A REPORT ON THREE SERIES OF EXPERIMENTS AND THE DESCRIPTION OF A SIMPLIFIED MODEL OF THE THIN WALL CYLINDER AND CONE BUCKLING MECHANISM

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SUMMARY

Three series of experiments were performed to investigate the nature of the buckling phenomenon. They were: a) influence of an increasing dimple on the buckling load of a cylinder; b) investigation of the strain distribution on one cylinder, after one or more waves were imposed on it; c) measurements of lateral deflections at buckling. The results of such experiments give some support to the idea of a buckling model, which can be of eventual interest to the thin wall cylinder and cone buckling problem, where a non-linear system (the buckled band) is considered in series with a linear system (the unbuckled portion of the cylinder, plus the testing-machine).

INTRODUCTION

The classical literature that deals with the mechanism of buckling of thin wall cylinders presents, in the post-buckling region, a non-linear relation between axial load and axial deflection, in such a way that large lateral deflections of the cylinder wall correspond to a large decrease of the load with which the cylinder reacts to an increasing imposed shortening. (Buckling is considered here in its engineering sense, i.e., collapse). These lateral deflections increase from zero to large finite values following a continuous curve, corresponding to (unstable or stable) equilibrium conditions. Therefore, this curve always starts at the classical value $K \simeq 0.6$.

These infinitesimal lateral deflections are always associated with membrane and bending stresses.

However, it is possible to imagine that the axial shortening imposed on the cylinder could be absorbed by its walls taking an almost developable polyhedral configuration (developable, except at its vertices), in which the membrane strain energy could be relatively small in comparison to the bending energy stored at the diamond borders. In this way the total strain energy (and the load) could possibly be smaller, and the resulting load vs. strain curve would become independent of the classical buckling load at $K \approx 0.6$. Buckling would be, in this way, a discontinuous process, with the cylinder jumping from a first mode (simple compression) of absorbing the axial shortening, to a second mode (a polyhedral buckled band, plus an unbuckled region of the cylinder). Thus, this second mode could be entirely independent of the first one.

In order to investigate this possibility, the experiments decribed below were performed, and some support to this approach was obtained. A very much simplified model of the cylinder and cone buckling phenomenon was studied, yielding simple and interesting results, which may eventually be useful. This model enables one to explain the buckling problem without the necessity of supposing a jump over the "energy hump".

SYMBOLS

A	semi-vertex angle of the cone
d	distance between recording points
E	Young's modulus
H	height of the cylinder
j	see figure 6
K	coefficient for the collapse of a cylinder
k	H/R
n	number of sides of the basic polygon of the buckled cross-section
n_o	value of n that minimizes the bending energy
P	axial force on the cylinder
Q	lateral force acting on the cylinder wall
R	radius
t	wall thickness
U	strain-energy
δ	axial shortening of the cylinder or cone
ϵ	ratio of the shortening to the height of the cylinder
ϵ_R	ratio of the shortening to the average radius of the buckled band (cone case)
η	lateral deflection of the cylinder wall

A

σ compressive stress

Subscripts:

b for bending

c for simple compression

l for linear system (unbuckled portion of the cylinder, plus the testing-machine)

EXPERIMENTAL RESULTS

Influence of a small increasing dimple on the collapse load of cylinders.- A constant lateral force Q was imposed on the wall of mylar cylinders, and the lateral deflections η at the loading point were measured when the axial load P on the cylinder increased. The collapse load was also measured. In this way, fig. 1 was obtained. It can be seen that dimples of depth to about one wall thickness do not affect the collapse load. Larger dimples cause one-wave initial buckling, which brings the collapse load to a lower level, where it remains practically unaffected by further very large imposed lateral deflections. When two or three initial waves appeared, they gave similar results. Cross-plots of these P,Q,η graphs were also obtained, and they show the distinct regions of one, two, and three initial waves, and collapse.

These graphs seem to show very strongly that for $R/t \cong 300$, collapse has a discontinuous nature. Also it appears that if an initial local imperfection is to affect collapse, the width of the initial imperfection has to be larger than a critical value.

Similar experiments were performed at various points around the cylinder, and they called attention to the influence of a non-uniform compression stress distribution on the one-wave initial buckling.

Strain measurements.- A mylar cylinder with $t = 0.010$ in., $R/t = 250$, and $H = 12$ in. was loaded with ~70% of the collapse load. Twenty-four half-diamond waves were imposed simultaneously on one section, simulating a buckled band (i.e., two dodecagons, out of phase). Properly placed strain-gages picked up the induced longitudinal and transverse, direct and bending strains. Because the strain-gage length and stiffness made it impossible to measure the crests of the bending-strain curves at the diamond borders, only qualitative conclusions were possible. However, it can be said that close to the polyhedral band, the bending energy is much larger than the change that occurs in the compression energy, when the waves are imposed on the wall.

When only one wave was imposed, and in spite of the experimental difficulty mentioned above, the bending energy was ~10 times the change of the compression energy, in a region close to the wave; and they were about equal at a section 4 in. away from the center of the wave.

It was also clear that most of this bending energy had to be provided by the elasticity of the loading device.

Recording of the lateral deflections at collapse.- The aim of these experiments was to try to record the cylinder wall movements at collapse, and to interpret them from the point of view of the continuity of the buckling process. In particular it was desired to detect any evidence of the existence during the "jump" of the unstable equilibrium stage, which corresponds to Tsien's "hump" problem. Eight mylar cylinders were studied, with $R \simeq 2.2$ to 4.8 in., $t = 0.007 \sim 0.010$ in., $H = 4.3$ in. Their ends were embedded in "cerrolow" metal. Ten small eye-pins were cemented to the wall, at the same height, and each one connected to a special wire potentiometer. The signals were recorded by a ten-channel recording oscillograph. Obviously, this arrangement provided an appreciable lateral support to the wall (the friction was ~8gr for each channel). Therefore, the height of the cylinder had to be kept small, and the load had to be applied a little off the center, in order that the buckled band would occur at the studied section.

About 50 recordings were performed, with very consistent results. Fig. 2 shows the recording paper of Exp. no. 190, with $R = 4.75"$, $t = 0.007"$, $d = 1.18"$. No initial waves were present, and the buckling occurred simultaneously along the studied section. It can also be seen from fig. 2 that 0.01 sec. after collapse, the cross-section was already a 10- or 11-sided polygon, and its shape changed to an 8- or 10-sided polygon after further shortening was imposed on the cylinder (the testing-machine was not immediately stopped). No evidence was found of an intermediate polygon between the circle and the 11-sided one.

Fig. 3 shows the recording paper of Exp. no. 152, with $R = 4.75"$, $t = 0.010"$, $d = 1.18"$. Initial waves can be seen at channels 2, 4, 5, 6, but apparently they had not much influence on the mode of collapse. It is interesting to note that all these channels reversed their senses at collapse. This is a strong indication that the mode of deflection at collapse is independent of initial small irregularities.

Fig. 3 also shows the cross-sections drawn according to the observed final configuration of the cylinder. The studied section stayed at half-height of the buckled band, showing a polygon with twice the number of sides of the basic one.

The results of the other experiments generally confirmed these conclusions.

A SIMPLIFIED MODEL FOR THE BUCKLING MECHANISM

Bending energy of the buckled band.- Following the lines presented above, the bending energy of the buckled band, corresponding to large lateral deflections, can be easily estimated if simplifying assumptions are made.

The simplest mode of buckling, which can be observed experimentally if a "stiff" machine is used, is obtained if the buckled portion of the cylinder or cone is considered to be a band of $2n$ triangles (fig.4). These triangles are half-diamonds, but the other halves are not considered, because they involve only relatively large radii of curvature. The bending energy is assumed to be stored at the borders of such triangles, and at the vertices, where the radii are small and assumed constant. Two non-dimensional parameters have to be empirically estimated, and they are N and γ as shown in fig. 4. The angles are estimated from the geometry.

The bending energy is minimized for n. Then, for a family of cylinders with $k = 4$, one obtains,

$$U_b = 21.8Et^3\epsilon_b^{1/3} \tag{1}$$

$$P_b = 7.3 \frac{Et^3}{H} \epsilon_b^{-2/3} \tag{2}$$

For simple compression:

$$U_c = k\pi R_o^2 tE\epsilon_c^2 \tag{3}$$

$$P_c = 2\pi ER_o t\epsilon_c \tag{4}$$

Two modes of absorbing axial shortenings.- The axial shortening imposed on the cylinder can be absorbed: a) by simple compression or b) by two systems in series: a non-linear system consisting of the buckled band, and a linear system consisting of the unbuckled portion of the cylinder (or cone), plus the testing-machine. The mechanism of the collapse would be, for this simplified model, the mechanism of "jumping" from the first to the second mode of absorbing the axial shortening.

The second mode is controlled by the equilibrium or minimum energy condition (after collapse):

$$P_c = P_b \tag{5}$$

This condition allows a graphical solution for $(U_c + U_b)$ after collapse, and the "jump" can occur when the $(U_c + U_b)$ curve crosses the U_c curve for the first mode alone.

Fig. 5 shows two solutions for the same cylinder. In a rigid-machine $K \simeq 0.34$. In a "soft" machine (assumed in this example 20 times more flexible than the cylinder alone, as in the case where a dynamometer is inserted) $K \simeq 0.11$.

The same process can be applied to cones, using the equations shown in fig. 4. A graph was made for the "soft" machine case, with results of the same order of magnitude as in ref. 1. The results of ref. 2 are about twice as large as the estimated ones. However, their trends coincide for A and Rm/t, while the graph over-emphasizes the influence of R_2/R_1. (Note that the experiments of ref. 2 apparently were not performed with a "soft" loading device).

For the internal pressure case, the same reasoning was applied, with waves disposed along one helix around the cylinder. The main parameter was found to be $\frac{p}{E}\left(\frac{R}{t}\right)^2$, and the results were of the same order of magnitude as the experimental ones, for small values of p, while the assumption of a developable surface remains acceptable. In this sense, the role of high internal pressures would be only indirect, changing the wave pattern from the almost-developable "diamonds" involving change of volume to the undevelopable sinusoidal surface, involving no change of volume, with large membrane strains. K increases accordingly, until attaining the limit, that is the classical value $K = 0.6$.

<u>A possible collapse mechanism</u>.- The minimum strain energy theorem requires that if a cylinder and a shortening are given, there is only one possible value n_o for the number of sides of the basic polygon. n_o decreases as the shortening increases. A graph can be made, relating the energy levels before and after collapse, the lateral wall deflections (bounded internally by j_i and externally by j_e), and n_o (fig. 6). The graph is self-explanatory, showing the regions where the second mode is impossible, possible but requiring external energy $\left((U_c + U_b) > U_c\right)$, or effective $\left((U_c + U_b) < U_c\right)$.

In terms of the "energy hump" idea, the cylinder would pass through an unstable equilibrium stage before reaching the stable stage. This unstable stage corresponds to a basic polygon with a larger number of sides (n_u) than the stable one (n_s). But, when changing from n_u to n_s, some points have necessarily to reverse their sense of motion, while others, which stayed motionless during the first stage, have to move now. It is more likely that these points, which start at or cross the $j = 0$ line during the second stage, do so without regard to the first stage.

Therefore, the strips or regions corresponding to these points would buckle directly into the stable lower energy level configuration, dragging the adjacent regions or strips directly to the n_s polygon. The process is possible, because the total amount of energy is available. In this way, the waves would initiate at certain points, and propagate laterally until completing the buckled band around the cylinder. But the intermediate steps (the growing waves) would only be dynamically possible. The process would be intrinsically discontinuous, as far as equilibrium positions are concerned. The waves would grow towards a well defined size and shape.

This explanation agrees with the experimental results mentioned before, where no evidence of intermediate polygons was found.

SUGGESTIONS FOR FUTURE RESEARCH

At the ITA Structures Laboratory, two series of experiments are being prepared: a) an investigation on the influence of the cylinder height and of the testing-machine stiffness on the collapse load; b) a correlation between lateral deflection, cylinder load and vertical displacement.

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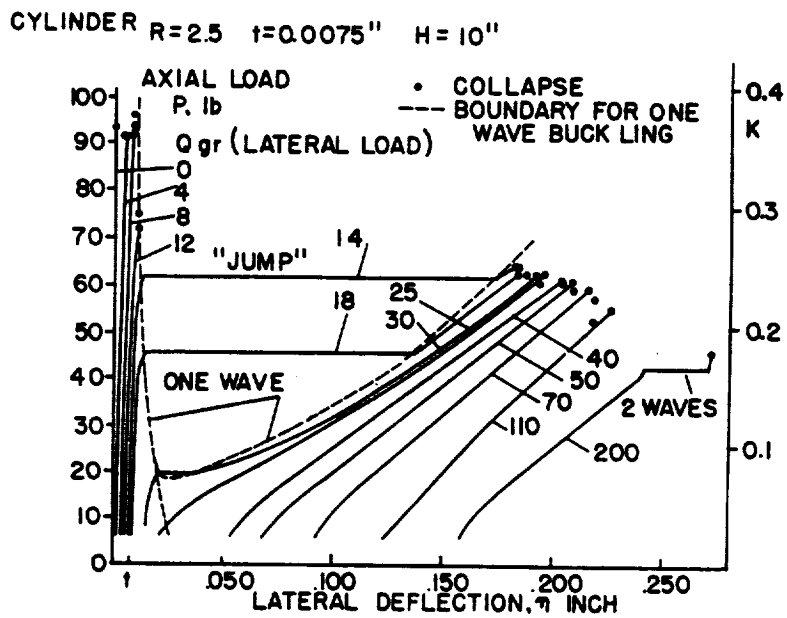

Figure 1.- Influence of an imposed dimple on the collapse load of a cylinder.

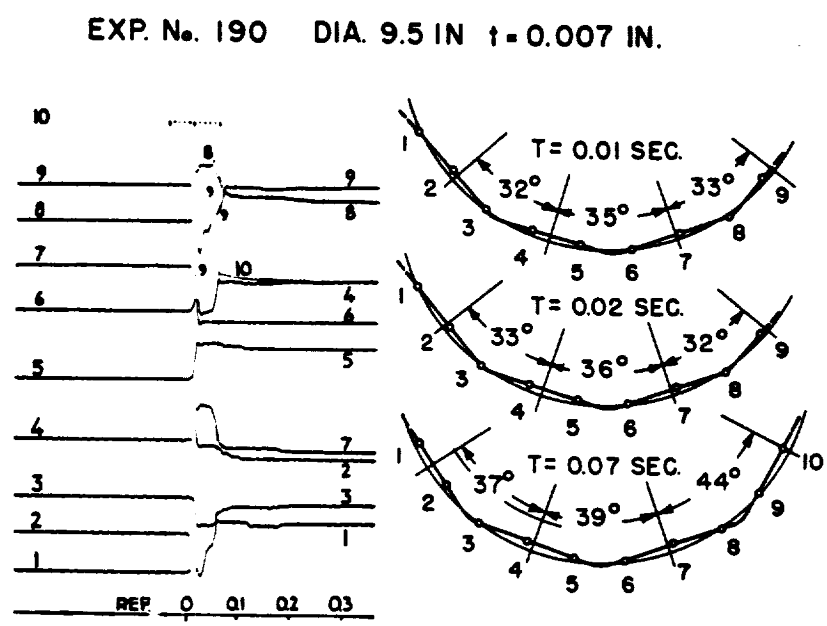

Figure 2.- Recording of lateral deflections at collapse.
Left: Recording paper. Right: Results.

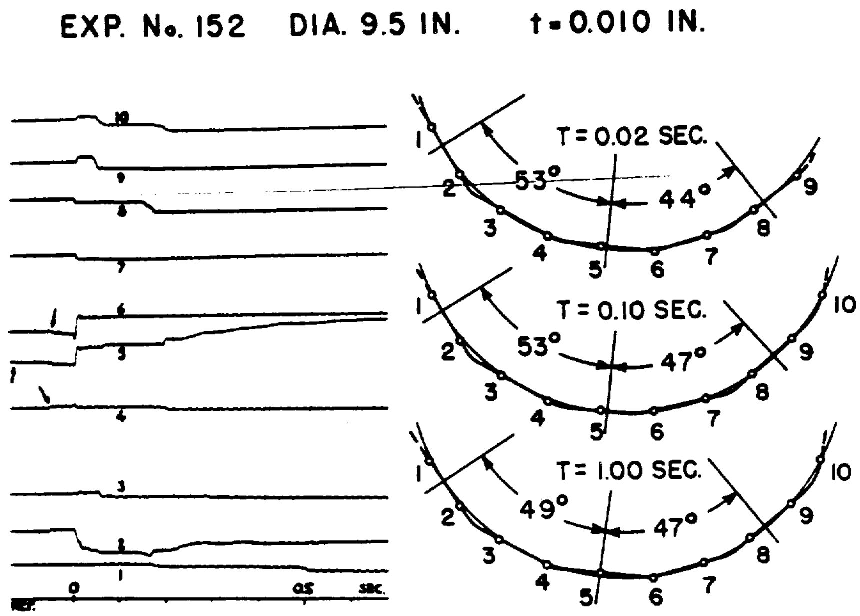

Figure 3.- Recording of lateral deflections at collapse.
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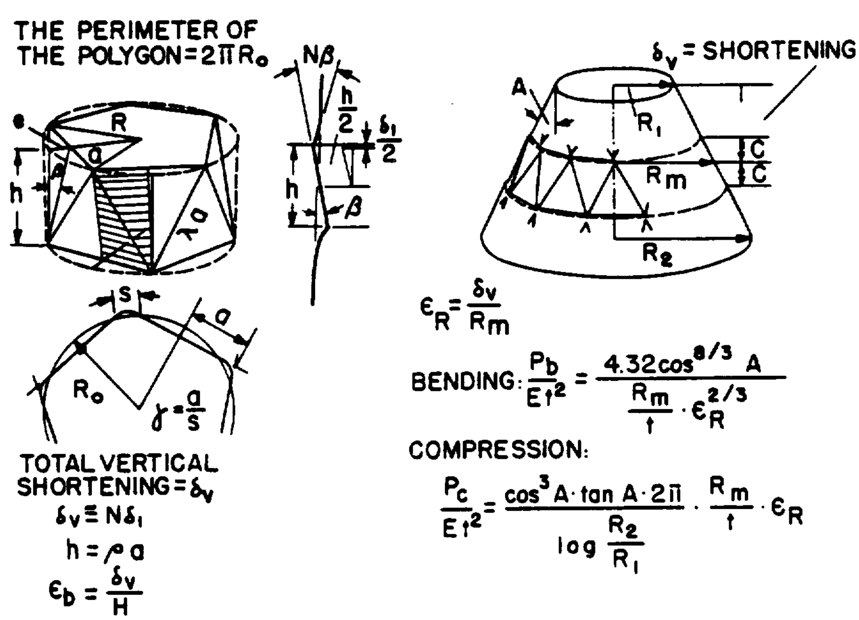

Figure 4.- A simplified model for the buckling problem.

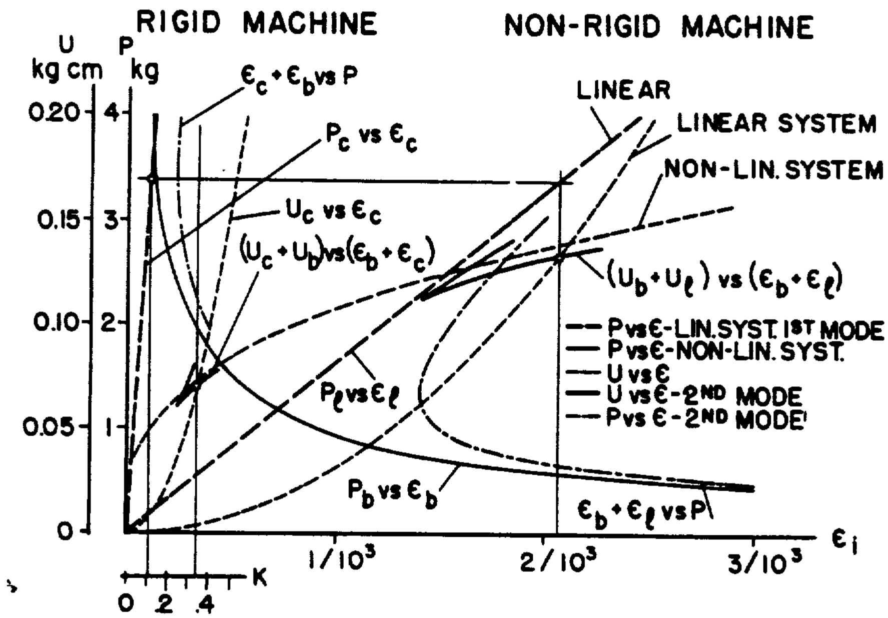

Figure 5.- Graphical calculation of the collapse load.

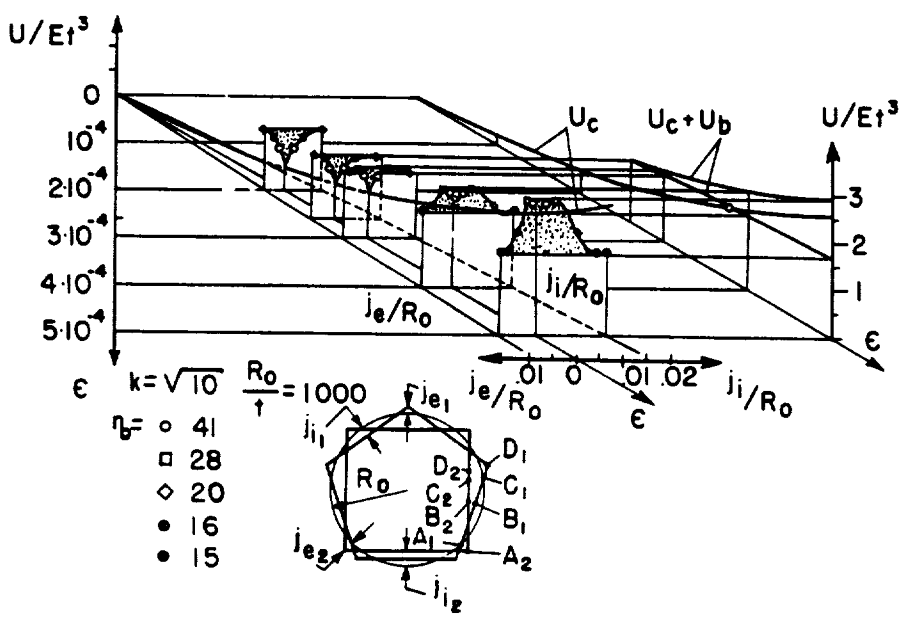

Figure 6.- Graph showing the changes of the energy level, lateral deflection, number of sides of the basic polygon, when a shortening is imposed on the cylinder.