# GENERAL INSTABILITY OF ORTHOGONALLY STIFFENED CYLINDRICAL SHELLS

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## SUMMARY

Earlier research at the National Aeronautics Research Institute (N.L.R.), Amsterdam, which forms the basis of recent work is reviewed. This early work refers to 2 schemes: the orthotropic shell and, in view of buckling modes where the half wave length is of the order of the ring distance, the shell with continuously distributed stringers and discrete rings. Linear theory is considered to be adequate for these structures, where the imperfections are small in comparison to the height of the ring sections. Recent developments account for pressure difference in addition to axial compression, for the correct stiffness matrix of skin panels in the post-buckling stage and for stringer bending due to hoop stresses in the skin, which are of importance as has been shown by the investigation of the post-buckling behaviour. Numerical data for the stiffness matrix of skin panels have been established. Numerical evaluation of the stability equation has not been performed as yet.

## INTRODUCTION

The general instability of cylindrical shells received attention in a former period in view of its importance for fuselages of large diameter. The critical condition then originates from fuselage bending. The complexity of the problem, caused by the large number of structural parameters, was a good reason for avoiding the complexity of the non-uniform load condition presented by bending. It might be expected that the easier problem of axial compression would yield valuable information for the actual fuselage problem in those cases where the circumferential half wave length of the critical mode would be much smaller than the radius of the cylinder. With certainty the critical bending stress would be greater than the critical stress in axial compression.

The occurrence of similar structures in missiles has given new impetus to the problem. Here again bending is the critical condition.

However with pressurized fuel tanks the effect of the pressure difference should not be disregarded.

In former calculations the way in which the lateral stiffness and the shear rigidity of buckled skin panels was accounted for was mainly a matter of guessing. The application to missile structures where thin skins are used calls for alleviation of this lack of knowledge. On the basis of earlier work by the N.L.R. on post-buckling behaviour the data needed for stability research have been established.

Another shortcoming of the former work on general instability has emerged recently from knowledge obtained on the tangential stiffness of buckled panels. In contrast to former opinions this stiffness proved to be quite great. It follows then however that the effect of deflection of the stringers caused by the hoop stresses of the skin should also be accounted for.

The present report reviews the various facets of the problem.

## SYMBOLS

| | |
|---|---|
| $a$ | radius of the cylinder |
| $\|a\|$ | stiffness matrix of the skin panel |
| $b$ | ring pitch |
| $c$ | $= c_s + c_r$ |
| $c_s$ | distance of centroid of stringer to skin, positive for inside stiffeners |
| $c_s$ | distance of centroid of ring section to skin, positive for rings inside |
| $\bar{e}$ | $\varepsilon/\varepsilon^*$ |
| $s$ | $\sigma/(E\varepsilon^*)$ |
| $t$ | skin thickness |
| $w$ | panel width |
| $A_r$ | area of ring section, including effective skin width |

| | |
|---|---|
| E | Young's modulus of elasticity |
| $I_s$ | moment of inertia of stringer section, including effective skin width |
| $\varepsilon$ | strain increment by buckling |
| $\varepsilon^*$ | $\pi^2 \left[3(1 - \mu^2)\right]^{-1} (t/w)^2$, critical longitudinal compression |
| $\mu$ | Poisson's ratio |
| $\nu$ | average number of rings per half wave length |
| $\sigma$ | increment of average stress by buckling |

## ADEQUACY OF LINEAR THEORY

The effect of imperfections with respect to the true cylindrical shape causes large discrepancies between the actual critical load of unstiffened cylindrical shells and the critical load predicted from linear theory. Finite deflections have to be considered and consequently non-linear equations are required to restore harmony between theory and test. The essential reason for this behaviour of the cylindrical shell is the coexistence of a symmetric and several asymmetric buckling modes. The actual buckling load is then highly sensitive to small imperfections - expressed as the difference between the local radial coordinate and the average radius - of the same order of magnitude as a fraction of the wall thickness t, or better the radius of gyration $t/2\sqrt{3}$. Correspondingly, if a sufficiently high internal pressure is applied - which restores the circular cross section and prevents buckling modes with circumferential waves - the critical load as predicted by linear theory proves to be correct.

In the case of the stiffened cylinder the necessity to account for non-linearity would certainly exist likewise if the imperfections would have the same order of magnitude as the radius of gyration of the ring section. However, since the imperfections of ring geometry can be kept small in comparison to the height of the ring section, it may be conjectured that imperfections have no major effect upon the critical load. Therefore it is considered that linear theory is adequate for the investigation of general instability of stiffened shells.

# REVIEW OF EARLIER WORK

## Structural Schemes

Applying linear theory the National Aeronautical Research Institute (N.L.R.), Amsterdam has analyzed the general instability of stiffened cylindrical shells under axial compression (ref.1). This investigation consisted of 2 phases. In the 1$^{st}$ phase the shell was assumed to be orthotropic, due to continuous distribution of the stiffnesses of the stringers and of the rings. In the 2$^{nd}$ phase the stringers were again continuously distributed, but the rings were discrete at constant pitch.

The reason to investigate this second scheme was that the longitudinal half wave length, as obtained from the orthotropic case, might well be about equal to or a small multiple of the ring pitch. So it seemed doubtful whether the orthotropic scheme would yield reliable results. No necessity existed to criticize the continuous distribution of stringers since the circumferential half wave length comprises a number of stringers.

The investigation took the following characteristics of the structural elements into account:
stringers and rings: longitudinal stiffness, bending stiffness in the plane normal to the shell, torsional stiffness, ring pitch and the excentricity of the center of gravity of the stiffenersections with respect to the skin;
skin: shear rigidity and effective width, which was added to the sections of stringers and rings.

In this way the buckling load parameter was a function of 7 structural parameters and 2 parameters for the buckling mode. The 7 structural parameters are characteristic for respectively shear stiffness of the skin, bending stiffness of the 2 systems of stiffeners, torsional stiffness of the 2 systems of stiffeners, ring pitch and excentricity of the stiffeners, the latter of which proved to be a combined parameter for the 2 systems of stiffeners. In the case of the orthotropic shell the 2 torsional stiffnesses combine into 1 parameter and since ring pitch has been removed 5 structural parameters remain.

## Results for Orthotropic Shells

The formula for the buckling load parameter is rather involved with its 5 + 2 parameters and minimization of this parameter with respect to the 2 mode parameters, so as to establish the critical load, is in general impossible.

Simplified expressions could be obtained by considering 5 classes of buckling modes, characterized by the various orders of magnitude of the ratio between the longitudinal and the circumferential wave lengths. Explicit formula for the critical load could be given for 4 of the 5 classes. Out of these 4 2 referred to short longitudinal wave length and small numbers of circumferential waves and 2 referred to long longitudinal waves and again small numbers of circumferential waves. Between these 2 groups the class of short longitudinal and circumferential wave lengths occurs. In this case, which usually is critical, the number of structural parameters reduces to 4, but an explicit formula for the critical load could not be given. However a rapidly converging procedure for the numerical determination of the critical load could be established.

An interesting result is that the effect of stiffener eccentricity with respect to the skin is of major importance. Compared to the case where the centroid of stiffeners falls in the plane of the skin outside stiffening increases the buckling load and reversely inside stiffening yields a reduction. In some cases the buckling load reduced to as low as $^1/3$ of the value for the case without excentricity.

## Results for Shells with Discrete Rings

The investigation of the structure with discrete rings could obviously be confined to those classes of modes in which the longitudinal wave length is short.

It should be remembered that the capacity of the skin to carry hoop stresses has been expressed as an effective width working together with the rings. Accounting for discrete rings the actual structure as far as the rings are concerned is fully recognized. However the behaviour of the skin is violated in that the distributed tangential force of the skin is concentrated at the rings. A more sophisticated scheme, where the concentrated effective skinwidth was replaced by an "effective skin thickness", was attempted but was cancelled because of the inherent analytical complexity.

For the infinitely long cylinder a formula could be obtained giving the relation between the 6 structural parameters (1 parameter vanished due to the restriction to short waves), 2 mode parameters (governing the displacements at the rings) and the buckling load parameter. A semi-graphical procedure could be evolved for establishing the critical load pertaining to a given set of structural parameters.

A result of major interest is that the solution obtained from orthotropic shell theory is surprisingly accurate. Figure 1 shows for the case of axially symmetrical buckling the reduction of the buckling load by the small average number $\nu$ of rings per half wave length. For $\nu > 2$ the error by assuming orthotropy is less than 1°/o. Therefore two rings on the half-wave length is mechanically aequivalent to ''many'' rings. Only for $\nu < 1,5$ the critical load is affected more than 5°/o by ring spacing. In the case of axially non-symmetrical buckling modes the results are similar. The error by orthotropy depends now also on the other structural parameters, but remains of the same order. A numerical example yielded for $\nu = 2$ 4°/o error and for $\nu = 1,6$ 6°/o.

Another result is that by increasing the ring stiffness the longitudinal half wave length decreases gradually until it reaches the value $\nu = 1$. Then general instability has degenerated into column failure of the stringers between the rings and the rings, though having finite stiffness, are aequivalent to infinitely stiff rings. The ring stiffness required for the exclusion of general instability can be expressed by a simple formula as far as axially symmetrical buckling modes are concerned

$$A_r \geq 4\pi^2 (a/b)^2 . (b/w) . (I_s/b^2) .$$

The prevention of general instability in axially non-symmetrical buckling requires unpractically large ring bending stiffness. Therefore in practice general instability is more critical than column failure of the stringers.

## RECENT WORK

### Scope

Recent developments on the basis of ref. 1 are meant to include load pressure difference, to improve the way in which the post-buckling behaviour of the skin panels is accounted for and to account for bending of the stringers due to hoop stresses of the skin.

### Post-buckling Panel Stiffness

In ref. 1 the post-buckling behaviour of the skin panels was accounted for by introducing effective widths for longitudinal and lateral stiffness and an effective shear rigidity. At the time when the investigation was done - 1942 - data existed only on the effective width in longitudinal compression; no data were available on effective width under arbitrary two-axial compression nor on the effective shear rigidity. These data have been established only recently and with the aim to be used for studying general instability (ref. 5).

The object is to know the stiffness matrix of skin panels in the post-buckling state with regard to incremental deformation,

$$\begin{vmatrix} s_1 \\ s_2 \\ s_3 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \cdot \begin{vmatrix} e_1 \\ e_2 \\ e_3 \end{vmatrix}$$

where the indexes 1, 2, 3 denote longitudinal direction, lateral direction and shear respectively. In the subcritical state the matrix is

$$|a| = \frac{1}{1-\mu^2} \begin{vmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{vmatrix}$$

In view of the limitation to general instability for load conditions without panel shear in the prebuckling state, we confine ourselves to panels where shear is absent in the initial state. Then symmetry considerations yield immediately $a_{13} = a_{23} = 0$. Further from Maxwell's principle $a_{ij} = a_{ji}$. So 4 elements remain to be established: $a_{11}$, $a_{12}$, $a_{22}$, $a_{33}$.

This has been done on the basis of a theoretical investigation by Koiter on the shear field of flat panels (ref.2). Koiter assumes the deflection pattern, given in fig.2, where f, L, m and $\alpha$ are parameters depending on the magnitude of the 3 overall strain components. The relations between the 4 parameters, the 3 overall strain components and the 3 average stress components have been evaluated in ref.3. When dealing with stability problems we need to know the stiffness matrix valid for small strain increments, with respect to a given initial state of strain. This matrix has been established recently. The results obtained will be discussed briefly.

Fig.3 shows $a_{33}$ as a function of the total strains $e_1$, $e_2$. The value for the unbuckled state $a_{33} = \left[2(1+\mu)\right]^{-1} = 0{,}3846$ is approached when the lateral strain $e_2$ is a large positive number. When lateral compression is added to longitudinal compression the shear rigidity decreases more and more and can even become negative. Conditions where $e_2$ is a large negative number will not occur in cylindrical shells.

However when $e_2$ is around zero the shear rigidity can have been reduced to 1/3 of the initial value. $a_{11}$ cannot present surprising results after what is known on the effective width. $a_{12}$ is usually a small negative number in the order of -0,05. As could be expected the lateral stiffness $a_{22}$ increases with increasing positive lateral strain. For $e_2$ in the order of 5 $a_{22}$ is in the order of 0,9, whereas $a_{22}$ is 1,10 in the unbuckled state. It is remarkable that with small negative $e_2$ in the order of -1 $a_{22}$ does not drop below 0,5 and even more surprising is that $a_{22}$ increases when the compression $-e_1$ increases under constant negative $e_2$. All this means that the lateral panel stiffness is an important factor to be considered in general instability investigations.

The foregoing results apply to initially flat panels, whereas the shell panels are shallow. Koiter (ref.4) established the relation between compressive strain and load in the initial post-buckling stage of curved panels. The parameter

$$\theta = (2\pi)^{-1} \left[12(1-\mu^2)\right]^{1/4} w(at)^{1/2} = 0,289\ w/(at)^{1/2}$$

proves to govern the initial stage of post-buckling behaviour. For $\theta < 0,64$ the slope of the load-strain curve is positive. However for a narrow panel like $w/a = {}^1/15$ (94 panels in the cylinder) and $w/t = 100$ $\theta$ amounts already to 0,75, where the slope is distinctly negative. The critical strain of the curved panel is however $1 + \theta^4$ times the critical strain of the flat panel of equal w/t and Koiter states: (see also fig.4) ''it would appear to be not too bold a conjecture that the behaviour of a narrow curved panel in the advanced postbuckling stage approaches the behaviour of a flat panel of equal width''. This conjecture finds support in the consideration, that the tendency to keep the extensional strain energy down has the effect of making the bulge towards the inside deeper than the bulge towards the outside. In the limit the buckling pattern appoaches the symmetrical configuration with respect to the chord plane. Then the incremental deformation starts from an initial state not different from the initial state of the flat panel. Consequently results obtained for flat panels may be used for analysis of general instability.

In one respect allowance should be made for the initial curvature. In determining the state of stress preceding general instability the stress-strain relations should account for the amount of lateral strain $(\varepsilon_2)_o = w^2/(24a^2)$, which stretches the curved panel and which is not accompanied by membrane stresses.

## Bending of Stringers by Hoop Stresses of the Skin.

General instability creates incremental hoop stresses $s_2$, proportional to the local deflection, which are not balanced by pressure difference and which result in a radial load q of the stringer. Therefore (when $s_2 > 0$) the stringer deflects towards the inside, thereby reducing $e_2$ and consequently $s_2$. This means that the flexibility of the stringers reduces the effect of the large lateral stiffness of the panel and that this flexibility has to be accounted for. A rigorous analysis would require that the discrete ring scheme should be adopted. Already, in the development of ref.1, the complexities encountered were such that the lateral stiffness of the skin was added to the discrete rings. Therefore at the present stage -where the addition of pressure difference and the true stiffness matrix of the skin panels have already increased the complexity of the equations- no attempt is being made to account rigorously for stringer flexibility. The analysis has been confined to the orthotropic shell, however with the introduction of a correction for stringer bending by hoop stresses. The way in which this correction has been introduced is as follows.

When bending by hoop stresses is neglected the radial displacement of the stringer is (in the orthotropic scheme) in any point equal to the deflection of the ring. Allowance should however be made for a difference in stringer and ring deflection due to stringer bending. This difference is nil at the points where the actual rings and the stringer intersect; the difference is maximal somewhere midway between the rings. Now this variable difference is replaced by its average value between 2 successive rings. This idea can be translated into a mechanical system, where the stringers are connected to the rings by springs and the stringers are assumed to be infinitely rigid with respect to q. With this scheme there is still a linear variation of q from one ring to the next one because of the change of ring and spring deflections from one ring to the next one. This difference is being neglected so that q is assumed to be constant throughout one bay. Before the average deflection can be established one further assumption has to be made with respect to the continuity of slope at the supports. When the half wave length of the buckling mode is much greater than the ring distance, the difference between the average q's of successive bays is small and the stringer is at its support practically clamped. Then the aequivalent spring stiffness is $720\,(n-1)EI_s/(nwb^4)$ per unit of area, where n/n-1 is the Vianello-correction accounting for combined bending and compression. The stiffness thus obtained is correct if the half wave length is very large in comparison to the ring distance. However in the other extreme case, where the half wave length is equal to the rib distance, the coefficient 720 has to be replaced by 120; moreover the Vianello correction is greater. So the clamped end assumption may underestimate the flexibility of the stringers quite considerably. This problem needs further consideration and a method will have to be devised by which instead of the coefficient 720 one is introduced which is a function of the wave length to ring pitch ratio.

### General Instability Equations.

The stability equations for the orthotropic scheme just discussed contain 4 displacements functions, 2 for displacements in the plane of the shell and 2 radial displacements of which one for the rings and one for the stringers. There are 4 equations available: 3 in the usual way from the equilibrium of an element of the orthotropic shell and one for the deflection of the spring between rings and stringers.

The solution of these equations for the infinitely long cylinder yields the buckling load parameter as a function of 2 mode parameters (for the 2 wave lengths) and 12 structural parameters: stiffness of the ring to extension, bending stiffness of stringers and of rings, torsional stiffness of stringers and of rings, excentricity of stringers and of rings, ''spring'' stiffness and 4 elements of the stiffness matrix of the skin panel. The complexity could be somewhat reduced by neglecting the torsional stiffnesses, which usually are not important, and the small matrix element $a_{12}$. Then still 9 structural parameters remain. This means that there is no hope to find as a result of much algebraic diligence an explicit formula for the critical load. There is however no difficulty to compute the critical load for a given set of numerical values of the structural parameters.

## FUTURE RESEARCH

Work still to be done is:

1. To find a better expression for the deflection of the stringers by hoop stresses, which takes into account the ratio between wave length and ring pitch.
2. To carry out numerical calculations for axial load and various pressure differences in order to study the effects of pressurization, of post-buckling behaviour of the skin and of stringer deflection by hoop stresses. Preferably these numerical calculations should be applied to structures and load conditions for which test results are available.
3. To investigate the possibility for analysis of the discrete ring scheme.

## REFERENCES

1. van der Neut, A.: The general instability of stiffened cylindrical shells under axial compression. Rep. S314, National Aeronautical Research Institute, Amsterdam; in Reports and Transactions Vol XIII, 1947, pg. S57-S84.
2. Koiter, W.T.: Het schuifplooiveld bij grote overschrijdingen van de knikspanning (The post-buckling behaviour of flat rectangular panels in shear and compression). Rep. S295, Nat. Aer. Res. Inst., Amsterdam, 1944, 86 pgs. (in Dutch).
3. Floor, W.K.G. and Burgerhout, T.J.: Evaluation of the theory on the post-buckling behaviour of stiffened flat rectangular plates subjected to shear and normal loads. Rep. S370, Nat. Aer. Res. Inst., Amsterdam; in Reports and Transactions Vol XVI, 1951, pg S9-S36.
4. Koiter, W.T.: Buckling and post-buckling behaviour of a cylindrical panel under axial compression. Rep. S476, Nat. Aer. Res. Inst., Amsterdam; in Reports and Transactions Vol XX, 1955, pg 71-84.
5. van der Neut, A.: The post-buckling stiffness of rectangular simply supported plates. Rep. VTH-113, Technological University, Delft, 1962, 15 pgs.

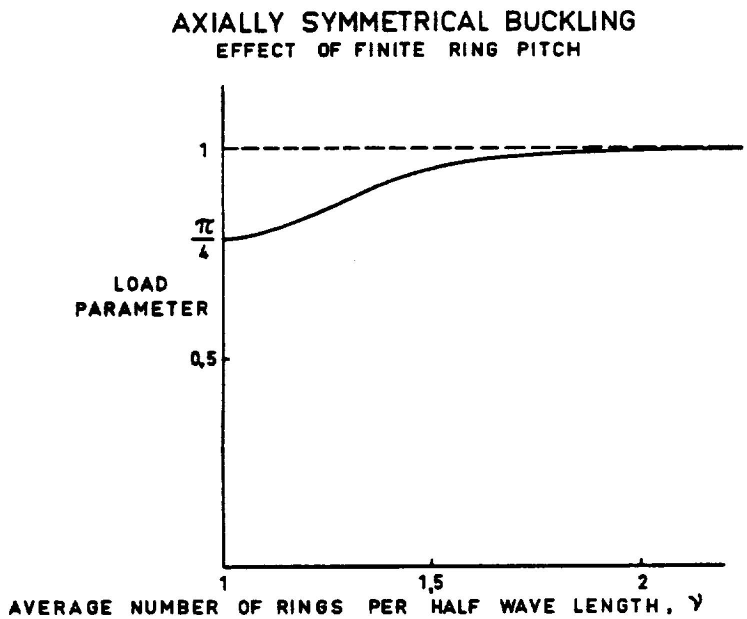


Figure 1

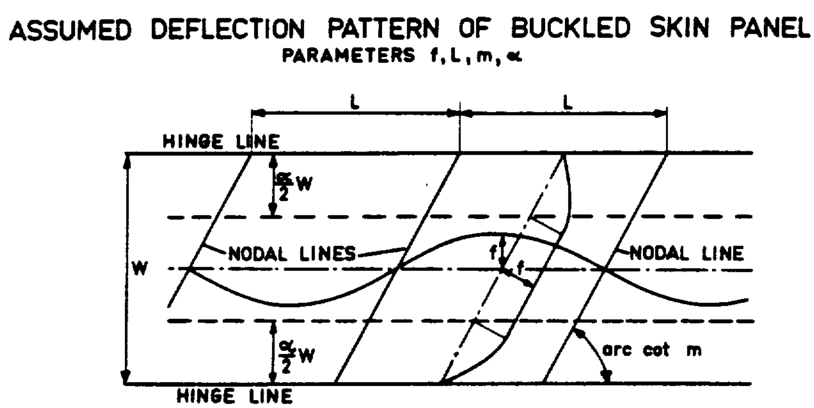


Figure 2

POST-BUCKLING PANEL RIGIDITY TO INCREMENTAL SHEAR

$a_{33} = E^{-1}\,\partial\sigma_3/\partial\varepsilon_3$ (UNBUCKLED $a_{33} = 0{,}3846$)

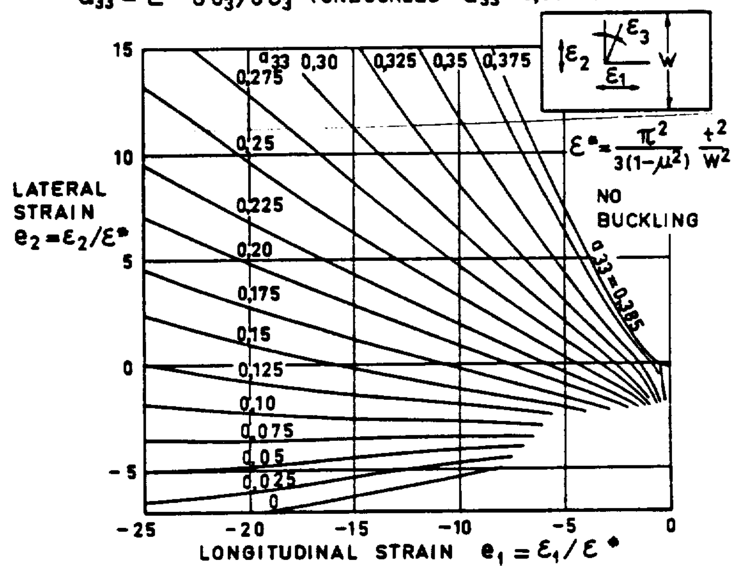


Figure 3

CONJECTURED POST-BUCKLING CURVES OF SHALLOW PANELS

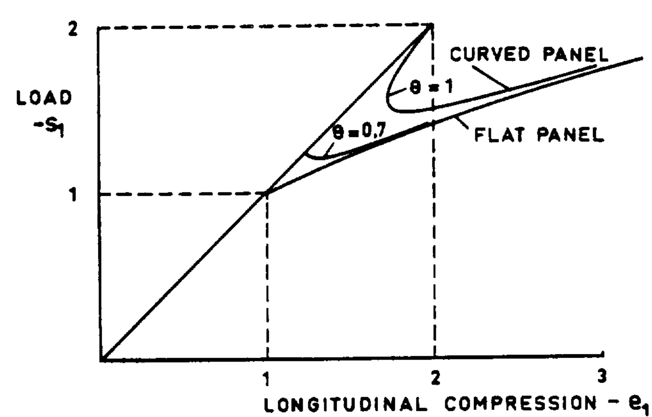


Figure 4