# ELASTIC STABILITY OF SIMPLY SUPPORTED CORRUGATED CORE SANDWICH CYLINDERS

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## SUMMARY

Theoretical buckling coefficients are obtained for the general instability of simply supported, corrugated core sandwich circular cylinders under combined loads with the core oriented parallel to the longitudinal axis of the cylinder. Buckling curves are presented for axial compression, external lateral pressure, torsion, and some typical interactions. The differential equations of equilibrium used to obtain the buckling equations were derived from the small deflection equations of Stein and Mayer which include the effect of deformation due to transverse shear. These equations are solved by Galerkin's equation. Remarks are made concerning the probable validity of the results of the small deflection theory for sandwich shells.

## INTRODUCTION

The solution for the general instability of corrugated core sandwich circular cylinders with the core oriented parallel to the axis of the cylinder is performed in a manner similar to the solution by Batdorf[1,2] for the general instability of homogeneous isotropic thin walled cylinders. In the solution presented in this report, and in Batdorf's solution, a differential equation obtained from small deflection theory is solved by Galerkin's[3] method. In the present report, the differential equations which were solved by Galerkin's method were obtained from the small deflection theory for curved sandwich plates by Stein and Mayer[4]. The elastic constants for corrugated core sandwich were derived from the basic corrugated sandwich geometry and material properties by Libove and Hubka[5]. The previous Stein and Mayer[6] solution for the general instability of corrugated core sandwich cylinders loaded under axial compression was performed in a similar manner. This report takes into consideration lateral internal and external pressure and torsion as well as axial compression.

The basic element of the idealized corrugated core sandwich consists of relatively thin isotropic facings which have negligible flexural rigidities about their own centroidal axes and a highly orthotropic core for which shear distortions are assumed to be admissible only in the plane perpendicular to the corrugation (circumferential). Furthermore, the bending rigidity of the core is assumed to be negligible in the transverse direction. Both the facings and the core are assumed to be elastic.

## DERIVATION OF BUCKLING EQUATION

It was assumed that the shear stiffness in the plane parallel to the corrugations (longitudinal) is infinite. The governing differential equations are given by equations (8) and (14) of reference 4. These equations include the influence of the transverse shear, $Q_y$. For simply supported edges, the boundary conditions on w and $Q_y$ are

$$w = \frac{\partial^2 w}{\partial x^2} = 0, \quad \frac{Q_y}{G_c c} = 0: \qquad x = 0, L$$

These conditions are satisfied by the assumed orthogonal functions

$$w = \sin \frac{ny}{2r} \sum_{s=1}^{\infty} a_s \sin \frac{s\pi x}{L} + \cos \frac{ny}{2r} \sum_{s=1}^{\infty} b_s \sin \frac{s\pi x}{L} \tag{1}$$

and

$$Q_y = \cos \frac{ny}{2r} \sum_{s=1}^{\infty} c_s \sin \frac{s\pi x}{L} + \sin \frac{ny}{2r} \sum_{s=1}^{\infty} d_s \sin \frac{s\pi x}{L} \tag{2}$$

where n is restricted to even positive integers equal to or greater than 4 and s is restricted to positive integers equal to or greater than 1.

The solution to the governing equations are obtained by use of Galerkin's method as described in reference 3. The Galerkin equations are:

$$\int_0^{2\pi r} \int_0^L Q_1 w V_1 \, dx dy = 0 \tag{3a}$$

$$\int_0^{2\pi r} \int_0^L Q_1 w V_2 \, dx dy = 0 \tag{3b}$$

$$\int_0^{2\pi r} \int_0^L Q_2 w V_1 \, dx dy = 0 \tag{3c}$$

$$\int_0^{2\pi r} \int_0^L Q_2 w V_2 \, dx dy = 0 \tag{3d}$$

where:

$$V_1 = \sin\frac{ny}{2r}\sin\frac{m\pi x}{L}, \quad V_2 = \cos\frac{ny}{2r}\sin\frac{m\pi x}{L}$$

The expression for $Q_1w$ is obtained by substituting each term of equations 1 and 2 into equation 8 of reference 6 and that for $Q_2w$ is obtained by substituting each term of equations 1 and 2 into equation 14 of reference 6. After the proper substitution and integration has been performed on equations (3a) and (3d), equation (3d) is solved for $c_m$, which is substituted into equation (3a). In a similar manner, after the proper substitution and integration has been performed on equations (3a) and (3c), equation (3c) is solved for $d_m$ which is substituted into equation (3b). With extensive simplification these equations become

$$A a_m - K_s \frac{8\beta n}{\pi m^2}\sum_s b_s \frac{ms}{m^2 - s^2} = 0 \tag{4}$$

$$A b_m + K_s \frac{8\beta n}{\pi m^2}\sum_s a_s \frac{ms}{m^2 - s^2} = 0 \tag{5}$$

where

$$A = \left\{ m^2 \frac{D_c}{D} + \frac{(m^2+\beta^2 n^2)^2[2J + m^2(1-\mu)]}{[2J + m^2(1-\mu) + 2\beta^2 n^2]m^2} + \frac{4Z^2 m^2}{\pi^4(\delta_1 m^4 + 2\delta_2 n^2 m^2 \beta^2 + \delta_3 \beta^4 n^4)} - K_c - K_p \beta^2 \frac{n^2}{m^2} \right\}$$

$$K_p = \frac{prL^2}{\pi^2 D} \qquad \delta_1 = 1 - \mu^2 \frac{1}{1 + \frac{EA}{E_c A_c}}$$

$$K_c = \frac{N_c L^2}{\pi^2 D}$$

$$K_s = \frac{N_s L^2}{\pi^2 D} \qquad \delta_2 = \frac{1+\mu}{1 + \frac{1+\mu}{1+\mu_c}\left(\frac{t_e}{A_c}\right)^2 \frac{E_c A_c}{EA}} - \mu\frac{EA}{EA + E_c A_c}$$

$$D = \frac{Eth^2}{2(1-\mu^2)}$$

$$\delta_3 = \frac{1}{1 + \frac{E_c A_c}{EA}}$$

$$J = \frac{G_c c L^2}{\pi^2 D}$$

$$\beta = \frac{L}{2\pi r} \qquad Z = \frac{L^2}{rh}(1-\mu^2)^{1/2}$$

Equations (4) and (5) can be solved explicity for $K_c$ and $K_p$ for each value of m and n. The minimum is determined by trial and error. In the case of the buckling coefficient $K_p$, it can be shown that the minimum occurs when m is one. The buckling coefficients for external pressure and for axial compression are shown in figures 2 and 3, respectively. Figure 4 shows the interaction between axial compression and lateral pressure for $Z = 10^3$.

## Solution for $K_s$; $K_p$ and $K_c$ Known

Each of equations (4) may be expressed in the form

$$A a_m - \lambda^{-1} B b_{sn} = 0 \tag{6}$$

Each of equations (5) may be expressed in the form

$$A b_m + \lambda^{-1} B a_{sn} = 0 \tag{7}$$

where n s are odd integers and

A is defined above

$$B = \frac{8\beta n}{\pi m^2} \sum_s \frac{ms}{m^2 - s^2}$$

$$\lambda^{-1} = K_s$$

In matrix form, these equations become

$$\lambda \left| a \right| = [A]^{-1} [B] \left| a \right| = [G] \left| a \right| \tag{8}$$

where $\lambda$ is a scaler, $|a|$ is a column matrix (the Fourier coefficients), and $[G]$ is a nonsymmetrical square matrix. The solution for $\lambda$ by matrix iteration is complicated by the fact that the cylinder will buckle at a load level which is independent of the direction of the applied shear. Therefore, the buckling coefficients and corresponding eigenvalues occur in pairs which are equal in magnitude but opposite in sign. When the matrix $[G]$ is formed, the upper right quadrant, $[G_1]$ , and the lower left quadrant, $[G_2]$ , are non-zero matrices; whereas the other two quadrants are null matrices. The matrix $[G]$ is also simplified by the fact that $[G_1]$ is equal to $[-G_2]$ . Making use of these relationships,

$$\lambda^2 \left| a_2 \right| = [-G_2][G_2] \left| a_2 \right| = [G_3] \left| a_2 \right| \tag{9}$$

The matrix $[G_3]$ can be formed from either of equations (6) or (7). An 8 x 8 matrix was formed and iterated to obtain the eigenvalue $\lambda^2$. The iteration continued until the scaler $\lambda^2$, remained constant to six significant figures. The buckling coefficient $K_s$ is the square root reciprocal of the eigenvalue $\lambda^2$.

For a given set of numerical values of the sandwich parameters, buckling coefficients are obtained for single values of n until the minimum value of the buckling coefficients is found. Figure (5) is a plot of $K_s$ as a function of Z for the special case of $K_c$ and $K_p$ equal to 0. Figure 5 is a plot of $K_s$ as a function of $K_c$ for Z equal to $10^3$. Figure 6 is a plot of $K_p$ as a function of $K_s$ for Z equal to $10^3$.

## COMPARISON WITH SOLUTION FOR HOMOGENEOUS ISOTROPIC THIN WALL SHELLS

The method of solution used for the corrugated sandwich cylinders is the same as the method of solution used for homogeneous isotropic cylinders in references (1) and (2). Therefore, if the parameters for a sandwich which is the equivalent of homogeneous sheet are substituted into the sandwich cylinder stability equations, the resulting equations should be the same as the equations presented in references (1) and (2). A sandwich with $G_c c = \infty$ and t=h is the equivalent of homogeneous sheet. For the case of t=h, the moment of inertia of the facing sheets about their own centroid cannot be neglected so that I is equal to $2/3\ t^3$. With this correction, the equation becomes

$$\left[\frac{(m^2+\beta_0^2)^2}{m^2} + \frac{12 Z_0^2 m^2}{\pi^4 (m^2+\beta_0^2)^2} - K_c - K_p \frac{\beta_0^2}{m^2}\right] a_m - K_s \frac{8\beta_0 n}{\pi m^2} \sum_s b_s \frac{ms}{m^2-s^2} = 0 \qquad (10)$$

where $\beta_0$ and $Z_0$ refer to the thin sheet parameters. This equation is the same as the equations given in references (1) and (2) for cylinders of homogeneous, isotropic thin sheet.

## VALIDITY OF RESULTS

The results of this analysis agree with the special case of the homogeneous isotropic thin wall shell. The considerable discrepancy between test data and the linear small deflection theory for the particular loading condition of axial compression has been reported frequently in the literature. In this case, test data may be on the order of only 15%[7] of that predicted by the small deflection theory. Analysis of test data indicates that the buckling coefficient is a function of r/t, whereas the small deflection theory does not indicate this dependence. The consequence of this comparison of data from unstiffened shells must on the surface lead to the conclusion that the small deflection theory does not accurately predict the behavior of sandwich shells for those cases in which the core shear distortion has a negligible effect. There is, however, some argument that the disagreement between theory and test should not be so great as for thin wall shells. This argument is that the sandwich shell behaves as if it had a relatively low r/t. That is, the effective thickness of the sandwich is relatively large and it might be supposed that the effects of initial imperfection are relatively small because of the rigidity of the wall. Although data are limited, this does not appear to be the real case. For example, March and Kuenzi[8], in their report of a large deflection theory for axial compression, show some test data. These data show relatively small scatter and the data fall within ± 30% of the large deflection predicted buckling loads.

For the case in which shear distortion becomes predominate, it is likely that the small deflection theory will provide good agreement with test data. In this case, the mode of failure (called a crimp) is one which has a very short wave length.

The above statements are relative to the specific case of axial compression. For the cases of external pressure and torsion, agreement between linear theory and test data is closer. Investigations of the interaction buckling coefficients from linear theory for thin wall shells indicates that the shape of the interaction agrees well with test data provided the non-dimensional buckling ratios are used instead of the buckling coefficients themselves.

The probable consequence of the comparisons with thin wall shell behavior are that the linear theory used herein will provide satisfactory solutions for a shell under torsion, will be reasonably accurate for external pressure, and will provide the correct shapes for interaction curves. In contrast, however, the theory will be inadequate for shells under axial compression when the shear distortion effects are negligible. This leads to a very real lack of data for sandwich shells under axial compression loads and under bending. In order to effectively describe the behavior of sandwich shells in axial compression, it will be necessary to perform an extensive series of tests in a systematic fashion. Very little of these data are avilable.

## REFERENCES


1. Batdorf, S. B.: A Simplified Method of Elastic-Stability Analysis for Thin Cylindrical Shells. NACA Rep. 874, 1947. (Formerly included in NACA TN's 1341 and 1342.)

2. Batdorf, S. B., Stein, Manuel, and Schildcrout, Murry: Critical Stress of Thin-Walled Cylinders in Torsion. NACA TN 1344, 1947.

3. Duncan, W. J.: Galerkin's Method in Mechanics and Differential Equations. R. & M. No. 1798, British A.R.C., 1937.

4. Stein, Manuel, and Mayers, J.: A Small-Deflection Theory for Curved Sandwich Plates. NACA Rep. 1008, 1951. (Supersedes NACA TN 2017.)

5. Libove, Charles, and Hubka, Ralph E.: Elastic Constants for Corrugated-Core Sandwich Plates. NACA TN 2289, 1951.

6. Stein, Manuel, and Mayers, J.: Compressive Buckling of Simply Supported Curved Plates and Cylinders of Sandwich Construction. NACA TN 2601, 1952.

7. Harris, Leonard A., Suer, Herbert S., Skene, William T., and Benjamin, Roland J.: The Stability of Thin-Walled Unstiffened Circular Cylinders Under Axial Compression Including the Effects of Internal Pressure. Jour. Aero. Sci., vol. 24, no. 8, Aug. 1957, pp. 587-596.

8. March, H. W., and Kuenzi, Edward W.: Buckling of Cylinders of Sandwich Construction in Axial Compression. Forest Products Laboratory Report No. 1830, 1957.

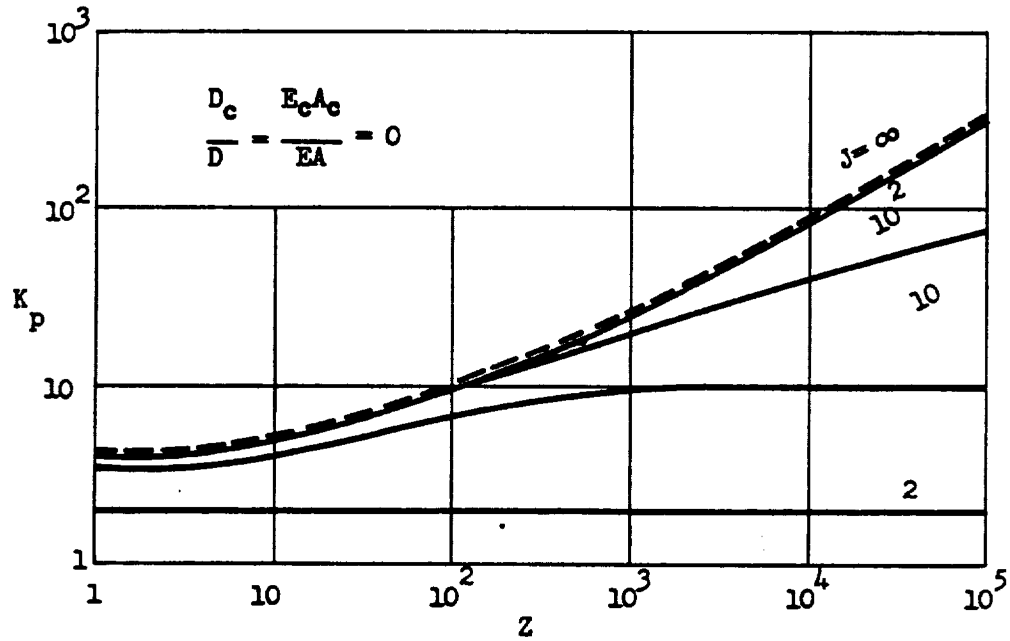


Figure 1.- Buckling coefficients for lateral pressure.

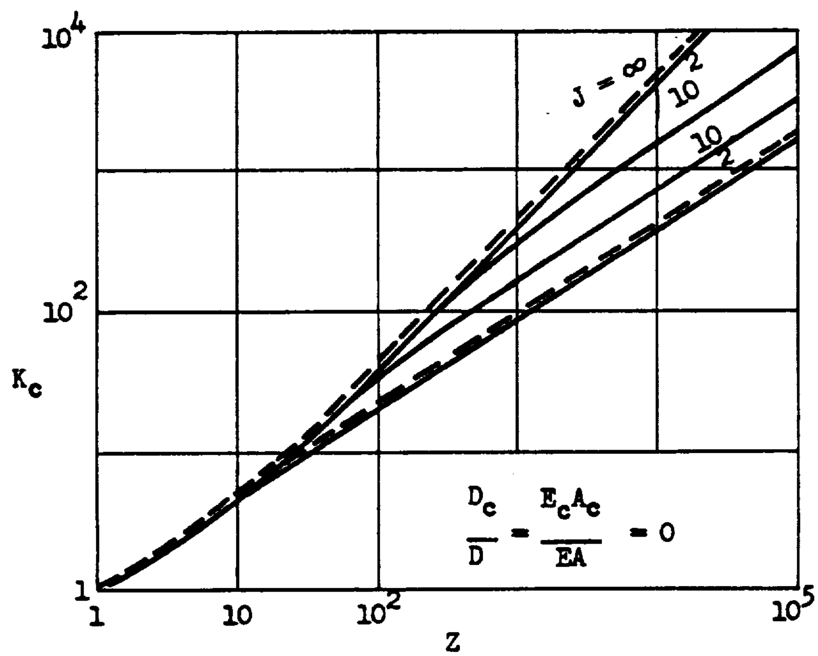


Figure 2.- Buckling coefficients for axial compression.

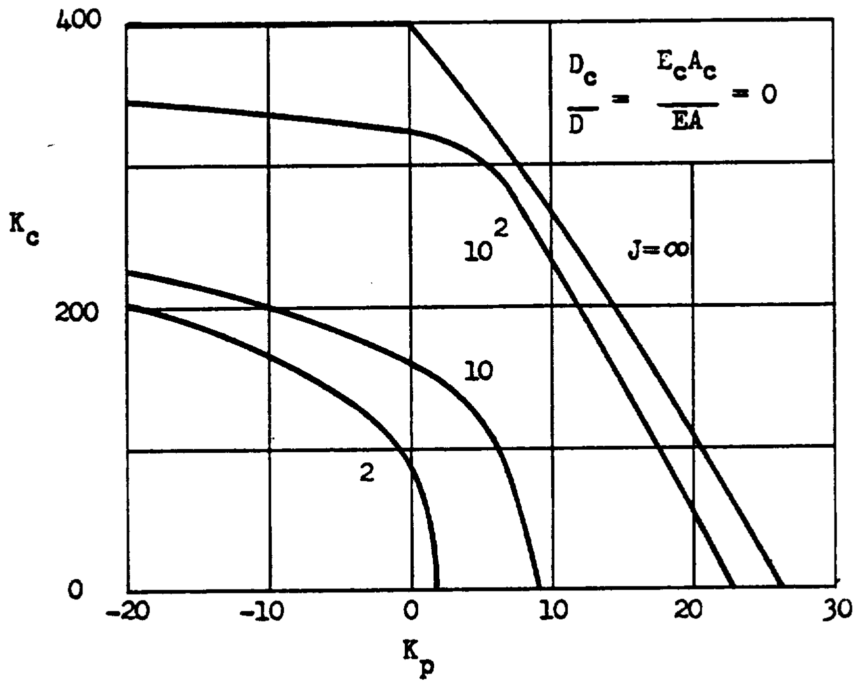


Figure 3.- Combined lateral pressure and axial compression ($Z = 10^3$).

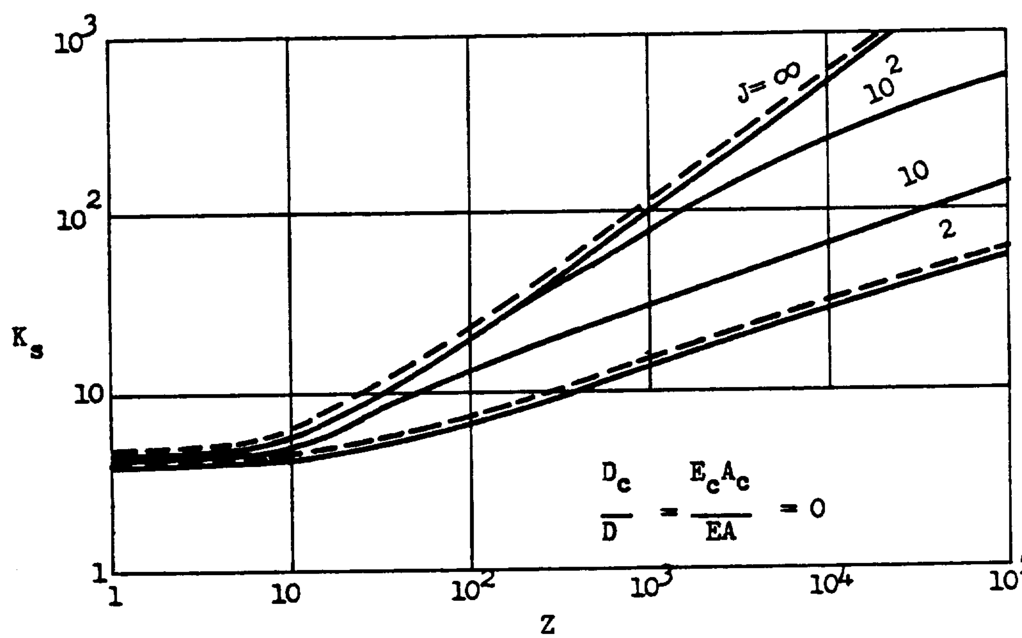


Figure 4.- Buckling coefficients for torsion.

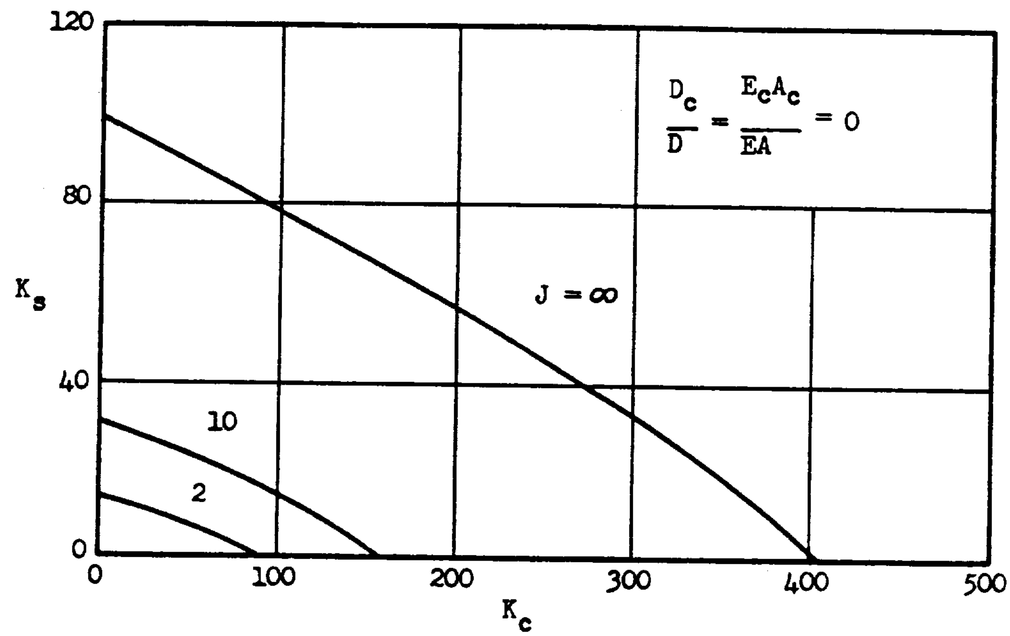


Figure 5.- Combined axial compression and torsion ($Z = 10^3$).

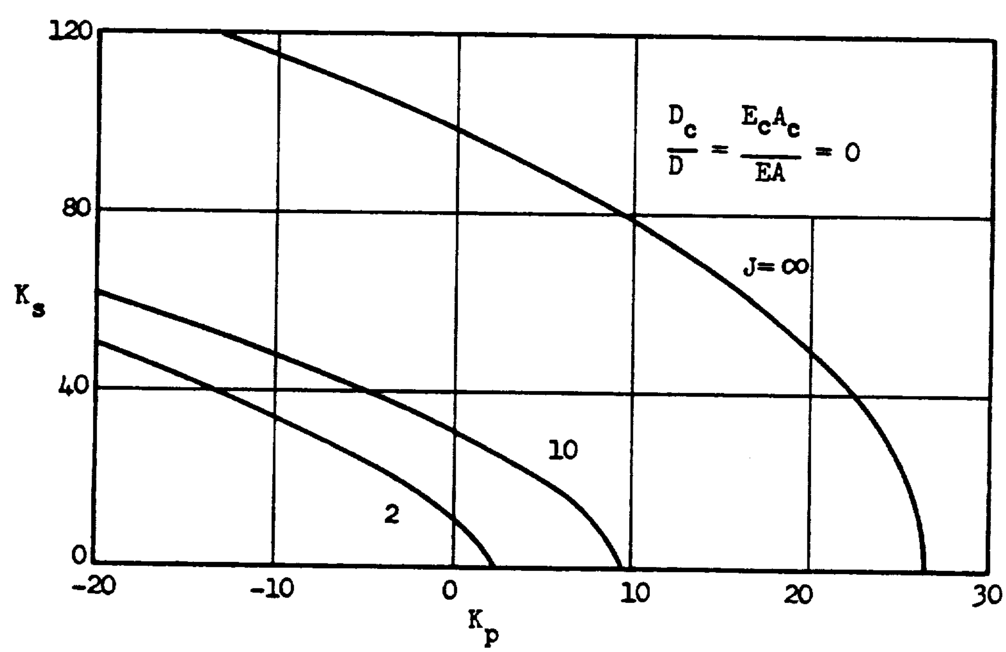


Figure 6.- Combined lateral pressure and torsion ($Z = 10^3$).