DEVELOPMENT OF DESIGN STRENGTH LEVELS FOR THE ELASTIC STABILITY OF MONOCOQUE CONES UNDER AXIAL COMPRESSION

By A. H. Hausrath and F. A. Dittoe

General Dynamics/Astronautics

L 3 1 0 9

SUMMARY

Design relationships have been derived for the determination of elastic buckling strength levels for unpressurized monocoque truncated cones under axial compression. Theoretical implications have been considered in the establishment of a semi-empirical analysis leading to the development of probability based design expressions. Data from 170 tests by various investigators were statistically evaluated for the expected mean, 90%, and 99% probability strength levels. Dispersion of data was found to be slightly less than that of monocoque cylinders. Non-linear effects of radius to thickness ratio or strength deterioration with length to radius ratio were not discernible.

INTRODUCTION

The elastic buckling of unpressurized monocoque right truncated circular cones under axial compression is analyzed statistically to establish design strength levels. The resulting prediction equations were derived from data in the ranges of semi-vertex angle from 2.87° to 75°, mid-height radius of curvature to thickness ratios of 98 to 4160, and slant height to mid-height radius of curvature ratios of 0.133 to 4.45. The resulting equations are intended for the design of cones where applicable in aerospace vehicles and other structures requiring high levels of structural reliability.

SYMBOLS

A	subscript indicating the "A" level.
"A" level	that level which would be exceeded by at least 99% of the entire population with 95% confidence; i.e., the confidence is 95% that at least 99% of the stability strengths of all

	cones in axial compression can be expected to exceed the "A" level.
B	subscript indicating the "B" level.
"B" level	that level which would be exceeded by at least 90% of the entire population with 95% confidence; i.e., the confidence is 95% that at least 90% of the stability strengths of all cones in axial compression can be expected to exceed the "B" level.
C	buckling coefficient; see eq. (2).
$\bar{C}$	mean value of C; see eq. (7).
E	material modulus of elasticity in compression.
e_i	theoretical frequencies for intervals in grouped data.
i	subscript indicating i^{th} value.
k	one-sided tolerance factor for the normal distribution; a function of sample size, probability level, and confidence level.
ℓ	slant height of conical frustum.
m	number of intervals in grouped data.
N	number of data values in sample.
n_i	observed frequencies for intervals in grouped data.
P_{cr}	axial compressive load, lbs., just prior to buckling.
$\bar{P}_{cr}$	expected mean value of P_{cr} .
$P_{cyl\,\infty}$	axisymmetric axial-buckling load of infinite cylinder with constant-thickness walls.
r	radius of cylindrical shell.
r_1	radius at small end of conical shell, measured in plane normal to axis of cone.
r_2	radius at large end of conical shell, measured in plane normal to axis of cone.
s	sample standard deviation; see eq. (8).

t	shell thickness.
α	semi-vertex angle of cone.
μ	mean value for a total population.
ν	Poisson's ratio.
ρ	radius of curvature.
$\bar{\rho}$	radius of curvature of conical shell at mid-height.
σ	standard deviation for a total population.
χ^2	statistical parameter indicating goodness-of-fit of data to a distribution function.

DISCUSSIONS

Justification for the Statistical Approach

Large variations in the actual buckling strengths of unpressurized truncated cones can be expected due to the unstable nature of pre-buckling load-deflection relationships for monocoque shells in compression. Cylinders, for example, tend to follow the familiar peaked load-deflection curve toward the bifurcation point and then into the post-buckling regime; however, to attain classical values of the buckling coefficient, all conditions (imperfections, eccentricities, end conditions, etc.) must be ideal. Since actual shells are subject to less than ideal conditions, the resulting buckling strengths will be not only significantly lower than theoretically indicated, but also widely dispersed. Existing data from cylinders are slightly more dispersed than from cones for axial compression and the response of spherical shells under external pressure shows similar scatter, probably for related reasons.

An examination of the test data presented in fig. 1 shows approximately equal scatter for all values of the semi-vertex angle. The cause of such variations in test results is not currently quantitatively defined. The high sensitivity to unmeasured or unmeasurable parameters makes an accurate analytical prediction of general instability strength improbable for the types of applied structure used in aerospace vehicles. Thus, in order to guarantee a reliable structure, statistical methods are used to establish practical design strength levels. These levels should not be considered as final. Analysis will eventually reveal the quantitative relationships between the buckling load and the above discussed conditions. This will permit a better evaluation of the test scatter and new statistical analyses resulting in more accurate prediction equations.

Designing at mean or typical strength levels acknowledges that 50% of structures so designed would fail before reaching design ultimate loads. A previous statistical analysis (ref. 1) indicates that 10% of cylinders designed to mean strength levels would fail at 75% or less of design ultimate strength in axial compression. A similar situation exists for cones.

Although the dispersion of test data for monocoque shells is generally recognized, mean expected strength levels are often erroneously advocated for design. "Eyeball" estimates of a lower bound to test data are also sometimes used, particularly when only a limited amount of test data are available. Unfortunately, the lower bound approach does not result in a quantitative evaluation of the reliability of the solution. Statistical analyses result in strength level estimates at desired levels of probability and confidence and thus make it possible to design to the required level of structural reliability.

This study endeavors to establish practical design relationships for the elastic stability of unpressurized monocoque cones under axial compression. The statistical levels adopted correspond to "A" and "B" levels as defined in SYMBOLS.

Test Data

The total number of pertinent tests known to the authors is 174. Refs. 2-6 contain data from 18 steel, 133 mylar, 15 nickel, and 8 aluminum cone tests. Although the preponderance of data are from the mylar specimens of ref. 2, the remaining data with four exceptions, fit into the mylar distribution satisfactorily, as may be seen in fig. 1. The value of mylar as a material for model stability test specimens was treated in ref. 2 where it was concluded that mylar was quite attractive for testing of this kind. A close scrutiny of the data reveals that the mylar results tend to be slightly higher than those of the metal specimens. Reasons for this may include experience gained in fabricating and testing a large number of mylar specimens and the low probability for local yielding of these specimens prior to buckling. The metal specimens were worked closer to their proportional limits and thus were more susceptible to failures precipitated by local yielding. In the cases of one aluminum and three nickel specimens, early failures may be directly attributed to local yielding. In the aluminum specimen, the failure load produced gross stresses in the cone wall above the material proportional limit while in the nickel tests there were questionable end conditions. These four tests were omitted from the statistical analysis but are shown in figs. 1 and 3 for reference purposes.

The remaining sample size of 170 consists of cones having $2.87^{\circ} \le \alpha \le 75^{\circ}$, $98 \le \bar{\rho}/t \le 4160$ and $0.133 \le \ell/\bar{\rho} \le 4.45$.

METHODS OF ANALYSIS

It is certainly to be recognized that from a statistician's point of view the sample population is far from ideal. The experiments were not statistically designed, there was no particular effort to randomize the combinations of geometrical parameters, and there are several obvious sources of bias in the sample population; however, distribution charts of $\ell/\bar{\rho}$ and α vs. $\bar{\rho}/t$ (tables 1 and 2) reveal some sampling in many of the usual ranges of interest.

The plot of the buckling coefficient C vs. $\bar{\rho}/t$ shown in fig. 1 discloses the most interesting implication that C is independent of $\bar{\rho}/t$. This finding is contrary to the conclusions of refs. 2, 3, 4, and 7 where functional relationships between C and some ρ/t were assumed similar to that for cylinders.

The theoretical result for cones presented in ref. 7 and affirmed in ref. 3 is

$$P_{cr} = P_{cyl\,\infty} \cos^2 \alpha \tag{1}$$

which may be written

$$P_{cr} = C\ 2\pi E t^2 \cos^2 \alpha \tag{2}$$

The well-known classical value for C is

$$C = \frac{1}{\sqrt{3(1-\nu^2)}} \tag{3}$$

which, for $\nu = 0.3$, is 0.605. However, experimental results for cylinders are such that $0.07 \lesssim C \lesssim 0.5$ and that C is strongly dependent upon radius/thickness ratios. The experimental evidence for cones is presently such that $0.194 \lesssim C \lesssim 0.478$ and that C is independent of radius/thickness ratios, at least for the range of data available. Additional influences of α or length effects are not discernible from plots of cone data.

An examination of the frequency distribution of C by means of a histogram (fig. 2) indicates near normalcy. If the variations in C could be attributed to eccentricities in loading and specimen geometry, local irregularities, etc., and if each of these could be assumed to occur in a random manner, then it can be shown that the values of C would be expected to be normally distributed.

The normal or Gaussian distribution function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \qquad (4)$$

so that

$$\int_a^\infty f(x)dx = \text{probability that } x > a \qquad (5)$$

In eq. (4), μ and σ are respectively the mean value and standard deviation for a total (infinite) population. As is the usual case, only estimates of μ and σ are available. These are the sample mean, $\bar{x}$, and the sample standard deviation, s, which were used to obtain the fitted normal curve shown in fig. 2 for comparison with the histogram.

An analytical evaluation of the conformity of the grouped data to the normal distribution was conducted using the χ^2 test. This test, discussed in most texts on basic statistics, consists of evaluating the parameter χ^2 from eq. (6) and comparing the result to tabulated percentage points for the χ^2 distribution. If the calculated value of χ^2 is less than the value tabulated for the applicable significance level and number of degrees of freedom, then there is no reason to reject the hypothesis that the data are from a normal population.

$$\chi^2 = \sum_{i=1}^{m} \frac{(n_i - e_i)^2}{e_i} \qquad (6)$$

In eq. (6), m is the number of intervals over which the summation takes place, n_i are the observed frequencies in the intervals, and e_i are the theoretical frequencies for the intervals from the fitted normal distribution.

The χ^2 goodness-of-fit test was conducted for the test data and resulted in the conclusion that at the 5% level of significance, the sample distribution is consistent with the hypothesis that the parent distribution is normal. The 5% level of significance, generally used for χ^2 tests for normality, was generously exceeded by the data. For the purposes of this paper, the values of C were thus assumed to be normally distributed.

The mean value of C and standard deviation were obtained from the 170 data values using eqs. (7) and (8).

$$\overline{C} = \frac{1}{N} \sum_{i=1}^{N} C_i \tag{7}$$

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\overline{C}-C_i)^2} \tag{8}$$

The "B" and "A" level estimates of C were then calculated from

$$C_B = \overline{C} - k_B s \tag{9}$$

$$C_A = \overline{C} - k_A s \tag{10}$$

where k_B and k_A are the proper probability tolerance factors. Values of k have been calculated using procedures outlined in chapter 1 of ref. 8 and tabulated in ref. 9. The values of k were calculated for the normal distribution such that the confidence is 95% that at least the desired proportions will exceed the "B" and "A" levels. For 169 degrees of freedom, $k_B = 1.465$ and $k_A = 2.592$.

The results of calculations using eqs. (7) through (10) are:

$$\overline{C} = 0.316 \tag{11}$$

$$s = 0.06077 \tag{12}$$

$$C_B = 0.227 \tag{13}$$

$$C_A = 0.158 \tag{14}$$

Plots of the expected mean, "B", and "A" levels are shown in figs. 1, 2, and 3 for comparison with available data.

RECOMMENDATIONS

The "A" and "B" levels are recommended for the practical design of unpressurized monocoque cones critical in buckling due to axial compression. The "A" level, eq. (14), is recommended for use for those structures the single failure of which could result in catastrophic loss or injury to personnel. The "B" level, eq. (13), may be used for structures not requiring the "A" level.

Application of eqs. (13) and (14) as coefficients for eq. (2) should be limited to cones having the following approximate geometries: $100 < \bar{\rho}/t < 4000$, $10^\circ < \alpha < 75^\circ$, and $0.25 < \ell/\bar{\rho} < 5$. Table 1 and 2 may be consulted to determine the number of known tests for a desired cone geometry. The stress level at the small end of the cone should be checked to preclude the possibility of an early failure precipitated by inelastic stresses.

Structural substantiation tests should be conducted on cones designed by the use of eqs. (2), (13), and (14) because of the influences of fabrication techniques, size, and end conditions in each particular design.

FUTURE RESEARCH

Additional testing of cones should be conducted in the sparsely populated ranges of tables 1 and 2. Of particular interest would be additional elastic tests in the high and low $\bar{\rho}/t$ and small α groups. These could permit a better understanding of the present experimentally indicated independence of buckling coefficient with radius/thickness ratio. The influence of end conditions on buckling of cones would seem to be greater than for cylinders and, if adequately evaluated, might be used to reduce the scatter of test data. Tests of larger and longer specimens should indicate the effects of size and length which are not discernible from existing data.

Accurate determinations of the compressive modulus of elasticity of specimen materials should be reported in all future stability test reports. Although difficult to obtain for thin gages, this information is helpful in statistically evaluating test data and may, in fact, be responsible for a significant amount of scatter in the existing data.

CONCLUDING REMARKS

For practical design purposes, reliable design buckling load levels may be established if sufficient data exist. Recent shell stability testing has greatly enhanced the fact that for unpressurized monocoque shells, existing theoretical solutions are unrealistic for design. The use of mean expected buckling strengths is also unconservative while lower bound estimates are of unknown reliability. Statistically determined allowable strength levels acknowledge the inevitable scatter of test data and permit the estimation of strengths at desired levels of probability and confidence. The application of statistics to other loading conditions and shell configurations would be desirable when enough data are available.

REFERENCES

1. Schumacher, J. G.: Development of Design Curves for the Stability of Thin Pressurized and Unpressurized Circular Cylinders. General Dynamics/Astronautics report no. AZS-27-275B, May 8, 1959, revised July 22, 1960.

2. Seide, P., Weingarten, V. I., and Morgan, E. J.: Final Report on Development of Design Criteria for Elastic Stability of Thin Shell Structures. Space Technology Laboratories, Inc. report no. STL/TR-60-0000-19425, Dec. 31, 1960.

3. Lackman, L., and Penzien, J.: Buckling of Circular Cones Under Axial Compression. Journal of Applied Mechanics, vol. 27, no. 3, Sept. 1960.

4. Homewood, R. H., Brine, A. C., and Johnson, A. E. Jr.: Buckling Instability of Monocoque Shells. AVCO Corporation report no. RAD-TR-9-59-20, August 18, 1959.

5. Lofblad, R. P. Jr.: Elastic Stability of Thin-Walled Cylinders and Cones with Internal Pressure Under Axial Compression. Mass. Inst. of Tech. ASRL TR 25-29, May 1959.

6. Lundquist, E. E., and Schuette, E. H.: Strength Tests of Thin-Wall Truncated Cones of Circular Section. N.A.C.A. report WRL-442, Dec. 1942.

7. Seide, P.: Axisymmetrical Buckling of Circular Cones Under Axial Compression. Journal of Applied Mechanics, vol. 23, no. 4, Dec. 1956.

8. Eisenhart, C., Hastay, M. W., and Wallis, W. A.: Selected Techniques of Statistical Analysis for Scientific and Industrial Research and Production and Management Engineering. McGraw-Hill Book Co., 1947.

9. Schumacher, J. G.: Statistical Determination of Strength Properties. General Dynamics/Astronautics report no. AZS-27-274A, Nov. 11, 1958, revised Dec. 29, 1959.

TABLE 1

DISTRIBUTION CHART $\bar{\rho}/t$ VS $\ell/\bar{\rho}$

$\bar{\rho}/t$	$\ell/\bar{\rho}$										
	0.12	0.25	0.5	0.75	1.0	1.5	2	3	5	≥8	Σ
200			1			5					6
300			2			2	1	3			8
450			4		6	17	7	4			38
650			2	3	3		1	5			14
900			1	2	3	11	5	5	1		28
1,400	2	5	14	8	6	12	2	4			53
2,000		3	6	4		3	2	2			20
3,000					2						2
4,000		1									1
Σ	2	9	30	17	20	50	18	23	1		170

TABLE 2

DISTRIBUTION CHART $\bar{\rho}/t$ VS α

$\bar{\rho}/t$	α (DEG.)								
	3	10	15	20	30	45	60	75	Σ
200			3	2		1			6
300				5	1	2			8
450		6		7	14	11			38
650		5		3	1		5		14
900	1	5		5	8	7	2		28
1,400		6		4	7	13	22	1	53
2,000				4	9	2	2	3	20
3,000						2			2
4,000								1	1
Σ	1	22	3	30	40	38	31	5	170

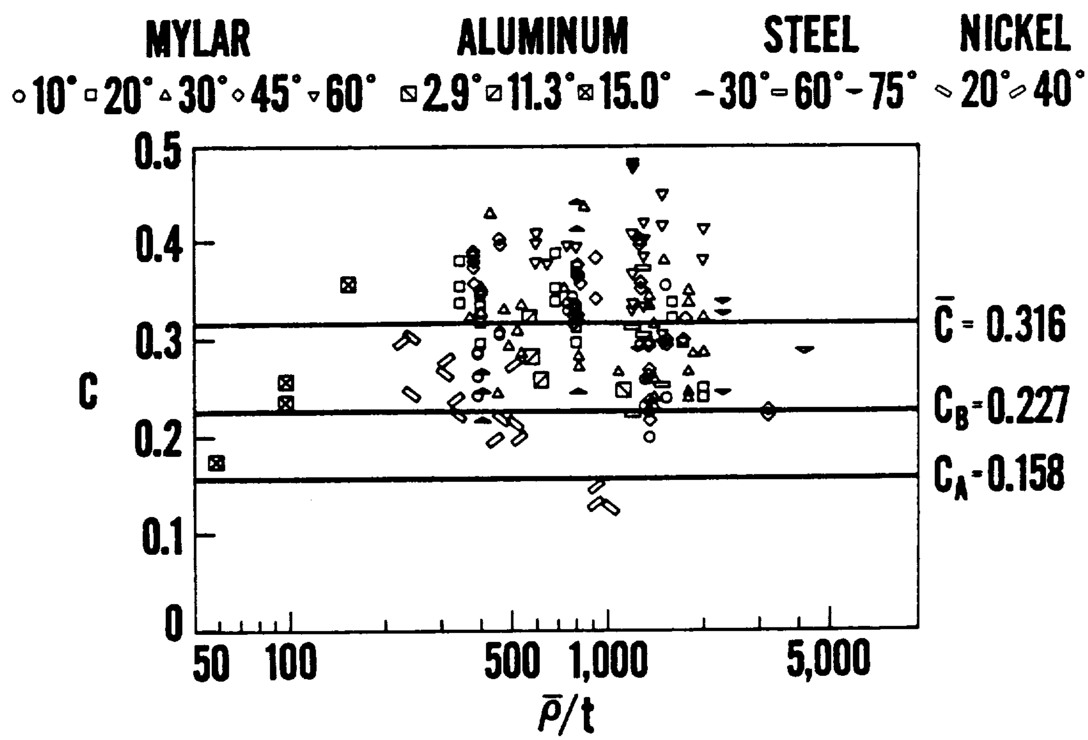

Figure 1.- Experimental evidence of the independence of buckling coefficient with respect to mean radius of curvature/thickness ratio in tested region.

CONE DATA

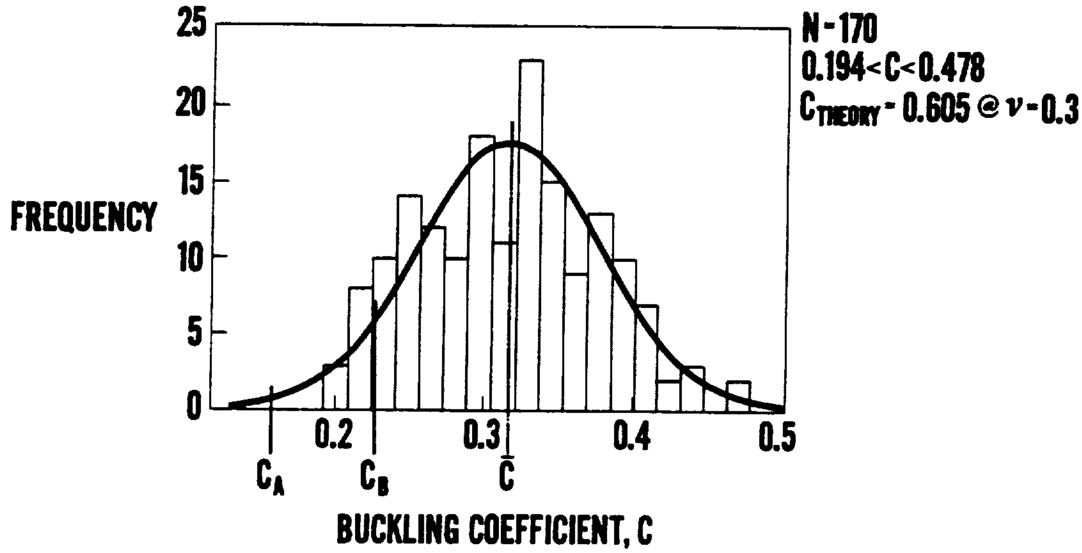

Figure 2.- Histogram of cone data and fitted normal curve.

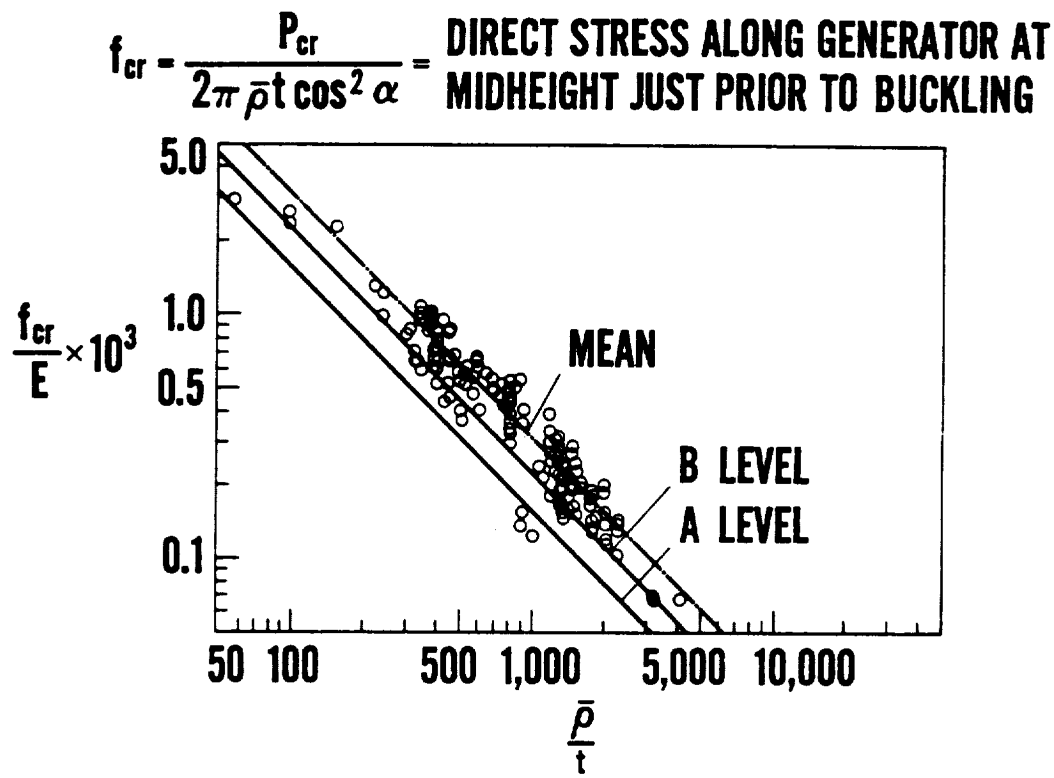

Figure 3.- Non-dimensional comparison of cone data with statistical levels.