ON THE INSTABILITY OF ORTHOTROPIC CYLINDERS

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SUMMARY

Preliminary results from experimental investigations on the buckling strength of ring-and-stringer stiffened aluminum cylinders and filament-wound glass-epoxy cylinders are compared with instability calculations based on small-deflection orthotropic cylinder theory. Correlation between experiment and calculation was reasonably good for the glass-epoxy cylinders; however, correlation for the ring-and-stringer stiffened cylinders can be achieved only when some of the basic wall stiffnesses for stiffened cylinders with buckled skin can be better defined. In particular, the wall shear stiffness and the wall bending stiffness in the circumferential direction need better definition.

INTRODUCTION

The test results reported herein were obtained from two separate test programs entailing instability failures of orthotropic cylinders subjected to compressive stresses. In the first program, bending tests were made on 77-inch-diameter cylinders stiffened with Z-section stringers and hat-section rings. The rings were purposely made small to induce "general instability" failures involving collapse of a section of wall large enough to include several stringers and one or more rings. The width-thickness ratio of the skin between stringers was large so that skin buckling occurred early in the tests. In the second program, 15-inch-diameter filament-wound glass-epoxy cylinders were tested in axial compression. Windings of the cylinders consisted of alternate layers of circumferential windings and helical windings such that orthotropic properties were achieved with orthogonal planes of elastic symmetry in the axial and circumferential directions.

Buckling data from these investigations are compared with instability calculations based on small-deflection orthotropic cylinder theory. Wall stiffnesses are needed for the calculations and a considerable portion of the presentation is allotted to the discussion of, and determination of, wall stiffnesses.

SYMBOLS

b distance between stringers in ring-and-stringer stiffened cylinders

D wall bending or twisting stiffness as indicated by subscript

E Young's modulus or extensional stiffness of wall or basic wall element as denoted by subscript; also used to denote equivalent value for Young's modulus of wall composite

G shear modulus or shear stiffness of wall or basic wall element as indicated by subscript; also used to denote equivalent value for shear modulus of wall composite

k volumetric ratio of glass in cylinder wall

M moment at buckling

R radius of cylinder

t skin thickness of ring-and-stringer stiffened cylinders

α helix angle measured from cylinder axis

$\bar{\epsilon}$ edge strain or unit shortening

ϵ_{cr} buckling strain

σ_{cr} buckling stress

μ Poisson's ratio for material or that associated with bending of orthotropic material, used with subscript to denote material or element and direction

μ' Poisson's ratio associated with extensional strains, used with subscript to denote element and direction

Subscripts:

e epoxy

g glass

x denotes axial direction of cylinder wall

y denotes circumferential direction of cylinder wall xy denotes twisting or shear stiffness of cylinder wall with respect to x- and y-directions

w denotes with-grain direction of basic wall element

c denotes cross-grain direction of basic wall element

wc denotes shear stiffness of basic wall element with respect to w- and c-directions

RING-AND-STRINGER STIFFENED CYLINDERS

Discussion of Pertinent Wall Stiffnesses

Values of some of the stiffnesses needed for the calculation of instability loads for ring-and-stringer stiffened cylinders are known with reasonable accuracy; others are known with very little accuracy. The structural behavior of longitudinally stiffened curved cylinders in compression was investigated in reference 1. Good correlation was achieved between calculation and experiment in predicting load-shortening curves for these cylinders which had a range of structural parameters encompassing the range of the present tests. This result implies that the extensional stiffness of the cylinder wall in the longitudinal direction can be predicted with reasonable accuracy; it also implies that the wall bending stiffness D_x can be estimated with reasonable accuracy. Reference 1 presents the necessary equations for evaluating E_x and D_x.

It can be shown from the study of reference 2 that Poisson's ratio μ_x' is small for sheet buckled in compression. Figure 1, which was constructed from numerical calculations made in reference 2, indicates this effect. Although these calculations were made for flat plates, the result is expected to have approximate application in the present tests because the study of reference 1 indicates that certain curved-plate results approach flat-plate results after skin buckling has been exceeded somewhat. The result indicated in figure 1 implies that μ_x', and hence μ_y' and μ_x and μ_y, can probably be taken equal to zero without introducing large errors into instability calculations.

Four stiffnesses, E_y, D_y, D_{xy}, and G_{xy}, remain about which little is known. Some effects of variations of these stiffnesses on the calculated buckling loads are discussed in subsequent sections.

Test Specimens and Test Procedure

The test specimens consisted of seven 77-inch-diameter cylinders stiffened with extruded Z-section stringers on the outer surface and with small, formed, hat-section rings on the inner surface. The stringers were 0.54 inch deep and the rings either 0.54 inch or 0.30 inch deep. The cylinders with 0.54-inch rings had a stringer-spacing—skin-thickness ratio b/t of 125, and those with 0.30-inch rings, $b/t = 200$. Further references to the cylinders will be made by specifying b/t of the cylinder wall. The skin, stringer, and rings were made of 7075-T6 aluminum alloy.

The cylinders were loaded in bending through a loading frame with the use of a hydraulic testing machine. The test setup was similar to that described in reference 3 for bending tests on ring-stiffened cylinders.

Test Results and Discussion

Test results are given in figure 2 along with calculated results made with the use of the stability equation of reference 4 and selected wall stiffness values. The circular test points represent cylinders with $b/t = 125$ and the square test points represent cylinders with $b/t = 200$. The calculation denoted by the solid curves was made with the use of the values discussed earlier for E_x, D_x, and Poisson's ratios, and with the use of rather arbitrary estimates of the stiffnesses E_y, G_{xy}, D_y, and D_{xy}. For this case G_{xy} was taken as the value given in reference 5 for flat plates, E_y was obtained by considering the effective width of buckled skin to be equal in the longitudinal and circumferential directions, D_y was taken as the value computed for the ring and that portion of skin equal to the width of the ring, and D_{xy} was computed on the assumption that local buckling of the skin could be neglected. This calculation is about 8 percent conservative for cylinders with $b/t = 125$ and about 25 percent conservative for cylinders with $b/t = 200$. (See fig. 2.)

Additional calculations were made to determine the influence of changes in some of the stiffnesses on calculated buckling load. In the several calculations made, only one stiffness at a time was varied from the set of stiffnesses used to construct figure 2. Results of the calculations are essentially as follows: (1) The calculated buckling moment was decreased 5 to 10 percent from that shown when the stiffness D_{xy} was taken to be zero. (2) The buckling moment was practically unchanged for a calculation made with the stiffness E_y calculated on the assumption that the buckled skin contributed nothing. (3) The buckling load was increased 5 to 10 percent over that shown for a calculation made with D_y calculated on the assumption that the skin was 100 percent effective in contributing toward the moment of inertia of ring and skin. Use of this assumption should not suggest that the skin is very effective insofar as it increases the stiffness D_y in this manner; rather the calculation was made to determine the sensitiveness of buckling to changes in D_y. The suggestion has been made (see ref. 6, p. 165) that the corrugation effect of buckles may increase the stiffness D_y. (4) An increase of 100 percent in the stiffness G_{xy} increased the buckling moment about 15 percent for the cylinders with $b/t = 200$; it increased the buckling moment for the cylinders with $b/t = 125$ about 15 percent for cylinders with large ring spacing and about 25 percent for cylinders with small ring spacings. The 100-percent change in shear stiffness may seem to constitute a rather large change; however, it is well within the limits of uncertainty for the shear stiffness of buckled sheet.

From these calculations it can be concluded that the main source of the discrepancy between calculation and experiment as indicated in figure 2 is probably not associated with the use of uncertain values for the stiffnesses E_y and D_{xy}; these stiffnesses were varied over their probable range of uncertainty with little influence on the calculated buckling moment. Hence, uncertainties in the values of D_y and G_{xy} remain as a probable cause of the discrepancy. If they are indeed the cause, the values of these stiffnesses would have to be increased more for highly buckled sheet than for moderately buckled sheet (corrugation effect) in order to bring calculation and experiment into agreement; the cylinders with $b/t = 125$ experienced instability at ratios of edge strain to local buckling strain of about half of those for the cylinders with $b/t = 200$. Corrugation effects such as these have been suggested in the literature. (See refs. 6 and 2.) Another cause for the discrepancy may be associated with the use of a stability equation based on small-deflection buckling theory. Theory is known to give erroneous results in the case of unstiffened cylinders, for instance. In an attempt to compensate for discrepancies from this source, a "correlation factor" was used in the calculations made. The factor was taken from reference 7 and has had only limited substantiation. Its value was approximately 90 percent for the cylinders with $b/t = 200$ and nearly 100 percent for the cylinders with $b/t = 125$. The discrepancy probably is not associated with the use of incorrect values of E_x and D_x because moment-shortening curves for the cylinders as determined by calculation and test were in reasonably good agreement.

One further calculation was made to determine the likelihood of some of the cylinders having failed by buckling between rings (panel instability) instead of in a general instability mode. The cylinders most susceptible to panel instability are those denoted by the square symbols at large ring spacings. Hence the calculation was made for those cylinders. It is indicated in figure 2 by the dashed curve and was made with the use of reference 1 and a column-fixity coefficient of 2.0. The use of other values of the coefficient would simply move the curve to the right or left. From this calculation it can be concluded that one of the test cylinders (the last square test point in fig. 2) may have failed by panel instability; the others probably did not. This result indicates that even very small rings evidently provide considerable restraint to the cylinder wall insofar as influence on the panel-instability mode of failure is concerned; the column-fixity coefficient of 2.0 is a considerable increase over the value of unity usually assumed in calculations for panel instability.

FILAMENT-WOUND GLASS-EPOXY CYLINDERS

Discussion of Pertinent Wall Stiffnesses

In contrast to the previous example of ring-and-stringer stiffened cylinders where some of the wall stiffnesses are known with reasonable accuracy, none of the wall stiffnesses are known very accurately for filament-wound cylinders. A basic element of the wall of glass-epoxy cylinders is one of unidirectional fibers embedded in epoxy. If the extensional stiffnesses of this orthotropic element were known, the extensional and bending stiffnesses of the wall could be calculated with the use of theory of elasticity. Two of the five elemental extensional stiffnesses are rather insensitive to distribution of fibers and epoxy and depend almost solely on the volumetric ratio of glass to glass plus epoxy; hence, they are known with reasonable accuracy. These stiffnesses are E_w (the extensional stiffness of the element in the direction of the fibers) and μ_w' (Poisson's ratio associated with contraction normal to the fibers caused by a load in the direction of the fibers). These stiffnesses can be written with good approximation as

$$E_w = kE_g + (1 - k)E_e$$

and

$$\mu_w' = k\mu_g + (1 - k)\mu_e$$

The remaining stiffnesses E_c and G_{wc} depend heavily upon the distribution of fibers and epoxy and were calculated with the use of a mathematical model similar to the one employed in reference 8 for monofilament laminates. The distribution of fibers and epoxy was taken into account in the calculations by a parameter which was chosen such that certain calculated wall stiffnesses agreed with stiffnesses measured in the course of the cylinder tests. Poisson's ratio μ_c' was found from the relationship $E_w\mu_c' = E_c\mu_w'$. Young's modulus and Poisson's ratio for glass and epoxy were required for the calculations. The values for glass were taken from the literature as $E_g = 10{,}500$ ksi and $\mu_g = 0.20$. Those for the epoxy were determined from compression tests on 1-inch by 1-inch by 4-inch blocks of the epoxy.

With the stiffnesses of the basic wall element known, the stiffnesses of helical and circumferential windings in any desired direction can be obtained with the use of equations of elasticity for orthotropic materials (see ref. 9), and then combined to give the extensional and bending stiffnesses of the cylinder walls. Typical calculated values of the extensional stiffnesses of a cylinder wall are given in figure 3 for the case of an equal number of helical and circumferential windings.

It is evident from figure 3 that the shear stiffness G_{xy} is rather insensitive to helix angle; it is small relative to the extensional stiffnesses E_x and E_y when compared with similar relationships for metals. Other observations that can be made are: (1) a large difference exists between the stiffnesses E_x and E_y for all but very small helix angles and (2) Poisson's ratios associated with extension of the cylinder wall are somewhat smaller than those associated with metals.

Test Cylinders and Test Procedure

The test cylinders were 15 inches in diameter and 15 inches long. The walls of the cylinders consisted of alternate layers of circumferential and helical windings. Each helical winding consisted of two half-windings, one at $+\alpha$ and the other at $-\alpha$ to the longitudinal center line of the cylinders. The walls of the cylinders were built up near each end with additional circumferential windings to prevent end failures. The radius-thickness ratio of the cylinders was about 145. Two epoxies were used in construction of the cylinders, Shell Epon 828 with curing agent D and Shell Epon 826 with curing agent CL. Young's modulus and Poisson's ratio for the two epoxies were nearly equal. The block tests indicated that they could be taken as $E_e = 450$ ksi and $\mu_e = 0.40$.

The cylinders were tested in axial compression between the platens of a hydraulic testing machine until buckling or failure occurred. Two tests were made on nominally identical specimens to determine the consistency of buckling loads. In some cases supplementary tests were conducted to obtain the volumetric ratio of glass in the walls, and photomicrographs were made to obtain an idea of the uniformity of construction. The volumetric ratio of glass appeared to be reasonably consistent from sample to sample within a cylinder and from cylinder to cylinder; it was taken as 0.65. The photomicrographs indicated some nonuniformity in wall construction although the overall thickness of the walls as determined by micrometer measurements was reasonably uniform.

Test Results and Discussion

The test results are given in figure 4 along with the results of an instability calculation made with the use of reference 4 and the calculated stiffnesses of figure 3. Because of the relatively low shear strength of epoxies, an additional calculation which represents shear failure of the cylinder wall from axial compressive stresses is shown. The shear failure calculation was made with the use of results from a compressive test on a 2.60-inch-diameter tube consisting of all circumferential windings. The dashed curve indicates the stress at which tubes that have an equal number of circumferential and helical windings are expected to fail in shear from an applied compressive stress. Cylinders which have an elastic buckling load not much greater than this stress are expected to fail at a somewhat lower stress because of the interaction between the two failures. That is, as a cylinder is loaded to loads somewhat less than the shear-failure load, epoxy shear stresses become large and the epoxy deforms plastically. This deformation causes a reduction in wall stiffnesses which, in turn, lowers the buckling load. The test cylinders with 45° and $67\frac{1}{2}^{\circ}$ helical windings showed evidence of this interaction at failure. Buckling was evident in that a buckle pattern covering the complete circumference of the cylinders and about half of the length of the cylinders was observed; and shear failure of the wall in the area of buckling was also observed. The shear-failure evidence was lacking in the tests on the cylinders with helical angles of 25°; these cylinders appeared undamaged when load was removed and the diamond-shape buckles, which had appeared suddenly at maximum test load, had disappeared. The appearance of the cylinders was somewhat deceptive, however, because the cylinders took less load on reloading than on the first loading. The test loads for these cylinders as well as those for other nominally identical cylinders were nearly equal. (See fig. 4.)

The relatively good agreement between the failing load for the cylinders having a helix angle of 25° with that computed on the basis of small-deflection buckling theory is attributed to the small radius-thickness ratio of the cylinders which was about 145. Previous studies on isotropic metal cylinders have indicated that good-quality cylinders with radius-thickness ratios of this magnitude will experience little deviation from theory, perhaps 15 percent (see, for instance, ref. 3) which is somewhat more than that indicated for the test cylinders.

One further observation can be made concerning the magnitude of compressive stress at failure of the tube with circumferential windings.

The applied stress of 10.9 ksi corresponds to a shear stress in the epoxy of about half that value or about 5.45 ksi. Tensile tests made on epoxy specimens by other investigators indicate that the epoxy has an ultimate tensile strength of about 9.5 ksi and failure occurs after considerable elongation (about 4 percent). The corresponding ultimate shear strength of the epoxy is expected to be about $\frac{9.5}{\sqrt{3}}$ or about 5.5 ksi, which is very close to the shear stress at failure of the tube in compression. Hence, on the basis of this single test, it appears that a tensile test of an epoxy specimen may be used to determine the compressive-failure strength of unidirectional glass-epoxy plates when loaded perpendicular to the direction of the fibers.

CONCLUDING REMARKS

Preliminary results from experimental investigations on the buckling strength of ring-and-stringer stiffened aluminum cylinders and filament-wound glass-epoxy cylinders are presented and discussed. Attempts at predicting test results are also given. Correlation between calculation and experiment for the glass-epoxy cylinders was reasonably good. However, the scope of the test results is extremely limited and additional testing is necessary to prove or disprove the theories set forth. Glass-epoxy cylinders have considerably different wall properties than those associated with metals; as a result, new failure modes may be important. In particular, shear stresses induced by the compressive load on the cylinder wall may cause wall failure. Additional testing is required to establish modes of failure and to establish the stiffness properties of the glass-epoxy walls.

The correlation obtained between calculation and experiment for the ring-and-stringer stiffened cylinders leaves considerable room for improvement. Wall stiffnesses - especially shear and circumferential bending - are known with insufficient accuracy, and "correlation factors" which delineate the discrepancy between theory and experiment for buckling of orthotropic cylinders are not adequately defined. Until these areas of weakness are strengthened, correlation between calculation and experiment cannot be adequately achieved. The tests indicate that even small rings offer considerable restraint to the cylinder wall in the panel-instability mode; but additional work is required to determine the quantitative effect of ring proportions on cylinder strength.

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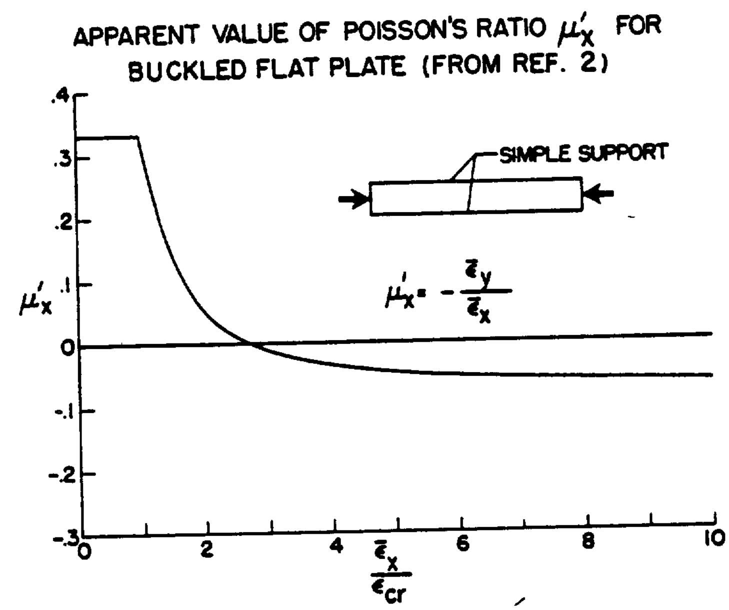

Figure 1

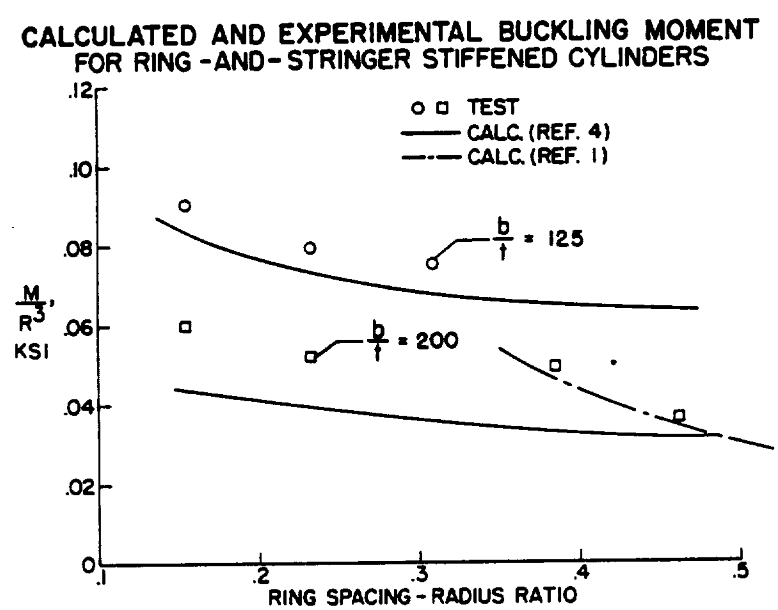

Figure 2

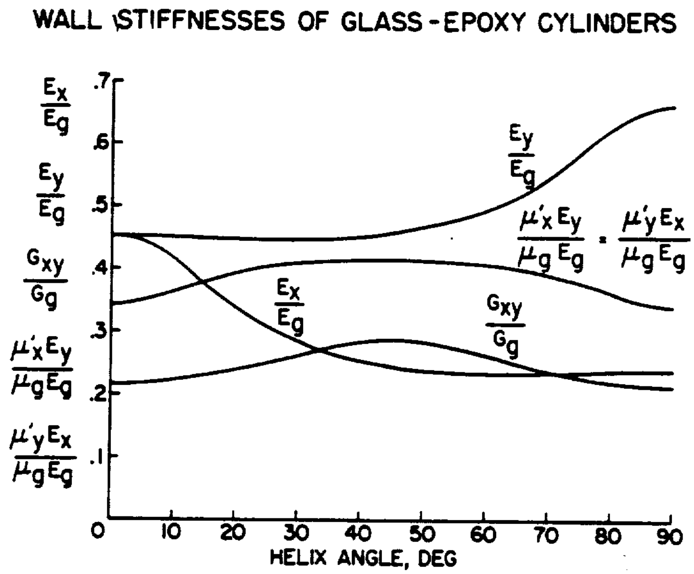

Figure 3

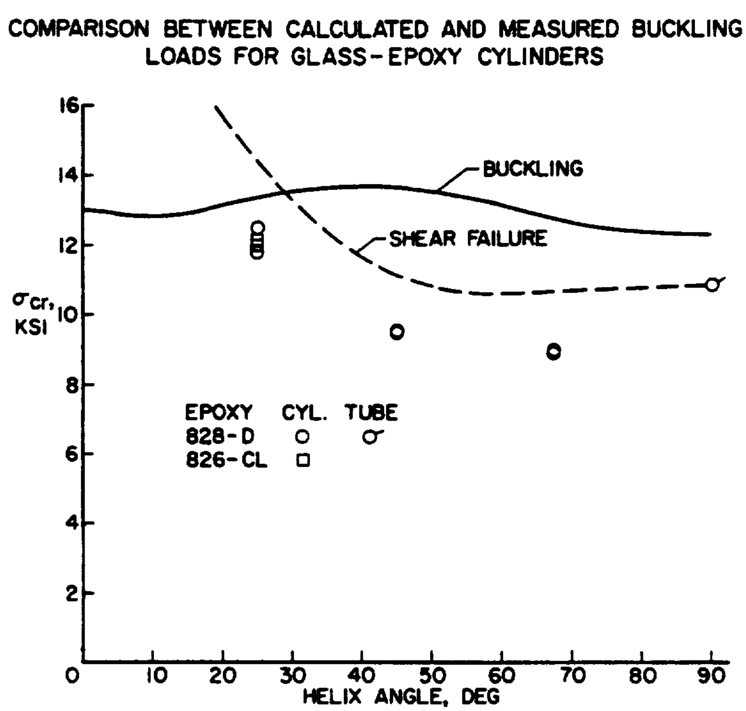

Figure 4