LOCAL CIRCUMFERENTIAL BUCKLING OF THIN CIRCULAR CYLINDRICAL SHELLS

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SUMMARY

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The problem of circumferential buckling of a thin circular cylindrical shell due to compressive hoop stresses which vary in the axial direction is examined. For extremely localised compressive hoop stress distributions resulting from thermal discontinuity effects, or from a uniform, radial line loading, the buckle pattern should also be localised. Simplified analyses into these two types of problem are considered which show that only a limited number of buckle deflection modes needs to be assumed.

INTRODUCTION

The compressive hoop stresses set up near the junction of a cylindrical shell heated axisymmetrically, and a cooler, stiffening ring or bulkhead, may be high enough to cause buckling of the shell. Similar stress distributions are caused by an axisymmetric radial line loading in an unstiffened shell. For both types of problem the localised nature of the high compressive hoop stresses suggests that the buckling mode may also be local.

This paper reviews the problems and discusses some simplified theoretical analyses which only consider the conditions in the shell close to the region in which the compressive hoop stresses are acting. No attempt is made, as in other published solutions, to represent the conditions over the entire length of the shell. By this means it is hoped to show that a realistic solution is obtained when using only a limited number of modes for the buckle deflection pattern.

SYMBOLS

a radius of shell

a_p, a_N, b_N deflection coefficients in radial displacement functions

C	a number
D	flexural rigidity
E	Young's modulus
h	shell thickness
L	shell length
L_β	"compressed" length of shell
P	radial line load
R	thermal buckling factor $(\alpha T)_c$
T	uniform shell temperature rise
w	shell radial displacement
x	axial shell co-ordinate measured from position of maximum hoop stress
y	circumferential shell co-ordinate
α	coefficient of thermal expansion
β	shell parameter $\left[3(1-\nu^2)/a^2h^2\right]^{\frac{1}{4}}$
λ	half wave length of buckling in circumferential direction
σ_y	circumferential membrane stress

Subscript:

c	critical

CIRCUMFERENTIAL THERMAL BUCKLING

Review

Hoff (ref.1) first investigated the stability of a simply supported cylindrical shell subjected to a uniform temperature rise. Infinite trigonometric series are used to represent the radial deformation of the shell and the axial stress distribution, and Donnell's simplified small deflection theory (ref.2) is used to obtain a solution in the form of an infinite determinant which can be truncated to give a solution to any desired degree of accuracy.

For clamped edged shells such a direct analysis is not possible and a solution of Donnell's equation requires the application of Galerkin's method (refs.3 and 4). In a more recent paper by Anderson (ref.5) a similar approach is made, for both clamped and simply supported edges, using the modified equation of equilibrium proposed by Batdorf (ref.6).

The result is shown in both references 1 and 5 that for long shells many terms are required in the radial deformation function to describe the buckle pattern accurately when it is expressed in the form

$$w = \sin\frac{\pi y}{\lambda}\sum_{p=1}^{\infty} a_p \sin\frac{\pi x}{L}\sin\frac{p\pi x}{L}, \qquad (1)$$

for clamped edges, or,

$$w = \sin\frac{\pi y}{\lambda}\sum_{p=1}^{\infty} a_p \sin\frac{p\pi x}{L} \qquad (2)$$

for simply supported edges.

For a uniformly heated shell attached to a rigid, non-expanding ring or bulkhead, the exact hoop stress distributions are shown in figure 1 and may be written as

$$\sigma_y/E\alpha T = \phi = e^{-\beta x}(\cos\beta x + \sin\beta x) \qquad (3)$$

for clamped edges, and

$$\sigma_y/E\alpha T = \theta = e^{-\beta x}\cos\beta x \qquad (4)$$

for simply supported edges.

It is evident that these stresses decrease rapidly away from the shell-bulkhead joint and, if the hoop stress distribution is represented by

$$\sigma_y = RE\sum_{m=0}^{\infty} S_m \cos\frac{m\pi x}{L}, \qquad (5)$$

many terms S_m would be required for adequate representation of σ_y over the entire length of the shell. Because of this it was suggested by the author (ref.7) that a more important parameter in this problem than the shell length would be that length of the shell near to each joint for which the hoop stresses are compressive (see figure 1), i.e. L should be replaced by L_β where for clamped edges

$$L_\beta = 3\pi/2\beta \tag{6}$$

and for simply supported edges,

$$L_\beta = \pi/\beta \ . \tag{7}$$

This concept was used in reference 4 where both the assumed buckle pattern and the hoop stress distribution were only represented over a region close to the shell-bulkhead joint. The small number of terms required to get convergence of the solution and the agreement which was obtained with an experimental investigation suggested that the line of approach was indeed valid.

Hemp (ref.8) has also recently considered this thermal buckling problem and the energy method of solution he presents requires only a consideration of the buckle deformations close to the shell bulkhead joint. In the method of reference 8 allowance is made for the presence of initial longitudinal curvature in the shell in the region of maximum compressive hoop stress. Preliminary calculations by the present author have shown that the effect of the initial curvature is stabilising. A more detailed investigation into this aspect is now in progress.

Analysis

Donnell's simplified equation (ref.2) may be written as

$$D\nabla^8 w + \frac{Eh}{a^2}\left(\frac{\partial^4 w}{\partial x^4}\right) + h\nabla^4\left\{\sigma_y \frac{\partial^2 w}{\partial y^2}\right\} = 0 \tag{8}$$

The clamped edged boundary conditions $w = \frac{dw}{dx} = 0$ at $x = 0$ and $x = \frac{NL_\beta}{2}$ are satisfied if we assume an expression for w of the form

$$w = \sin\frac{\pi y}{\lambda}\sum_{p=1}^{\infty} a_p \sin\frac{2\pi x}{NL_\beta}\sin\frac{2p\pi x}{NL_\beta} \quad . \tag{9}$$

In this expression the parameter N defines the buckle pattern length in terms of the parameter, L_β. In equation (5) L is replaced by L_β and in a given problem the coefficients S_m characterising the particular stress distribution are inserted. The substitution of the modified equation (5) and equation (9) into equation (8) yields an equation which is not satisfied identically by any choice of the coefficients a_p. Recourse is therefore had to Galerkin's method of solution. The resultant stability determinant containing terms in R and λ is then analysed to find the value of λ which makes R a minimum. Various approximations are made by truncating the determinant and increasingly more terms are retained until convergence on the minimum value of R is found.

In reference 4 it was assumed that N = 2 and equation (5) could be replaced by

$$\sigma_y = \frac{RE}{2}\left[1 + \cos\frac{2\pi x}{L_\beta}\right] \qquad 0 < x < \frac{L_\beta}{2} \tag{10}$$

Adequate convergence was obtained from only a 3 x 3 determinant the results of which are plotted in figure 2. The corresponding results for a simply supported shell are also shown and it is seen to have a critical buckling temperature 20% lower than for the clamped edged shell. This relatively close agreement is not surprising because although in the region of the shell-bulkhead joint clamping gives a more rigid support to the shell, the compressive hoop stresses are larger and act over a 50% greater length of shell than for the simply-supported shell. These two effects tend to cancel each other.

It is now thought that a more realistic approach would have been to vary N in the range $N \geqslant 1$ and so determine R_{min} as a function of N. However, a first order solution using the above method with N = p = 1 produces the following simple result for clamped edges which is in excellent agreement with figure 2, viz

$$(\alpha T)_c = 6.45\frac{h}{a} \tag{11}$$

The corresponding coefficient from reference 5 for a long, clamped edged shell is 3.6. However, for long shells the size of the determinant needed in reference 5 for convergence was greater than the computer programme allowed (greater than 13 x 13). This fact coupled with the relative simplicity of the present method and the correlation obtained with experiment leads the author to think that the present approach is probably more valid.

For a non-uniform, axial shell temperature distribution and for a flexible, expanding bulkhead the method above applies directly provided the shell stress distribution is correctly represented in the analysis.

This approach has been used in reference 4 in correlating the theory with the results of an experimental investigation. The steel shell concerned had a radius-thickness ratio of 2540 which for clamped edges gives a theoretical uniform shell buckling temperature of 230°C. For the measured temperature distribution at buckling the theoretical maximum shell temperature was derived as 324°C whereas the experimental value was 300°C. This is considered to be fair agreement. A photograph of the buckled pattern is shown in figure 3.

CIRCUMFERENTIAL BUCKLING DUE TO A RADIAL LINE LOADING

Analysis

For an axisymmetric, radial line loading the compressive hoop stress distribution is similar to that in the thermal buckling problem of the clamped edged shell (equation (3)) and may be approximated by

$$\sigma_y = \frac{P\beta a}{4h}\left(1 + \cos\frac{2\pi x}{L_\beta}\right), \quad 0 < x < \frac{L_\beta}{2} \quad . \qquad (12)$$

The buckle deflection function is assumed to be

$$w = \sin\frac{\pi y}{\lambda}\left[a_N\left(1 + \cos\frac{2\pi x}{NL_\beta}\right) + b_N\left(1 - \cos\frac{4\pi x}{NL_\beta}\right)\right], \qquad (13)$$

$$0 < x < \frac{NL_\beta}{2} \quad ,$$

where the second term in b_N is a correction term.

The results obtained from the various analyses, using Donnell's simplified equation as described earlier, can be expressed in the form

$$P_c = CEh\left(\frac{h}{a}\right)^{3/2}, \tag{14}$$

and are summarised below.

N	a_N	b_N	C
1	YES	YES	1.41
1	YES	NO	1.44
2	YES	NO	0.80
3	YES	NO	1.11
4	YES	NO	2.19

Comparison of the two results for N = 1 suggests that the effect of the correction term b_N is small. Also it is seen that the minimum value of C corresponds to a value of N = 2 when the buckle deflection pattern extends over a length twice that for which the hoop stresses are compressive. It should be pointed out that for $N > 1$ no allowance was made for the presence of the small tensile hoop stresses in the region $\frac{3\pi}{4\beta} < x < \frac{7\pi}{4\beta}$ and there is some error in the assumed distribution for σ_y in equation (12).

There are very few other analyses into this problem in the literature but in reference 9 a solution is quoted for a long shell having the value $\frac{a}{h} = 100$. The result quoted for P_c is

$$P_c = 4.2 \times 10^{-4} Eh, \tag{15}$$

whilst from this present paper a coefficient of 8 is found. For $100 < \frac{a}{h} < 1000$ there is an approximate 2:1 relationship between equation (14) with C = 0.8 and the result of reference 9.

It is worth noting that the analysis in reference 9 attempted to represent the conditions over the entire length of shell and required four terms in the buckle deflection function and twelve terms to represent the stress distribution. Thus the point can again be made that for local loading problems considerable simplification occurs and reasonable accuracy is retained by only considering the local buckling problem.

CONCLUSIONS

Analyses have been discussed which consider the problem of circumferential buckling of thin cylindrical shells. It has been shown that when the compressive hoop stresses are localised considerable simplicity results by also assuming the buckling mode to be local. Reasonable correlation with an experimental result has been obtained for a thermal buckling problem.

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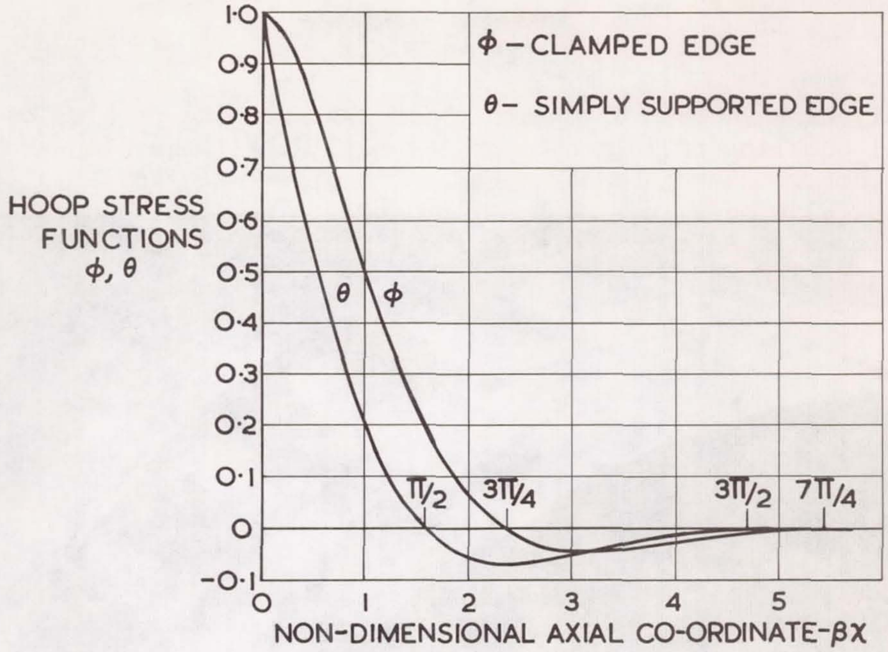

Figure 1.- Hoop stress functions for a cylindrical shell. Uniform shell temperature rise.

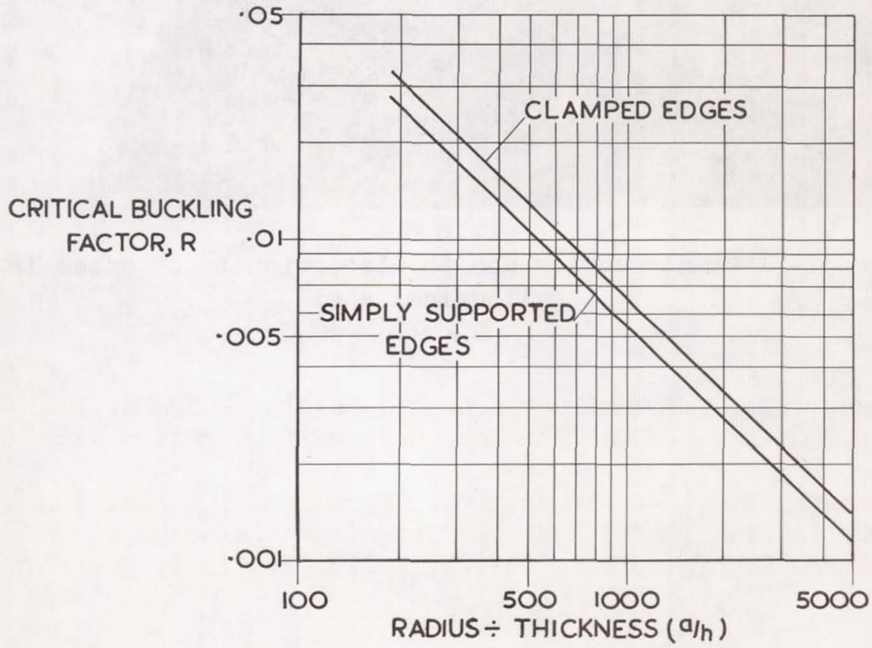

Figure 2.- Critical buckling factor for cylinders subjected to uniform shell temperature rise.

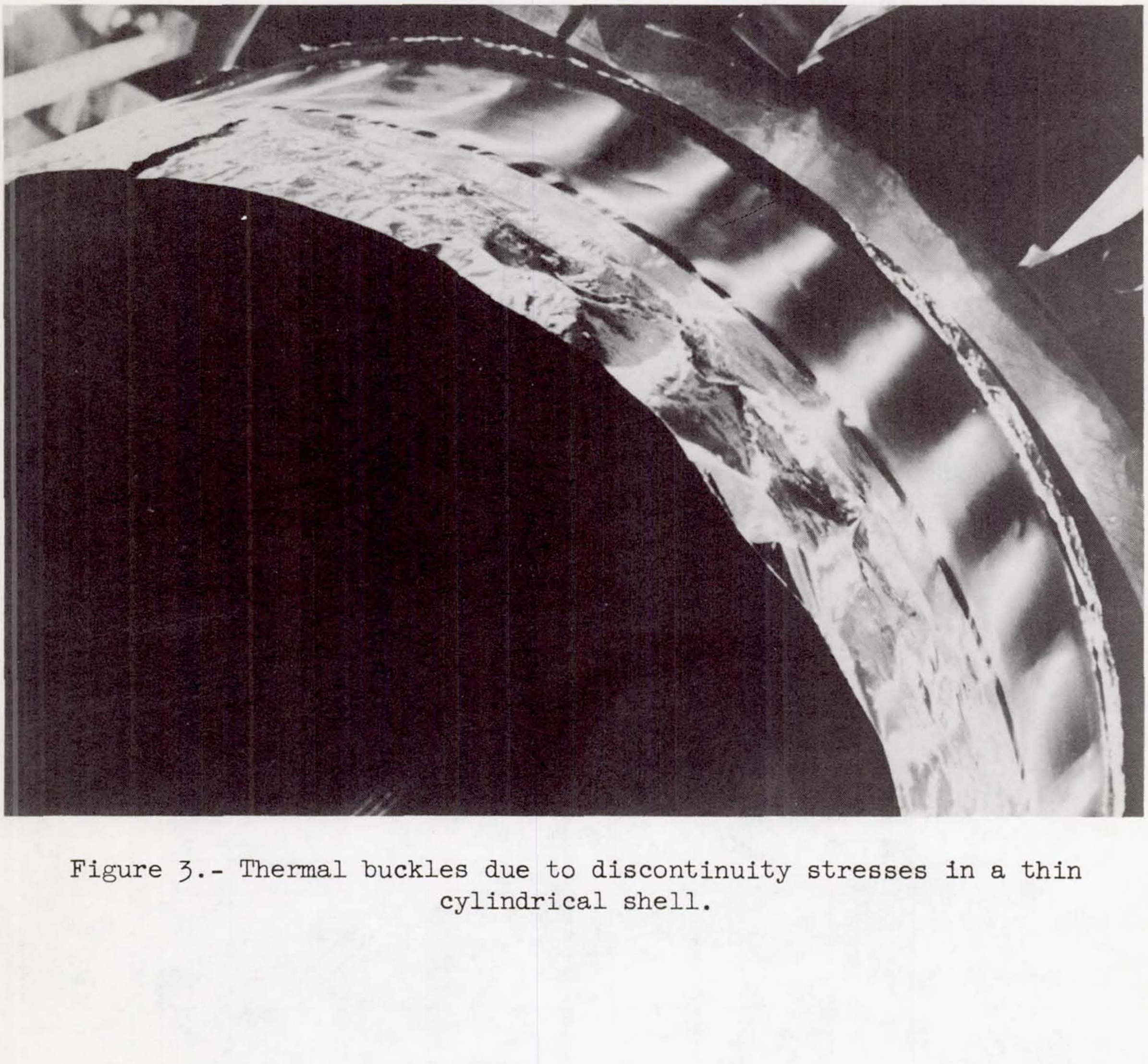

Figure 3.- Thermal buckles due to discontinuity stresses in a thin cylindrical shell.