THE MEMBRANE APPROACH TO BENDING INSTABILITY OF PRESSURIZED CYLINDRICAL SHELLS

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SUMMARY

Recent theoretical and experimental research is briefly described to trace the development of deformation and the occurrence of collapse in pressurized circular cylindrical membranes under applied moment loading. The collapse of pure membrane cylinders is then compared with instability of pressurized cylindrical shells. This approach leads to a better understanding of the behavior of pressurized cylinders under bending loads. The results suggest possibilities for further research utilizing the membrane approach.

INTRODUCTION

The fuel and oxidizer tanks of large launch vehicles are often highly pressurized circular cylindrical shells and, during the launch phase of flight, may be subjected to large bending moments from aerodynamic and inertia loads. The strength of pressurized cylinders in bending is thus an important consideration in launch vehicle design, and considerable work has been done on this problem from the usual standpoint of buckling of shells with finite wall bending stiffness. (See, for example, refs. 1, 2, and 3.) In the present paper, it is shown that the study of pure membrane cylinders having no wall bending stiffness but maintaining their shape only by virtue of internal pressure leads to an explanation of pressurized cylinder behavior. With wall bending stiffness accounted for in an approximate way, a simple formula for the strength of pressurized cylinders in bending is obtained.

SYMBOLS

E	Young's modulus
F	applied axial compressive load

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K	curvature of axis of circular cylinder due to bending moment
M	applied bending moment
P	total axial force (due to internal pressure and externally applied load)
p	internal pressure
p^*	pressure parameter, $\sqrt{12(1 - \mu^2)}\, \frac{p}{E}\left(\frac{r}{t}\right)^2$
r	radius of circular cylinder
t	thickness of cylinder wall
μ	Poisson's ratio

Subscript:

c	collapse

BENDING OF CYLINDRICAL MEMBRANES

Experiments on Membrane Cylinders

As one phase of a study of the behavior of membrane structures, tests have been made at the NASA Langley Research Center on 6-inch diameter cylinders made of 1/2 mil Mylar $\left(\frac{r}{t} = 6{,}000\right)$. A photograph of one such specimen is shown in figure 1. This specimen is loaded by internal pressure, bending, and axial tension. Note that wrinkles of very short wavelength have appeared on the right side where compressive stress due to bending moment has canceled the tensile stress due to pressure and axial load.

The behavior of these specimens in bending is illustrated in figure 2. In this figure the ordinate is the applied bending moment divided by the moment required to reduce the stress to zero at one point on the cross section. The abscissa is a dimensionless parameter representing the curvature of the axis of the cylinder. The circles and squares are test points obtained on a specimen similar to that shown in figure 1.

Theoretical Behavior

The solid-line curve in figure 2 is a result of the theory presented in reference 4. This theory describes the behavior of membranes in a partly wrinkled condition with the assumption that, when one of the principal stresses in a membrane becomes zero, many small wrinkles will occur in a direction normal to the direction of zero principal stress. Correspondingly, the membrane is assumed to carry no stress whatsoever normal to the wrinkles. According to reference 4, the theoretical behavior of a membrane cylinder is as follows: For moments such that $\frac{2M}{Pr} < 1$, the cylinder bends like an elastic beam and the response is linear (fig. 2). At $\frac{2M}{Pr} = 1$ wrinkling begins along one generator of the cylinder and, as the moment is increased further, the extent of the wrinkled region increases circumferentially. The moment-curvature curve becomes nonlinear after wrinkling begins, and its slope (representing the bending stiffness of the cylinder) decreases as moment increases. Only when the wrinkled region has progressed all the way around the cylinder does the slope become zero so that instability (collapse) of the cylinder theoretically occurs. The collapse moment $\left(\frac{2M}{Pr}\right) = 2$ is approached asymptotically. Notice that the theoretical collapse moment of a membrane cylinder is precisely twice the initial wrinkling moment.

Correlation of Tests and Theory

In the membrane-cylinder theory presented in reference 4, moments due to externally applied axial load are neglected. Thus, in order for this theory to be compared with experiment, the test specimens had to be shortened until such moments were negligible. The test points shown in figure 2 (obtained from a specimen having a length-radius ratio of about unity) were found to be essentially independent of the proportions of axial load contributed by internal pressure and by externally applied load. Therefore, the previously mentioned requirement of negligible moment due to axial load was satisfied. Nevertheless, the test points in figure 2 are somewhat above the curve from reference 4 and show a dependence on the magnitude of the total axial load.

In an attempt to account for the disparity between membrane theory and the tests, a modification was made in the theory by assuming that over the wrinkled portion of the cylinder the skin could carry a small uniform compressive stress normal to the wrinkles. When this compressive stress was taken to be $\frac{0.6Et}{r}$ (the buckling stress for a highly pressurized cylinder in axial compression) the dash-dot curves shown in figure 2 were obtained. These curves are seen to agree well with the test results.

The need for the modified theory to predict these results for pressurized Mylar cylinders with $\frac{r}{t} = 6{,}000$ testifies to the importance in precollapse behavior of even very small wall bending stiffness in pressurized cylinders.

If the modified theory is projected to collapse as in figure 2, it predicts a collapse moment greater than the theoretical collapse moment for a true membrane cylinder - specifically, the predicted moment is twice the moment at which one point on the cylinder first reaches a compressive stress of $\frac{0.6Et}{r}$. This ideal condition is not reached in practice, however, and collapse actually occurs before the wrinkles can envelop the whole circumference of the cylinder. For example, in the Mylar test specimens of figure 2, collapse was observed at moments slightly above the upper test points when the uniform pattern of small wrinkles (fig. 1) abruptly shifted to a few deep wrinkles concentrated in one or two zones. (In some instances, these deep wrinkles assumed a pattern resembling elongated diamond buckles.) It is of some interest, then, to determine just when bending collapse of practical pressurized cylinders does occur in relation to the collapse moments predicted by the unmodified and modified membrane theories. This comparison has been made in reference 5 and is reviewed briefly in the following section.

COLLAPSE OF THIN-WALLED PRESSURIZED CYLINDERS IN BENDING

Presentation of Shell Data in Membrane Terms

Experimental data on instability of pressurized cylindrical shells in bending are compared in figure 3 with the theoretical collapse moment for pressurized cylindrical membranes. In this figure the ordinate is the ratio of the shell collapse moment to the theoretical initial wrinkling moment of a cylindrical membrane and the abscissa is a dimensionless pressure parameter which is proportional to the internal pressure and the square of the shell radius-thickness ratio.

The theoretical collapse moment for a cylindrical membrane (twice the initial wrinkling moment) is shown as the horizontal dashed line in figure 3. The solid-line curve shows the collapse moment according to the modified membrane theory, in which, instead of zero stress, the cylinder walls are assumed capable of carrying a uniform compressive stress of $\frac{0.6Et}{r}$ normal to the wrinkles over the entire circumference. It is expected that this curve would represent an upper limit for the collapse moment of cylindrical shells; its equation is simply

$$M_c = p\pi r^3\left(1 + \frac{4}{p^*}\right) \tag{1}$$

The test points represent failure of pressurized Mylar and 7075-T6 aluminum-alloy cylinders in bending $\left(291 < \frac{r}{t} < 1,333\right)$ obtained by several investigators (refs. 1 and 2). These cylinders failed at moments greater than the membrane collapse moment by virtue of the bending stiffness of the cylinder walls; but their collapse moments lie below the solid curve representing the limiting moment. On the average, somewhat less than half of the difference between the limiting moment and the membrane collapse moment is realized in the tests. In many cases failure was accompanied by a change from wrinkles to diamond shape buckles.

Contribution of the Membrane Approach

In the study of the bending of cylindrical membranes, a clear distinction is made between wrinkling and collapse (fig. 2). A membrane cylinder can carry moments twice the wrinkling moment and does not lose stiffness suddenly when first wrinkling occurs. Similarly, for a pressurized shell, the limiting moment (the solid-line curve of fig. 3) is twice the moment which initially induces a compressive stress of $\frac{0.6Et}{r}$ in an extreme fiber. According to classical linear theory (refs. 6 and 7), buckling of practical pressurized cylinders in bending occurs when a compressive stress only slightly higher than $\frac{0.6Et}{r}$ is reached in an extreme fiber. It appears from figure 3, then, that for p^* greater than about 2, considerable moment-carrying ability remains in a shell beyond this classical buckling moment. Thus, the classical buckling phenomenon is revealed as a comparatively mild local instability analogous to initial wrinkling in a membrane cylinder, and the test points of figure 3 apparently represent a second instability analogous to membrane collapse. For p^* less than about 2, these two instabilities are not distinguishable. This distinctive behavior of cylindrical shells in bending has also been pointed out in references 2 and 7.

An extremely complex analysis would be required for a detailed description of the shell behavior subsequent to classical buckling. On the other hand, a good lower boundary to the test data in figure 3 over a wide range of p^* is given by the simple empirical expression

$$M_c = p\pi r^3\left(1 + \frac{1}{p^*}\right) \tag{2}$$

which retains the form of the upper limit expression, equation (1).

FUTURE RESEARCH

Additional research is needed to explore further the membrane approach to the study of pressurized shell behavior. The test data available at present are largely based on very small cylinders of foil-like materials. Unfortunately, the data which are presented in figure 3 for cylinders of sheet gages are limited to very low values of the pressure parameter p^* $(p^* < 3)$. Test data are needed for shells made of sheet gages at large values of the pressure parameter. Plasticity, a consideration which has been ignored in this report, may play in important role in such an investigation.

The importance of the combined bending-axial-compression load condition in launch vehicles suggests that pressurized cylinders under these loads should also be investigated from the membrane point of view. Theoretically, for short cylinders the extension of figure 3 to the case of combined axial load F and bending moment M is simple; the only requirement is that the quantity M_c be replaced by $M_c + F_c r$, where M_c and F_c are the applied moment and axial compressive load which jointly cause collapse. However, for cylinders of practical length the added moments due to the axial compression load may have to be taken into account. In addition, test data are needed for the combined load condition for both foil and sheet gages.

CONCLUDING REMARKS

Recent research results on bending of pressurized membrane cylinders, when viewed together with experimental collapse data on pressurized thin-walled cylindrical shells, have clarified the bending behavior and failure of such shells. Additional research would be beneficial in exploiting the membrane approach to the study of the instability of pressurized shells. Experimental data are needed on large pressurized cylindrical shells of sheet gage at large values of the pressure parameter. Such tests should include combined axial and bending moment loading as well as pure moment loading.

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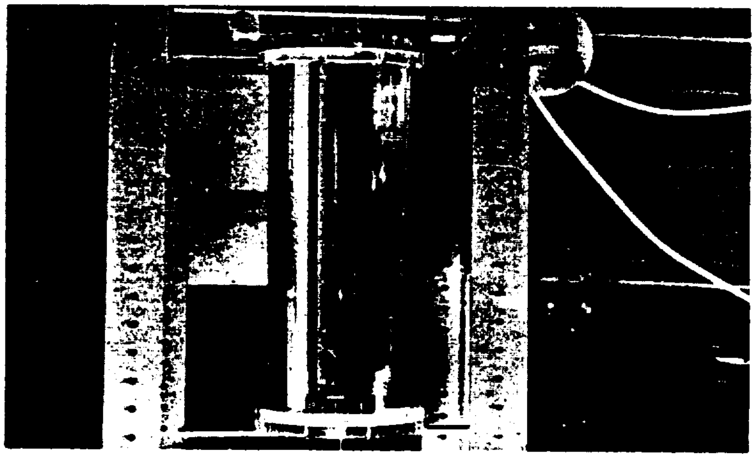

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Figure 1.- Thin pressurized Mylar cylinder in combined bending and tension.

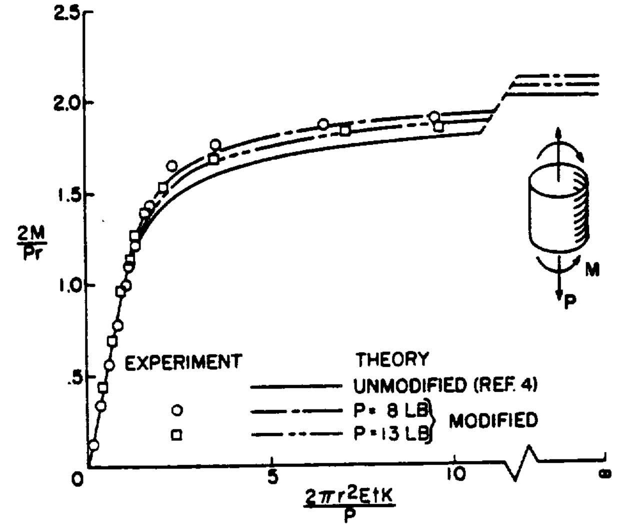

Figure 2.- Moment-curvature relationship for Mylar cylinders in bending. r/t = 6,000.

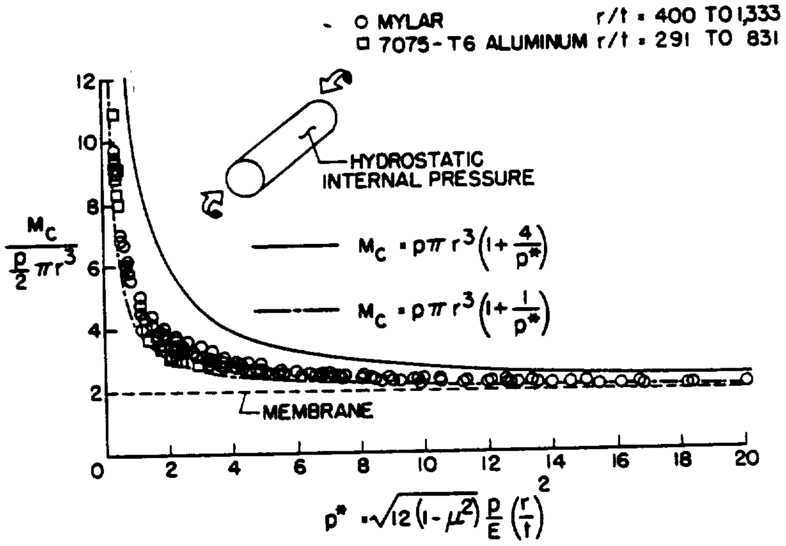

Figure 3.- Bending strength of pressurized cylinders.