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BUCKLING OF LAYERED ORTHOTROPIC AND SANDWICH CYLINDRICAL SHELLS IN AXIAL COMPRESSION

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SUMMARY

Results of modifications of large-deflection theory for isotropic materials to shells constructed of orthotropic layers and sandwich are presented. Experimental evaluation of the buckling of a rather extensive series of plywood cylinders and of a few curved panels of sandwich construction shows reasonably good agreement with buckling predicted by theory.

INTRODUCTION

It has long been recognized that the buckling of long, thin-walled, cylindrical shells of isotropic material under axial compression is not predictable by the classical small-deflection theory which assumes buckling of the cylinder walls with a radial displacement of the form (ref. 1):

$$w = F \sin n\theta \sin\frac{m\pi x}{L}$$

Buckling stresses about 40 percent of those given by the classical theory were predicted by von Karman and Tsien (ref. 2) who assumed buckling in diamond-shaped waves with an inward deflection in the interior of each diamond. Their analysis was based on large-deflection theory and an energy method.

In this paper, the results of applying the method of von Karman and Tsien to shells of layered orthotropic and sandwich construction are compared with experimental evaluation of plywood cylinders and curved panels of sandwich construction. The inward radial deflection of the cylinder wall, w, was chosen to be of the diamond-shaped form (ref. 3):

$$\frac{w}{r} = g + \delta \cos^2\left(\frac{\pi y}{b} - \frac{\pi x}{a}\right)\cos^2\left(\frac{\pi y}{b} + \frac{\pi x}{a}\right)$$

and appropriate modifications were included in the analyses to account for orthotropic layers and for low shear modulus in the sandwich core.

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LAYERED ORTHOTROPIC CYLINDERS

The analysis of plywood cylinders under axial compression by March (ref. 3) was applied to layered orthotropic materials in general. From this the buckling stress of such a cylinder under axial compression is given by the formula

$$p_{cr} = K\sqrt{E_b E_1}\,\frac{h}{R} \tag{1}$$

where h is cylinder wall thickness, R is radius of neutral axis of cylinder wall, and

$$K = \frac{1}{12\sqrt{3}}\,(M_4 Q)^{1/2} \tag{2}$$

$$Q = 4M_3 - 9\,\frac{M_2^2}{M_1} \tag{3}$$

Values of M_1, M_2, M_3, and M_4 are dependent on the elastic properties of the shell wall and the buckle aspect ratio (ratio of circumferential dimension to axial dimension). It was found by computation that relative minimum values of K were very closely approximated by assuming the buckle aspect ratio to be $(E_2/E_1)^{1/4}$. Introduction of this simplification and introduction of nondimensional elastic parameters similar to those used by Thielemann (ref. 4) results in

$$\left.\begin{aligned} M_1 &= 1 + A^2 + \frac{8A^2}{A^2 + 9AB + 81} + \frac{8A^2}{81A^2 + 9AB + 1} + \frac{34A^2}{A^2 + AB + 1} \\ M_2 &= 1 + \frac{16A^2}{A^2 + AB + 1},\quad M_3 = 1 + M_2,\quad M_4 = 6 + 2C \end{aligned}\right\} \tag{4}$$

where

$$A = \left(\frac{E_a E_2}{E_b E_1}\right)^{1/2},\quad B = \left(\frac{1}{G_{ab}} - \frac{2\mu_{ba}}{E_b}\right)\left(E_a E_b\right)^{1/2},\quad C = \left(E_1\mu_{21} + 2\lambda G_{12}\right)\left(E_1 E_2\right)^{-1/2} \tag{5}$$

and

$$E_a = \frac{1}{h}\int_{-h/2}^{h/2}\frac{E_x}{\lambda}dr,\quad E_b = \frac{1}{h}\int_{-h/2}^{h/2}\frac{E_y}{\lambda}dr,\quad G_{ab} = \frac{1}{h}\int_{-h/2}^{h/2}G_{xy}dr$$

$$E_1 = \frac{12}{h^3}\int_{-h/2}^{h/2}\frac{E_x}{\lambda}r^2dr,\quad E_2 = \frac{12}{h^3}\int_{-h/2}^{h/2}\frac{E_y}{\lambda}r^2dr,\quad G_{12} = \frac{12}{h^3}\int_{-h/2}^{h/2}G_{xy}r^2dr$$

$$\mu_{ba} = \frac{1}{hE_a}\int_{-h/2}^{h/2} E_x\mu_{yx}dr \text{ (approx.)}, \quad \mu_{21} = \frac{12}{h^3E_1}\int_{-h/2}^{h/2} E_x\mu_{yx}r^2dr \text{ (approx.)}$$

$$\lambda = 1 - \mu_{yx}\mu_{xy}$$

where E is modulus of elasticity, G is modulus of rigidity associated with shear distortion of the plane of the shell wall or layer, μ is Poisson's ratio, x is axial direction of cylinder, and y is circumferential direction. Poisson's ratio, μ_{yx}, is defined as the ratio of contraction in the x direction to extension in the y direction associated with tension in the y direction.

Computation of Q values for three B values and a range of values in A produced the curves shown in figure 1. The point for isotropic cylinders is also shown on figure 1 and the buckling stress formula for the isotropic cylinders reduces to

$$p_{cr} = 0.242\ E\ \frac{h}{R} \tag{6}$$

Formula (1) was solved for previously reported experimental determination of the buckling stresses of plywood cylinders (ref. 5) of several constructions of yellow birch and yellow-poplar veneers. The data considered were limited to the linear elastic range by selecting only the shells that buckled at stresses no greater than 90 percent of the compressive proportional limit stress of the plywood. Measured elastic properties were used to compute the stiffness parameters A, B, and C and these values substituted into expressions (4) to obtain K values. The computed stresses are compared with the experimental values in figure 2. Although there is considerable scatter of individual points, the agreement between experiment and theory can be regarded as reasonably close for the type of problem considered.

SANDWICH CYLINDERS

The analysis for sandwich cylindrical shells in axial compression was carried out by March and Kuenzi (ref. 6) by applying the analysis for plywood cylinders (ref. 3) and accounting for effects of shear deformation in the core of a sandwich by an approximate "tilting" method as used by Williams, Leggett, and Hopkins in their analysis of flat sandwich panels (ref. 7). The analysis of reference 6 includes sandwich with orthotropic facings and cores and presents buckling coefficients for sandwich with isotropic facings on certain orthotropic cores. Buckling coefficients have also been determined for sandwich with certain orthotropic facings (ref. 8).

The facing stress at axial buckling load of a sandwich cylinder of thin isotropic facings of equal thickness on an isotropic core is given by the formula

$$p_{Fcr} = \frac{4N}{5Q_1} E \frac{h}{R} \tag{7}$$

where h is the sandwich thickness, R is the mean radius of the cylinder, E is the modulus of elasticity of the facings and

$$N = \frac{5M_1Q_1}{4\eta} + \frac{5}{6Q_1\ (1 + Q_2)} \left[\frac{M_2\eta + M_3\eta^2V}{1 + M_4\eta V + M_5\eta^2V^2} + Q_2M_2\eta\right] \tag{8}$$

$$Q_1 = 2\left[\frac{3(1 - \mu^2)}{\frac{c^2}{h^2} + \frac{c}{h} + 1}\right]^{1/2},\quad Q_2 = \frac{1}{3}\left(\frac{1 - \frac{c}{h}}{1 + \frac{c}{h}}\right)^2$$

$$V = \frac{ctE}{3(1 - \mu^2)\ hRG_c},\quad \eta = n^2\,\frac{h}{R}$$

$$M_1 = \frac{1}{12z^2} + \frac{2z^2}{3(1+z^2)^2} - \frac{3\left[\frac{1}{64z} + \frac{z^3}{4(1+z^2)^2}\right]^2}{\frac{1+z^4}{128} + \frac{z^4}{16(z^2+9)^2} + \frac{z^4}{16(9z^2+1)^2} + \frac{17z^4}{64(1+z^2)^2}}$$

$$M_2 = 3z^2 + 2 + \frac{3}{z^2},\quad M_3 = \frac{1}{8}\ (9z^4 + 70z^2 + 9)\ \left(\frac{1}{z^2} + 1\right)$$

$$M_4 = \frac{27}{8}\ (z^2 + 1),\quad M_5 = \frac{1}{8}\ (9z^4 + 70z^2 + 9)$$

where c is core thickness, t is facing thickness, μ is Poisson's ratio of facings, G_c is modulus of rigidity of core, n is number of buckles in a circumference, and z is buckle aspect ratio. Minimization of formula (8) for N with respect to η and z results in curves presented in figure 3. The value of N = 1 at V = 0 represents a cylinder wall with a "shear-rigid" core that has no extensional stiffness and reduces to the buckling stress of an isotropic shell with spaced facings. The curve for c/h = 1, extending from the short straight line toward the lower right, represents the shear instability of the sandwich wall and resultant buckling load per unit circumference of hG_c (approx.).

Theoretical values of N were computed by formula (8) for previously reported experimental determination of the buckling of sandwich curved panels under axial compression (ref. 6). It was found by calculation that the panels were larger than the size of one theoretical buckle and therefore may represent the performance of a complete cylinder. The sandwich was of aluminum facings on end-grain balsa wood cores and on soft cork board cores. Core modulus of rigidity values ranged from 15,000 psi. to 320 psi. Facing stresses at buckling were well below compressive proportional limit stresses. Experimental values of N were computed by solving formula (7) equated to the experimental facing stress at buckling. The experimental and theoretical values of N are compared in figure 4. Although many of the points are on the lower portion of the line, thus representing sandwich with "soft" cores, there is reasonably close agreement between experimental and theoretical values of N.

CONCLUDING REMARKS

Comparison of experimental and theoretical results indicates that a large-deflection theory incorporating buckles of diamond-shape describes the behavior of long, thin-walled, cylindrical shells of layered orthotropic and sandwich construction in axial compression.

Application of the theory for layered orthotropic shells to general buckling of stiffened shells would be of interest. Further experimental evaluation of complete sandwich cylindrical shells is necessary.

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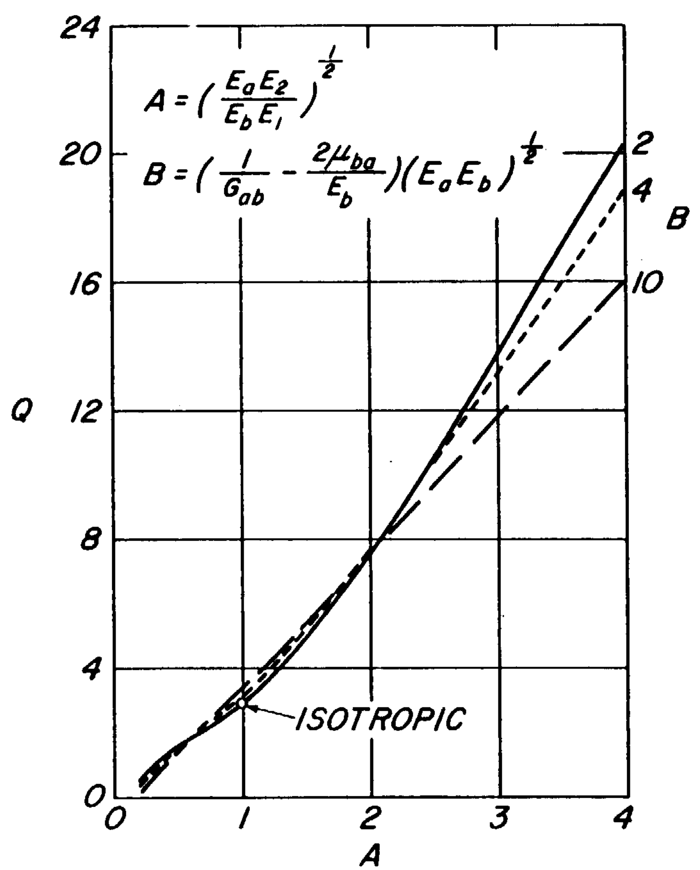

FIGURE 1. - VARIATION OF Q WITH A AND B FOR LAYERED, ORTHOTROPIC, CYLINDRICAL SHELLS IN AXIAL COMPRESSION.

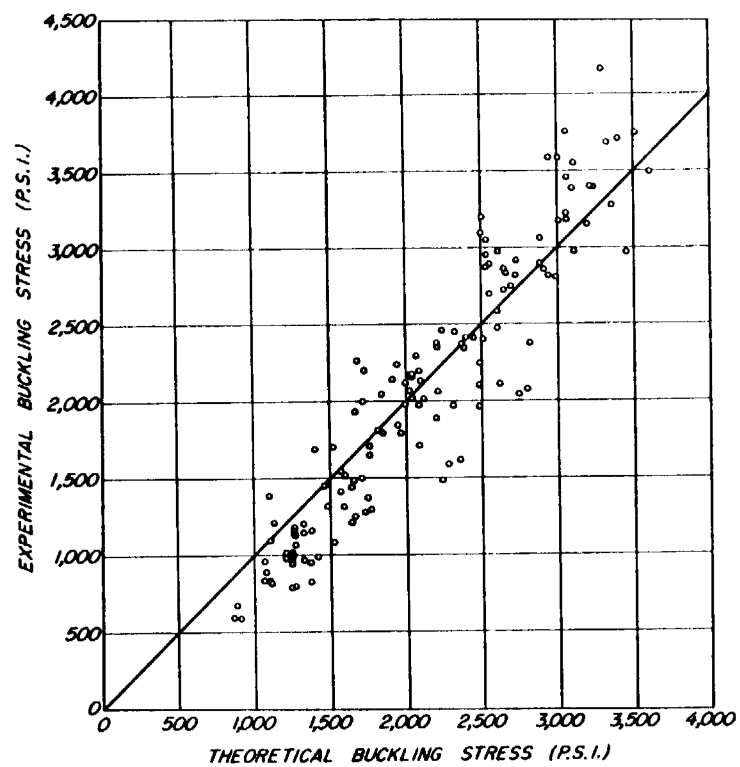

FIGURE 2.- COMPARISON OF EXPERIMENTAL AND THEORETICAL BUCKLING STRESSES OF LONG, THIN-WALLED, CYLINDRICAL SHELLS OF PLYWOOD IN AXIAL COMPRESSION.

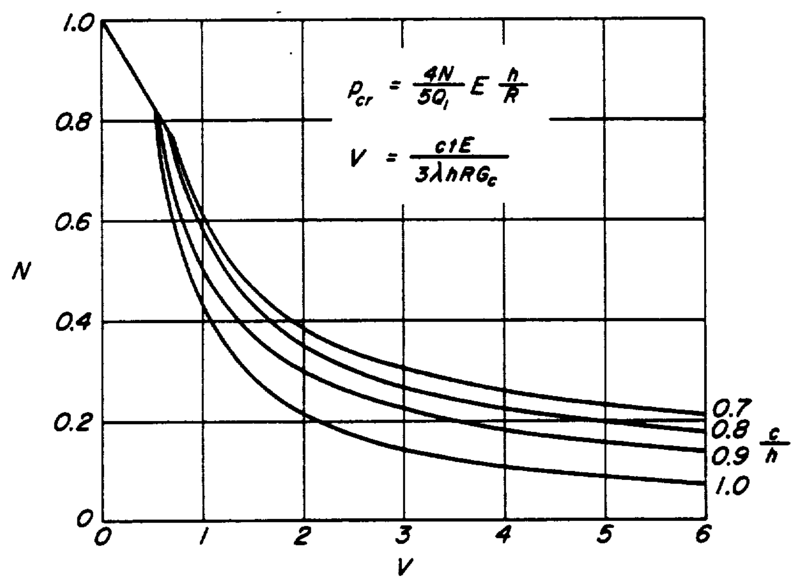

FIGURE 3. - N VALUES FOR BUCKLING OF THIN - WALLED, CYLINDRICAL SHELLS OF ISOTROPIC SANDWICH IN AXIAL COMPRESSION.

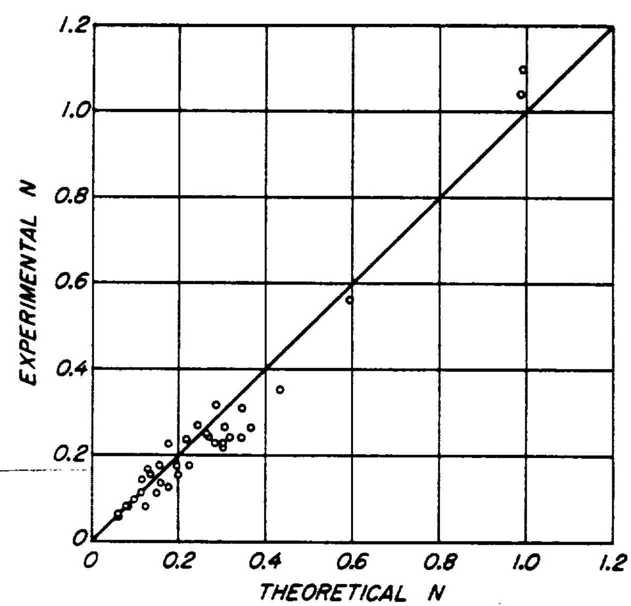

FIGURE 4.- COMPARISON OF EXPERIMENTAL VALUES OF N WITH THEORETICAL VALUES FOR BUCKLING OF SANDWICH CYLINDRICAL SHELLS IN AXIAL COMPRESSION