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STABILITY PROBLEMS IN MISSILE STRUCTURES

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SUMMARY

A number of specific buckling problems relating to missile propellant tanks and transition sections are enumerated. In some cases approximate techniques used by missile designers to determine stability are mentioned. The inadequacies of these approximate methods and the unavailability of appropriate usable information is stressed.

Furthermore, the mathematical complexities of nonlinear buckling problems are considered. Some possibly fruitful areas of mathematics to simplify these complexities are discussed with the hope that research in these areas will provide a better means through which the analyst may supply the designer with more realistic and accurate theoretical data.

INTRODUCTION

The desire to reduce structural weight in missiles and space vehicles in order to increase payload has resulted in thin-walled shell construction subject to elastic and inelastic buckling as one primary mode of failure. In many instances an adequate buckling analysis, or method of analysis, or experimental information is not available. Consequently, very crude but hopefully conservative approximations or idealizations are employed for analysis of the design. Often designers even avoid entirely particular lightweight configurations because of the complete lack of experimental or theoretical information on potential instability problems. The results of the above situations may be either the choice of a design which may not be near optimum or, perhaps even worse, a very expensive static test or flight test failure resulting in costly delays and vehicle modifications.

This paper discusses a number of missile shell stability problems which can be categorized among those which cause difficulty for designers. The specific problems are considered under the classifications of tank dome problems, tank barrel problems, or transition section problems. Many of these stability problems may be formulated as boundary value problems in nonlinear partial or ordinary differential equations; however, the solutions of these equations are extremely difficult to obtain or to approximate.

Three distinct sets of ideas are entertained in regard to simplifying the methods of solution for these problems. The first ideas are in regard to high-speed digital computer means of determining parameters resulting from Ritz-type methods used with stationary energy principles. The second set of notions concerns the behavior of approximation to integral equation formulations of shell buckling problems rather than to differential equation formulations. The last group of ideas explore in a very preliminary fashion the possibility of converting the boundary value problems of shell stability into initial value problems by methods similar to those of Ambarzumian or those of "invariant imbedding" used in transport theory. Perhaps then as initial value problems simpler methods may be found to determine stability or multiplicity of solutions.

STABILITY OF TANK DOMES

Elliptical Domes Under Internal Pressure

The thin-walled elliptical shell (hemi-ellipsoidal shell of revolution) whose ratio of length of major to length of minor axis is $\sqrt{2}$ is often chosen as a missile tank dome primarily on the premise that such domes give the shallowest rise and hence minimize transition section length and weight without introducing the possibility of dome circumferential buckling. Rattinger has clearly shown by experiment that flatter elliptical domes will buckle under internal pressure.

Without entertaining the idea that such buckling might not be an adverse phenomenon, two important assumptions underlie the choice of a "root-two" elliptical dome. The first is that linear membrane analysis tells all that the designer needs to know. This implies that the displacements of the dome under load and the effect of edge restraints have little bearing on the dome stresses or stability. The second assumption is that the loading is a constant internal pressure.

Within the framework of linear membrane theory Baltrukonis has shown that for "root-two" elliptical domes under axisymmetric internal hydrostatic pressure compressive circumferential stresses do arise. These stresses appear in the neighborhood of the attachment points of the dome; the extent to which they continue into the dome depends upon the pressure gradient in relation to the total pressure. The fact that a great number of missiles with "root-two" domes which have been flight tested and have not experienced detectable buckling due to this effect may imply any of the following:

1. the domes have sufficient stiffness to withstand the compressive stress without buckling, or

2. the pressure gradient in relation to the maximum pressure is sufficiently small so that the compressive region is confined to the shell boundaries where attachments provide restraint against buckling, or

3. the displacements of the shell are large enough to change substantially shell curvatures and thus invalidate linear theory.

The nonlinear behavior of very thin elliptical domes under internal pressure is still a quantitative unknown. Tests indicate bursting pressures of 20% to 40% over that which linear membrane theory predicts. This phenomenon seems to be due to large curvature changes in domes under high pressure. These observations are offered as part justification for the third reason given above.

The successes of past designs of given geometry, thickness, and "pressure gradient-maximum pressure" relations unhappily do not insure the success of future designs which may incorporate variable thickness "root-two" domes of larger than conventional diameters and with different relationships between pressure gradient and maximum pressure. Consequently, the problem of buckling of elliptical domes under axisymmetric internal hydrostatic pressure still remains an open question.

When a propellant tank is in the horizontal position and containing a propellant, Baltrukonis has again shown membrane hoop compression under the nonaxisymmetric hydrostatic pressure. Here again is the source of a significant stability problem.

Shallow Spherical Domes Under Internal Pressure

In order to decrease the total length of a missile, decrease its structural weight, and raise its fundamental frequency of lateral vibration one tries to shorten as much as practical transition sections between propellant tanks. To accomplish this shortening some designers have considered using shallow spherical shells for tank domes. Reasoning from linear membrane theory they conclude that the lateral component of the longitudinal membrane stress at the dome-tank juncture must be resisted by a sufficiently stiff ring frame so that buckling of the tank walls and dome is avoided. The ring frame is customarily designed with a stiffness sufficient to prevent first mode "out-of-roundness" buckling were the ring free and under a uniformly distributed line load in the plane of the ring. The intensity of this line load is determined from the lateral component of the membrane stress in the dome. Consequently, the increased weight due to the heavy ring required under the above assumptions does not give the shallow spherical dome design a decided advantage over the elliptical dome design.

The author is not at all certain that for a shallow dome without a ring frame buckling will occur as first mode out-of-roundness. Most certainly large nonlinear local bending effects will occur as internal pressure is increased. The dome and tank wall curvatures will change measurably before buckling occurs. The deformed shape may tend to approach that of a shallow ellipse for which the mode of buckling is not in general first mode out of roundness. The presence of a ring frame may tend to reduce the amount of displacement at the edge of the dome, but the mode of buckling would seem to be dependent upon the stiffness of the ring frame. Hence, the nature of the stability of a shallow spherical shell under internal pressure and attached to a cylindrical tank wall with or without a ring frame of given stiffness is still an unresolved problem.

It seems heuristically reasonable that for sufficiently stiff rings "out-of-roundness" first mode buckling is most likely to govern the ring design. A pertinent suggestion has been to stabilize the ring against such buckling by a series of lightweight tension rods much like the spokes on a bicycle. It is believed that these rods would inhibit first mode buckling by providing restrain against outward motion of the ring. If such stabilization is possible, then a lighter ring may be used designed against buckling at some higher mode. Perhaps a weight-saving may be realized. Thus, the problem of the stability of a shallow spherical dome attached to a cylindrical shell edged by a ring frame which itself is stabilized by tension rods (perhaps prestressed) provides a challenging and useful area of investigation for the shell analyst. Further, weight optimization including the ring and tension rod weights with stability as the failure criterion would be most useful.

Domes for Segmented Tanks

In order to reduce the circumferential stress in propellant tank barrels the segmented tank, whose cross-section resembles the shape of a scalloped round doily, was devised. Domes for such tanks pose a stability problem for the analyst. For relatively flat domes a small radius of curvature is required at the transition from the dome to the segments of the barrel, hence one may legitimately anticipate circumferential compression in these areas when the tank is subjected to internal pressure. These transition segments are not shells of revolution but more closely resemble sections of a football. A computational complexity is thus encountered in determining the linear membrane stresses as well as the buckling behavior of these football segments. Certainly, the solution of this problem would be useful to designers.

Domes Under Static External Pressure

Several environmental and test conditions introduce the occurrence of static or quasistatic external pressures on domes. Such mishaps as rapid emptying of propellant tanks during simple hydrostatic testing without proper vent valve opening, air transport of tanks from higher to lower altitudes with unopened vent valves, and geysering of cryogenics during tank filling have all inadvertently caused situations of external pressures on domes. The frequency of buckling failures due to such mishaps have prompted designers to consider using various "failsafe" valves, which would permit rapid pressure equalization under such conditions. The design of such valves requires the knowledge of the buckling pressure of the domes. The problem of static external pressure on domes arises also in regard to recoverable boosters and in missile tanks in which a common dome separates two varieties of propellant.

Under some of the conditions mentioned above buckling occurs without rupture of the dome but with plastic deformations. Internal pressure may be used to "pop" the domes back into shape. Whether or not such a procedure is harmful to the integrity of the dome has not yet been determined.

Thus, dome buckling under external pressure is a significant large deflection problem complicated by the fact that in practice domes are not always shallow. Judging from the discussions in the literature on the nonlinear solution of the shallow spherical shell problem one may say with assurance that the buckling of deeper domes provides an extreme challenge. Perhaps different mathematical and new numerical schemes are necessary to eek out the solutions to such problems.

Domes Under Dynamic External Pressure

In a number of applications tank domes are subjected to rapidly applied external pressures. For example, in-flight ignition of a higher stage engine in the neighborhood of a lower stage tank dome causes a sudden dynamic blast to impinge on the dome below. In order to prevent damage to the lower stage which may be intended to be recovered or which might, if damaged, explode and cause a disruption of the higher stage the dome must be able to withstand the dynamic load. As the outstanding work of Ezra and Foral of the Martin Company in Denver has shown, for blast loads of short duration a peak external pressure considerably greater than the static buckling pressure can be withstood before permanent buckling is experienced. However, high speed motion pictures by Foral show considerable motion and large displacements due to high pressure impulses. An understanding of the dynamic response under such loads and the relation of this dynamic response to the permanent buckling or tearing of the shell is not at all apparent. Just what stability means under such dynamic conditions is not at all clear. Here again is a complicated nonlinear problem to test the acuity of the analyst.

Concentrated Loads on Domes

The weight of many missile components is often supported by relatively heavy trusses in the transition section areas. These trusses are usually fastened to ring frames in the walls of the transition sections. Should these components be mounted directly on domes in the transition areas, truss weights could be saved. This practical consideration leads one to be concerned with the effects of concentrated loads on domes.

In testing of some shallow domes under single concentrated loads one of two different types of behavior is usually observed. For thicker domes plastic yielding in a significant neighborhood around the concentrated load is observed. For thinner domes no noticeable plastic yielding takes place; however, the concentrated load causes snapthrough or oil canning. Aside from desiring to know the load which causes snapthrough either local or general on shallow or deep domes, one should also like to know the values of those physical parameters which mark the separation point between the two different phenomena; i.e., elastic snapthrough or local yielding.

Explosively Formed Domes

Explosively forming from flat sheets large diameter domes for missile application has been suggested as a useful and economical manufacturing scheme. Tests on small scale models show that for sufficiently thin sheets of material used in explosively forming domes, wrinkles or buckles can form in the neighborhood of the apex of the dome and also around the supporting edge of the dome. Although methods for preventing these wrinkles are being devised, better understanding of the forming of these wrinkles would certainly be useful.

STABILITY PROBLEMS RELATING TO TANK BARRELS

Problems of Barrel Design

The choice of the type of barrel design to be used in missile propellant tanks is determined by several important considerations. Primarily the function of the tank barrel is to contain the propellant; thus, it must be able to withstand the internal pressures caused by the propellant and the tank pressurization system. Another function relegated to the tank barrel is transmitting the axial thrust of the engines to the payload; thus, the barrel must be able to withstand the compressions developed. It is the resistance of this compressive loading that often leads to varieties of barrel designs.

A popular choice is the cylindrical tank with integral longitudinal stringers and hoop frames flexibly fastened to the ~~stringer flanges.~~ The stringers are usually designed to some sort of column buckling and crippling criterion which in the hands of a good analyst appears adequate. Whether or not the interior hoop frames provide an aid to stability is an unanswered question. On the basis of a variety of tests with internal pressure the presence of the frames and whether they are attached to the shell or the stringers does not seem to affect the buckling load significantly. A more adequate resolution of this problem is desirable.

For larger diameter rockets requiring relatively less internal pressure honeycomb construction for tank barrels will reduce tank weights. Honeycomb, however, in its customary design is adversely affected by thermal stresses. When the outer skin is heated and the inner skin cooled, the core material is placed under tension stresses which bring about separation between the core and the face sheets. A design which permits periodically spaced creases or ripples in the inner face sheet of a sandwich construction will tend to permit contraction of the inner face and expansion of the outer face without extremely high core tensions. The ripples will merely be pulled flat. The problem of the stability of honeycomb cylinders with such ripple inner face sheets has not at all been examined. Here, too, is a fruitful area of research for the structural analyst in the area of nonlinear buckling of sandwich cylinders.

Concentrated Load on Tank Barrels

Among the causes of concentrated loads on tank barrels one can include

1. struts and straps used in fastening booster rockets in cluster configurations,
2. handling loads in transport of missiles.

Significant problems in shell stability arise from these concentrated loads. Internally pressurized thin shells with identical longitudinal struts of varying cross section equally spaced about the circumference and supporting concentrated loads pose a problem of stability. Not only the behavior of these struts under longitudinal loads, but also under concentrated normal loads is significant. Study of the stability and deformation of such design with two, four, or six longerons would be most useful.

Straps or linkages which may be used to hold clusters of booster rockets together impose concentrated loads. Usually the strength of the barrel shell is ignored in the direct carrying of these loads and interior rings or support structures are assumed to carry the loads. If the degree to which the shell can resist concentrated tangential loads is better assessed, the internal support structure can be lightened or perhaps even eliminated.

The interior hoop frames mentioned in the section entitled, Problems of Barrel Design, are useful in preventing tank collapse due to handling loads. Precisely to what degree they are useful is not known. They are customarily designed by no really rational method. Knowing how these rings interact with the shell structure and maintain stability would be useful.

Dynamic Loads on Tank Barrels

Rapidly applied dynamic loads on tank barrels may arise from three important sources; the launch of a rocket vehicle from an underground silo, a nearby detonation, or a nearby upper-stage ignition. As in the case of dynamic loads on domes just what structural stability means and whether it should be considered in analysis under such loads is still an open question.

STABILITY PROBLEMS IN TRANSITION SECTIONS

Longerons in Transition Sections

The tremendous thrusts of rocket engines in liquid propellant missiles are usually resisted by heavy longerons which distribute these thrusts to cylindrical panels by means of shearing action. The required length of longeron is dictated by the buckling strength of the attachedd panels. This buckling strength is usually assessed as that of a flat plate of approximately the same dimensions as the curved panel. An adequate stability analysis of curved panels with variable section longerons does not seem to be available.

In order to save weight and shorten transition sections, some designers run the longerons into the tank barrel area. The stability behavior of a longeroned "transition section" and "tank barrel" ensemble is indeed complex and even good approximations to this behavior have not been forthcoming although certain designs employing this principle have been successful.

Should the longeron be made in two sections spliced in some way at the "tank-transition section" junction a noticeable decrease in bending stiffness of the longeron may be realized at this splice. This reduction in bending stiffness will tend to make the configuration more susceptible to buckling. The effect on stability of such a splice should be studied.

Other Transition Section Problems

Transition sections between tanks and between missile and payload must transmit high compressive forces and are designed entirely on stability considerations. To complicate the analysis of such sections is the presence of access doors. The stability of the section often depends strongly on the fastening and load-carrying capability of these doors, complicated by other loading environments such as transonic buffeting.

Still another interesting problem relating to transition sections is that of local elastic buckling on the compressive side of a laterally vibrating missile. The amount of bending stiffness reduction in the missile cross section due to local buckling is not well known. Determining this would enable the analyst to determine the decrease in the fundamental structural frequency of the missile due to such local elastic buckling during large lateral vibrations.

SOME AREAS OF MATHEMATICAL RESEARCH

With Donnell's idea that the buckling of a cylindrical shell under axial load is essentially a nonlinear phenomenon and with the clear confirmation of this notion by Von Karman and Tsien the shell analyst is forced into a mathematical discipline which is both very complicated and still in its infancy. Many shell stability problems can be considered as boundary value problems in the theory of nonlinear partial differential equations. This author would like to mention three different mathematical areas in which he feels research would be of benefit to the shell analyst.

Extremizing Nonlinear Functions

In the work of Von Karman and Tsien and more recently of Kempner concerning the postbuckling behavior of thin cylindrical shells the method of approximate solution of the nonlinear problem stems from the Principle of Stationary Potential Energy. Ignoring the important question of existence of a solution, analysts often employ a Ritz-type procedure. An approximate displacement function containing a number of arbitrary parameters is generated; compatibility in terms of this function is satisfied. The remaining task is to determine the values of the parameters which minimize or at least make stationary the potential energy function.

Research to determine adequately convergent numerical schemes applicable to high-speed digital computers to minimize or extremize nonlinear functions of many parameters would certainly be fruitful to the shell analyst. Methods of steepest descent seem most promising and should be investigated further.

Integral Equation Formulations

Budiansky appears to be the only author to have employed the discipline of integral equations to shell buckling. His formulation of the shallow shell problem under uniform pressure led to nonlinear Hammerstein-like integral equations defined over a finite domain with a rather complicated kernel of Kelvin functions with singularities at its boundaries. An equivalent formulation with linear kernels but an infinite domain is also possible.

Since generally speaking the boundary conditions of the problem are already incorporated into the integral equation formulations, such questions as existence and uniqueness of solutions or numbers of nonunique solutions should be simpler for integral equation formulations than for differential equation formulations. Also, the rapidly developing field of functional analysis should provide many powerful theorems useful in the study of integral equations. Since nonsingular linear problems in integral equations lend themselves to numerically better behaved approximate solutions than do the equivalent differential equation formulations, one might hope for the same advantage in singular, nonlinear problems. For all these reasons the author would like to encourage the mathematically inclined shell analyst to become interested in and do some research in nonlinear integral equations.

Invariant Imbedding

Recent work by Bellman, Kalaba, and Wing in the mathematics of transport theory, has led to a method of solving boundary value problems called "invariant imbedding" which is an extension of the work of Ambarzumian. A primary objective of the method is the conversion of boundary value problems to initial value problems which seem to be well suited to high-speed digital computation. Although this method has not been applied to shell theory, preliminary investigations indicate that it has good potential. The invariant imbedding technique may be very useful in determining shell stability and should be examined more carefully.