SOME SHELL STABILITY PROBLEMS IN MISSILE AND SPACE VEHICLE ANALYSIS

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SUMMARY

A brief discussion is presented of three structural shell stability studies in progress at Lockheed Missiles & Space Company: the stabilizing influence of solid propellants, the effect of cushion stiffness on buckling under cushion loading, and the snap-through buckling of axially loaded cylinders. Recent analytical results are compared with test data. Also included is a brief description of a few shell stability problems of present interest to LMSC for which satisfactory methods of analysis are not available.

INTRODUCTION

One of the consequences of the critical importance of weight saving in missile and space vehicle design is that most major structural components consist of thin-walled structural shells. In terms of the usual classifications of shell theory, most of the shells in such applications are not only thin but are extremely thin. This thinness, of course, promotes buckling and large deflections. Consequently, a substantial portion of the structural research effort in any missiles and space company must be devoted to shell stability investigations.

Research and methods-of-analysis studies are presently in progress at Lockheed Missiles & Space Company on a broad range of shell stability problems. This report briefly discusses three of these problems: the stabilizing influence of solid propellants, the effect of cushion stiffness on buckling under cushion loading, and the snap-through buckling of axially loaded cylinders. The first of these concerns analyses in which the propellant is assumed, as a first approximation, to act as a soft elastic foundation. The accuracy of small-deflection buckling theory in such analyses and the suitability of the Winkler-foundation assumption are considered. The second study pertains to the design of large-diameter cylindrical shell structures to resist handling and storage loads. A support load occasionally must be introduced into a cylindrical shell in an unstiffened region of the shell surface. The possibility of buckling under loads applied by cushioned supports and the effect of cushion stiffness on the buckling strength of the structure are discussed. The third problem, snap-through buckling of axially loaded cylinders, is very well-known. New theoretical results are presented which, for the first time, agree with experimental data for the minimum postbuckling equilibrium load.

Finally, in spite of the research effort devoted to shell stability studies, what may be called the "methods gap" continues to grow. A brief description also is presented of a few shell stability problems of present active interest to LMSC for which satisfactory methods of analysis are not available.

THREE SHELL STABILITY STUDIES

Influence of Soft Elastic Cores

The use of solid-propellant fuels in rocket motor cases has led to widespread interest in the stabilizing influence of soft elastic cores on the buckling strength of circular cylindrical shells. As a consequence, a number of papers on core-stabilized cylinders have appeared in the literature in recent months. A notable example is the recent paper by Seide on buckling under uniform lateral pressure and axial compression (ref. 1).

Axially-symmetrical loading. - An extensive study of core influence has been in progress at LMSC for several months. As a part of this study, an analysis has recently been completed for the buckling of core-filled cylinders subjected to axially-symmetrical loading (ref. 2). A stability equation is derived for a simply-supported cylinder subjected to axially-symmetrical lateral pressure of arbitrary axial distribution, combined with a central axial force. The analysis is an extension of a corresponding empty-cylinder analysis in reference 3, and the treatment of the core effect is based on Seide's general elastic-core analysis in reference 1.

Numerical results are presented in reference 2 for certain lateral pressure distributions of interest in motor-case analysis. For a cylinder subjected to an axially-symmetrical band of pressure, it is found that the magnitude of the buckling pressure is independent of both band location and cylinder length, unless the band is located near one end of the cylinder or the cylinder is extremely short. Furthermore, for sufficiently wide pressure bands, the magnitude of the buckling pressure is also independent of the bandwidth.

As would be expected, the buckling pressures for wide pressure bands are found to be the same as those given in reference 1 for relatively long core-filled cylinders pressurized over their entire length. These buckling pressures also may be shown to be the same as those given by the following relatively simple equation derived in reference 4 for a wide ring of rectangular cross section, filled with a soft elastic core, and subjected to uniform external pressure:

$$\frac{p}{1 + E_c\, r/\left[(1 - \mu_c)\, Et\right]} = n^2 \frac{D}{r^3} + \frac{1}{n} \frac{E_c}{2(1 - \mu_c^2)} \qquad (1)$$

where:

$D = Et^3/\left[12(1 - \mu^2)\right]$

E, E_c = Young's modulus, ring and core

μ, μ_c = Poisson's ratio, ring and core

r = ring radius

t = thickness

n = number of circumferential waves in buckle pattern

The physical significance of individual factors in the analysis is much more evident in equation (1) than in the more complex, cylinder equations. The second term in the denominator on the left side of the equation represents the prebuckling influence of the core, and usually is negligibly small. The first term on the right side represents the ring bending stiffness, and the second term the core effect during buckling. For $E_c = 0$, equation (1) reduces to the well-known Donnell solution for an empty ring.

The Winkler-foundation assumption. - Equation (1) was derived in terms of the assumption that the core in the ring is an elastic medium in a state of plane stress. It is important to note that the alternative assumption that the core acts as a Winkler foundation, i.e., as a set of uncoupled springs, may be shown to lead to a factor $(1/n^2)$ instead of $(1/n)$ in the last term in the above equation. In the range of practical stiffnesses for solid propellants, n is much greater than unity. Therefore omission of shear coupling in the foundation (as was done, for example, in refs. 5 and 6) greatly underestimates the stabilizing influence of the core, and may lead to results which are grossly conservative.

Accuracy of small-deflection theory. - Both Seide's analysis and the analysis in reference 2 are based on small-deflection buckling theory. As is well-known, this theory does not always yield results in agreement with test data. However, it is generally recognized that, for empty cylindrical shells, the agreement is reasonably close for loadings in which the prebuckling membrane stresses are predominantly circumferential. Seide has shown in reference 1 that reasonably close agreement also may be obtained for core-filled cylinders, for the case of uniform lateral pressure loading. An additional comparison for core-filled cylinders is shown in figure 1 for a case of nonuniform lateral pressure, namely, for a circumferential band of pressure. The theoretical results are based on reference 2, and the test data are from reference 7. In view of the fact that it is quite difficult to determine an appropriate experimental value for Young's modulus of the core material, the agreement between theoretical and experimental values again may be said to be reasonably close.

Elastic Cushion Loading

Cushioned support-saddles pressing against the unsupported lateral surface of a large-diameter cylindrical shell structure induce compressive hoop stresses in the shell wall which can cause the shell to buckle. However, the cushions also act as an elastic foundation which tends to stabilize the shell wall. The stabilizing restraint is similar to that derived from a soft elastic core. Consequently, the stiffness of the support-saddle cushioning material is a strong factor in determining the magnitude of the applied pressure at which buckling failure may occur under cushion loading. The allowable pressure is lower for relatively soft cushions, with fluid-pressure loading constituting a limiting case corresponding to cushions of zero stiffness.

Unsymmetrical loading. - Both axially symmetrical and unsymmetrical (i.e., transverse) cushion loading are of practical interest at LMSC. Of course, if the circumferential distribution of the pressure applied to a cylindrical shell is such that a significant amount of circumferential bending is induced in the shell wall from the outset, the shell may fail by local bending or local beam-column action rather than by buckling. However, axial symmetry of the applied pressure is not a necessary condition for bifurcation instability. A band of pressure circumferentially distributed according to the relation:

$$p = p_{max} (1 + \cos \varphi)/2 \qquad (2)$$

where φ is the circumferential coordinate, causes the cylinder cross sections to translate and decrease in diameter, but does not induce significant circumferential bending prior to buckling. An analysis of bifurcation instability under the pressure distribution of equation (2) was carried out as part of the present study, and is reported in reference 8. Results of the analysis indicate that the maximum allowable fluid pressure for the unsymmetrical loading treated in reference 8 is somewhat greater than that for the corresponding axially-symmetrical case, but the difference is not great. Therefore, subsequent studies of bifurcation instability under cushion pressure have been limited to axially-symmetrical loading in the present investigation.

Axially-symmetrical loading. - The most recent result of the axially symmetrical loading investigation is a pilot study of a core-filled circular ring subjected to external pressure applied by soft elastic cushions (ref. 4). The equation determined for the buckling pressure in that analysis is:

$$\frac{p}{1 + E_c r/\left[(1 - \mu_c)\,Et\right]} = n^2\frac{D}{r^3} + \frac{1}{n}\left[\frac{E_c}{2(1 - \mu_c^2)} + \frac{R(n)\,E_s}{2(1 - \mu_s^2)}\right]$$

$$R(n) = \frac{\left(1 + \frac{t_s}{r}\right)^n + 1}{\left(1 + \frac{t_s}{r}\right)^n - 1} \qquad (3)$$

where the subscript s denotes cushion, and the remaining symbols are defined above. For $E_s = 0$, this equation reduces to the relation given in equation (1) for fluid pressure loading. Equation (3) indicates that the stability of the structure may be increased by increasing the stiffness of the cushion material or by decreasing the cushion's thickness.

The Axially Loaded Cylinder

A study of the snap-through buckling of axially loaded circular cylindrical shells has been in progress at LMSC for quite some time. This is a well-known problem which has challenged investigators in the field of structural shell stability analysis for many years. The continuing interest in the problem is indicated by the relatively large number of references to it in the collected abstracts of papers for the present symposium.

Large-deflection analysis. - The first phase of the LMSC study of this problem has recently been completed, and is reported in reference 9. This phase treats the theoretical postbuckling behavior of isotropic cylinders which are free from initial geometric imperfections. General interest in this aspect of the problem was first stimulated by the findings of von Kármán and Tsien in 1941 (ref. 10). Results of their analysis indicate that if large displacements are considered, the load can be shown to drop sharply from the bifurcation point given by small-deflection theory to a relatively low minimum in the postbuckling range. Their analysis was later improved by other investigators, among whom, Kempner (ref. 11) gave the most accurate solution. However, the ratio given in reference 11 between the minimum postbuckling equilibrium load and the classical, small-deflection buckling load is $P/P_{CL} = 0.30$. The corresponding experimental values reported by Thielemann in reference 12 are only 0.10 to 0.12

In the LMSC analysis, as in the previous analyses, the Rayleigh-Ritz procedure is used to represent the elastic system in terms of a countable number of degrees of freedom. The Newton-Raphson iteration method was employed to obtain solutions to the nonlinear equation system, and an IBM 7090 computer was used in the numerical work. The number of degrees of freedom in the Rayleigh-Ritz analysis was successively increased until no significant change occurred in the magnitude of the minimum postbuckling equilibrium load.

Results of the analysis of reference 9 are shown in figure 2. The expression assumed for the radial displacement component is of the form:

$$w = \sum \sum a_{ij} \cos (i\, m\, \pi\, x) \cos (j\, n\, \pi\, y) \qquad (4)$$

where m,n are wavelength parameters, x,y are the axial and circumferential coordinates, and i,j are integers. Curve A in figure 2 represents the case in which all coefficients a_{ij} except a_{20}, a_{11}, a_{02} are set to zero, and coincides with the Kempner solution. By successively including additional degrees of freedom in the displacement function, equation (4), the results represented by curves B, C, and D are obtained. It may be seen that:

- the magnitude of the minimum postbuckling equilibrium load decreases as the number of degrees of freedom is increased,

- the rate of decrease diminishes as the number of degrees of freedom is increased,
- the minimum value found for the greatest number of degrees of freedom, Case D, is approximately $P/P_{CL} = 0.11$ and,
- this value is in close agreement with the corresponding experimental values (0.10 to 0.12) reported by Thielemann.

Reference 9 also presents information on the energy levels associated with various postbuckling equilibrium configurations.

Experimental investigation. - Of course, the objective of investigations of postbuckling behavior such as those in references 9 through 11 is to furnish information which may contribute to the ultimate establishment of an adequate buckling criterion. It is believed that extensive additional testing will be necessary before such a criterion can be formulated. Unfortunately many of the early tests of axially loaded cylinders were designed to determine simply the load level at which snap-through occurs. It is now widely recognized that experimental evidence is needed on a much broader range of questions, such as the influence of initial geometric imperfections of known form, the response to dead-weight loading, changes in buckle pattern during loading and unloading in the postbuckling region, etc. A test program designed to furnish certain information of this kind has been initiated at LMSC. It is believed that the results of such tests, together with continued theoretical effort, can ultimately serve as a rational basis for design of the axially loaded cylinder.

Pressurized and core-filled cylinders. - The analysis reported in reference 9 of theoretical postbuckling behavior under axial compression may be extended with relatively little difficulty to internally pressurized cylinders and to cylinders filled with a soft elastic core. This work also is in progress. Results in reference 12 and elsewhere indicate that the load range within which buckling is possible for internally pressurized cylinders (i.e., the range between the classical buckling load and the minimum postbuckling equilibrium load) diminishes with increasing internal pressure. The same tendency also may be expected with soft elastic cores. Hence, the choice of a buckling criterion in these cases may well be less critical.

FUTURE RESEARCH

Although the number of papers in the literature on structural shell stability has increased to a substantial volume, the present growth of aerospace technology is creating new problems at an even greater rate. A few shell stability problems which are of present interest to LMSC and for which satisfactory methods of analysis are not available are briefly listed below.

Methods of analysis for the influence of solid propellants on the buckling strength of motor cases are needed in which the propellant is treated as a viscoelastic rather than an elastic medium. Results are required for an entire class of load distributions and load-time profiles. Another broad class of problems of particular interest is that of structural instability under dynamic loading. Included in this category are impulsive loading of only a few microseconds duration. Within the field of ordinary static buckling analysis, insufficient information is available on the buckling of shells whose principal radii of curvature are functions of one or both of the shell coordinates, e.g., a deep ellipsoidal dome subjected to, say, uniform external pressure.

A particularly troublesome task for the structures analyst is the assessment of the effect of a large cutout on the buckling strength of a cylindrical or conical shell. Information seems to be lacking for both reinforced and unreinforced openings and for monocoque and stiffened shells.

The increasing use of filament-wound motor cases has given renewed emphasis to the need for information on the buckling of both orthotropic and bi-layered cylinders. Similarly, the immense size of booster fuel tanks for recently proposed vehicles has increased the interest in methods of analyzing cylinders of both orthotropic and sandwich construction.

Finally, in shell stability investigations, relatively greater emphasis must be given to experimental research. Although it is often more difficult to obtain funding for testing than for analysis, the uncertain reliability of small-deflection buckling theory makes the role of experimental data especially critical in shell applications.

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COMPARISON OF THEORY AND TESTS FOR BUCKLING UNDER CIRCUMFERENTIAL BAND OF PRESSURE

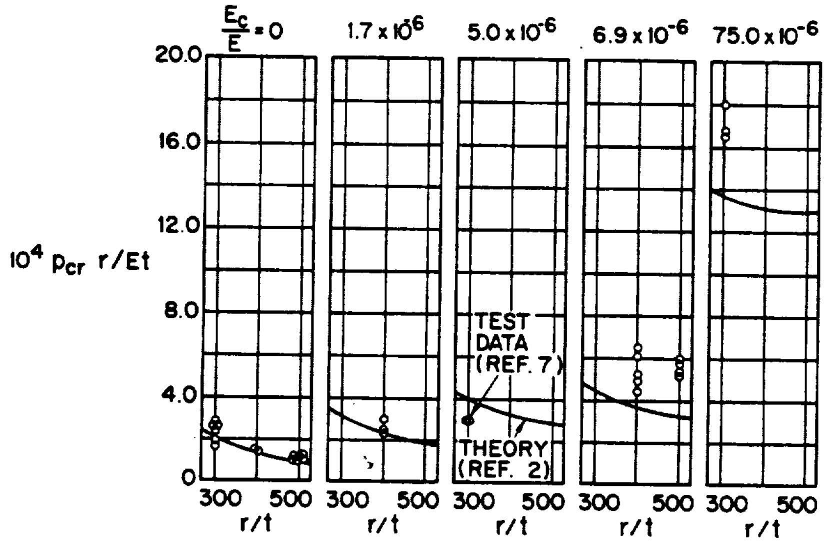

Figure 1

THEORETICAL LOAD-DISPLACEMENT CURVES FOR THE AXIALLY LOADED CYLINDER (FROM REF. 9)

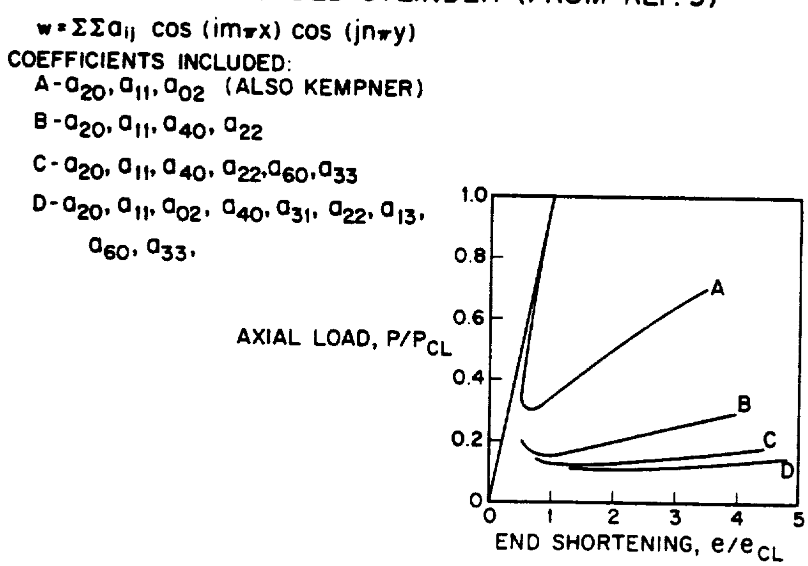

Figure 2