PROBLEMS ASSOCIATED WITH THE DESIGN OF LARGE SHELL STRUCTURE

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SUMMARY

An account of the approach used in design of a large sandwich construction nose cone is given. The areas in which analytical methods are needed are discussed and a theoretical procedure is proposed.

INTRODUCTION

Many of the space exploration probes and satellites are housed on the nose of large missiles. These payloads require protection during the launch phase and the missile requires fairing to reduce aerodynamic drag. Thus, large jettisonable nose panels are needed in many applications. The panels must possess integral stiffness since there is little or no room for supporting structure. These requirements have resulted in the design of large "split-cone" sections of sandwich construction. Satisfactory methods of analysis do not exist for this type of structure.

SYMBOLS*

x, r, z, p	Linear coordinates
θ, ϕ	Angular coordinates
a, b, c, d, L, L', B	Dimensions
u, v, w, q, v', s	Displacements
$I, J, F, G, N, M, K, K', H$	Displacement parameters

A (with proper subscript)	Coefficients of displacement parameters
f, g, i, j, k, l, m, n, h	Integers or subscripts

*See also figures 1 and 2.

DESIGN APPROACH

The task under consideration is to design a nose fairing for a 10 foot diameter missile. The purpose of the fairing is to reduce vehicle drag, and to protect the payload from heat and airloads during the boost phase of the flight. A cone-on-cylinder configuration is chosen for the proposed design. The 10 foot diameter cylinder portion is 10 feet long. The conical portion is 16 feet long with a taper of 14.5° per side. There are no internal frames or bulkheads forward of the base. To facilitate jettisoning in flight, the fairing is fabricated in halves. The half shells are then held mated by a few explosive fasteners along the vertical split lines.

The air pressure or loads act in a crushing direction and produce axial and hoop compressive stresses in the shell wall. Since the airload is not uniform, shear and bending are also present. In addition the shell must be able to withstand thermal stresses resulting from rather severe aerodynamic heating.

Environmental conditions indicate the use of fiberglass sandwich for shell wall construction. The designer must then attempt to establish the required core and laminate thickness to provide the required strength and stability. He can satisfy these by the use of conventional stress equations but he soon finds that these apply to complete cylinders of solid sheet material. The cylinders treated are considered very long or assumed held round at intervals. Only a small amount of information exists on sandwich cylinder allowables, and in each case the author points out the need of further investigation and testing to establish valid design allowables.

In this particular design the engineer does not have a complete cylinder. He has two half cone-cylinders, since moment continuity is lost at the split lines. The knee area, or juncture between the cone and cylinder, acts as a stiffening ring to some degree. It divides the shell into two distinct bays which might buckle independently if the juncture is stiff enough. The designer finds very little information on the required stiffness of reinforcing rings for cases of this nature.

To determine the elastic buckling pressure of the conical portion of the half shell, the designer will again be forced to use complete cylinder formulas and use assumed values for effective length and radius. The loss in strength due to the split line and the absence of a rigid ring cannot be adequately predicted.

The airloads on a ring (a unit length of shell) can be divided into a uniform crushing component and an asymmetrical component. The uniform component is reacted by hoop compression in the ring and no bending is present. The asymmetrical component induces a shear reaction in the shell, and a ring flattening effect is expected. A quick analysis shows that a shear reaction of the classical VQ/I type produces no ring bending with the asymmetrical airload. However, further checking shows that only slight variations in the distribution of the applied pressure or the reacting shear will produce very high computed ring flattening deflections and bending moments. Thus the stiffness required to prevent flattening becomes a major unknown.

In most such design projects, the designer is aware of the vehicle performance penalty for unnecessary structural weight. Yet he is faced with a schedule and budget that does not permit developmental testing. A nose cone failure could waste a missile and launch effort costing millions of dollars. If a designer is to have a measure of confidence in his product, under these circumstances, the finished hardware is sure to be unnecessarily heavy.

For example, previous analyses of this type panel have treated the problem on an "equivalent-thickness" basis. The sandwich panel is reduced to an equivalent thickness of solid metal and compared with cylinder data of the same effective length, radius, and mean radial pressure distribution. Margins of safety of 50% have been allowed to account for the effects of the split line. Such procedures are believed to result in excessive sandwich core weight.

The cone-on-cylinder shell discussed here is believed to be useful enough to be worthy of further research. The author knows of four existing satellite programs which utilize this shape, and several more that are under consideration.

ADDITIONAL THEORETICAL INVESTIGATION

A limited investigation was initiated to determine if the critical buckling load of the nose cone could be found by theoretical means. For this purpose a structural idealization of the nose cone was made considering its physical features. The configuration was a sandwich shell composed of a spherical nose cap, a conical section, and a cylindrical section. The shell is split into two halves to facilitate jettisoning; the edges of the half shells are reinforced along the parting line by fairly stiff beams. The shell is supported at its base by the missile body and just below its spherical cap by stout bulkheads.

Only the cone-cylinder sections need be considered in a stability analysis. At the juncture of the cone and cylinder the discontinuity angle of the shell plus a local thickening of the sandwich acts as a stiffening ring. Since it could not be decided initially if this stiffening was sufficient to cause the cone and cylinder to buckle as individual elements, provision for buckling of the combination was included in the analysis. It was also decided to include the discontinuity of bending moment along the shell parting line although the effect of this now appears of somewhat minor consequence. It was felt that an essential feature in this analysis should be the low transverse shear stiffness of sandwich construction.

Before attempting an analysis, the load-temperature-time history for the nose cone was considered. It was found that the design loads occurred at near room temperature and were followed at some later time by the peak thermal gradient during which the applied loads were nearly zero. The distribution of pressure and temperature is practically uniform over the conical and cylindrical segments under consideration. Although the design pressures can occur at a slight angle of incidence, it is felt that this affects the bending only since the instability is primarily a function of the mean radial pressure distribution.

Having made the above idealizations, some time was spent in a brief review of applicable literature. Papers on thin cones and cylinders, and on sandwich cylinders were reviewed. From the survey it became apparent that the only feasible analytical approach would be an approximate one based upon an energy method. The inherent hazard of such an approach is that it leads to unconservative values of critical loads if the expression for deformation does not admit the true buckled shape (reference 1, page 90). Several authors state that satisfactory solutions for cylinders subjected to hydrostatic pressure can be obtained with small deflection theory and others suggest that for sandwich construction snap-through type buckling is unlikely (sec. 1.1, ref. 1, p. 498; ref. 2, p. 49; ref. 3, p. 2). Although no such statements were found for cones, there appears no reason for their behaving differently from cylinders as long as the cone angle is not large. With these assumptions it is possible to follow the procedure of reference 4, which also makes use of reference 5, in computing the critical buckling load for the nose cone. The detailed steps taken and the equations are much too lengthy for presentation here so only a brief outline of principles will be shown. The geometry of the cone is shown in figure 1.

Strain components for the cylindrical portion of the shell can be expressed in terms of displacements as equations 12.56 of reference 6. However, in addition to u, v, and w, terms involving transverse shearing strains $\gamma_{\theta x}$ and $\gamma_{\theta z}$ omitted in equations 12.47 (ref. 6) must be included in the last three equations of 12.57 (ref. 6) to account for the low shear modulus of the sandwich core. Similar strain expressions for the cone may be derived in terms of displacements s, v', and q and shear strains γ_{rp} and $\gamma_{\theta p}$.

The normal deflection (q, w) assumed for each half shell was the sum of the three shapes shown in longitudinal section in figure 2.

The expressions used are:

$$w = (F)\ (M) + (G)\ (L) + (N)\ (K)$$

$$q = (F)\ (I) + (H)\ (J) + (N)\ (K')$$

Where:

$$F = \sum_{f} \left[- \overbrace{\frac{6}{3\pi-16} \frac{A_f}{f}}^{*} + \overbrace{\frac{8\sqrt{2}}{3\pi-16} \frac{A_f}{f} \operatorname{Sin} \left|\frac{\theta}{2} + \frac{\pi}{4}\right|}^{*} + A_f \operatorname{Sin} f\theta \right]$$

$$G = \sum_{g} \left[- \overbrace{\frac{6}{3\pi-16} \frac{A_g}{g}}^{*} + \overbrace{\frac{8\sqrt{2}}{3\pi-16} \frac{A_g}{g} \operatorname{Sin} \left|\frac{\theta}{2} + \frac{\pi}{4}\right|}^{*} + A_g \operatorname{Sin} g\theta \right]$$

$$N = \sum_{n} \left[- \overbrace{\frac{6}{3\pi-16} \frac{A_n}{n}}^{*} + \overbrace{\frac{8\sqrt{2}}{3\pi-16} \frac{A_n}{n} \operatorname{Sin} \left|\frac{\theta}{2} + \frac{\pi}{4}\right|}^{*} + A_n \operatorname{Sin} n\theta \right]$$

$$M = \sum_{m} A_m \operatorname{Sin} \frac{m\pi(x-c)}{(d-c)}$$

$$L = \sum_{\ell} A_{\ell} \operatorname{Sin} \frac{\ell\pi(x-c)}{(d-c)} \operatorname{Sin} \frac{\pi(x-c)}{2(d-c)}$$

$$K = \sum_{k} A_k \operatorname{Sin} \frac{k\pi(x-b)}{(d-b)}$$

$$H = \sum_{h} \left[- \overbrace{\frac{6}{3\pi-16} \frac{A_h}{h}}^{*} + \overbrace{\frac{8\sqrt{2}}{3\pi-16} \frac{A_h}{h} \operatorname{Sin} \left|\frac{\theta}{2} + \frac{\pi}{4}\right|}^{*} + A_n \operatorname{Sin} h\theta \right]$$

*These terms are zero for f, g, h, n = even, i.e., a continuous shell.

$$I = \sum_i A_i \operatorname{Sin} \frac{i\pi(r-B)}{(L-B)} = \sum_i A_i \operatorname{Sin} \frac{i\pi(x-b)}{(c-b)}$$

$$J = \sum_j A_j \operatorname{Sin} \frac{j\pi(r-B)}{(L-B)} \operatorname{Cos} \frac{\pi(r-B)}{2(L-B)}$$

$$= \sum_j A_j \operatorname{Sin} \frac{j\pi(x-b)}{(c-b)} \operatorname{Cos} \frac{\pi(x-b)}{2(c-b)}$$

$$K' = \sum_k A'_k \operatorname{Sin} \frac{k\pi(r-B)}{(L'-B)} = \sum_k \frac{1}{\operatorname{Cos}\phi} A_k \operatorname{Sin} \frac{k\pi(x-b)}{(d-b)}$$

The above expressions for q and w were found to satisfy all boundary conditions provided shear strains $\gamma_{\theta x}$, $\gamma_{\theta z}$, γ_{rp}, $\gamma_{\theta p}$, and Poissons ratio effects in the strain component expressions previously derived are neglected.

The strain energy of the sandwich cylinder and core, due to deflections q, w, are determined similarly to the method of reference 4. It was found more convenient to take the reference surface at the core mid-height. An expression of similar form to that of equation 47 (ref. 4) is found for the strain energy of the shell, and the flexural energy of the edge beams of the nose cone is added to the term corresponding to $B_{15'}$ of reference 6.

In applying the method of minimizing the total potential, only the change of load potential during buckling need be found. Part of this change comes from the induced radial compression in the nose cone and part from the axial force on the shell. The former can be found using the trick of reference 1, page 288. The equivalent load is the quantity on the right side of equation (a). The quantity in parenthesis is replaced with the radial curvatures from the expressions for strain components in the nose cone. Radial displacement terms can be eliminated by consideration of inextensible buckling. Longitudinal displacements can be expressed in terms of the "tilting" factors and neutral surface ordinates used for strain energy expressions. For the second part of the change in external energy, the average shortening of an elemental length of genetrix can be taken as

$$1/2 \left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] \text{ or } 1/2 \left[\left(\frac{\partial q}{\partial r}\right)^2 + \left(\frac{\partial v'}{\partial r}\right)^2\right]$$

where terms in v, v' must be again expressed in terms of "tilting" factors and neutral surface ordinates.

The change in total potential is taken as the sum of load potential and strain energy. Arranging terms involving "tilting" factors and neutral surface ordinates on the right-hand side, these parameters can be eliminated by minimization and result in a right side expression similar to that of equation 59 (ref. 4).

Further minimization of the change in total potential with respect to A_cA_m, A_gA_e, A_fA_i, A_hA_j, and A_nA_k results in five simultaneous equations containing q_{cr} (pressure) and the integers f, g, h, i, j, k, l, m, and n. These equations are much too lengthy to show here. They also are much too complex to solve manually and should be programmed onto a large computer.

Due to the time schedule of the particular nose cone being designed, there was insufficient time to attempt solution on the IBM 7090 computer available at this facility. However, this would be an ideal tool to use in this analysis. It is completely feasible and practical to obtain an IBM solution to the equations. The problem is common enough throughout the industry to warrant this research and the majority of the aerospace companies who would use such data have facilities for processing a computer program.

CONCLUDING REMARKS

Tying such a theoretical analysis development program to an experimental investigation of sandwich nose cone models would make an excellent research program. For instance, it is strongly suspected from the tests of thin metal models that the k-mode displacements are unnecessary in the analysis and would affect large simplifications in the analysis by being omitted. At the same time, there also exists the possibility that tests of sandwich shells might reveal that some other displacement mode is required although this does not appear likely from tests to date. In either event, completion of the analysis and test corroboration would give us a very useful design tool.

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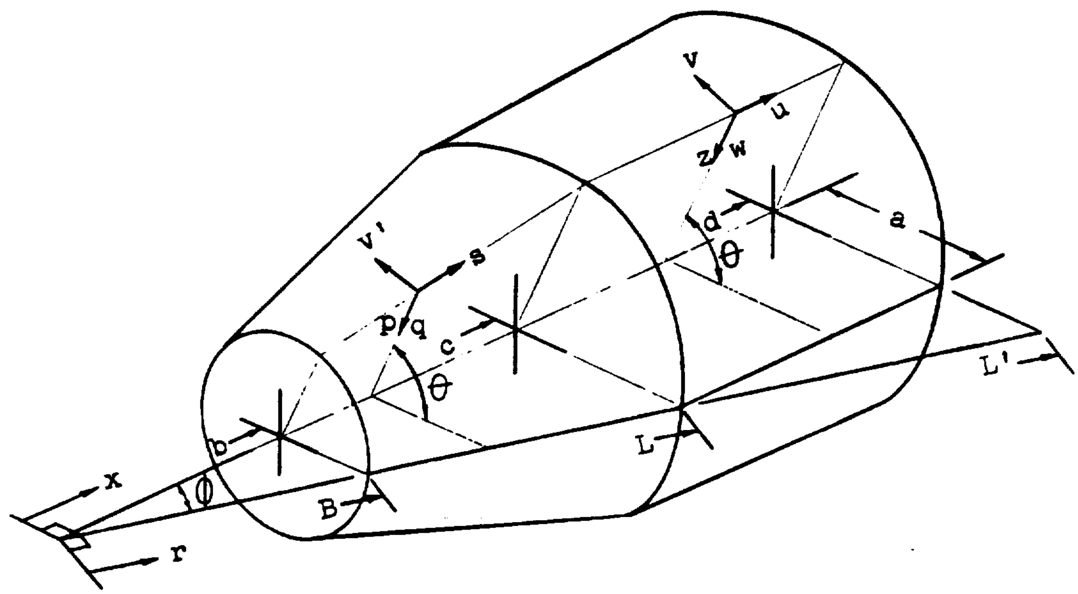

Figure 1.

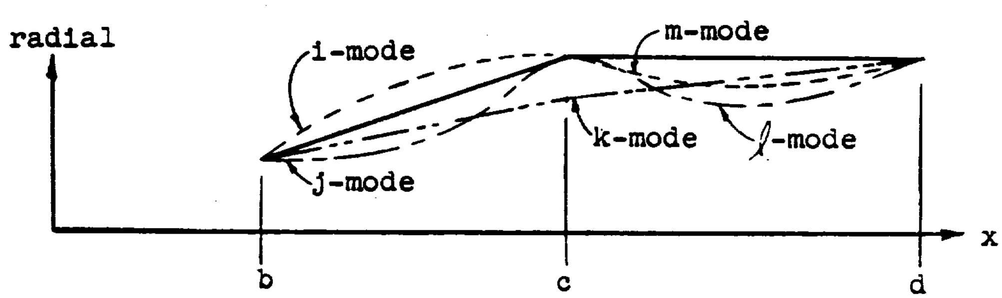

Figure 2.