INVESTIGATION OF YIELD COLLAPSE OF STIFFENED CIRCULAR CYLINDRICAL SHELLS WITH A GIVEN OUT-OF-ROUNDNESS

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SUMMARY

The effect of out-of-roundness on the yield collapse strength of ring stiffened circular cylindrical shells under hydrostatic pressure has been determined experimentally. On the basis of the experimental data and theoretical stress distributions, empirical relations have been developed which, in conjunction with the theoretically predictable yield collapse strength of round ring-stiffened circular cylindrical shells, permits the prediction of the yield collapse strength of shells with out-of-roundness.

INTRODUCTION

Procedures for predicting the yield collapse strength of ring-stiffened circular cylindrical shells composed of materials having a non-linear stress-strain curve have been developed by the authors in Reference 1.

Out-of-roundness in the shell and stiffening rings may result from manufacturing procedures, relief of residual stresses with time and severe transient or static loading of stiffened shell structures. The need to determine the yield collapse strength of ring-stiffened shells having out-of-roundness and the absence of an available technique for evaluating the strength of out-of-round stiffened shells lead to the present investigation the results of which are presented herein.

Theoretical Treatment

A survey of existing theoretical treatments of out-of-round stiffened circular cylindrical shells was made and excellent contributions were found in References 2 to 5 and particularly in Reference 6. Of all those treatments none are strictly applicable to the case at hand.

There are several theoretical complexities involved in the problem. A stiffened shell with out-of-roundness under hydrostatic pressure undergoes displacements which introduce nonuniform load distribution on the stiffener. As a result of this, the direct stresses in the stiffener are not constant, and the extent of this variation is not known. The determination of bending stresses in the stiffener is a complex matter because the moment and the effective stiffener change with θ (see Figure 1), shell dimensions and pressure level. The variation in loading and effective stiffener introduce large difficulties in the determination of the stiffener's elastic stability. It is needless to say that the difficulties are multiplied when non-linear and/or elasto-plastic strains exist in the stiffened shell.

Adopted Approach

In spite of all the above-mentioned complications, a meaningful approximation of the strains in the stiffener can be obtained by a procedure based on the following reasoning. The presence of the stiffener in a circular cylindrical stiffened shell under pressure develops in the shell disturbances which die out in a short distance from the stiffener. This distance is a function of the thickness of the shell and its radius. The differential equation for axisymmetric deformations of circular cylindrical shells, as given in Reference 7, is

$$w = e^{-\beta x}\left[C_3 \cos \beta x + C_4 \sin \beta x\right] \tag{1}$$

where

$$\beta^4 = \frac{3(1-\nu^2)}{a^2h^2}$$

a = Radius of Shell

h = Thickness of Shell

ν = Poisson's Ratio

Using equation (1), the shear in a ring-stiffened shell is obtained and the distance from the ring at which it reaches its first zero, for the shell shown in Figure 1, is shown in Figure 2. If the stiffened shell is cut where the shear first becomes zero, then the ring and the shell, as shown in Figure 3, will act as a ring of T cross section. Using this simple approach, bending stresses have been computed by employing the well known bending moment expression

$$M = qr_o \frac{w_o}{1-(q/q_{cr})} \tag{2}$$

where

q = The loading on the T cross-section ring

r_o = Radius of the ring

w_o = Maximum out-of-roundness

q_{cr} = Buckling load of the T cross-section ring

To determine the direct stresses in the stiffener, the shear forces existing at the stiffener-shell junction for a perfectly round stiffened cylindrical shell were obtained and applied to the stiffener. The stiffener was considered to be a thick ring, and the Lamé solution was used to compute the direct stresses in the ring.

Models Tested and Results Obtained

Model data are given in Table 1. All models were made out of 7075-T6 aluminum alloy. A typical compression stress-strain curve is shown in Figure 4. They were machined out of solid stock to a tolerance of ± 0.002 inch in the cos 2θ out-of-round mode. Strains were read, by means of SR-4 strain gages, at θ = 0°, 90°, 180°, and 270°, in the circumferential direction. Typical strain reading curves are given in Figures 5 and 6, and they are compared with the theoretically predicted strains and those obtained from perfectly round stiffened shells.

In Table 1 the collapse pressures are given for the out-of-round models as well as the perfectly round ones for the sake of comparison The ratio

$$\frac{q_f}{\sigma_y} = \frac{\text{Collapse Pressure}}{\text{Yield strength of material used based on 1-2\% offset}}$$

was used to denote the non-dimensional collapse pressure.

Discussion of Experimental Data and Their Utilization

From the typical strain records shown in Figures 5 and 6, it is clear that the deviation of the ring strains is greater than the deviation of the mid-bay strains in the case of out-of-round shells. Thus, it is not surprising to find that the mode of failure was that of stiffener collapse rather than that of free plastic flow at mid-bay for which the perfectly round models were designed. The agreement of experimental and theoretical strains in the stiffener and particularly of the maximum strain in it can be of use in determining the pressure which will collapse the stiffener.

The presence of the stiffener, in out-of-round stiffened shells under pressure, results in shell displacements which are non-axisymmetric. It appears that the non-axisymmetric displacements which propagate almost without decay along the shell are directed such that the mid-bay out-of-roundness is reduced and the spread in circumferential mid-bay strains is narrowed.

When the collapse of out-of-round stiffened shells results from failure of the stiffener in the elasto-plastic range, the maximum stress produced in the stiffener is responsible for the creation and spread of plasticity. The establishment of the extent of the plastic region necessary to produce an elasto-plastic stability failure of the stiffener is not easy. Using the experimental data and/or the theoretical predictions which agree very well, the following scheme has been devised in which the pressure producing maximum elastic strains in the stiffener of perfectly round and out-of-round models can be manipulated to predict the collapse strength of aluminum stiffened shells. If P_p is the pressure producing a strain in the stiffener of a perfectly round, efficiently designed stiffened shell equal to the proportional limit, and P_o is the pressure producing a maximum strain in the stiffener of an out-of-round stiffened shell equal to the proportional limit, then it is apparent that the reduction in strength can be correlated with the difference between these two pressures.

An examination of the experimental data disclosed that the reduction in strength can be approximated by the use of a reduction factor, R_f. On this basis, the reduced strength for a given out-of-roundness is given by:

$$\left.\frac{q_f}{\sigma_y}\right|_o = \left.\frac{q_f}{\sigma_y}\right|_p (1 - R_f)$$

where

$$R_f = \frac{1}{2}\left[\frac{P_p - P_o}{P_p}\right]$$

For the out-of-round models tested, only Models 1-O, 3-O, and 3-H-1-O collapsed, and experimental reduction factors are obtained. Results based on the above procedure, using theoretical and experimental pressure-maximum stiffener strains relations are given in Table 2.

The empirical scheme proposed predicts ultimate strengths, which for the shells tested, differ from the experimentally predicted strengths by no more than 3 per cent.

REFERENCES

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TABLE 1. MODEL DATA

Models	Outside Diameter Inches	Shell Thickness Inches	Stiffener Spacing Inches	Depth of Stiffener Inches	Width of Stiffener Inches	Number of Bays	Relative Stiffness Factor**	$\Delta = \frac{w_o}{h}$	σ_y: 1-2% Offset, psi	Maximum Recorded Strain in the Stiffener, Micros	q_f Collapse Pressure psi	q_f/σ_y
3-D	10	0.550	7.5	1.375	0.550	3	0.382	0	72,500	5,500	9,750	0.1345
1-0	10	0.550	7.5	1.375	0.550	3	0.382	0.0455	78,000	8,150	9,350	0.120
3-0	10	0.550	7.5	1.375	0.550	3	0.382	0.0910	78,000	10,200	9,100	0.1165
3-H	10	0.550	3.75	0.973	0.389	3	0.225	0	72,500	10,700	10,300	0.142*
2-0	10	0.550	3.75	0.973	0.389	3	0.225	0.0910	78,000	17,200	10,050	0.129*
4-0	10	0.550	3.75	0.973	0.389	3	0.225	0.0455	78,000	19,500	10,000	0.1285
3-H-1	6	0.330	2.25	0.584	0.233	7	0.225	0	72,500	9,150	10,200	0.141
3-H-1-0	6	0.330	2.25	0.584	0.233	7	0.225	0.0910	72,500	16,250	9,000	0.124

* Model believed to be at imminent failure. Pressure was greatest that could be obtained during test.
** Quantity first introduced by the authors in Reference 1.

TABLE 2. REDUCTION IN COLLAPSE STRENGTH

Model	1-0	3-0	3-H-1-0
R_f Based on experimental strains	0.12	0.13	0.18
R_f Based on theoretical strains	0.8	0.13	0.16
R_f Based on actual collapse pressures	0.11	0.14	0.12

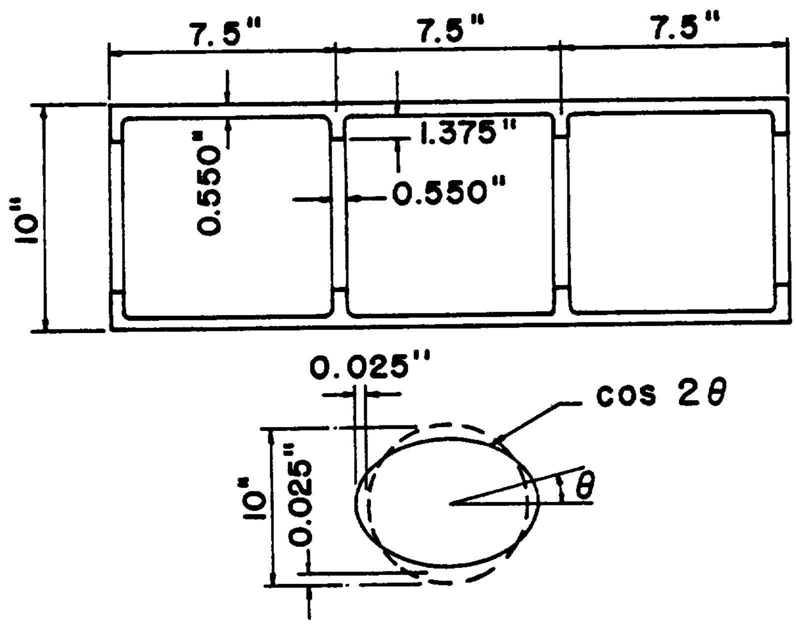

Figure 1.- Dimensions and initial out-of-roundness for model 1-0.

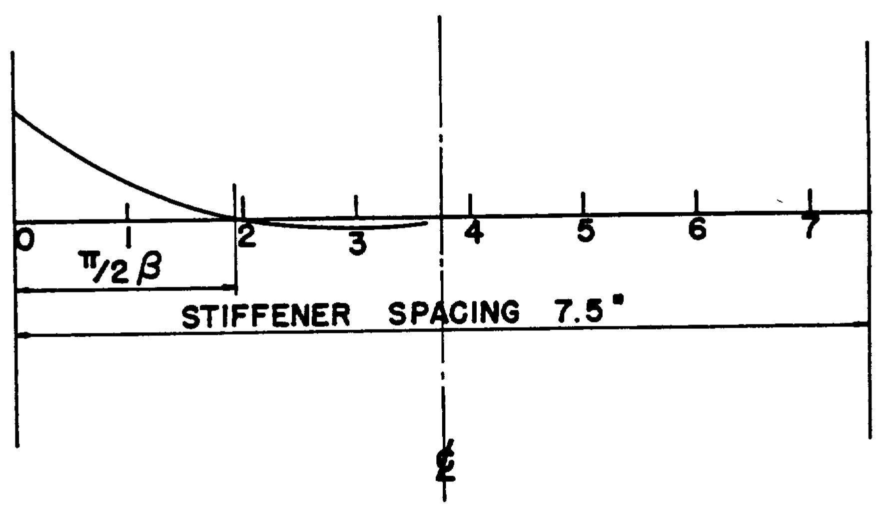

Figure 2.- Shear variation in shell of figure 1.

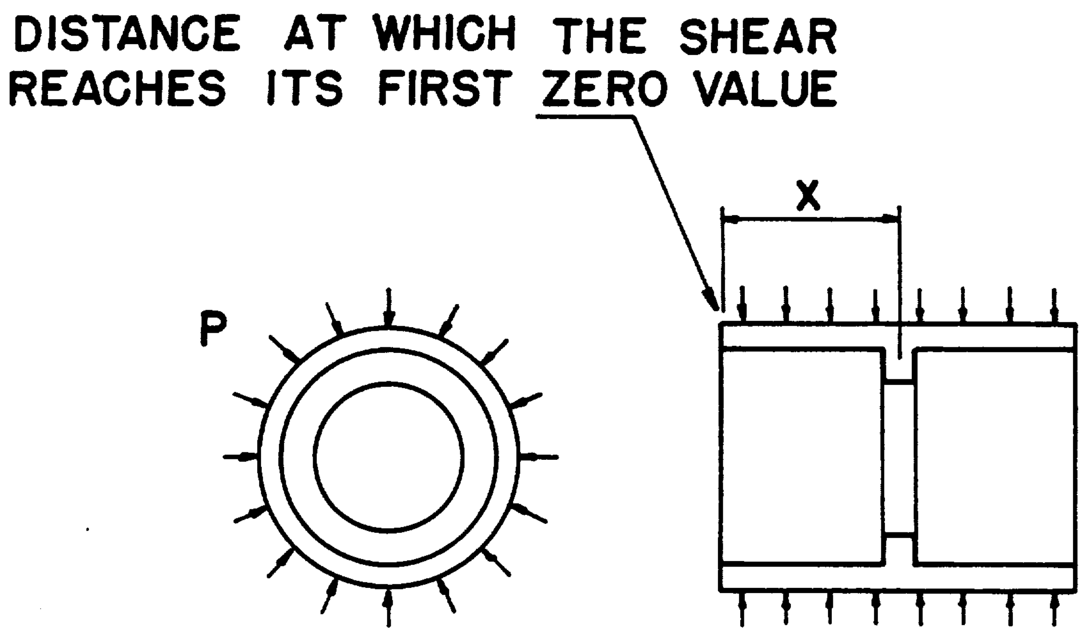

Figure 3.- Sketch of the T cross-section ring.

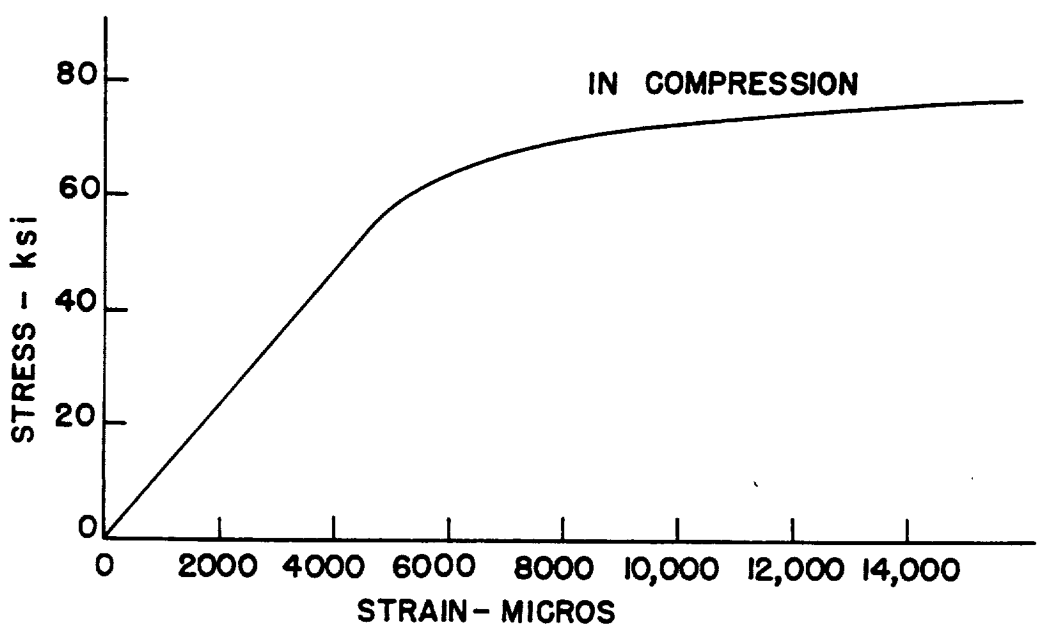

Figure 4.- Typical compression stress-strain curve of 7075-T6 aluminum alloy.

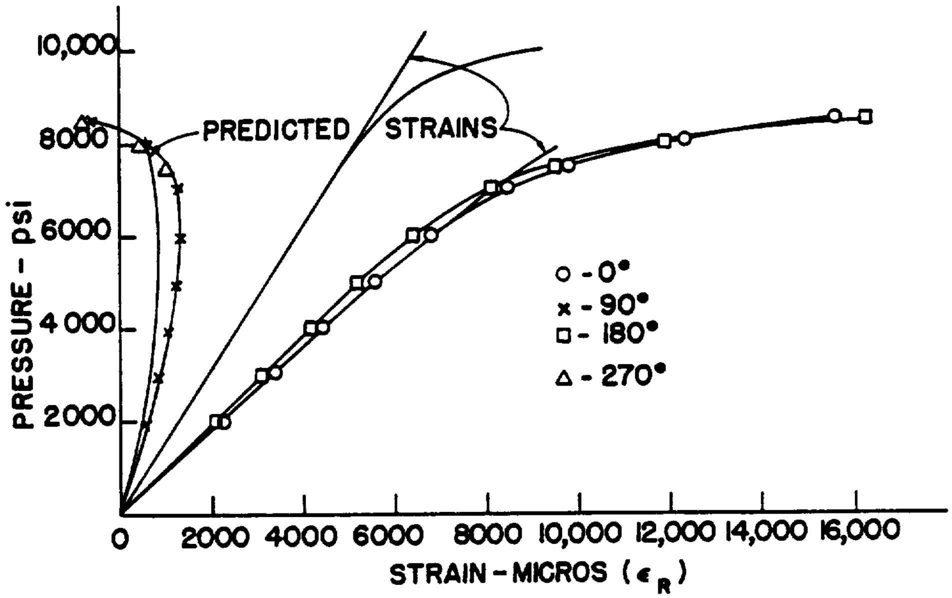

Figure 5.- A typical variation of experimental ring strains in perfectly round and out-of-round stiffened shells and their comparison with the theoretically predicted strains.

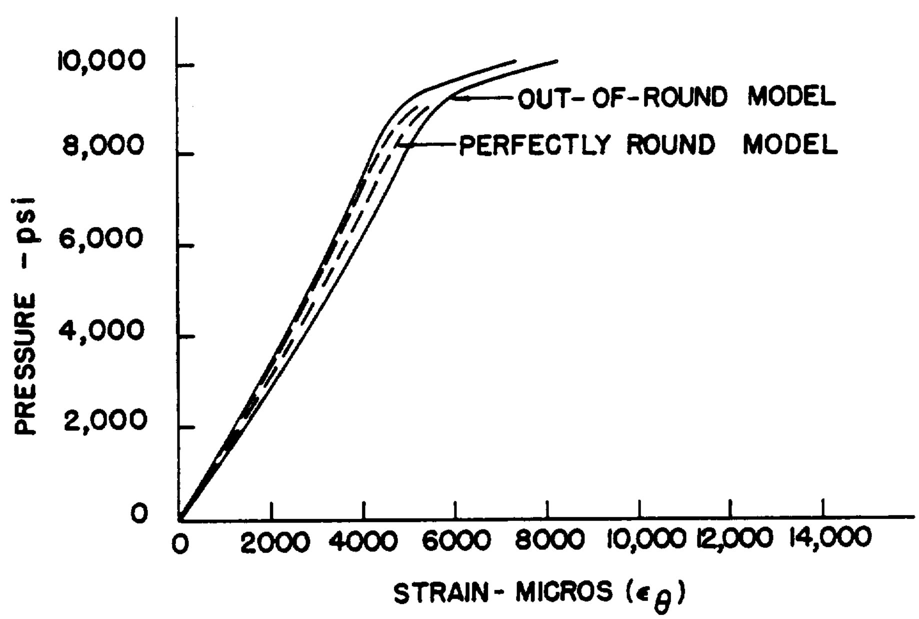

Figure 6.- A typical range of variation of mid-bay circumferential strains in perfectly round and out-of-round ring-stiffened circular cylindrical shells.

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