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TECHNICAL NOTE

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ONE-DIMENSIONAL HEAT CONDUCTION THROUGH THE SKIN OF A VEHICLE UPON ENTERING A PLANETARY ATMOSPHERE AT CONSTANT VELOCITY AND ENTRY ANGLE

By William R. Wells and Charles H. McLellan

Langley Research Center
Langley Station, Hampton, Va.

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SUMMARY

Closed-form solutions of the one-dimensional heat-conduction equations for the flow of heat into a plate with a laminar boundary layer have been obtained for a configuration entering a planetary atmosphere with constant velocity and negative entry angle. The atmospheric density was assumed to obey an exponential law and the temperature was assumed constant initially. The solution is in the form of a Fourier series expansion which, for most practical applications, can be approximated by retaining only one term of the expression. The solution applies to the initial part of the entry before the maximum heating conditions are encountered.

INTRODUCTION

One of the serious problems experienced during entry of a vehicle into a planetary atmosphere is high convective aerodynamic heat-transfer rates. In some instances when the heat is absorbed in the skin, large thermal gradients can develop through the skin. Normally the heat conduction through a material with a time-dependent heat rate, such as that encountered during entry of a vehicle into the atmosphere, is difficult to analyze and can best be handled by step-by-step numerical processes which are generally time consuming or require the use of automatic computers. (See refs. 1 and 2.) A few simple conduction analyses have been performed for special types of entry in which the boundary conditions make it possible to obtain closed analytical expressions, such as those given in reference 3. The temperature time history of the skin of a vehicle entering an exponential atmosphere has been analyzed in reference 4; in this analysis a constant entry angle and laminar flow are assumed. The obtained solution for the skin of arbitrary thickness, however, contains terms which generally have to be treated by numerical integrations. The additional assumption of a constant velocity during the initial entry phase has been assumed in the present analysis. From these assumptions a simple but useful closed-form algebraic expression has been obtained for the temperature time history of the initial phase of the entry which generally does not include the phase in which the maximum heat rate occurs (about eight-tenths of the entry velocity according to ref. 5). This analysis will however be applicable to a significant part of many entry trajectories.

SYMBOLS

$B = \dfrac{-k_1 C}{K_m\sqrt{R}}\, e^{-\beta h n}\left(\dfrac{V}{\sqrt{gr}}\right)^m$ °F/ft

C dimensional constant, 17,600 Btu/ft$^{3/2}$-sec

$C_m = \dfrac{1}{\sqrt{K_m \rho_m c_m}}$ $\dfrac{\text{ft}^2\text{-sec}^{1/2}\text{-°F}}{\text{Btu}}$

c_m specific heat of material, Btu/lb-°F

g local value of gravitational acceleration, ft/sec^2

h height of entry above surface of planet, ft

i imaginary unit, $\sqrt{-1}$

j integer

K_m thermal conductivity, Btu/sec-ft-°F

k_1 ratio of local heat flux to stagnation-point heat flux

m,n dimensionless constants, 3.15 and 0.50, respectively

$N_m = \dfrac{\cosh\sqrt{\dfrac{\omega}{\alpha_m}}(\tau - x)}{\sinh\sqrt{\dfrac{\omega}{\alpha_m}}\,\tau}$

q heat flux, Btu/ft^2-sec

r distance from planet center, ft

R	radius of curvature at a stagnation point, ft
s	variable in Laplace transform
T	temperature, $^{\circ}F$
L/D	lift-drag ratio
$\overline{T}$	transformed temperature function
$\bar{u}$	ratio of velocity component normal to radius vector to circular orbital velocity
V	resultant velocity, ft/sec
W/C_DA	ballistic parameter, lb/ft^2

$$W = \frac{k_1 C}{\sqrt{R}} e^{-\beta hn}\left(\frac{V}{\sqrt{gr}}\right)^m \frac{Btu}{ft\text{-}sec^{3/2}}$$

x	distance normal to surface, ft
y	altitude, ft

$$Y = \frac{e^{-(\beta nV \sin \gamma)t}}{\sqrt{-\beta nV \sin \gamma}} \sec^{1/2}$$

z	complex variable
α_m	thermal diffusivity, ft^2/sec
β	atmospheric density decay parameter, $\frac{1}{23500}$ ft^{-1}
t	time, sec
ρ	atmospheric density, $slug/ft^3$
ρ_m	density of material, lb/ft^3
τ	thickness of plate, ft
γ	flight-path angle relative to local horizontal direction, positive for climbing flight and negative for descent

$\omega = -\beta nV \sin \gamma \ \sec^{-1}$

Subscripts:

o reference value

∞ free stream (ambient atmosphere)

i initial condition

s stagnation point

ANALYSIS

Assumptions and Definitions

For the present analysis, the following assumptions are made:

(1) The atmosphere varies exponentially. (See ref. 5.)

(2) The flow is laminar and the heat rate of any station on the body is proportional to the value at a stagnation point. (See refs. 5 and 6.)

(3) The velocity is sufficiently high so that the heating rate can be considered independent of the surface temperature.

(4) The skin (being analyzed) has a zero heat rate on the back (unexposed) side.

(5) Specific heats for the material are constant.

(6) The vehicle descends through the atmosphere at a constant velocity and flight-path (or entry) angle.

(7) The flow is one dimensional with radiation neglected.

Assumptions (1) to (3) allowed Chapman (ref. 5) to write the following equations for ρ_∞ and q:

$$\rho_\infty = \rho_o e^{-\beta y} \tag{1}$$

$$q = \frac{k_1 C}{\sqrt{R}}\left(\frac{\rho_\infty}{\rho_o}\right)^n\left(\frac{\bar{u}}{\cos \gamma}\right)^m \tag{2}$$

with

$$\bar{u} = \frac{V \cos \gamma}{\sqrt{gr}}$$

Assumption (6) allows y to be written as

$$y = Vt \sin \gamma + h \tag{3}$$

The constants k_1, C, m, n, and β are given in references 5 and 7. Substitution of equations (1) and (3) into equation (2) gives

$$q = \frac{k_1 C}{\sqrt{R}} e^{-\beta hn} \left(\frac{V}{\sqrt{gr}}\right)^m e^{-\beta nVt \sin \gamma} \tag{4}$$

For convenience the following definitions are made:

$$B = \frac{-k_1 C}{K_m \sqrt{R}} e^{-\beta hn} \left(\frac{V}{\sqrt{gr}}\right)^m \tag{5}$$

$$\omega = -\beta nV \sin \gamma \tag{6}$$

Equation (4) therefore can be expressed as

$$q = -BK_m e^{\omega t} \tag{7}$$

The applicability of these assumptions is shown in figures 1 to 3 for several entry conditions. Numerical integration of the equations of motion has been used for the calculation of the time history of velocity, entry angle, and altitude. These values in turn have been used with equation (2) and the values of the 1959 ARDC atmosphere (ref. 8) in determining the heating rate q_s shown as solid-line curves in figures 1 to 3. The main test for the validity of the assumptions is whether the velocity and entry angles are constant and how the deviations affect the heating rate. Values of q_s obtained from equation (7) (dash-line curves) are therefore included in the figures. For initial values of velocity of 20,000 ft/sec, entry angle of -20° at 400,000 feet, and W/C_DA of 120 lb/ft^2, it can be seen from figure 1 that the assumptions are good for the first 40 seconds which include altitudes down to 120,000 feet. Increasing the value of W/C_DA to 1,000 lb/ft^2 in figure 2 increases the time to about 43 seconds and brings the altitude down to 90,000 feet.

In order to illustrate a condition in which these assumptions cannot be expected to apply, the same calculations have been made for a very shallow entry angle. It can be seen in figure 3 that small changes in the entry angle can represent large percentage changes and consequently appreciable changes in the altitude and heating rate. This discrepancy results from assumption 6 which assumes that the flight-path angle remains constant throughout the analysis. The usefulness of the analysis can be extended to these shallow entry angles by using a mean value of the angle for the part of the entry that is being considered. To illustrate this approach, curves have been included in figure 3 for $\gamma = -1.92^{\circ}$. The heat rate as computed from equation (7) is shown as dash-line curves. An improvement in the agreement with the solid-line curve can be obtained by changing the mean value of γ to -2.10° which gives from equation (3) an altitude which agrees with the corresponding solid-line curves for altitude at $t = 200$ seconds. This mean value of γ when substituted into equation (7) gives the improved curve appropriately marked in figure 3.

Transformation and Solution of Basic Equations

The transient temperatures of the surface are governed by the Fourier equations:

$$\alpha_m \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{8}$$

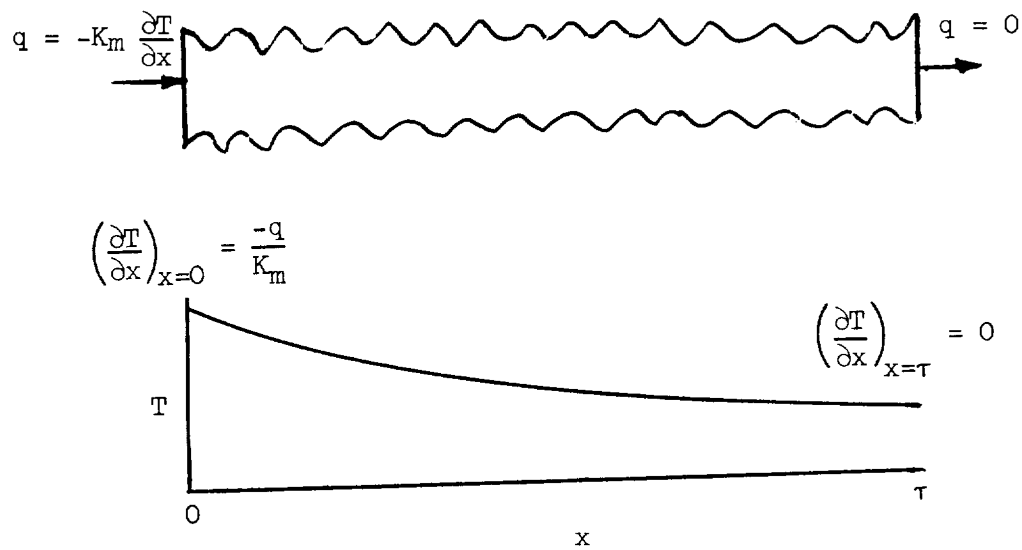
and

$$q = -K_m \frac{\partial T}{\partial x} \tag{9}$$

From the previous assumptions the boundary conditions for the Fourier heat-conduction equation are

$$\left.\begin{aligned}
\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} &= \frac{-k_1 C}{K_m \sqrt{R}}\left(\frac{\rho_\infty}{\rho_o}\right)^n\left(\frac{\bar{u}}{\cos\gamma}\right)^m \\
\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=\tau} &= 0 \\
T(x,0) &= T_i
\end{aligned}\right\} \tag{10}$$

These conditions are illustrated by the following sketch which shows T as a function of x:



Equations (8) and (9) with boundary conditions (10) specify the solution to the heat-flow problem. This boundary valued problem can be solved efficiently by the use of operational mathematics, the method of the Laplace transform in particular. (See ref. 9.) If the Laplace transform of $T(x,t)$ with respect to t is denoted by

$$\overline{T}(x,s) = \int_0^\infty e^{-st}T(x,t)dt \tag{11}$$

then equation (8) and the transformed boundary conditions become, respectively,

$$\alpha_m \frac{d^2\overline{T}}{dx^2} - s\overline{T} = -T_i \tag{12}$$

$$\left(\frac{d\overline{T}}{dx}\right)_{x=0} = \frac{B}{s - \omega} \tag{13}$$

and

$$\left(\frac{d\overline{T}}{dx}\right)_{x=\tau} = 0 \tag{14}$$

The solution to the total linear differential equation (eq. (12)) is simply

$$\overline{T}(x,s) = C_1 e^{\sqrt{\frac{s}{\alpha_m}}x} + C_2 e^{-\sqrt{\frac{s}{\alpha_m}}x} + \frac{T_i}{s} \tag{15}$$

The constants C_1 and C_2 are determined by conditions (13) and (14) to be

$$C_1 = \frac{-e^{-\sqrt{\frac{s}{\alpha_m}}\tau}\sqrt{\frac{\alpha_m}{s}}\ \frac{B}{s - \omega}}{2\ \sinh\left(\sqrt{\frac{s}{\alpha_m}}\ \tau\right)} \tag{16}$$

$$C_2 = \frac{-e^{\sqrt{\frac{s}{\alpha_m}}}\sqrt{\frac{\alpha_m}{s}}\ \frac{B}{s - \omega}}{2\ \sinh\left(\sqrt{\frac{s}{\alpha_m}}\ \tau\right)} \tag{17}$$

Then,

$$\overline{T}(x,s) = \frac{T_i}{s} - B\sqrt{\alpha_m}\left\{\frac{\cosh\left[\sqrt{\frac{s}{\alpha_m}}(\tau - x)\right]}{\sqrt{s}(s - \omega)\sinh\left(\sqrt{\frac{s}{\alpha_m}}\ \tau\right)}\right\} \tag{18}$$

A series solution, which converges quite rapidly for large values of t, for the inverse transform of equation (18) can be found from the inversion integral from the theory of complex variables. From this the temperature is given as

$$T(x,t) = \frac{1}{2\pi i}\lim_{\psi \to \infty}\int_{\phi - i\psi}^{\phi + i\psi} e^{zt}\overline{T}(x,z)dz \tag{19}$$

where both ϕ and ψ are any real positive numbers. Equation (19) can be evaluated by simply taking the sum of the residues of the expression

$$e^{zt}\left\{\frac{T_i}{z} - \frac{B\sqrt{\alpha_m}\ \cosh\left[\sqrt{\frac{z}{\alpha_m}}(\tau - x)\right]}{\sqrt{z}(z - \omega)\sin\left(\sqrt{\frac{z}{\alpha_m}}\ \tau\right)}\right\}$$

The poles of this expression are at $z = 0$, ω, and $-\alpha_m \frac{j^2\pi^2}{\tau^2}$ where $j = 1, 2, \ldots$. The residues of these poles are, then,

$$T_i + \frac{B\alpha_m}{\omega\tau}$$

$$\frac{-B\sqrt{\alpha_m}e^{\omega t}\cosh\left[\sqrt{\frac{\omega}{\alpha_m}}(\tau - x)\right]}{\sqrt{\omega}\sinh\left(\sqrt{\frac{\omega}{\alpha_m}}\tau\right)}$$

and

$$\frac{\frac{2B\alpha_m}{\tau} e^{\frac{-\alpha_m j^2\pi^2 t}{\tau^2}} \cos\left(\frac{j\pi x}{\tau}\right)}{\omega + \frac{\alpha_m j^2\pi^2}{\tau^2}} \qquad (j = 1, 2, 3, \ldots)$$

Therefore,

$$T(x,t) = T_i + \frac{B\alpha_m}{\omega\tau} - \frac{B\sqrt{\frac{\alpha_m}{\omega}}e^{\omega t}\cosh\left[\sqrt{\frac{\omega}{\alpha_m}}(\tau - x)\right]}{\sinh\left(\sqrt{\frac{\omega}{\alpha_m}}\tau\right)}$$

$$+ \frac{2B\alpha_m}{\tau}\sum_{j=1}^{\infty} \frac{e^{\frac{-\alpha_m j^2\pi^2 t}{\tau^2}} \cos\left(\frac{j\pi x}{\tau}\right)}{\omega + \frac{\alpha_m j^2\pi^2}{\tau^2}} \tag{20}$$

In general, most entry conditions are such that the last term (series on the end) of equation (20) and the term $\frac{B\alpha_m}{\omega\tau}$ contribute very little to the temperature of the skin for values of t of 1 second or more and can be neglected. This fact can be verified by simply evaluating a few of the terms for any particular entry condition. The resulting expression for the temperature is

$$T(x,t) = T_i - \frac{B\sqrt{\frac{\alpha_m}{\omega}}e^{\omega t}\cosh\left[\sqrt{\frac{\omega}{\alpha_m}}(\tau - x)\right]}{\sinh\left(\sqrt{\frac{\omega}{\alpha_m}}\,\tau\right)} \tag{21}$$

Working Charts

The expression for the temperature rise $T - T_i$ from equation (21) is

$$T - T_i \equiv \Delta T = \frac{-B\sqrt{\frac{\alpha_m}{\omega}}e^{\omega t}\cosh\left[\sqrt{\frac{\omega}{\alpha_m}}(\tau - x)\right]}{\sinh\left(\sqrt{\frac{\omega}{\alpha_m}}\,\tau\right)} \tag{22}$$

This expression can be put into a form which is convenient for application by making the following definitions:

$$\Delta T = T - T_i \equiv WYN_mC_m \tag{23}$$

where

$$Y = \frac{e^{\omega t}}{\sqrt{\omega}} = \frac{e^{-(\beta nV \sin \gamma)t}}{\sqrt{-\beta nV \sin \gamma}} \tag{24}$$

$$N_m = \frac{\cosh\left[\sqrt{\frac{\omega}{\alpha_m}}(\tau - x)\right]}{\sinh\left(\sqrt{\frac{\omega}{\alpha_m}}\,\tau\right)} \tag{25}$$

$$C_m = \frac{1}{\sqrt{K_m\rho_m c_m}} \tag{26}$$

and

$$W = -BK_m = \frac{k_1C}{\sqrt{R}}\, e^{-\beta hn}\left(\frac{V}{\sqrt{gr}}\right)^m \tag{27}$$

In equation (23), only Y and N_m depend on the position and time variables of the problem; W and C_m are entirely dependent on the entry conditions V and γ and on the material under consideration. If plots are made of Y as a function of ω for various values of t and of N_m as a function of $\tau\sqrt{\frac{\omega}{\alpha_m}}$ for various values of $\frac{x}{\tau}$, rapid prediction of ΔT could be performed by use of these plots and a knowledge of the entry conditions and the properties of the material. Such charts are shown in figures 4 and 5.

Several significant facts can be obtained from figure 4:

(1) At values of $\tau\sqrt{\frac{\omega}{\alpha_m}}$ less than about 0.3, the gradients are very small through the skin (or plate) and the skin can be considered as a thin plate.

(2) Above a value of $\tau\sqrt{\frac{\omega}{\alpha_m}}$ of about 2, the effect of thickness of the plate on the surface temperature is negligible.

(3) The back-side temperature decreases very rapidly as $\tau\sqrt{\frac{\omega}{\alpha_m}}$ increases above 1; at $\tau\sqrt{\frac{\omega}{\alpha_m}} = 3$ the back-side temperature rise is only 10 percent of the heated-side temperature rise and decreases to only 0.1 percent at $\tau\sqrt{\frac{\omega}{\alpha_m}} = 7.6$.

CONCLUDING REMARKS

Closed-form expressions as approximate solutions of the one-dimensional heat-conduction equations have been derived for a configuration initially at constant temperatures entering a planetary atmosphere with constant velocity and entry angle. This solution is for the part of the entry before the maximum heat-transfer rates are reached. The atmospheric density is assumed to be exponential and the boundary layer to be laminar at entry. A series expression exists for the temperature history. It is noted, however, that in general most entry conditions are such that the only terms in the expression that contribute are the initial temperature and the term involving the hyperbolic functions. For this type of entry the expression was written in such form that convenient working charts could be presented. An example of the use of these charts is presented also.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 11, 1962.

APPENDIX

ILLUSTRATION OF USE OF WORKING CHARTS

In order to illustrate the application of the expressions developed in this report, a typical entry heating condition is considered. It is assumed that the temperature distribution through copper skins of different thicknesses is required for a vehicle entering the atmosphere at an altitude of 400,000 feet, a velocity of 20,000 feet per second, and an entry angle of -20°. The skin is assumed to be electrolytic copper which has a thermal conductivity of 0.0548 Btu/ft-sec-°F at 1,000° F. The radius of the blunt nose is assumed to be 1 foot. The constants of the problem are as follows:

$V = 20{,}000$ ft/sec

$h = 400{,}000$ ft

$\gamma = -20^{\circ}$

$R = 1$ ft

$K_m = 0.0548$ Btu/ft-sec-°F

$\rho_m = 521$ lb/ft^3

$\rho_o = 0.00238$ slug/ft^3

$C = 17{,}600$ (from ref. 5)

$m = 3.15$ (from ref. 5)

$n = 1/2$ (from ref. 5)

$\frac{1}{\beta} = 23{,}500$ ft (from ref. 5)

$c_m = 0.104$ Btu/lb-°F

$$C_m = 0.580\ \frac{\text{ft}^2\text{-sec}^{1/2}\text{-}^{\circ}\text{F}}{\text{Btu}}$$

$\alpha_m = 0.001011$ ft^2/sec

$g = 32.2$ ft/sec^2

$r = 21,300,000$ ft

This example treats the heating of two stations on the body: station 1 which is at the stagnation point and station 2 which is at a point that has a heat flux of 1/10 that at the stagnation point.

Substitution of the values of k_1, C, K_m, R, β, h, n, m, V, g, and r into equations (5) and (27) and the values of β, n, V, and γ into equation (6) gives the following values for B, ω, and W:

$B = -28.2$ °F/ft

$\omega = 0.1455$ sec^{-1}

$W = 1.546$ Btu/ft-sec$^{3/2}$

From equation (23) ΔT is given as

$$\Delta T = WYN_mC_m = 0.897YN_m$$

Then, depending on the particular time and station, ΔT can be determined once N_m and Y are obtained from figures 4 and 5, respectively.

Figures 6 to 8 show the results of these calculations. Figures 6 and 7 show the temperature-rise variation with time for specific values of x/τ for the two body stations under consideration and two plate thicknesses. These curves are valid for times up to about 40 and 43 seconds for vehicles having values of W/C_DA of 120 and 1,000 lb/ft^2, respectively, as seen from figures 1 and 2. Figure 8 gives the variation of the temperature ratio $\frac{\Delta T}{(\Delta T)_{x=0}}$ with x/τ for several plate thicknesses. It is interesting to note that these curves do not depend on time.

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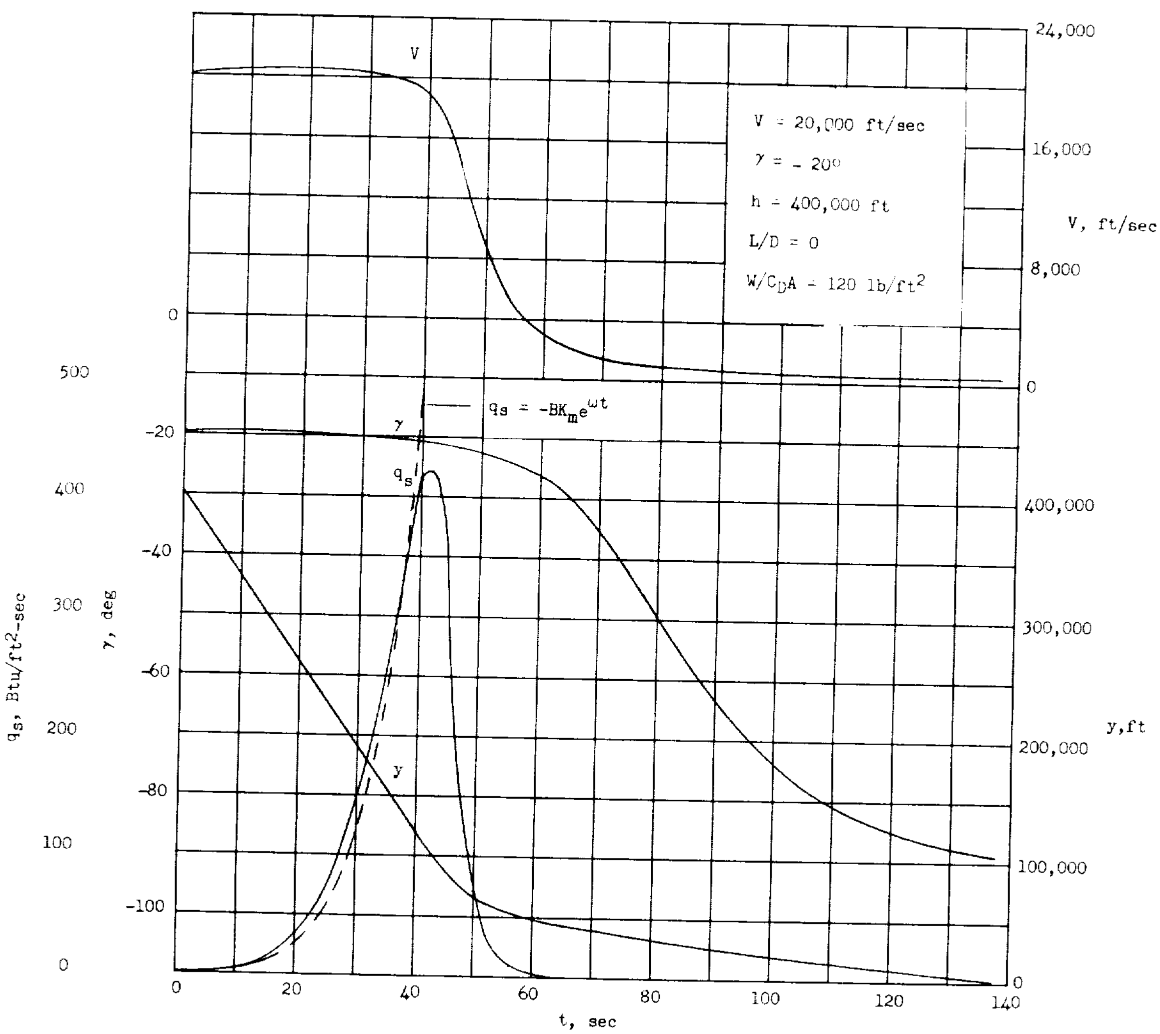

Figure 1.- Time history of velocity, entry angle, stagnation heat-transfer rate, and altitude for a high initial flight-path angle and a low ballistic parameter.

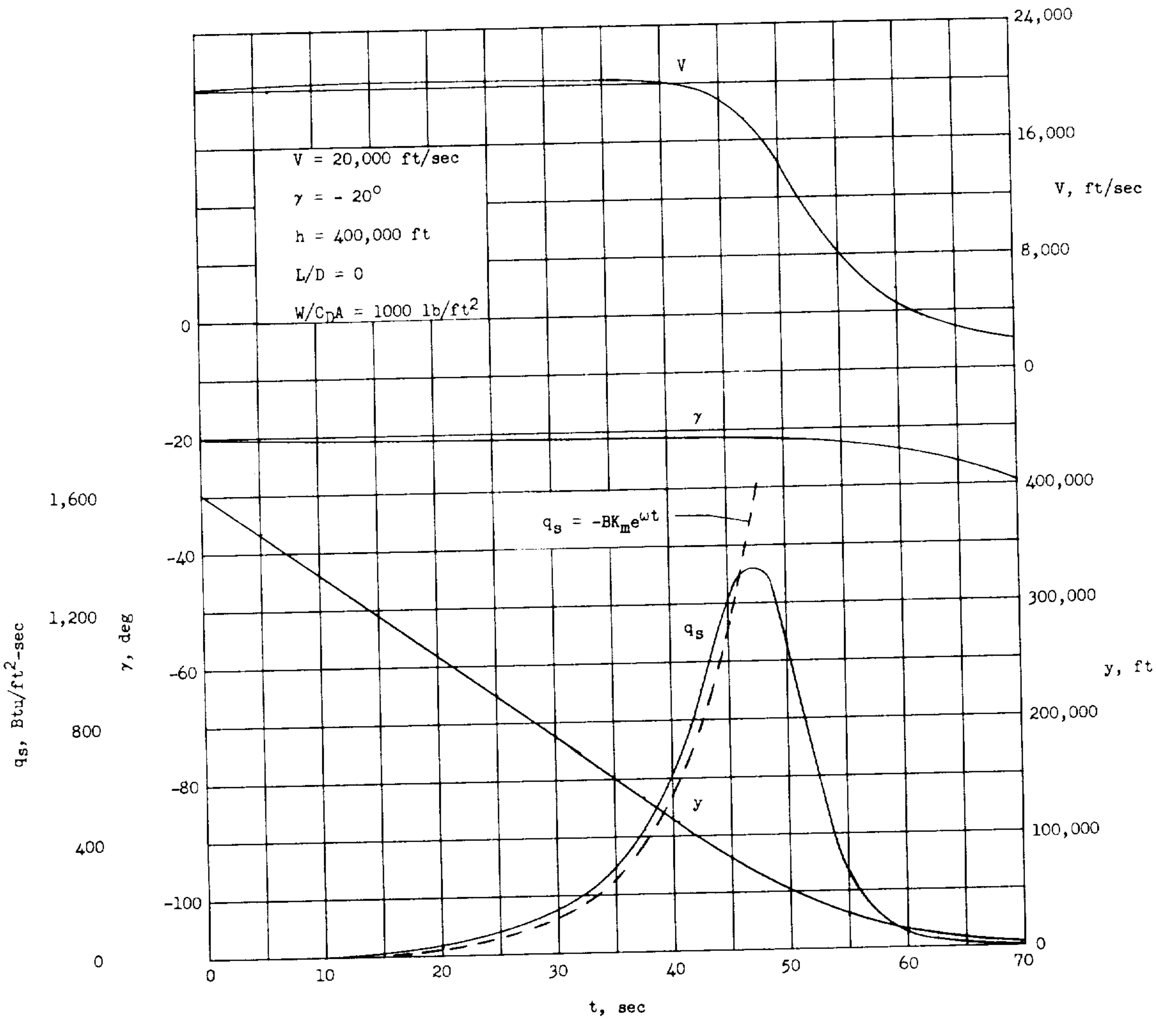

Figure 2.- Time history of velocity, entry angle, stagnation heat-transfer rate, and altitude for a high initial flight-path angle and high ballistic parameter.

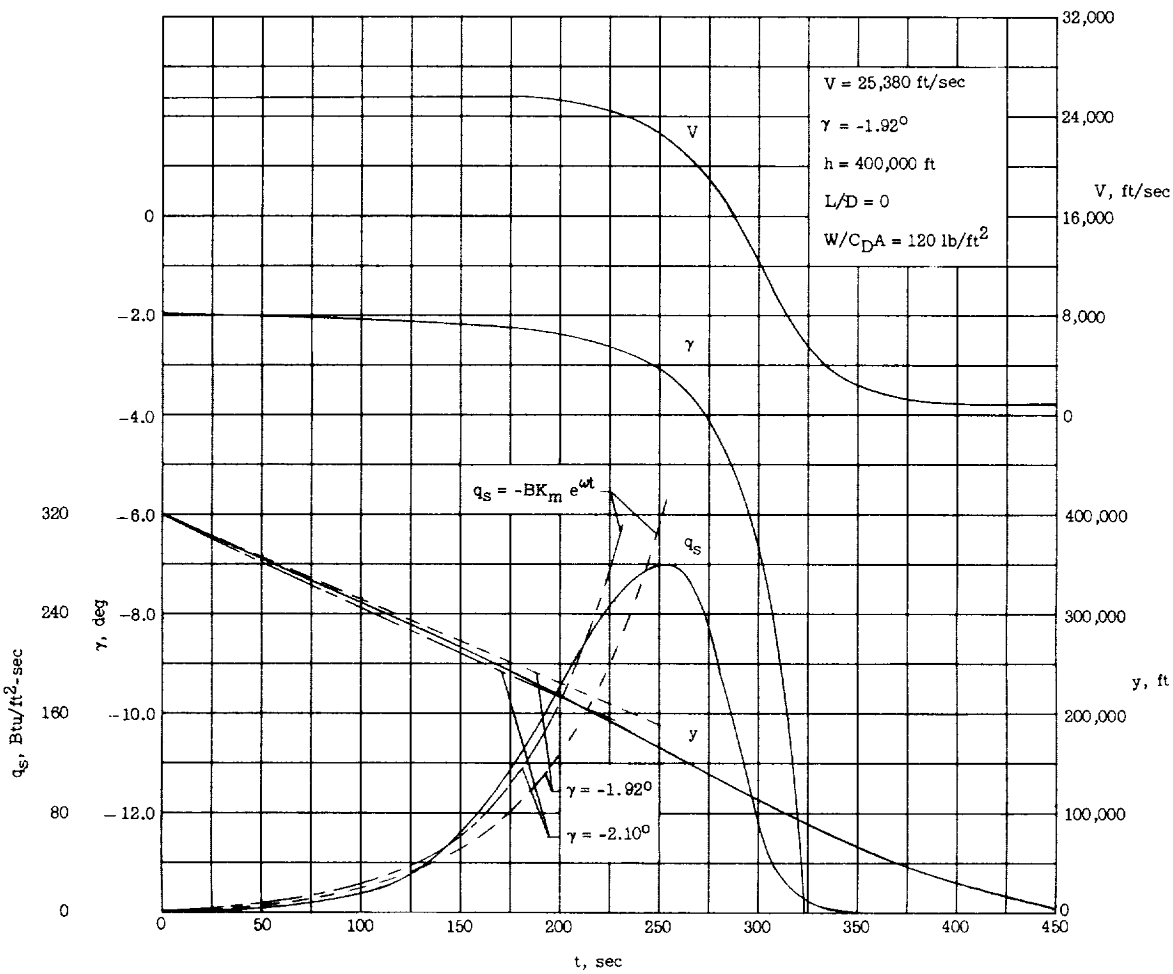

Figure 3.- Time history of velocity, entry angle, stagnation heat-transfer rate, and altitude for a low initial flight-path angle and low ballistic parameter.

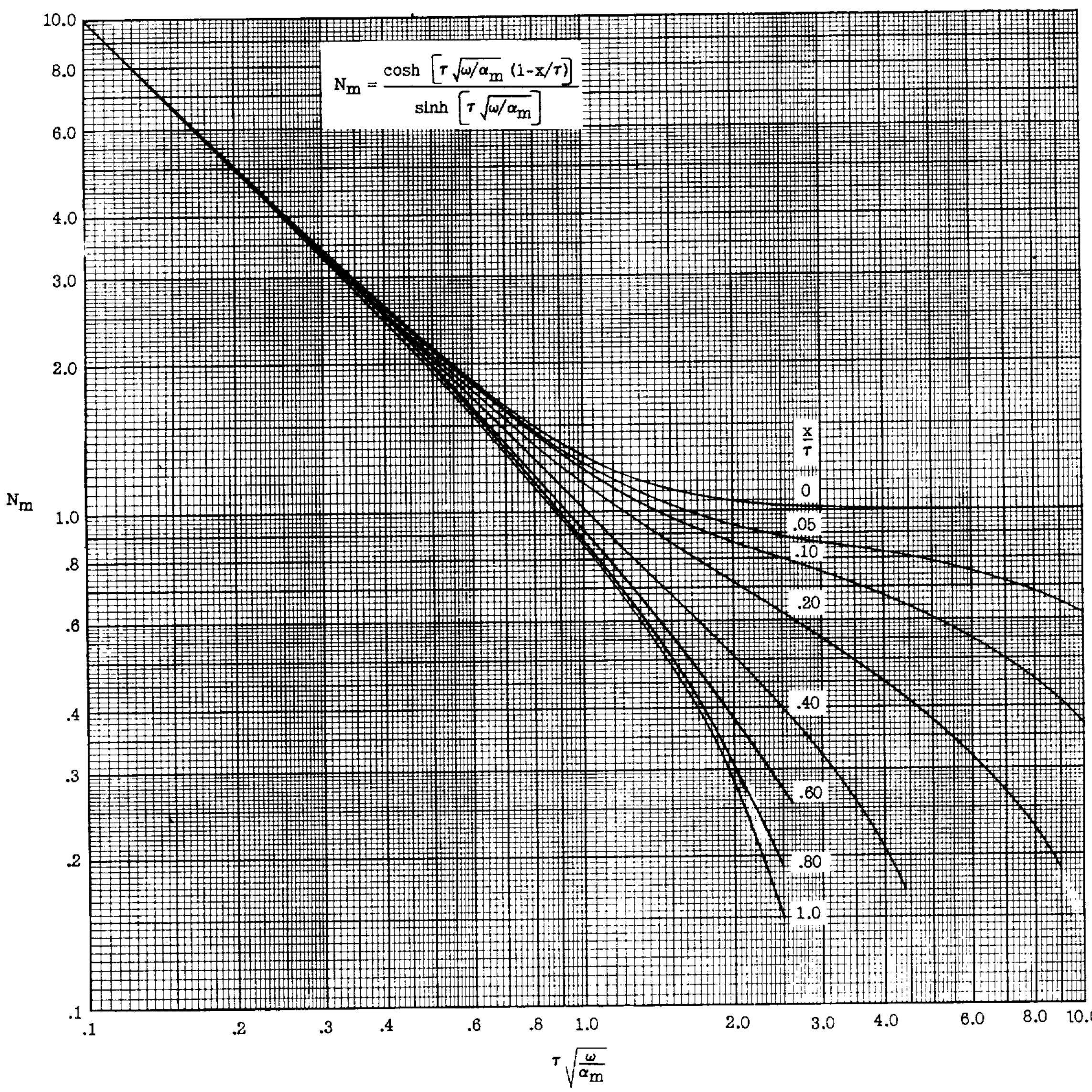

Figure 4.- Values of N_m for different values of x/τ and the parameter $\tau\sqrt{\frac{\omega}{\alpha_m}}$.

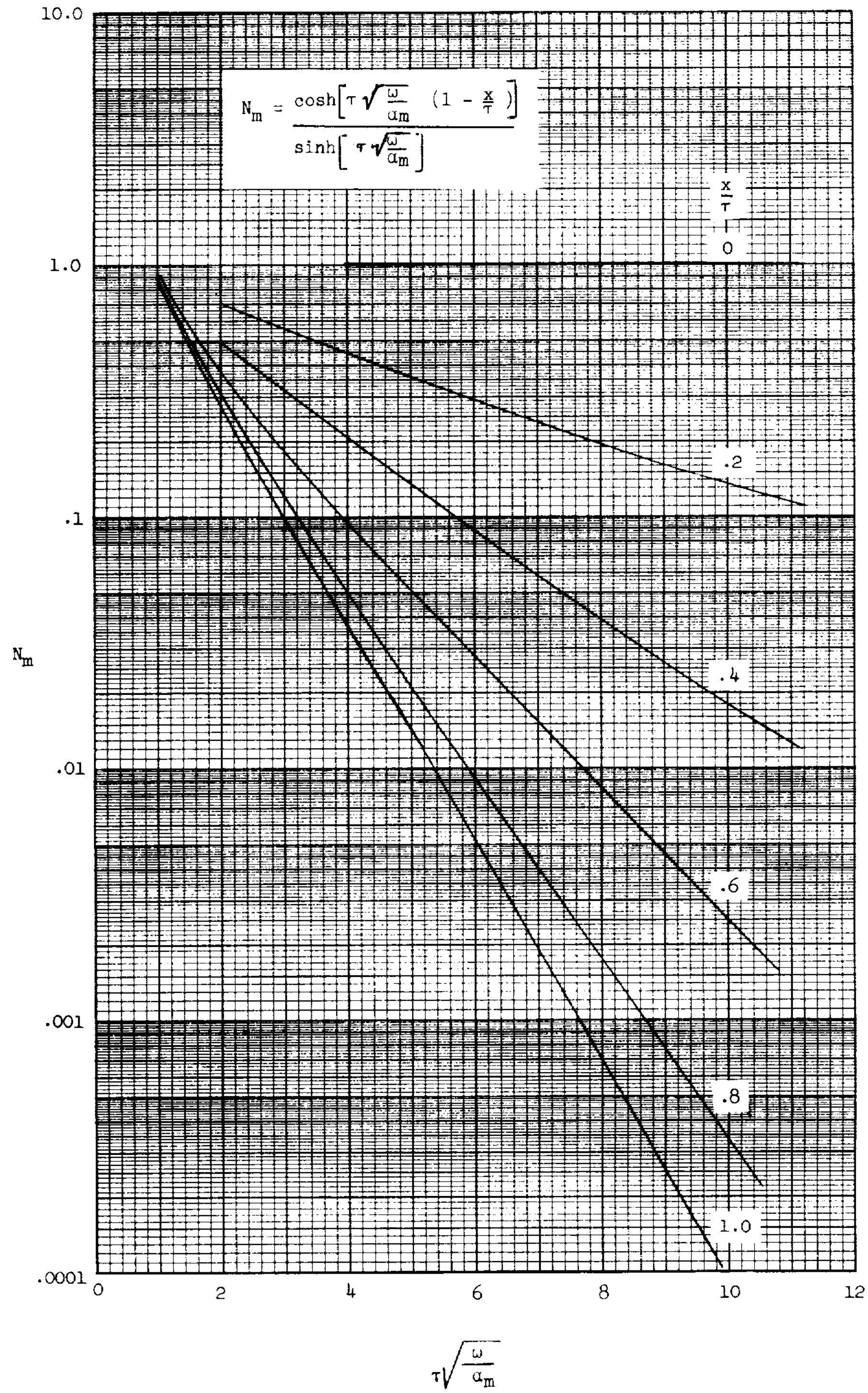

Figure 4.- Concluded.

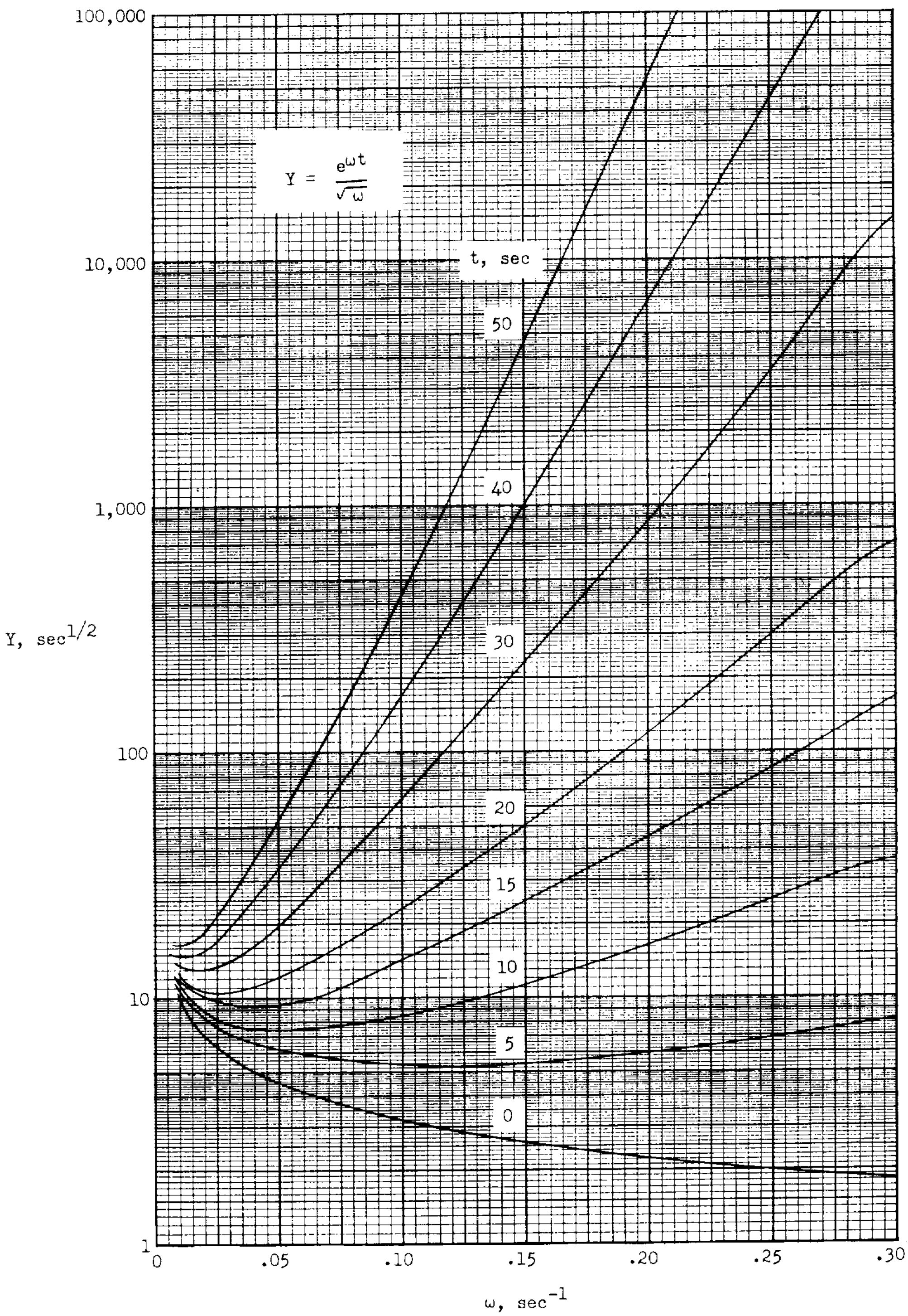

Figure 5.- Variation of Y with ω for various values of t.

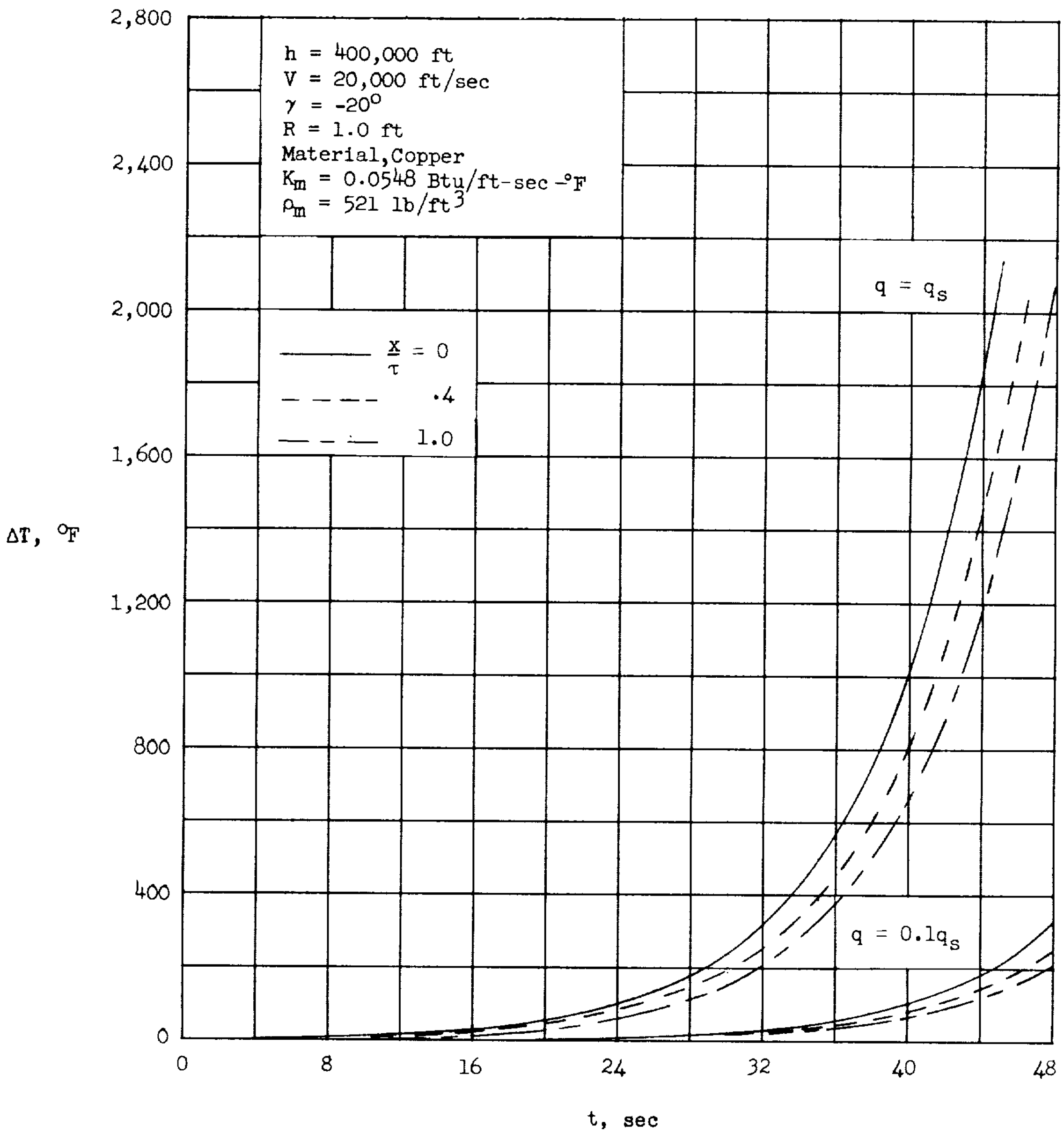

Figure 6.- Temperature-rise variation with time for a plate thickness τ of 1 inch.

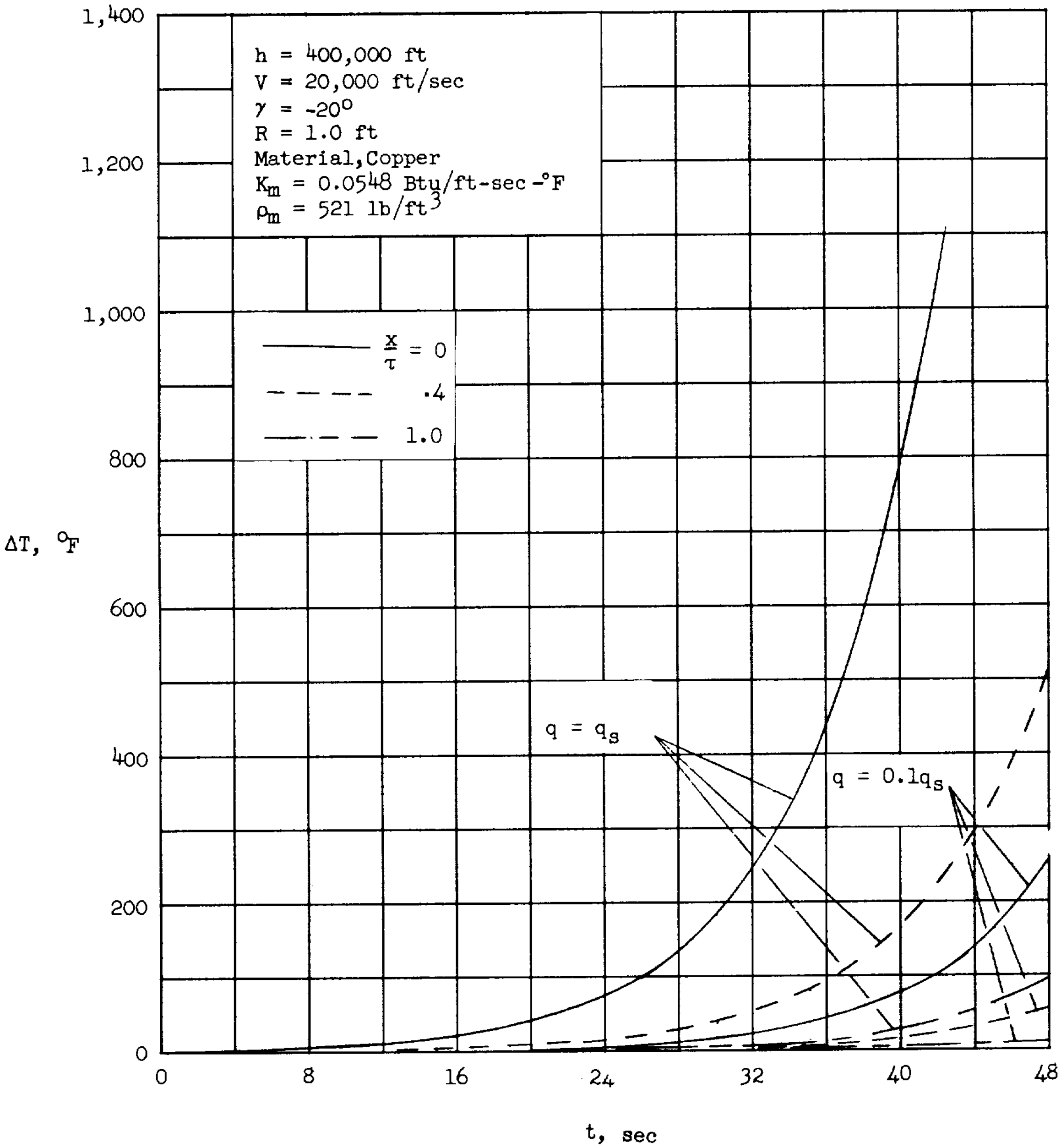

Figure 7.- Temperature-rise variation with time for a plate thickness of 4 inches.

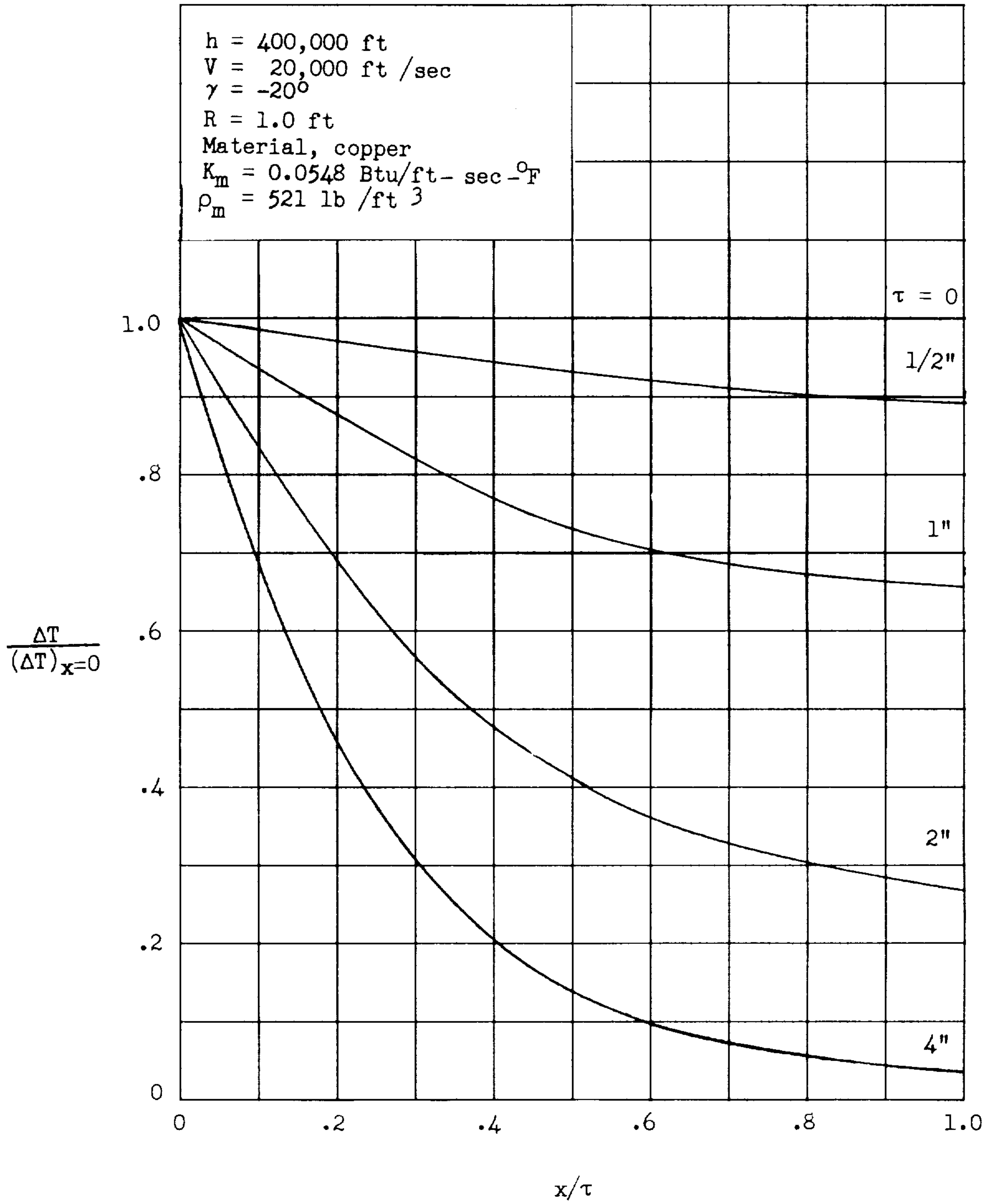

Figure 8.- Variation of ratio of temperature rise at any point in plate to that at the surface of plate with x/τ.