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THERMAL BUCKLING OF CYLINDERS

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SUMMARY

Several theoretical and experimental investigations on the buckling of cylinders due to both axial and circumferential thermal stresses are reviewed. Differences that exist among the various results are discussed and areas of future work are indicated.

INTRODUCTION

In recent years numerous investigators have obtained theoretical solutions to various problems on the buckling of cylinders due to thermal stress and in some cases have obtained the interaction of thermal stresses with stresses due to externally applied loads. In the solution of these problems thermal stresses caused by arbitrary temperature distributions are generally represented by a Fourier series and then a Fourier series solution of the buckling problem is obtained. For a specific temperature distribution, the Fourier coefficients for the thermal stress are evaluated and the buckling conditions are determined. It can be seen that any varying stress distribution problem could be treated in the same manner; thus, the work that has been done is not necessarily limited to thermal stress problems.

Two general cases have been considered in the literature. The first case, which was investigated in references 1 to 3, is buckling of a cylinder due to circumferential stresses that vary in the axial direction. Circumferential thermal stresses are caused mainly by restraint of thermal expansion in the vicinity of the cooler rings. The interaction of circumferential thermal stress with uniform axial compression is treated in reference 3. Buckling of a cylinder due to axial stresses that vary in the circumferential direction was considered in references 4 and 5. Such a stress distribution results when a cylinder is heated nonuniformly around the circumference. For combinations of these cases, in which both circumferential and axial stresses are varying, no theoretical solution is available, but certain conclusions based on the results of the separate analyses can be inferred.

In the theory applied throughout references 1 to 5, either the Donnell equation or the modified Donnell equation (see ref. 6) was used.

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Some of the more significant aspects of the various solutions are discussed in this paper.

SYMBOLS

E	Young's modulus
L	cylinder length between rings
r	cylinder radius
t	cylinder wall thickness
T_{max}	maximum cylinder temperature
ΔT	temperature difference between rings and skin
x	axial coordinate
α	coefficient of linear thermal expansion
γ	ratio of axial stress to the classical cylinder buckling stress, $\frac{\sigma\sqrt{3(1 - \mu^2)}r}{Et}$
μ	Poisson's ratio
σ	axial stress
σ_x	maximum bending stress
σ_{Tx}, σ_{Ty}	thermal stress in axial and circumferential directions, positive in tension
τ	temperature buckling coefficient, $\alpha \Delta T \frac{r}{t}$
φ	circumferential coordinate

BUCKLING OF CYLINDERS DUE TO CIRCUMFERENTIAL STRESSES THAT VARY IN THE AXIAL DIRECTION

When a ring-stiffened cylinder is heated rapidly, cooler rings cause circumferential thermal stresses which are a maximum in compression over the rings and generally decay to zero away from the rings. The interaction of these thermal stresses with uniform axial compression to cause buckling is shown in figure 1. It is assumed that the rings and skin are both at constant temperatures, the difference in these temperatures being ΔT which is plotted as the ordinate in the form of the temperature buckling coefficient τ. The abscissa γ is the ratio of the axial compressive stress to the classical buckling stress for a cylinder under uniform compression.

The solid curve from reference 3 applies to a cylinder of many bays with each bay simply supported. A similar problem was considered in reference 1 for the case of heating alone, except that the cylinder was considered to be one-bay long. The value of τ from reference 1 indicated by the circle symbol at γ equal to zero is about $2\frac{1}{2}$ times the value obtained for a cylinder with many bays. This difference does not at first appear reasonable since the equations used in references 1 and 3 lead to the same stability determinant. However, as shown in figure 2 the thermal stress distribution is not the same for the two problems. In figure 2 the theoretical circumferential thermal stress σ_{Ty} is plotted in nondimensional form against $\frac{x}{L}$. The stress distribution for a cylinder of many bays is given by the solid curve while the dashed curve applies to a one-bay simply supported cylinder. The extent of the region of compressive stresses is greater for the cylinder of many bays than for the one-bay cylinder. The reason for the difference between the two curves is that, in a cylinder of many bays, the radial expansion of the cylinder wall is symmetric about a ring which effectively clamps the ends of each bay as far as circumferential thermal stress is concerned. For a given temperature, the average thermal stress in a cylinder of many bays is over twice the average thermal stress for a cylinder of one bay. If the results of references 1 and 3 are compared on the basis of average stress at buckling, the cylinder of one bay is found to have an average stress only 20 percent greater than a cylinder of many bays, even though, as shown in figure 1, the buckling temperature for a cylinder of one bay is over $2\frac{1}{2}$ times that for a cylinder of many bays. On the basis of this explanation, the result is reasonable and is similar to that found for buckling of flat plates where the average stress at buckling increases somewhat as the stress is concentrated more toward the edges.

The curves in figure 1 indicate that a significant portion of the classical buckling stress can be applied without lowering the buckling temperature. This behavior can be explained by examining again the average circumferential stress in the cylinder. For a given temperature, it can be shown (see fig. 8 of ref. 3) that the addition of axial compression reduces the average circumferential thermal stress. For a simply supported cylinder (solid curve in fig. 1) the reduction in average thermal stress compensates for the destabilizing effect of the axial compression and results in essentially no change of the buckling temperature. For a clamped cylinder (long-dash—short-dash curve in fig. 1), this compensation is even greater so there is a net temperature increase for buckling. At high values of γ, the axial stress becomes the dominant factor in buckling and the temperature rise at buckling quickly drops to zero at γ equal to 0.95. This 5-percent reduction in buckling stress from the classical value is caused by circumferential stresses produced by the restraint of Poisson's expansion in the vicinity of the rings.

An analysis of the buckling temperature of a clamped end cylinder is also given in reference 2. The result of this analysis is indicated by the square symbol in figure 1 and is seen to be almost twice the value obtained in reference 3 (long-dash—short-dash curve at γ equal to zero). The stress distribution and the boundary conditions were the same in the two references; however, the eighth-order Donnell equation was used in reference 2 whereas the fourth-order modified Donnell equation was used in reference 3. A preliminary check of the results of reference 2 indicates there may be some numerical errors in the calculations; however, the correct numerical results obtained from the Donnell equation would still be significantly higher than those obtained in reference 3. Batdorf (ref. 6) indicated that the use of the Donnell equation in combination with a Galerkin solution, as was done in reference 2, could lead to incorrect results for clamped cylinders. The discrepancy is possibly due to divergent series resulting from differentiating the deflection function 8 times. Additional information on the accuracy of the Donnell equation for clamped cylinders is obtained from calculations made for a clamped cylinder loaded to produce a uniform circumferential stress, which is a limiting case of the more general thermal stress problem. For this case the result obtained from the Donnell equation is as much as 50 percent higher than either the result obtained by Sturm (ref. 7) or the result obtained from the modified Donnell equation. The reason for the differences between the two equations has not been completely explained, but it does appear that the Donnell equation gives incorrect results for clamped cylinders and that the modified equation should be used.

It is well known that experimental buckling results are lower than values calculated by using small-deflection theory for cylinders in axial compression. However, for circumferential compression, theoretical results are in agreement with tests. Thus, one could expect good agreement between theory and experiment at lower values of γ where circumferential compression is dominant but at higher values of γ the theory will be unconservative. The test points in figure 1 at τ equal to zero are for room-temperature bending tests, reported in reference 8, of 7075-T6 aluminum-alloy cylinders and indicate the magnitude of the reduction in buckling stress from the classical value for $\frac{r}{t}$ equal to 300. In reference 3 it is proposed that γ be reduced at any value of τ by the same percentage as the reduction at room temperature. This procedure results in the dashed curve in figure 1. The two test points for the heating tests are from the results of reference 9 for 2024-T3 aluminum-alloy cylinders loaded in pure bending and then heated. Reasonable agreement is shown between the tests and the dashed curve.

BUCKLING OF CYLINDERS DUE TO AXIAL STRESSES THAT VARY IN THE CIRCUMFERENTIAL DIRECTION

If a cylinder is heated such that the temperature varies around the circumference, nonuniform axial thermal stresses will arise. The effect of these thermal stresses on the load-carrying ability of the cylinder can be inferred from the analyses given in references 4 and 5. It was shown in reference 4 that, based on small deflection theory for pure bending of a cylinder, the maximum compressive stress for practical cylinder proportions is essentially the classical buckling stress for a cylinder in uniform compression. For most cases of nonuniform heating, the axial thermal stress distribution in a cylinder is similar to a bending-moment stress distribution in the vicinity of maximum compressive stress. Thus, the theoretical maximum axial compressive thermal stress at buckling can also be taken as the classical buckling stress for uniform compression. An exception occurs if the region of compressive stress is very small in the circumferential direction. For example, in reference 10 a theoretical buckling stress slightly higher than the classical value is indicated for a cylinder heated along a very narrow longitudinal strip.

In reference 5, buckling of a cylinder under a varying axial stress is also investigated. The stress distribution is represented by a Fourier cosine series and the resulting stability determinant is evaluated with the aid of certain simplifying assumptions. It is shown that the maximum stress for a bending distribution ($\cos \varphi$) or the distribution given by the next higher term ($\cos 2\varphi$) approaches the classical value as the size of the determinant is increased. It is of interest to note that with the use of the same simplifying assumptions the stability determinant for these two cases can be put in closed form, and the maximum stress at buckling can be shown to be precisely the classical buckling stress for uniform compression when the size of the determinant approaches infinity.

It has been mentioned previously that theoretical buckling stresses for cylinders must be reduced to correspond to values observed in tests. Thermal stress distributions are more likely to be similar to bending stress distributions than uniform compression so that the experimental results obtained in bending tests of cylinders should be a good estimate of the maximum thermal stress at cylinder buckling. Experimental buckling stresses for uniform compression, which are slightly lower than the results obtained in bending tests, would provide a conservative estimate of the thermal buckling stress.

Even though the maximum stress at cylinder buckling has been specified, it is still somewhat of a problem to determine the temperature at which this stress occurs. The usual elementary analysis of thermal stress will not be sufficient in many cases as is illustrated in figure 3 where the longitudinal and circumferential variation of axial thermal stress is shown for a typical heating condition. Figure 3 was obtained from the results of reference 11 where an analysis is given of the axial thermal stress present in a cylinder with a nonuniform temperature distribution. The dashed curves represent the elementary thermal stress in each bay, and they differ considerably from either the experimental points or the solid curves, which were calculated by the theory of reference 11. The thermal stresses shown in figure 3 were the result of heating a cylinder on one side only over the central portion of the cylinder. Strains were not measured in the areas directly under the heating lamps but, in the other regions of the cylinder, stresses determined from measured strains agree reasonably well with theoretical values.

As previously mentioned, the results of references 4 and 5 indicate that, if the classical buckling stress for uniform compression is acting over only a portion of the circumference of a cylinder, buckling still occurs. Hence, it is reasonable to assume that the interaction curve obtained in reference 5 for uniform heating and uniform compression would also apply to stresses acting over a portion of the cylinder circumference. Using this assumption and the results of the various buckling analyses that have been previously discussed, the experimental results of reference 11 can be correlated with theory as shown in figure 4. In figure 4, the maximum cylinder bending stress σ_x is plotted against maximum cylinder temperature T_{max}. The upper curve is the result that would be obtained if there were no thermal stress and the bending strength was reduced in proportion to the reduction of Young's modulus with temperature. The middle curve was calculated from reference 3 for a cylinder that is uniformly heated. The curve represents the interaction of a uniform compressive stress with the circumferential thermal stresses present due to the cooler rings. The cylinders represented by the test points on figure 4 were not heated uniformly but were heated as indicated in figure 3; this heating caused axial thermal stresses in addition to the load-induced stress. The maximum compressive bending stress was at the bottom of the cylinders which was also the point of maximum heating and high compressive thermal stresses; therefore, this region was critical for buckling. The load-carrying ability of the cylinder is indicated by the lower curve which represents the allowable load-induced stress and was obtained by subtracting the calculated axial thermal stress at the bottom of the cylinder from the value given by the middle curve. The data exhibit scatter which is about the same as observed in the results of room-temperature tests and is in reasonable agreement with predicted values.

AREAS OF FUTURE RESEARCH

For the problem of buckling due to circumferential thermal stress, experimental results have been obtained for the case in which significant amounts of axial compression have also been present. Further research is needed in this area to determine experimental buckling results for cylinders that buckle due to temperature alone. In addition the load-carrying ability of the temperature-buckled cylinder would be of interest.

Axial thermal stresses could be a significant factor in buckling of longitudinally stiffened cylinders which might be used as an interstage structure of launch vehicles. Therefore, it would appear that extension of the method of reference 11 for calculating thermal stresses to include stiffened cylinders with the possibility of local buckling between stiffeners should be an area for future research. Also the effect of a varying axial stress on the buckling behavior of a longitudinally stiffened cylinder should be investigated in order to determine whether there is any significant difference from the case of uniform compression.

CONCLUDING REMARKS

The work of several investigators on the buckling of cylinders under varying axial and circumferential thermal stresses has been reviewed. It has been shown that the severity of the circumferential thermal stress is strongly dependent on the boundary conditions. The interaction of circumferential thermal stresses with axial compression is also discussed. A method of incorporating these results with the observed reduction in axial buckling stress from the theoretical value is indicated and is shown to agree with experimental results.

For cylinders that are heated nonuniformly, the axial thermal stress cannot in general be predicted by elementary theory. A method is available to predict these stresses which can be added to the load-induced stresses to obtain the complete axial stress distribution. Buckling can then be determined by assuming that the maximum axial stress is equal to the buckling stress that would be obtained in a cylinder heated uniformly and loaded by bending or compression.

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INTERACTION OF CIRCUMFERENTIAL THERMAL STRESS WITH AXIAL COMPRESSION, $\frac{L^2}{rt} = 75$

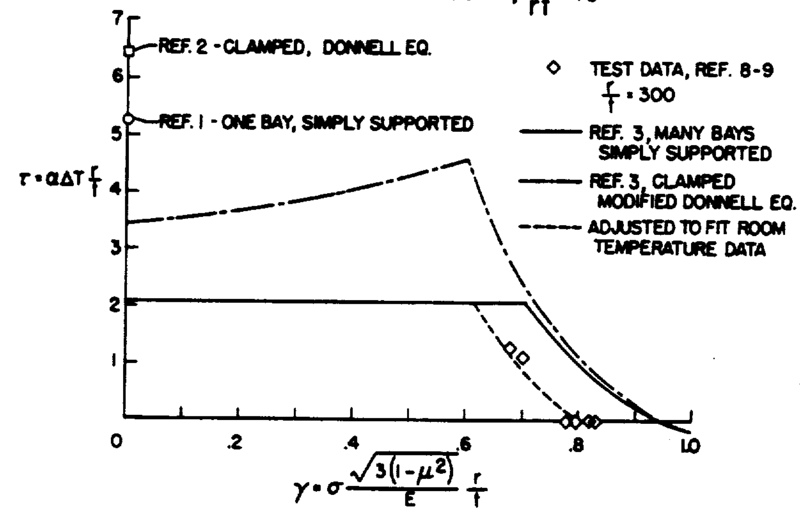

Figure 1

VARIATION OF CIRCUMFERENTIAL THERMAL STRESS ALONG BAY LENGTH

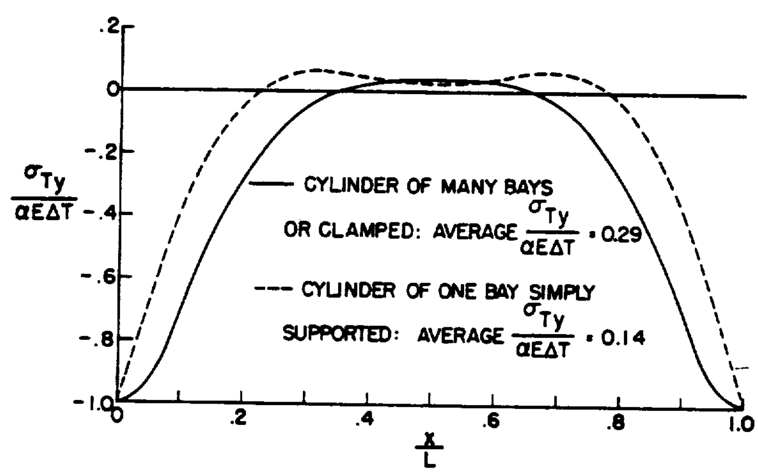

Figure 2

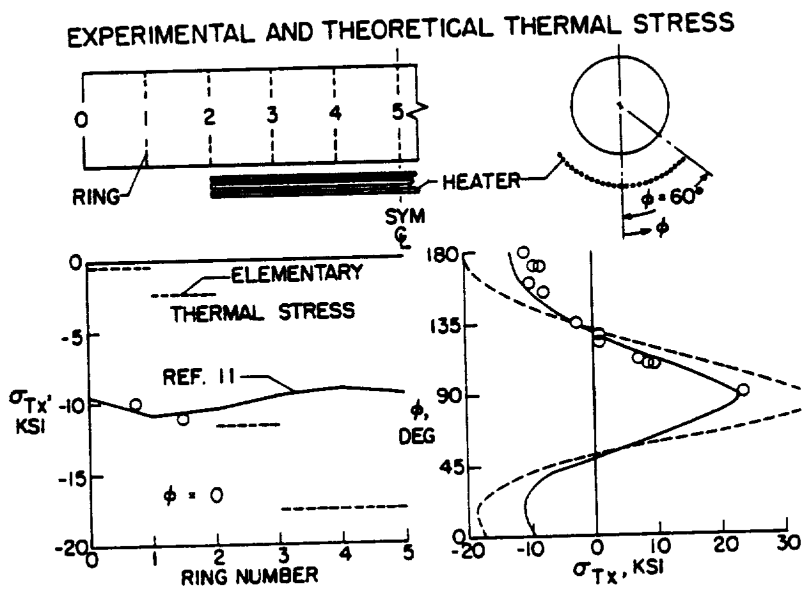

Figure 3

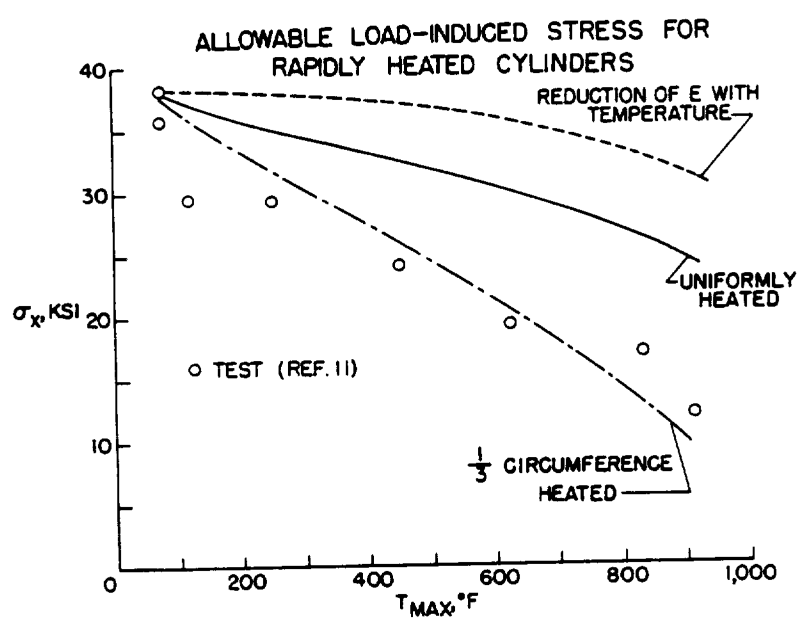

Figure 4

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