THE EFFECT OF INITIAL IMPERFECTIONS ON THE BUCKLING STRESS OF CYLINDRICAL SHELLS

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SUMMARY

Techniques have been developed for making essentially "perfect" thin cylindrical shells and for making shells with definite types of initial deformations. The "perfect" shells give buckling stresses much higher than have previously been obtained. Initial deformations of the form $\Delta R = a_o \sin \frac{\pi x}{L}$ have been tested and ranges of positive a_o were found in which there was no lowering of the buckling stress. Negative values of a_o caused a decrease in the failure load. A theoretical solution which indicated the same trends as were found experimentally has been carried out.

L
3
1
5
9

INTRODUCTION

Ever since the great disagreement between theory and experiment on the buckling stress of axially compressed cylinders was discovered efforts have been made to determine the cause of this disagreement. One approach was to refine the theoretical analysis to determine 1) whether or not the lowest eigenvalue has been established and 2) whether initial deformations might have an effect on the buckling stress. These studies showed that, if the cylinder was assumed perfect and, if end effects were neglected, the buckling stress was given by $\sigma_{cr} = Et/R\sqrt{3(1 - \mu^2)}$ which, for $\mu = 0.3$ becomes $\sigma_{cr} = 0.605\ Et/R$. Secondly, the work of Donnell and others indicated that there was probably a considerable effect from initial deformations. In these studies it was assumed that the initial deformations had essentially the wave form of the post buckling mode and no attempt was made to determine the effect of other types of initial deformations.

The experimental investigations that were carried out were never as good as the theoretical studies. The specimens contained eccentricities and discontinuities from seams and other fabrication techniques, had unknown internal stresses, and the testing methods in most cases were relatively crude. For example, the actual stress distribution in the specimen was seldom measured during test. For the above reasons it was felt desirable to carry out carefully controlled tests on specimens whose imperfections were accurately known and to correlate the data obtained with new theoretical studies. The work was carried out under NASA research grant no. NsG-18-59.

SYMBOLS

a_o	maximum amplitude of initial deformation, inches
D	plate stiffness $= Et^3/12(1 - \mu^2)$ lb inch
E	Young's modulus, psi
F	stress function defined as $F_{xx} = N_y$, $F_{yy} = N_x$, $F_{xy} = -N_{xy}$
K	buckling stress coefficient in the equation $\sigma_{cr} = KEt/R$
L	length of shell, inches
N_x, N_y, N_{xy}	membrane forces, lb/inch
R	shell radius, inches
ΔR	radial value of initial deformation, inches
t	shell thickness, inches
w	radial displacement of shell, inches
x	axial distance along the shell, inches
y	circumferential distance around shell, inches
μ	Poisson's ratio
σ_{cr}	buckling stress of the shell, psi

Subscripts:

xx, xy, yy	denote $\partial/\partial x^2$, $\partial/\partial xy$, $\partial/\partial y^2$

THE EXPERIMENTAL PROGRAM

The objectives of this program were:

1) to develop a method of making cylindrical shells which were as nearly perfect as possible;
2) to establish testing techniques in which the stress distribution was controllable and measurable;
3) to determine the axial buckling stress of these nearly perfect shells
4) to add known imperfections to the shells and determine the effect of these imperfections on the buckling stress; and
5) to attempt to isolate the most damaging forms of initial deformations so they could be avoided in manufacture whenever possible.

To date, items 1) through 3) have been completed and some work has been carried out in 4).

The method of manufacturing of the cylindrical shells is not new with GALCIT (see reference 1) but a number of refinements were required in order to obtain the uniformity and accuracy desired. Basically it consisted of plating a copper shell on an accurately machined wax mandrel and then melting the mandrel out of the shell. For a shell 8 inches in diameter and 10 inches long, a steel cylinder 7 inches in diameter and 13 inches long was used as a core (this provided a means of water cooling the wax to harden it). On this core was cast a layer of wax consisting of a two to one mixture of refined paraffin and Mobile Cerese Wax 2305. This was cooled, machined to the proper shape, and sprayed with silver paint thinned with toluene. The plating was done in a Cupric Fluoborate, $Cu(BF_4)_2$, bath with a 15 inch diameter copper anode which was bagged in fine mesh Dynel fabric. During plating the mandrel was rotated and the bath was given additional agitation by forcing air through it. Using voltages less than 10 volts and current densities up to 55 amperes per square foot, the plating time was approximately 20 minutes per 0.001 inches of plate.

After plating, the ends of the cylinder were removed (some thickness increase occurred just at the ends), the wax was carefully melted out, and the residual wax and silver paint was removed with benzene. The shell thickness used in this series of tests was approximately 0.0045 inches. The thickness variation in any shell could be held to less than 3 per cent. In the radial direction, the desired radius could be held to $\pm$ t/2 or approximately $\pm$ 0.0025 inches.

The specimen was mounted (using Cerrobend) to a load measuring ring instrumented with 24 strain gages which gave the stress distribution in the shell. The assembly of specimen and loading ring were then placed in a very stiff, controlled displacement testing machine in which the displacement was controlled by three fine pitch screws (one turn of the screw gave 0.025 inches displacement). These could either be operated together or could be adjusted individually to correct for nonuniformity in stress distribution. See figure 1.

Two types of axially symmetric initial deformations were tested, a sine wave in which $\Delta R = a_o \sin \pi x/L$ and a constant curvature form given by $\Delta R = a_o \left[2(2x/L) - (2x/L)^2\right]$. Values of a_o/t ranging from -20 to +45 were tested.

EXPERIMENTAL RESULTS

Figure 2 shows the results of this first test series in which σ_{cr}/σ_{cl} is plotted against a_o/t. These are compared with the Kanemitsu and Nojima value which for the L/R and the R/t corresponding to these specimens gives $K = 0.17$. The following important features can be seen:

1. That with proper care in manufacturing and testing, values of the buckling stresses can be obtained which are much higher than those usually found.

2. That, for the displacement forms tested, small departures from initial straightness lower the buckling stress and that the effect for inward displacements is greater than that for outward displacements.
3. That if the outward displacements are increased the value of the buckling stress again rises until it reaches essentially the same value as that for the initially straight cylinder.
4. That the constant curvature and sine wave shapes give essentially the same values of σ_{cr} for the larger values of a_o/t.

The drop in the value of σ_{cr} for $0 < a_o/t < 10$ and the return to the "perfect" value for $a_o/t > 20$ cannot be explained at this time. Although considerable effort was made to keep the stress uniform around the circumference of the cylinder, variations of $\pm$ 10 per cent were common and this probably accounts for some of the scatter in the experimental results. Since the classical theoretical solutions have assumed edges which were free to expand radially and, since the edges of the present cylinder were rigidly clamped, it may be that the lowering of the value of σ_{cr} below the classical value may be due in part to this fact.

THEORETICAL SOLUTION

The theoretical approach was as follows:

1) Determination of the stresses and deflections in the shell that occur during the loading and before buckling.
2) Consideration of the stresses and deflections occurring during buckling as small perturbations about step 1) and linearizing the resulting equations.
3) Solving the eigenvalue problem obtained in 2) for the minimum eigenvalue.

To simplify the problem, the shallow shell equations of Marguerre (reference 2) were used. These give

$$\nabla^4 F = Et\left[w_{xy}^2 - w_{xx}w_{yy} + 2w_{o_{xy}}w_{xy} - w_{o_{xx}}w_{yy} - w_{xx}w_{o_{yy}}\right] \tag{1}$$

$$D\nabla^4 w = F_{yy}(w_{xx} + w_{o_{xx}}) - 2F_{xy}(w_{xy} + w_{o_{xy}}) + F_{xx}(w_{yy} + w_{o_{yy}}) \tag{2}$$

where w_o is the initial deviation from the flat plate and F is the stress function. For the circular cylindrical shell, w_o is given by $w_o = (y^2 - y^2)/2R$. Using this equation for w_o one arrives at the set of equations originally derived by Donnell and used in his studies of initial imperfections (reference 3). For the initial deformations of the present tests the equation for w_o becomes

$$w_o = \frac{1}{2R}(y^2 - y_o^2) - a_o \sin\frac{\pi x}{L} \quad (3)$$

which when substituted into equations 1 and 2 gives

$$\nabla^4 F = Et\left[w_{xy}^2 - w_{xx}w_{yy} - a_o\left(\frac{\pi}{L}\right)^2 \sin\frac{\pi x}{L}\, w_{yy} - \frac{1}{R}w_{xx}\right] \quad (4)$$

$$D\nabla^4 w = F_{yy}\left(w_{xx} + a_o\left(\frac{\pi}{L}\right)^2 \sin\frac{\pi x}{L}\right) - 2F_{xy}w_{xy} + F_{xx}w_{yy} + \frac{1}{R}F_{xx} \quad (5)$$

Letting F^* and w^* be the solutions of the axially symmetric problem and $\bar{F}$ and $\bar{w}$ be the perturbation values occurring during buckling, we have $F = F^* + \bar{F}$ and $w = w^* + \bar{w}$. Substituting into 4 and 5 and linearizing by neglecting higher order terms one obtains a set of equations in $\bar{F}$ and $\bar{w}$ which can be solved for the eigenvalues. The boundary conditions used were: 1) The ends of the shell are free to expand radially and, while so doing, they are simply supported; 2) for the axially symmetric problem u^* and w^* (deflection in axial and radial directions) must be independent of y and v^* (circumferential displacement) must be zero: 3) Axial motion of the ends of the shell (usually a periodic function in y) is permitted.

Carrying out the remainder of the mathematics leads to the dotted curve shown in figure 2 showing that, for $a_o > 0$ no decrease in buckling stress should occur and that for $a_o < 0$ an appreciable drop in load carrying ability should be observed. Comparing this with the experimental data we find that the trends are correct but that the reduction in σ_{cr} for $a_o < 0$ is actually less than that predicted theoretically. Also, it was found that the number of circumferential waves was higher than the theory predicts and, if the experimental value for this wave number were to be used in the theoretical solution the agreement was better.

FUTURE PROGRAM

The continuation of this program calls for

1) An attempt to reduce the scatter in the experimental data by striving for a more uniform axial stress distribution in the shell.
2) A detailed study of the effect of the clamped end of the shell on the buckling stress - possibly trying to make a shell with a negative Poisson displacement so that it would be straight at the buckling stress.
3) Extension of the experimental program to other forms of initial deformation.
4) Consideration of other R/t ratios.
5) A more sophisticated theoretical treatment of the problem.

REFERENCES

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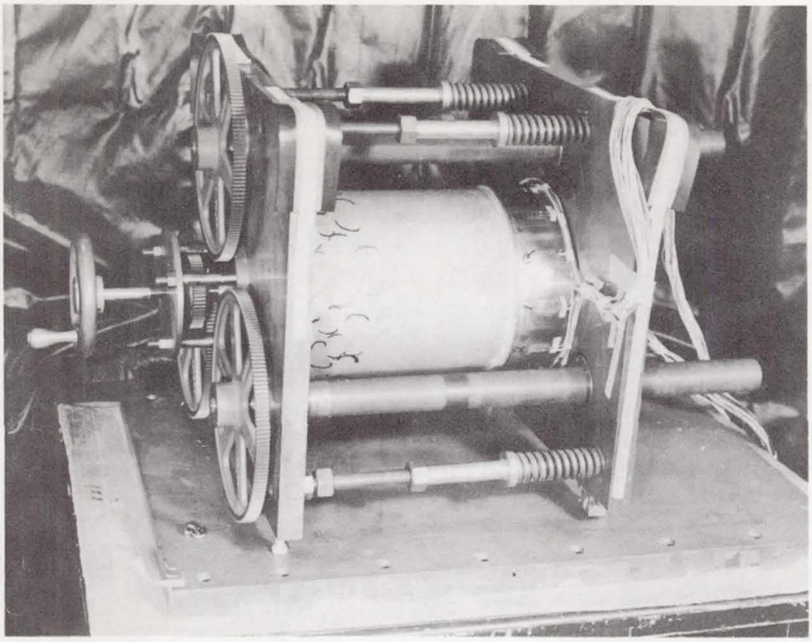

Figure 1.- Testing machine, specimen, and load ring.

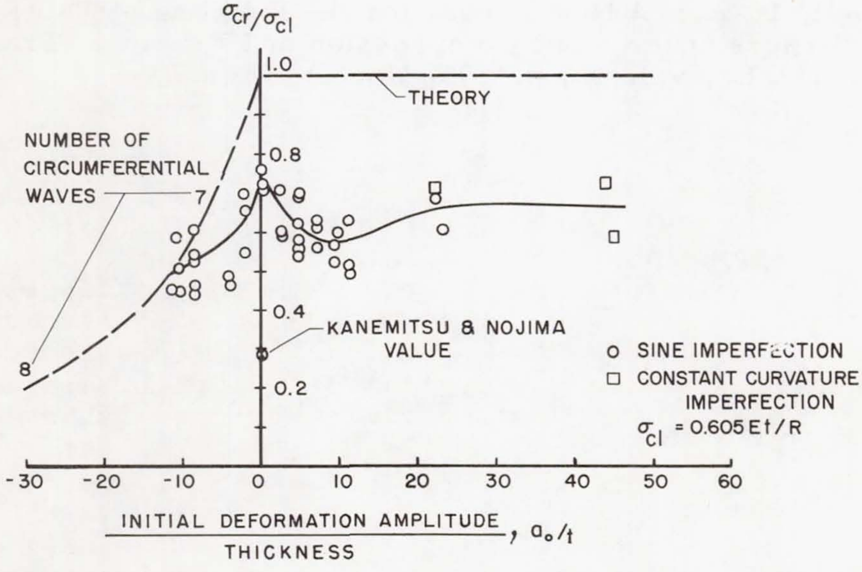

Figure 2.- Buckling stress variation with initial imperfection amplitude.