15)$$

$$\begin{aligned} n(4,2)M(4,2) + n(56,26)M(56,26) + n_n M_n = \rho \\ \frac{n_n}{n(4,2)} = 4/13 \end{aligned} \quad (16)$$

Eqs. (15) and (16) may be solved for $n(4,2)$ and $n(56,26)$ for a given set of ρ and T . Writing

$$\begin{aligned} n(56,26) &= 10^{28+t} \\ \rho &= \rho_7 \times 10^7 \\ C(T_9) &= 32.08 + 1.39 \log T_9 - \frac{34.62}{T_9} \\ F(\rho, T_9) &= C(T_9) - 27.146 - \frac{13}{14} \log(10.7 \rho_7) \\ \sigma &= 10^{\frac{1}{14} [t - \log(10.7 \rho_7)]} \end{aligned} \quad (17)$$

then we have:

$$\rho = \frac{F(\rho, T_0)}{1 - \nu^{14}} \quad (18)$$

Eq. (18) may be solved by a graphical method.

Judging from the form of Eq. (18) ν has a very strong dependence on ρ and T . Thus the transition between Fe^{56} and He^4 is very rapid. Fig. 3 reproduces the abundance of Fe^{56} as a function of temperature. The transition temperature regime is around $\Delta T = 1$. The values of ρ and T for which $\rho_{\text{Fe}} = \rho_{\text{He}}$ is plotted in Fig. 4.

The heat of transition is around 2.2 Mev per nucleon. The thermal energy is around 0.3-0.5 Mev per particle. During the transition process a drastic gravitational contraction may take place. This may cause instability inside the star. Burbidge et al. argued that this instability may trigger a supernova².

In the next section we shall study the problem of hydrostatic stability. It is possible to formulate the instability caused by iron-helium mathematically.

V. Hydrostatic Stability of a Star

This problem has been widely treated¹⁰. Here we shall follow Dyson's approach¹¹.

The equations of hydrostatic equilibrium may be written as

$$\frac{dV}{dM} = v \quad (19)$$

$$\frac{dP}{dM} = - \frac{GM}{3Vr} \quad (20)$$

where

$$V = \frac{4\pi}{3}r^3 \quad (21)$$

M = mass enclosed inside a sphere of radius r

v = specific volume of a gram of matter

The total energy of a star τ is:

$$\tau = E + E_{gr} \quad (22)$$

$$E = \int_0^m \epsilon(M) dM \quad (23)$$

$$E_{gr} = -G \int_0^m [r(M)]^{-1} M dM \quad (24)$$

where ϵ is the specific internal energy, E is the total internal energy and E_{gr} is the gravitational energy. A small perturbation of the star is described by moving each mass-shell M radially

outward by a displacement δr ; subject to the restriction

$$\delta r = 0 \quad \text{at } M = 0 \quad (25)$$

For an adiabatic motion we have exactly

$$\left(\frac{d\epsilon}{dV}\right)_{Ad} = -P \quad (26)$$

$$\left(\frac{d^2\epsilon}{dV^2}\right)_{Ad} = -\left(\frac{dP}{dV}\right)_{Ad} = \frac{P\gamma}{V} \quad (27)$$

where $\gamma = \frac{C_p}{C_v}$ and is the ratio of specific heats at constant pressure C_p and constant volume C_v . Various forms of γ will be derived in the appendix.

We now calculate the change in total energy T produced by the displacement, up to the second order in δr . We find

$$\delta T = \delta E + \delta E_{gr} \quad (28)$$

$$\delta E = \int_0^m dM \left[-P \delta V + \frac{P}{2\gamma} (\delta V)^2 \right] \quad (29)$$

$$\delta E_{gr} = G \int_0^m M dM \left[\frac{\delta r}{r^2} - \frac{(\delta r)^2}{r^3} \right] \quad (30)$$

$$\delta V = \frac{d}{dM} (\delta V) \quad (31)$$

$$\delta V = 4\pi (r^2 \delta r + r (\delta r)^2) \quad (32)$$

$$\delta T = \delta' T + \delta^2 T \quad (33)$$

$$\delta' T = \int_0^m dM \left[-P \frac{d}{dM} (4\pi r^2 \delta r) + \frac{GM}{r^2} \delta r \right] \quad (34)$$

After the first term is integrated by parts, $\delta^1 \tau$ vanishes identically by virtue of Eq. (19). This is expected since the star is in equilibrium.

To see whether the equilibrium is stable we examine the second order terms. We have

$$\delta^2 \tau = \frac{1}{2} \int dV \left[\frac{4}{3} \frac{dP}{dV} g^2 + r \left(\frac{dg}{dV} \right)^2 \right] \quad (35)$$

$$g = \delta V = 4\pi r^2 \delta r \quad (36)$$

The star is stable if $\delta^2 \tau > 0$ for every $g(V)$ and unstable if $\delta^2 \tau < 0$ for some $g(V)$. The star is certainly unstable if for a uniform contraction with $g = kV$. Thus

$$\int \left(\gamma - \frac{4}{3} \right) P dV \leq 0 \quad (37)$$

is a sufficient condition for instability. To have stability, it is necessary to have $\gamma > \frac{4}{3}$ in some regions of the star.

Writing $g = Vf$ and integrating the first part of (35), we have

$$\delta^2 \tau = \frac{1}{2} \int P dV \left[\frac{4}{3} V^2 \left(\frac{df}{dV} \right)^2 + \left(\gamma - \frac{4}{3} \right) \left(\frac{dg}{dV} \right)^2 \right] \quad (38)$$

This shows that it is necessary for instability to have in some part of the star.

The iron-helium transformation has an $\gamma \sim 1.13$.*

In the next section we shall apply the stability argument (37) to pre-supernova stars.

* See appendix

VI Speculation on the Structure of Pre-supernova Stars

Ever since the first supernova was recorded in human history in 1054 A.D.¹², learned scholars could not but wonder at the implications of such glorious events¹³. Today we are still almost as ignorant as our ancestors when such an event was entered into historical records merely as a guest star. We are just beginning to feel we can reconstruct such events on a piece of paper by reason and by correlating the properties of matter we have learned up-to-date. We are still quite far from the detailed knowledge of such events.

Nevertheless, we look into this matter with almost the same point of view as the Chinese Calendrical Computer Royal* who predicted that the appearance of this particular supernova will mean plenty of crops to the people¹⁴. He could be right if supernovae are what we think they are today. They bring us all the heavier elements, including carbon, nitrogen and oxygen which are the main chemical composition of crops. The Computer Royal was wrong only in his wording. The supernova (now the crab Nebula) may bring crops to beings, but not to the people on earth, nor would his prediction be true in any foreseeable future. The

* His name is Yang Wei-Te. Although his duty was to set up the calendars, occasionally he interpreted the "symbols of Heavens" to give advice to the emperor.

dynasty he served had long since been succeeded by others, and others by still others, and the solar system implicitly promised by him is yet to come.

We start our investigation by assuming: (1) all pre-supernova stars have internal temperature $T > 10^9 \text{ }^\circ\text{K}$, (2) energy produced in all nuclear processes is small as compared to the thermal energy, (3) general relativistic effect may be neglected, (4) hydrostatic equilibrium is maintained at all times.

The assumption that $T > 10^9 \text{ }^\circ\text{K}$ is probably all right. The structure and evolution of stars up to $2 \times 10^8 \text{ }^\circ\text{K}$ have been investigated and such stars could not possibly become supernovae. Cameron⁸ suggested that at $T \sim 8 \times 10^8 \text{ }^\circ\text{K}$, when carbon burning starts a new instability very similar to the "helium flash"^{6,7} may cause a star to become a supernova. However, his arguments do not apply to the case when the center is only slightly degenerate. Then the assumption that $T > 10^9 \text{ }^\circ\text{K}$ may be realized.

The general relativistic effect could be neglected so long as $\rho < 10^{10} \text{ g/cm}^3$. The correction due to general relativistic effect will be only a few percent even at $\rho = 10^{10} \text{ g/cm}^3$.

The nuclear energy available from the transmutation of

Si^{28} to Fe^{56} is only 0.1 mev per nucleon.¹⁵ Since the internal energy of electrons under prevailing conditions is around 0.3 mev per electron, one may neglect the contribution from $\text{Si}^{28} \rightarrow \text{Fe}^{56}$ reactions. In a more general treatment we may wish also to include the energy production in such reactions.

So far we have considered the core only. In the outer envelope of the star the possibility of nuclear processes leading to large amount of energy production (e.g. , 3α -reaction or even hydrogen burning) may not be excluded. The evolution of the core, as is seen from Table II, is sufficiently rapid. The total energy that may flow in and out of the core during the entire period of the evolution of the core is small compared with either the total energy lost to neutrinos or the increase of thermal energy of the core due to contraction. Thus the envelope will not produce a noticeable effect on the core. On the other hand, the rapid evolution of the core will produce an effect on the envelope that is not negligible. This effect needs special treatment, and we do not propose to study it now. In all, what we shall propose to study from the core will shed us light upon the evolution of the envelope on the one hand, and on the other hand such

study may tell us if $\text{Fe}^{56} \rightarrow \text{He}^4$ conversion is sufficient to cause a star to become a supernova.

Finally, the assumption of perfect hydrostatic equilibrium is the most flawless one. The characteristic time for evolution in no case falls short of 10^4 seconds, and the time for free fall (a state completely devoid of hydrostatic equilibrium) is less than 1 second.

In the following we shall derive the equations that will govern the evolution of the core. First we examine the four equations that govern the general structure of a star:

$$\frac{dP}{dr} = - \rho \frac{GM}{r^2} \quad (39)$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad (40)$$

$$\frac{dL_r}{dM} = \xi \quad (41)$$

$$\frac{dT}{dr} = - \frac{L_r}{4\pi r^2 \lambda} \quad (42)$$

where ξ is the energy generated per unit mass, L_r is the radial energy flux, λ is the thermal conductivity. P is supposed to be a known function of ρ and T . ξ is also supposed to be a known function of ρ and T .

$$P = P(\rho, T) \quad (43)$$

$$\xi = \xi(\rho, T) \quad (44)$$

Suitable boundary conditions have to be added in order

that Eqs. (39)-(42) may yield a unique solution. General discussions of such boundary conditions may be found in standard textbooks¹⁶.

Equation (39) merely states the condition for hydrostatic equilibrium and therefore should remain as it is. Eq (40) actually defines M as a function of ρ and r. Eq. (41) and Eq. (42) require special attention in our case.

In Eq. (41) ξ assumes the following form

$$\xi = \frac{d\epsilon_{gr}}{dt} + \frac{dU}{dt} \quad (45)$$

$$\frac{d\epsilon_{gr}}{dt} = - \left[\rho \frac{dv}{dt} + \frac{d\epsilon}{dt} \right]_{M=constant} \quad (46)$$

$$\frac{dU}{dt} = \overset{the}{\text{rate of energy loss due to neutrino emission}}$$

ϵ = internal energy per gram of matter.

That $M=const.$ in (46) means we follow a specific mass element.

In (45) we have neglected all nuclear processes. $\frac{dU}{dt}$ is $> 10^6$ ergs/grm-sec. The magnitude of $\frac{dL_r}{dM}$ may be estimated as follows:

No stable stars have been observed with a luminosity $> 10^5 L_{\odot}$. The maximum rate of energy generation per gram of matter may be safely taken to be less than

$\frac{10^5 L_{\odot}}{M_{\odot}} \sim 10^5$ ergs/grm-sec. Thus $\frac{dU}{dt} \gg \frac{dL_r}{dM}$ in general. This means $\frac{d\epsilon_{gr}}{dt} \gg \frac{dL_r}{dM}$. A natural simplification for Eq. (41)

is therefore to put $\frac{dL_r}{dM} = 0$.

Thus

$$\left[- \frac{P dv}{dt} - \frac{d\epsilon}{dt} - \frac{dU}{dt} \right]_{M=\text{const.}} = 0 \quad (47)$$

Equation (47) may not be integrated with respect to t , the time in general, since $\frac{dU}{dt}$ is an explicit function of v and T . Eq. (47) replaces Eq. (41) .

Equation (42) is then incomplete in the sense that in our approximation $\frac{dL_r}{dM} = 0$, L_r may be taken as zero throughout the star. Eq. (42) relates the temperature gradient to the energy flux. It thus represents an equation of cause and effect. At $r + dr$ the value of T is determined by its value at r and the energy flux L_r . The assumption $\frac{dL_r}{dM} = 0$ essentially isolates each point from the other as far as Eq. (42) is concerned. Thus Eq. (42) should not be present. Eq. (39), (40), and (47) plus suitable boundary conditions should determine the structure of the star completely.

Is this logically possible? What we have done is to assume that during the entire lifetime of the core (after $T = 10^9 \text{K}$) -- around 3000 years -- the total radiation energy that flows in and out of a mass element dM is negligible as compared with its total thermal energy. The energy that flows in and out of a gram of matter is less than

$(10^4 \text{ ergs/gram-sec} \times 3000 \text{ years}) \approx 10^{16} \text{ ergs/g.}$ The total energy content is in between $10^{17} - 10^{18} \text{ ergs/g.}$ Our assumption seems to be justified. Thus, at each mass point the thermal energy content is determined by its past history only. This is expressed in Eq. (47) with the constraint set up by Eqs. (39) and (40).

Equation (47) may be further simplified. Since

$$\left[-P \frac{d\mathcal{V}}{dt} - \frac{d\mathcal{E}}{dt} \right]_{M=\text{const.}} = -T \frac{ds}{dt}_{M=\text{const.}} \quad (48)$$

where s is the entropy, we have

$$T \frac{ds}{dt} \Big|_M = u(\rho, T) \quad (49)$$

where $u(\rho, T) \equiv -\frac{dU}{dT}$.

The boundary conditions are:

$$\text{At } t = 0, \quad \rho = \rho_0(r), \quad T = T_0(r) \quad (50)$$

$$M = m, \quad \rho = \rho_m, \quad T = T_m \quad (51)$$

where m is the mass of the core which is not necessarily constant but must be a known function of t . ρ_m and T_m are supposedly known functions of m and t . The boundary conditions (50), (51) replace the outer envelope of the star. In general m is not a constant and may not be a known function either. Yet if we assume during the few thousand

years of the final stage of stellar evolution the addition of mass to the core is negligible, then m , ρ_m , T_m may all be taken to be constant. The boundary may be defined such that $L_r=L_v$.

After $\rho(r)$ and $T(r)$ are solved we may eliminate r and plot ρ against T . The core will be represented by a line segment in the ρ - T plane. As time elapses this line segment will move and will change its shape in general. This motion marks the evolution of the core. Eq. (37),

$$\int (\gamma - \frac{4}{3}) PdV \leq 0 \quad (37)$$

may be used to check the hydrostability of the star. The shape of the line segment will change once it enters the iron helium transition regime.

To summarize, the structure of pre-supernova stars as described in the earlier part of this section is contained in the following set of differential equations

$$\frac{dP}{dr} = - \rho \frac{GM}{r^2} \quad (52)$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad (53)$$

$$\left(\rho \frac{\partial v}{\partial t} + \frac{\partial E}{\partial t} \right)_M = \frac{dU}{dt} - \frac{dE_n}{dt} \quad (54)$$

the boundary conditions are

$$t = 0, \quad \rho = \rho_0(r) \quad T = T_0(r) \quad (55)$$

$$M = m, \quad \rho = \rho_m \quad T = T_m \quad (56)$$

when $\frac{dE_n}{dt}$ is the rate of nuclear energy release.

Some general results may be obtained without solving Eqs. (39), (40) and (49). We consider two cases:

(i) The temperature is the same throughout the core, i.e., an isothermal core. This is the most unstable situation. The structural curve is a vertical line in the ρ -T plane. The 50 percent iron curve is also very steep (almost a vertical line) as can be seen from Eq. (18). (Fig. 4). Thus the core will have a $\gamma \sim 1.13$. The integral in Eq. (37) is certainly negative.

(ii) The density is the same throughout the core. In other words, the core is entirely supported by the temperature gradient. This is the most desirable situation for stability although a core of this type does not exist. Such a core is convectively unstable. The structural curve is a horizontal line in the p-T plane. The portion of horizontal curve that will lie in the $\gamma \sim 1.13$ zone is of the order ΔT where ΔT is the iron-helium transition temperature regime. The model we shall discuss in the next section corresponds roughly to this situation.

VII An Analytically Soluble Model

In this section we shall construct a model which is analytically soluble. Eq. (49) may be integrated when

$$u(p,T) = T f(s) \quad (57)$$

when $f(s)$ is an arbitrary function of the entropy s . Then,

$$\int \frac{ds}{f(s)} = t + C(M) \quad (58)$$

where $C(M)$ is a function of M only. (In the following, we shall use M as the independent variable. The reason for this choice is obvious because of the form of Eq. (48).)

We now consider the case of a perfect gas sphere, and let $f(s) = u_0$, where u_0 is a constant, then we have

$$P = \rho RT \quad R = \text{gas constant} \quad (59)$$

$$\epsilon = \frac{3}{2} RT \quad (60)$$

$$f(s) = u_0 \quad (61)$$

In order to reduce $C(M)$ to unity, we further let

$$\rho = T^{3/2} \quad \text{at } T = 0 \quad (62)$$

then for arbitrary t $\rho(M) = T(M)^{3/2} \exp\left[\frac{u_0}{R}(t - t_0)\right]$ (63)

is the solution of Eq. (47). Thus, from Eqs. (54) and (63)

$$P \propto \rho^{5/3}$$

The structure is described by that of a polytropic gas sphere of index 1.5.

The solutions are:

$$P = P_c(t) \left[\theta_{3/2}\left(\frac{r}{r_0}\right) \right]^{5/2} \quad (65)$$

$$\rho = \rho_c(t) \left[\theta_{3/2}\left(\frac{r}{r_0}\right) \right]^{3/2} \quad (66)$$

$$T = T_c(t) \theta_{3/2}\left(\frac{r}{r_0}\right) \quad (67)$$

where $P_c(t)$, $\rho_c(t)$ and $T_c(t)$ are functions of t only. P_c , ρ_c , T_c are values of P , T , at $r=0$ since $\theta_{3/2}(0)=1$. $\theta_{3/2}(X)$ is the Lane-Emden function of polytropic index 3/2. And

$$r_0^{-1} = \left\{ \frac{5/2}{4\pi G} \frac{P_c(t)}{[\rho_c(t)]^2} \right\}^{1/2} \quad (68)$$

Also $\theta_{3/2}(0) = 1$, $\theta'_{3/2}(0) = 0$.

For $r < r_0$, $\theta_{3/2}$ may be expanded as

$$\theta_{3/2}\left(\frac{r}{r_0}\right) = 1 - \frac{1}{6}\left(\frac{r}{r_0}\right)^2 + \frac{3/2}{120}\left(\frac{r}{r_0}\right)^4 - \dots \quad (69)$$

$\theta_{3/2} = 0$ at $r=r_1 = 3.65r_0$. This may be taken as the natural

boundary of the star.

It remains to determine the forms of $P_c(t)$, $\rho_c(t)$ and $T_c(t)$. To solve them, the boundary conditions at $M=m$ must be used. In this particular model we may replace the boundary conditions by the law of evolution of a perfect gas sphere:

$$\frac{\rho_c(t)}{\rho_c(0)} = \left[\frac{T_c(t)}{T_c(0)} \right]^3 \quad (70)$$

Thus we obtain:

$$P_c(t) = P_0 \exp \left[\frac{8}{3} \frac{u_0}{R} (t-t_0) \right] \quad (71)$$

$$\rho_c(t) = \rho_0 \exp \left[\frac{2u_0}{R} (t-t_0) \right] \quad (72)$$

$$T_c(t) = T_0 \exp \left[\frac{2}{3} \frac{u_0}{R} (t-t_0) \right] \quad (73)$$

The evolution of this model star is now completely determined. The structure as plotted on the $\log \rho - \log T$ plane is a straight line of slope $3/2$. The top of this line moves upwards along the direction of slope 3. (Figure 5)

When the curve reaches the iron-helium transition regime the structure of the star will change. In fact, if it does not change, the star will be stable against iron-helium phase change. Let the transition regime extend over a temperature range $\Delta T_0 = 1$. Take $T_0 = 8$, then when the phase change is just about to finish at the center, we have

(the iron helium phase change occurs inside a sphere of radius $r = \bar{\xi} r_0$)

$$\theta_{3/2}(\bar{\xi}) = \frac{\Delta T_9}{T_9} = 1/8 \quad (74)$$

Then

$$\int (\delta - \frac{4}{3}) P dV = 4\pi P_c(x) \left[-(\frac{4}{3} - 1.13) \int_0^{\bar{\xi}} (\theta_{3/2})^{5/2} \xi^2 d\xi + \frac{1}{3} \int_{\bar{\xi}}^{\xi_1} (\theta_{3/2})^{5/2} \xi^2 d\xi \right] > 0 \quad (75)$$

where $\xi = \frac{r}{r_0}$, $\xi_1 = \frac{r_1}{r_0}$, $r_1 = 3.65 r_0$. $r = r_1$ is the natural boundary of the star defined as $\theta_{3/2}(r_1) = 0$.

This star is stable against the phase change. The fault, dear Brutus, is perhaps not in our stars, but in ourselves. For we have assumed that the structure does not change in the iron-helium transition regime.

The equations of state $P(\rho, T)$ and $\epsilon(\rho, T)$ depend on the composition. The pressure P is increased since the gas is mainly composed of Fe^{56} which decomposes into $13 He^4 + 4n$. The number of electrons is not changed substantially. Thus the number of particles is increased by around a factor of 2 if we maintain the same ρ and T . ϵ also increases as the equi-partition energy per particle remains constant while the number of particles is increased, and energy is required to dissociate Fe^{56} . We assume P is not substantially changed and ϵ has a linear dependence on the temperature. These

assumptions are not particularly valid, but would do for our model. The linear dependence of f on T is used in order that an analytical solution is possible. Thus we assume the form of $P(f, T)$ is still unchanged, but

$$\epsilon(f, T) = C T + D \quad (76)$$

where C and D are constants to be determined such that

$\epsilon(P, T)$ is a continuous function at the boundary of transition zone. C and D may be determined if we know the width of the transition zone and the value of γ . We assume $\gamma = 1.13$ and $\Delta T_g = 1$. Then,

$$\gamma = \frac{R + C}{C} = 1.13 \quad (77)$$

$$C = 7.67 R \quad (78)$$

$$D = 6.17 RT \quad (79)$$

Thus

$$\begin{aligned} \epsilon &= \frac{3}{2} RT + 6.17 (T - T_0) R \\ &= 7.67 RT - 6.17 RT_0 \end{aligned} \quad (80)$$

inside the transition temperature regime, and

$$\epsilon = \frac{3}{2} RT \quad (81)$$

outside the transitions temperature regime. Then Eq. (47)

admits the following solution:

$$\rho = T^{7.67} T_b^{6.17} \exp \left[\frac{u_0}{R} (T - T_0) \right] \quad (82)$$

since ρ is continuous at the boundary. Combining Eq. (82) with P, we have

$$P \propto \rho \left(1 + \frac{1}{7.67} \right)$$

inside the transition regime. Thus, when the center is inside the transition regime, the structure is that of a polytropic gas sphere of polytropic index 7.67. Outside the transition zone the simple Lane-Emden function of index 3/2 with no singularity at the center is no longer valid. For the details of such solutions, Emden's original textbook¹⁷ and Chanchaseklier's textbook¹⁸ are recommended.

The actual iron-helium transition curve in the $\log \rho$ - $\log T$ plane has a slope ~ 15 . The width is around $\Delta T_9 = 1$. The structural curve for a gas of $n=7.67$ has a slope of 7.67 in the $\log \rho$ - $\log T$ plane. A simple geometrical construction of the structural curve inside the transition zone (assuming the center just completes such transition) gives us the part of the star that is inside the transition zone:

$$\begin{aligned} \Delta \log \rho &\approx 1 \\ \Delta \log T &\approx \frac{1}{8} \end{aligned} \quad (83)$$

A large portion of the core (from the center where $\rho = \rho_c$ to around $\rho \approx \frac{1}{10} \rho_c$) lies in the $\gamma = 1.13$ region. A

numerical integration of Eq. (37) with the following form of structure:

Inside the transition zone

$$\begin{aligned} P &= P_c(\xi) (\theta_{1.67})^{8.67} \\ \rho &= \rho_c(\xi) (\theta_{1.67})^{7.67} \\ T &= T_c(\xi) (\theta_{1.67}) \end{aligned} \quad (84)$$

Outside the transition zone

$$\text{The same (but with a cut-off at } \xi \sim 1000) \quad (85)$$

gives a negative result although with a somewhat narrow margin. In Eqs (84) and (85) θ_n is the Lane-Emden function of polytropic index n .

We note from Fig. 5 that our model star evolves into a supernova via homologous transformations. This result is not general. We have assumed a form of neutrino energy loss function that makes the entropy increase linearly with temperature. In general this is not to be expected.

This model illustrates the method we propose to study the structure of pre-supernova stars. Problems involving gravitational contraction must be solved by numerical methods in general. In this model because of the simple form of the neutrino emission function (Eq. (57) and (61)), the evolution is homologous. In this case an analytical solution is apparent.

To study the general case a more detailed knowledge about matter under such extreme conditions as described earlier in this paper is necessary. Among them are: the equation of state, the adiabatic exponent γ , the equilibrium state of elements, the equilibrium among neutrons and protons and electrons under high density conditions.

VIII. CONCLUSIONS

In this paper we outlined the general properties of matter at extreme temperatures. From these properties, we arrive at the following conclusions:

- a) When $T > 10^9$ °K optical radiation inside stars in general may be neglected against neutrino radiation. Most of the neutrinos come from the annihilation process of neutrino production.
- b) From the virial theorem (Sect. III), stars can evolve only by radiating energy. In the final stage of stellar evolution ($T > 10^9$ °K), the rate of evolution is only limited by neutrino emission process. Therefore the characteristic time for evolution is the same as the relaxation time for cooling by emitting neutrinos.
- c) The stellar structural equation may be simplified by neglecting optical radiation altogether. The star becomes a transparent body: the rate of energy dissipation is the same as the neutrino production rate. A simple model is constructed to illustrate the evolution of such stars.

We have considered only the case in which the hydrostatic instability is due to the iron-helium phase change at $T \sim 8 \times 10^9$ °K. No attempt is made here to correlate this supernova mechanism to any of the two types of supernova observed.

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Appendix*

By definition, γ is

$$\gamma = - \frac{V}{P} \left. \frac{dP}{dV} \right|_{Ad} \quad (A-1)$$

where the subscript "ad" means the process is adiabatic.

Let the two thermodynamic variables be x , y , and write:

Pressure: $P = P(x, y)$

Specific Volume: $v = v(x, y)$

Specific energy: $\epsilon = \epsilon(x, y)$

Then $dP = P_x dx + P_y dy \quad (A-2)$

$$dv = v_x dx + v_y dy \quad (A-3)$$

the adiabatic condition $Pdv + d\epsilon = 0$ then gives

$$\epsilon_x dx + \epsilon_y dy = -P(v_x dx + v_y dy) \quad (A-4)$$

$$\left. \frac{dy}{dx} \right|_{Ad} = - \frac{\epsilon_x + P v_x}{\epsilon_y + P v_y} \quad (A-5)$$

Combining Eqs. (A-2), (A-3), (A-5) we have

$$\gamma = - \frac{v}{P} \frac{(P_x \epsilon_y - \epsilon_x P_y) + P(P_x v_y - P_y v_x)}{v_x \epsilon_y - v_y \epsilon_x} \quad (A-6)$$

*Part of the material has been taken from Reference (11).

In particular, let $x = P$, $y = v$, we have

$$\gamma = \frac{Pv + v \frac{\partial \epsilon}{\partial v}}{P \frac{\partial \epsilon}{\partial P}} \quad (\text{A-7})$$

Assuming ϵ and Pv are functions of temperature alone. Then

Eq. (A-7) reduces to

$$\gamma = 1 + \frac{\Delta(Pv)}{\Delta \epsilon} \quad (\text{A-8})$$

Using the numbers quoted in Reference (5) we find

$$\begin{aligned} \Delta(Pv) &= 2.53 \times 10^{17} \text{ erg/g} \\ \Delta \epsilon &= 19.8 \times 10^{17} \text{ erg/g} \\ \gamma &= 1.13 \end{aligned} \quad (\text{A-9})$$

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Table Captions

Table I. Comparison among P_P , P_F , and P_D at various densities and temperatures.

P_{rel} = Pressure of a perfect gas without radiation pressure P_r .

P_F = Pressure of a Fermi gas including contributions from pairs.

P_D = Pressure of a completely degenerate Fermi gas.

$$\rho_0 = 6 \times 10^6 \text{ g/cm}^3$$

The discrepancy between P_F and P_P at lower densities is due to the presence of pairs. When the density is high the discrepancy is less. In this table we demonstrate that at $\frac{E_F}{kT} \approx 6$ the assumption of a perfect gas is not a very bad one.

Table II. Physical Quantities of an evolving star, $T_9 = T/10^9$ °K. ϵ

is the specific energy per gram of matter.

$$\tau_{\text{cooling}} = \epsilon / -\frac{d\epsilon}{dt}$$

ρ and T are calculated from Eq. (12). $L_0 = 3.8 \times 10^{33}$ ergs/sec.

Table I

T (°K)	$\frac{\mu}{kT} (\approx \frac{E_F}{kT})$	$\log(\beta/\beta_0)$	$\log P_{rel}$	$\log P_F$	$\log P_D$	$\log P_r$
10^9	3.6	-2	-2.28	-2.29		-2.72
	6.12	-1	-1.28	-1.25	-1.6	
	10.2	0	-0.28	-0.024	-0.08	
	19.4	1	+0.72	1.34	1.30	
3×10^9	0.4	-1	-0.81	-0.38		-0.82
	2.3	0	0.19	0.24	-0.08	
	6	1	1.19	1.41	1.30	
	13.28	2	2.19	2.71	2.67	
	28.8	3	3.19	4.04	4.03	
5×10^9	0.61	0	0.42	0.71	-0.08	0.24
	3.02	1	1.42	1.51	1.30	
	7.8	2	2.42	2.74	2.67	
	17.1	3	3.42	4.04	4.03	

Table II

T (BK)	1	2	3	5
ρ (g/cm ³)	6×10^6	5×10^7	1.7×10^8	7×10^8
$-\frac{d\epsilon}{dt}$ (ergs/g-sec)	1.67×10^6	2×10^9	2×10^{10}	1.5×10^{13}
ϵ (ergs/g-sec)	10^{17}	2×10^{17}	3×10^{17}	5×10^{17}
L_ν/L_\odot	10^6	10^9	10^{10}	10^{13}
τ (sec) cooling	6×10^{10}	10^8	1.5×10^7	3×10^4
τ (sec) collapse	~ 1	~ 1	~ 1	~ 1

Figure Captions

- Fig. 1. The rate of energy loss due to the annihilation process (1), $\log \left(-\frac{dU}{dt} \right)$ is plotted against T for different values of $\log N_0$, where N_0 is the characteristic number density for electrons (Ref. (4)). $\log N_0 = 0$ corresponds to a density of $3 \times 10^6 \mu_e \text{ g/cm}^3$ where μ_e is the molecular weight for electrons. $\frac{dU}{dt}$ is measured in units of $\text{ergs/cm}^3\text{-sec}$ and T in units of $10^9 \text{ }^\circ\text{K}$. Numbers attached to curves are values of $\log N_0$.
- Fig. 2. The rate of energy loss of (1) as a function of density. $\log \left(-\frac{dU}{dt} \right)$ is plotted against $\log N_0$ for different values of T_9 . T_9 is measured in $10^9 \text{ }^\circ\text{K}$ units.
- Fig 3. $F_e^{56} \rightarrow H_e^4$ phase change. The density of F_e^{56} is plotted as a function of temperature. The total matter density is 10^5 g/c.c.
- Fig. 4. The density for which $\rho_{Fe} = \rho_{He}$ is plotted against T_9 . To the left of the curve the main composition of matter is mainly iron and to the right helium.

Fig. 5. The evolution of the model star (Sect. VIII). The structures of this star at $t = t_0, t_1, \dots, t_n$ are as shown. The center moves along the dotted straight line, whose equation is $\rho/\rho_0 = (\tau/\tau_0)^3$.

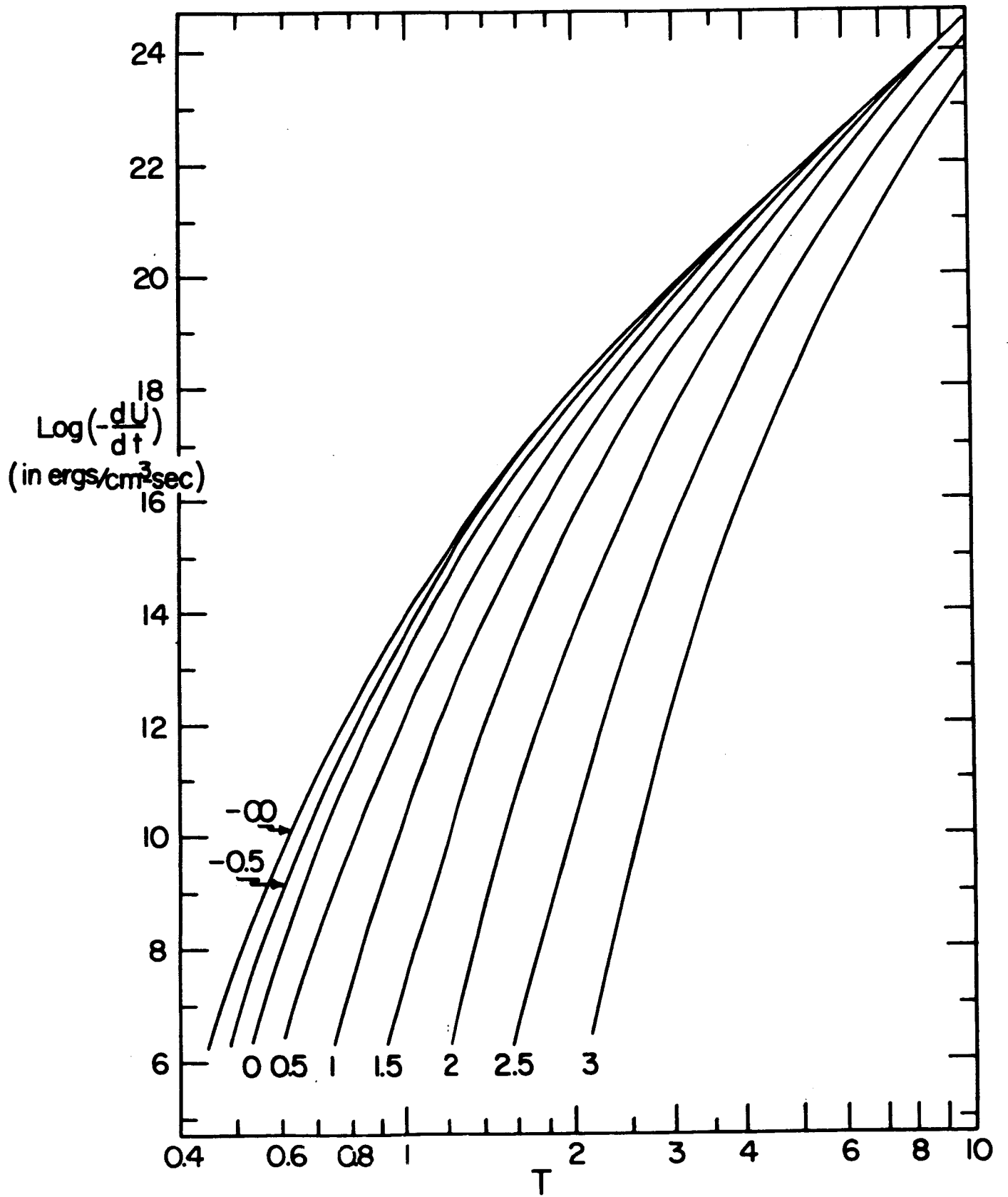


Fig. 1

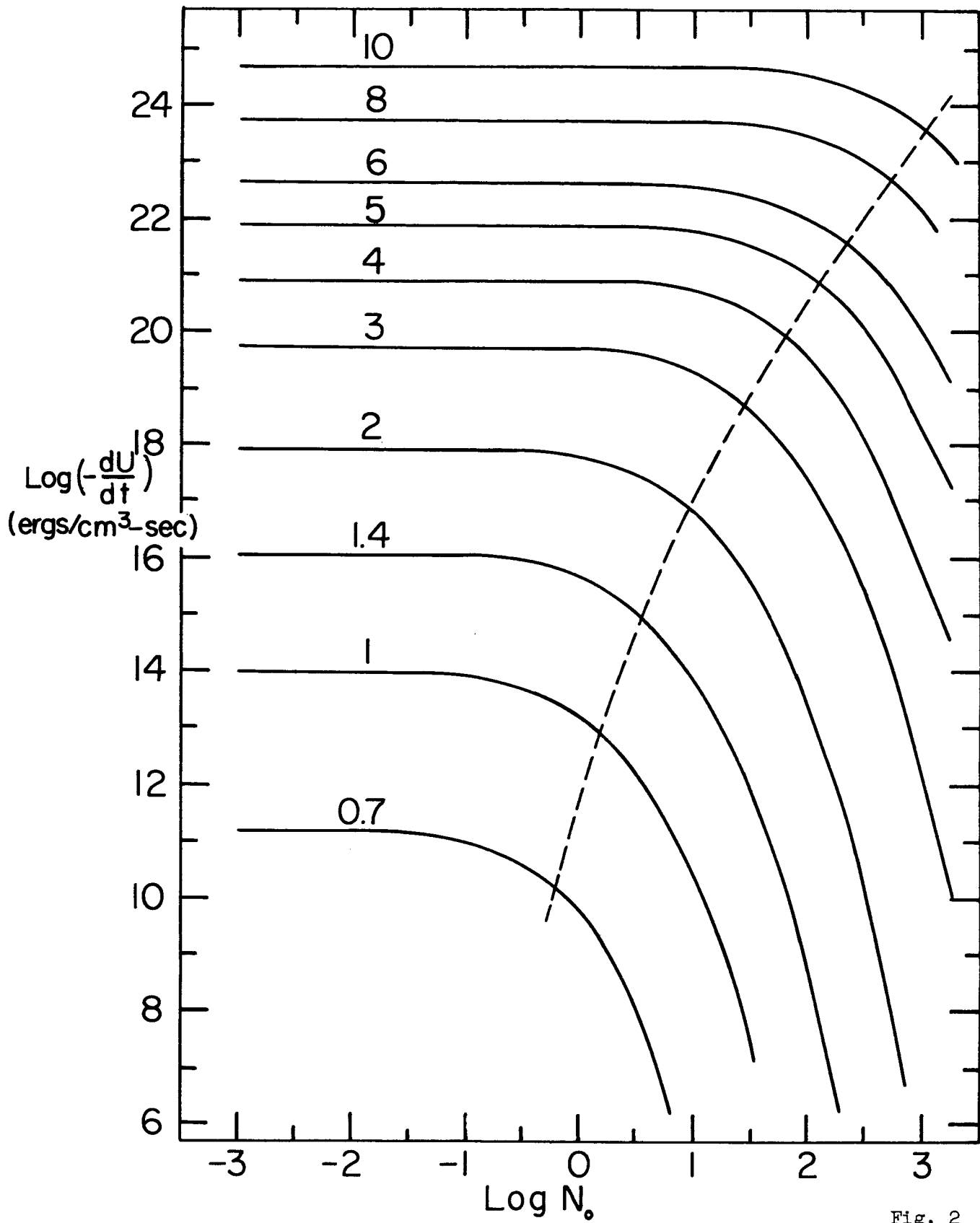


Fig. 2

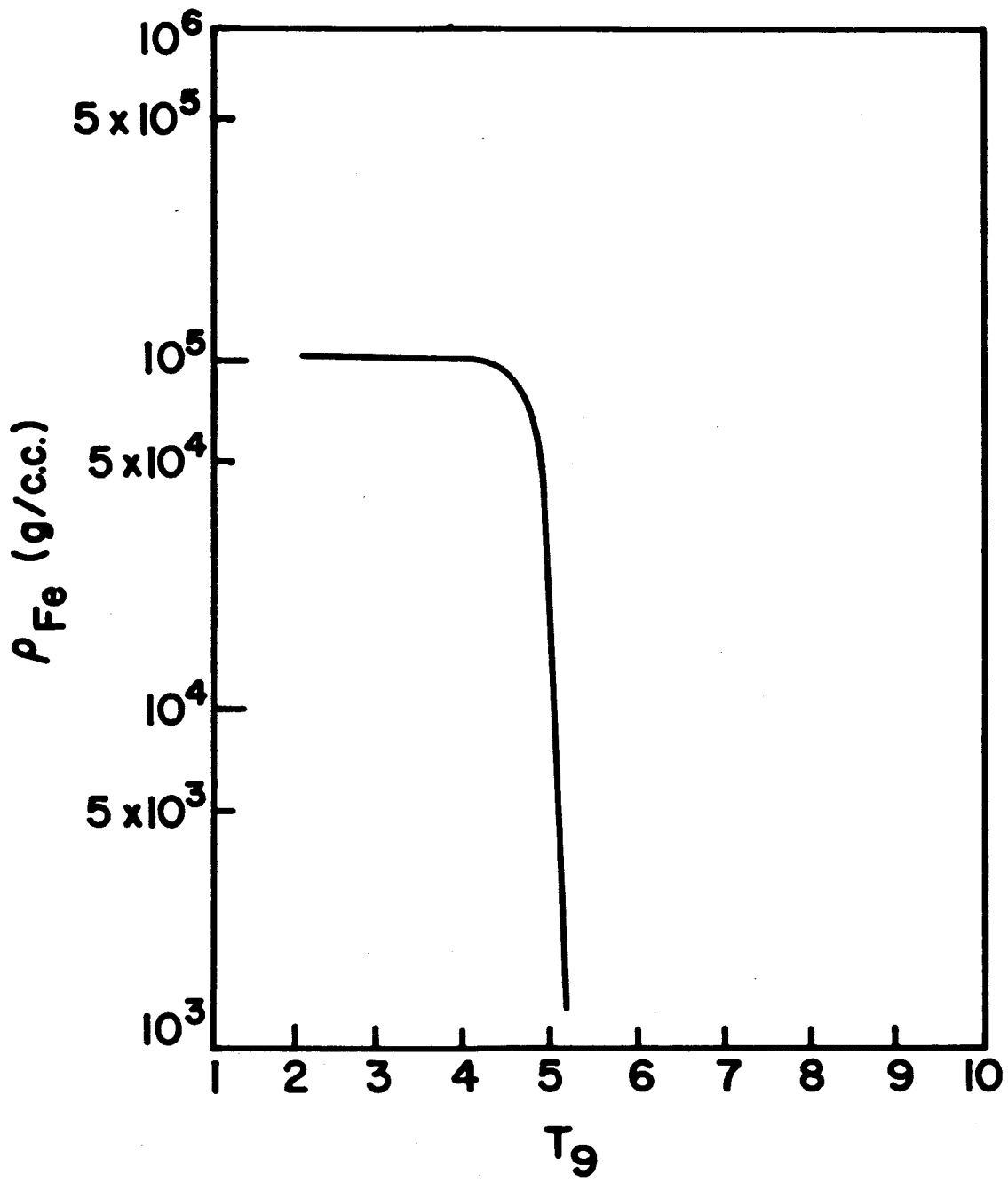


Fig. 3

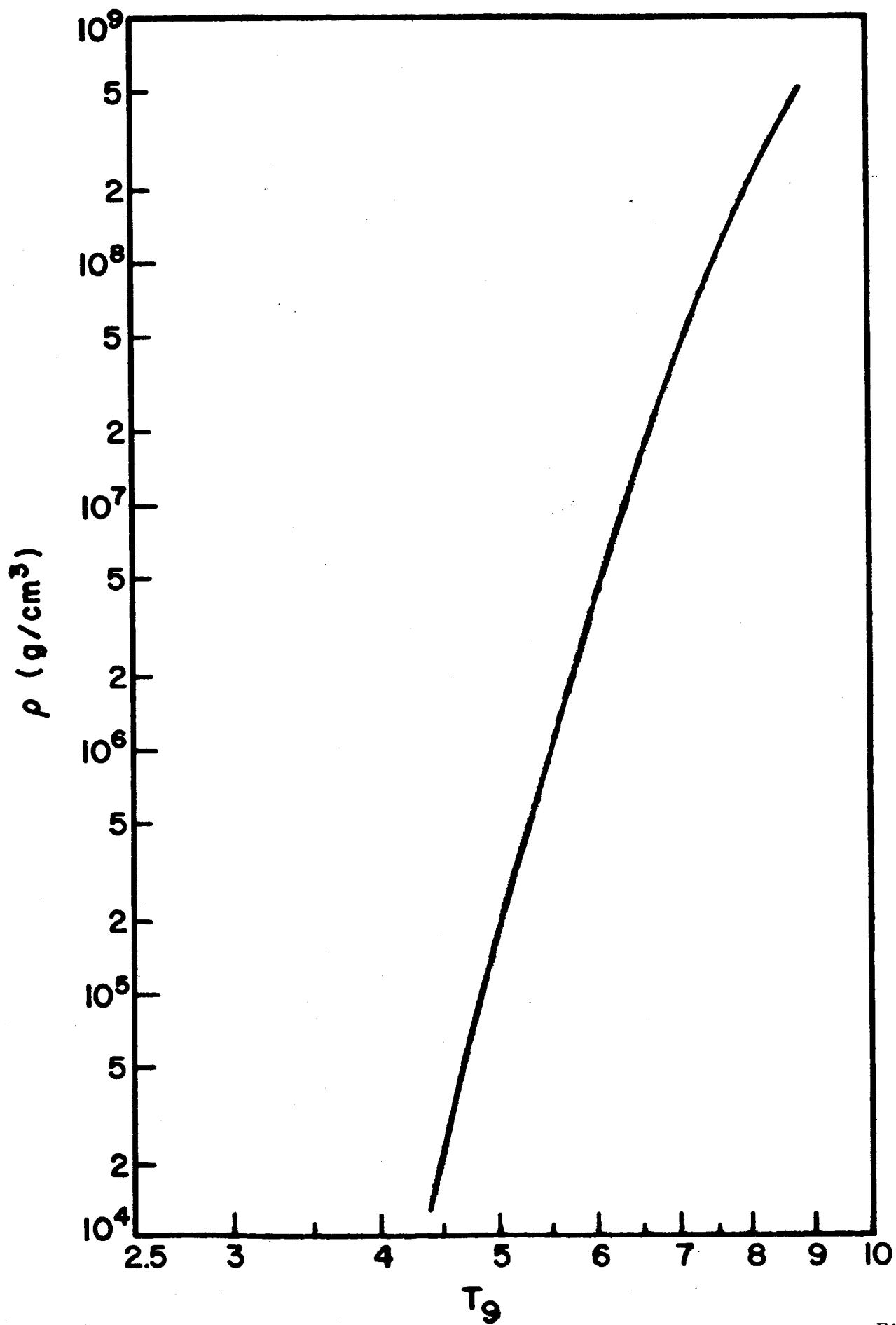


Fig. 4

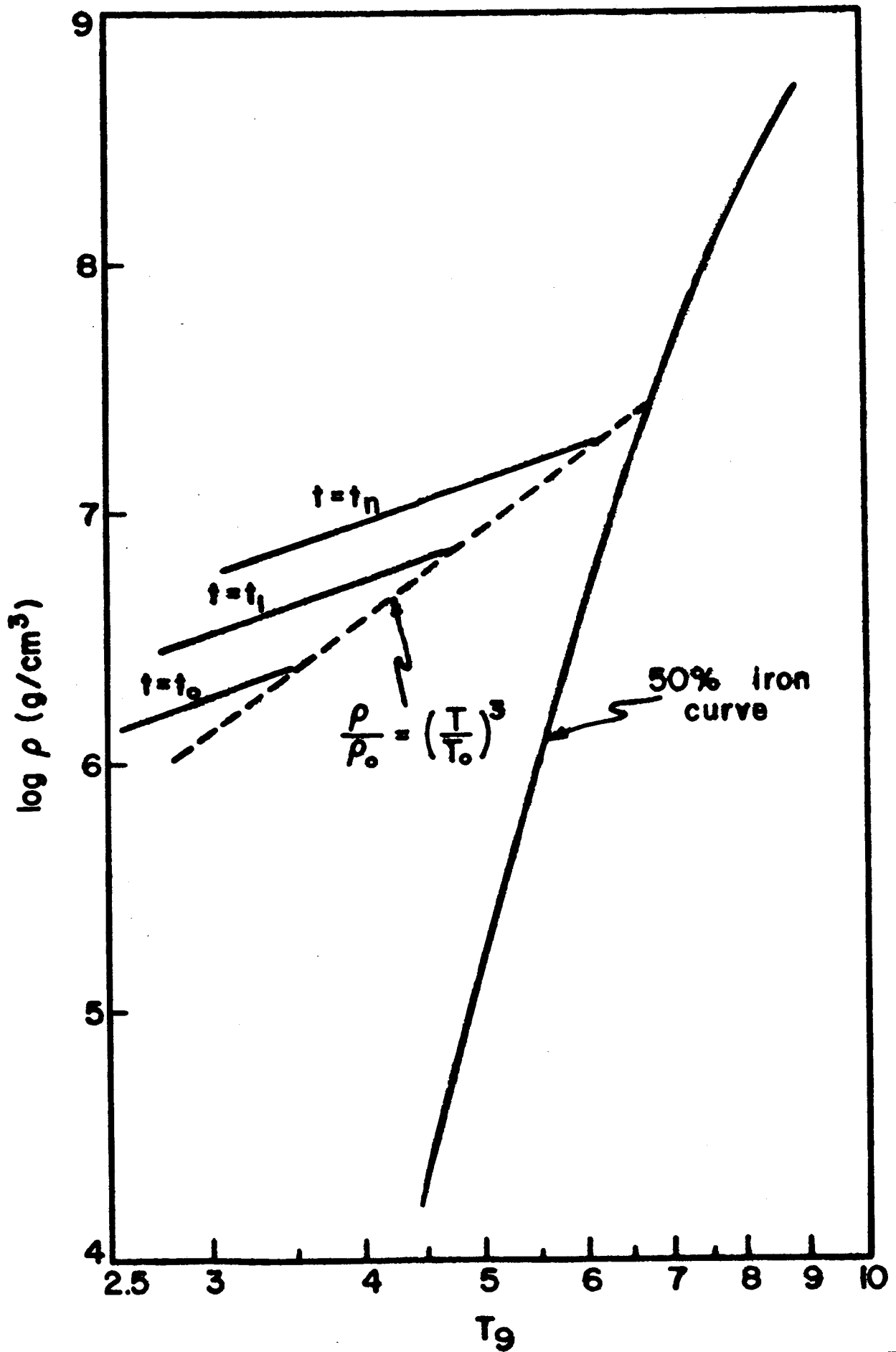


Fig. 5