# DESIGN AND TESTING OF HONEYCOMB SANDWICH CYLINDERS UNDER AXIAL COMPRESSION

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## SUMMARY

Experimental results for 36" diameter honeycomb cylinders fabricated with thin (0.010") aluminum faces and cores prove that it is quite feasible to stabilize thin faces so they can be loaded beyond the yield point. The effect of initial imperfections and the various modes of failure are discussed.

## INTRODUCTION

A recent series of tests by the Douglas Aircraft Company on 36" diameter cylinders has established that with proper design and manufacturing techniques, it is possible to stabilize very thin (0.010") metal skins so that they will carry compressive loads well into the inelastic region. After a brief development period no trouble was experienced at joints, and failures occurred in the middle of the cylinders. The failures occurred by wrinkling of the faces rather than by general instability.

## SYMBOLS

$A_o$ amplitude of imperfection

d diameter of a circle inscribed in a honeycomb cell

E Young's modulus of elasticity of face

$E_t$ tangent modulus of face

$E_{sec}$ secant modulus of face

$E_c$ Young's modulus of elasticity of core

$F_c$ ultimate strength of core

$G_c$ shear modulus of core in a plane normal to the faces and parallel to the load

$I$ moment of inertia

$K$ constant

$k$ spring constant

$L$ length of column

$N_{cr}$ axial compressive buckling load (#/in)

$R$ mean radius of curvature of cylinder

$T$ thickness of core

$t$ thickness of monocoque cylinder

$t_f$ thickness of one face of sandwich

$\epsilon$ strain

$\eta$ plasticity reduction factor ($\eta$ = 1 in elastic region)

$\rho$ radius of gyration

$\sigma$ stress

$\sigma^*$ stress from stress-strain diagram

$\sigma_{cr}$ face buckling stress

$\sigma_f$ predicted failing stress with imperfection

$\sigma_{mc}$ face monocell buckling stress

$\sigma_{wr}$ face wrinkling stress

# MODES OF FAILURE

## Review of Theoretical Analyses

In the literature, several solutions are presented for buckling of sandwich cylinders under axial compression (Ref. 1). Adequate analyses exist for most modes of failure including small and large deflection theories with, and without initial imperfections.

Solutions to the governing partial differential equations are obtained by assuming a particular mode of failure and then solving for the critical load corresponding to that particular mode of failure. For sandwich cylinders, many modes of failure are possible, each corresponding to a certain critical load. The designer must proportion the shell to obtain the minimum weight construction for a given load by choosing the proper combination of face thickness, core cell size, depth, and shear rigidity for a given cylinder diameter and axial load. The role of initial imperfections and their effects on the different possible modes of failure must be understood. The initial imperfections which occur during manufacturing, especially near end fittings and openings, will in most practical applications limit the efficiency attainable from sandwich constructions.

## GENERAL INSTABILITY

General instability corresponds to over-all buckling of the shell. For sandwich construction the effect of shearing rigidity of the core may be extremely important. The small deflection theoretical buckling load of a cylinder with transverse shear deflections included (Ref. 2) is compared to predicted loads obtained by neglecting shear effects in Figure 1. The postbuckling load predicted by a large deflection theory (Ref. 3) is also plotted (Figure 2). For low core shear moduli, initial buckling occurs by the faces sliding with respect to each other (crimping). As the core modulus is increased, buckling strength increases until the core is sufficiently rigid to prevent the faces from sliding and then the classic over-all buckling occurs. The core rigidity required to force buckling in this "rigid core" region is a function of the $Et_f/R$ of the cylinder. Many experiments have yielded unsatisfactory information because the radius (R) and face thickness ($t_f$) have been scaled down causing the cylinder to be in the soft core region where only a small fraction of the expected strength could be obtained. Obviously, for maximum strength, the cylinder must be designed to be in the rigid core region.

The classic small deflection critical buckling stress (Ref. 4) is, when $G_c \geq \frac{Et_f}{R}$ ,

$$\sigma_{cr} = \frac{N_{cr}}{2t_f} = \frac{\eta_{ET}}{R} \left[ 1.05 - \frac{\eta Et_f}{1.8\ RG_c} \right] \qquad (1)$$

with

$$\eta = \frac{\sqrt{E_t\ E_{sec}}}{E} \qquad (2)$$

The plasticity reduction factor $\eta$ for cylinders under axial compression is obtained from Gerard (Ref. 5).

When the core shear modulus is low, i.e., $G_c \leq Et_f/R$, failure occurs by shear instability and the critical load is

$$N_{cr} = T\ G_c. \qquad (3)$$

ROLE OF IMPERFECTIONS IN SMALL DEFLECTION THEORY

Assuming an imperfection consisting of sine waves around the circumference of a cylinder and considering small deflection theory, the effect of imperfections in monocoque cylinders is determined by a factor of the form

$$1 \Big/ \sqrt{1 + K\ \frac{R}{t}\ \frac{A_o}{t}}$$

while for sandwich cylinders the equivalent expression is

$$1 \Big/ \sqrt{1 + K\frac{R}{T}\ \frac{A_o}{T}}\ .$$

It is significant that for general instability, the effect of initial imperfections in sandwich construction is related to the total thickness, T. Sandwich thicknesses are much greater than monocoque thicknesses and the same magnitude of imperfection is much less serious in sandwich construction.

## LARGE DEFLECTION THEORY

Large deflection theories are more complete and predict not only the initial buckling load (Figure 2, point A) but also the maximum postbuckling load a cylinder can sustain (Figure 2, point B). The nonlinear behavior of monocoque cylinders has been extensively studied (Ref. 6) and it is generally recognized that the minimim point B in Figure 2 corresponds to radial displacements of the cylinder of approximately five sheet thicknesses. If initial imperfections of this magnitude exist, as they often do for foil gages, failure will indeed occur at these low loads. Donnell and Wan (Ref. 7) have described the failure mechanism as a function of the magnitude of the assumed initial imperfection and as expected higher loads are obtained for more nearly perfect cylinders. Often designers choose the postbuckling stress (point B) rather than the initial buckling stress (point A). This may be exceedingly conservative for monocoque cylinders of low R/t and for sandwich cylinders.

## LOCAL BUCKLING

In attempting to stabilize a thin foil gage so that it will accept loads stressing it up to the yield point, one must consider ways in which the sheet may fail by local buckling.

## MONOCELL BUCKLING

A form of local buckling is monocell buckling, namely, buckling of individual cells within the honeycomb core. Norris and Komers (Ref. 8) give the monocell buckling criterion

$$\sigma_{mc} = 0.9\,\eta\,E\left(\frac{t_f}{d}\right)^{3/2} \qquad (4)$$

In the inelastic range, the plasticity reduction factor for flat plates (Ref. 9) is used, and is given by

$$\eta = \frac{E_{sec}}{E} \quad . \tag{5}$$

The designer must choose a core cell size to prevent monocell buckling. This can be accomplished without difficulty by choosing from the wide variety of commercially available cores.

## WRINKLING

With a cell size chosen to prevent monocell buckling, the faces can be considered to be continuously supported by an elastic medium, i.e., the core. Local failures of the faces can occur independent of the dimensions of the cylinder if the core does not have sufficient ability to stabilize the faces. Since the failures will be highly localized, curvature effects can be neglected and analyses developed for flat plates can be utilized.

Non-symmetric failure can occur if the core does not have sufficient shear rigidity to force the two faces to act as an integral unit. The analysis presented previously for general instability included failure by shear instability (crimping) but neglected the flexural resistance of the individual faces about their own neutral axes. Hoff and Mautner's (Ref. 10) analysis for flat plates using energy techniques shows that thin sandwich panels may buckle in the "skew-ripple" unsymmetric mode. The critical load per inch for sandwiches with thin cores is

$$N_{cr} = 2\ t\ \sigma_{cr} = 1.18\ t_f^{\ 3/2} \sqrt{\frac{E\ E_c}{T}} + .774\ TG_c \tag{6}$$

This should be compared to the expression obtained before for the "soft" core region by neglecting the resistance of the faces to flexure, namely

$$N_{cr} = TG_c \quad . \tag{7}$$

As the core modulus approaches zero, the only load carrying capability is obtained from the flexural resistance of the faces.

For some material properties and dimensions, symmetrical wrinkling may occur at lower loads. Depending on the assumptions made, one can obtain a variety of expressions for critical loads. The simplest approach neglects the shear effects of the core and treats the faces as a beam on an elastic foundation (Ref. 11). The spring constant is obtained by considering the midplane of the core as an axis of symmetry and then treating the core material as an elastic spring. Upon substitution into the expression for buckling of a beam on an elastic foundation,

$$N_{cr} = 2\sqrt{k\,EI}\ ,$$

one obtains

$$N_{cr} = 1.73\sqrt{\frac{E\,E_c}{T}\,t_f^{\;3}}\ . \tag{8}$$

This indicates that lower loads are obtained for thicker cores. As the core thickness is increased, other factors must be considered.

Upon examining Figure 3, one realizes that neighboring core strips move different distances. Since the strips are physically attached, shear stresses develop and cause the displacements to die off with increasing core depth. Assuming that the displacements die off linearly with depth, Hoff (Ref. 9) using energy techniques obtained

$$\sigma_{wr} = .91\left[\,E\,E_c\,G_c\,\right]^{1/3}. \tag{9}$$

In terms of $N_{cr}$, equation (9) becomes

$$N_{cr} = 1.82\,t_f\left[\,E\,E_c\,G_c\,\right]^{1/3}.$$

Williams (Ref. 12) assumed an exponential decay of displacements and in Equation (9) obtained 0.85 instead of 0.91. Hoff's solution (Ref. 10) by a theory of elasticity approach gave lower values which were functions of the core depth to face thickness ratio. For thick cores, the coefficient was (0.873)(0.91) = 0.794. Many authors have recommended using a coefficient of 0.50 as it gives reasonable agreement with experimental data, but it should be realized that this implies symmetrical wrinkling.

The references mentioned provide criteria for determining whether a given sandwich is "thick" or "thin," but the criteria have been left out of this brief summary. To recapitulate, unsymmetrical "skew-ripple" occurs for cores with low shear moduli, whereas symmetrical wrinkling occurs for cores with high shear rigidity and/or thick cores.

For inelastic wrinkling, a suitable plasticity reduction formula is applied. Since curvature effects are neglected, it is reasonable to use the flat plate reduction factor,

$$\eta = \sqrt{\frac{E_t}{E}} \ . \tag{10}$$

For large plastic strains, the tangent modulus, $E_t$, changes so rapidly with increasing load that the same value of critical load is obtained from Equation (9) independent of the coefficient assumed.

## EFFECT OF INITIAL IMPERFECTIONS ON WRINKLING

The equations derived for buckling predict the critical load in terms of the elastic constants and provide no information on the strength properties required. The required core compressive strength and the adhesive strength depends on the initial imperfections formed during manufacture or induced in service. Knowing the type and magnitude of the imperfection, one can determine the required strengths. However, by assuming various imperfection values, one obtains a good appreciation for the serious effects produced by small imperfections. One also realizes the importance of using high strength cores and adhesives to insure that the failures occur by buckling before the local stresses exceed the allowable values. Since imperfections are always present, expecially around joints, openings, closing members, etc., it is imperative to understand their significance and to minimize their importance by using high strength adhesives and cores.

Yuseff's (Ref. 11) analysis of wrinkling assumes imperfections having the same shape and wave length as the wrinkle pattern. The reduction in strength due to imperfections is easily visualized by comparing the results to the theoretical wrinkling stress without imperfection. Yuseff's equation 22 may be written

$$\frac{\sigma_f}{\sigma_{wr}} = \frac{1}{1 + 1.39 \frac{(E_c \ G_c)^2}{\eta E \ F_c} \ \frac{A_0}{t_f}} \tag{11}$$

Using the properties of the test cylinders Figure 4 shows the reduction in predicted failing stress as a function of the amplitude of the initial imperfection for an elastic failure ($\eta = 1$).

The reduction in wrinkling strength due to the presence of an initial imperfection can be minimized by using a sandwich core with high compressive strength and a high strength adhesive.

## COMPARISON BETWEEN TEST AND THEORY

Axial compression tests were run on eight 7075-T6 aluminum sandwich cylinders with faces 0.010 inches thick (Ref. 13). The outside cylinder diameters were 36 inches and the length of each was 30 inches. The cores in each case were aluminum Hexcel Al 3/16-5052 .001 P oriented with the ribbon direction parallel to the load. The core thicknesses were 0.125 inches, 0.188 inches, and 0.400 inches. Table I and Figures 5 and 6 show test results.

The first three cylinders fabricated were unsatisfactory and failed at low stress levels. This was due to poor detail design of the end doublers and unbonded regions. The original doubler was not scalloped and acted as a constraint preventing the cylinder from expanding laterally when loaded axially. This constraint caused local bending, introducing an initial imperfection, and caused wrinkling type failure at low stress levels ($\approx$ 30,000 psi). Scalloping the doubler relieved the problem by preventing highly localized bending. One can easily obtain an appreciation of the problem by computing the free radial expansion of the cylinder. The hoop strain is caused by Poisson's ratio

$$\epsilon_h = \mu \epsilon_L$$

and the radial displacement $\delta$

$$\delta = R\, \epsilon_h = R\, \mu \epsilon_L \,.$$

At 62,000 psi the axial strain is 0.0068 in/in. and the radial expansion is 0.0408 inches. If this expansion is prevented by a rigid ring, premature failure occurs due to highly localized bending.

In order to prevent localized bending, a 0.010 inch scalloped doubler was used in subsequent tests.

To visualize the comparison of the test results to the theoretical buckling stresses predicted by various formulas, one can place the concepts on an elementary level. Consider the buckling of a perfect pin-ended column. The critical buckling stress is

$$\sigma_{cr} = \frac{\pi^2 E_t}{\frac{L^2}{\rho^2}} = K E_t$$

At a given tangent modulus (i.e., at a given strain in the column), the column can support a stress without a stability failure provided that the stress is less than $K E_t$. In Figure 7, $\sigma = K E_t$ represents resistance to buckling instability failure. Also plotted in Figure 7 is $\sigma^*$ vs. $E_t$ where $\sigma^*$ and $E_t$ are obtained from a conventional stress-strain diagram of a uniaxial tension specimen. The curve $\sigma^*$ vs. $E_t$ represents the ability to carry load assuming no instability failure. As the column is loaded, the strain in the unbent column determines the tangent modulus and the stress. As loading increases from zero, the strain increases, the stress increases, and the tangent modulus decreases. Eventually the stress reaches a level where an instability failure occurs, namely, the intersection of the two curves. Hence, the intersection represents the predicted buckling load.

The principal outlined above for obtaining the critical stress of a pin-ended column also applies for finding the critical stress of a sandwich cylinder for each of the various assumed modes of failure. Thus, one can compute for each cylinder (assuming no imperfections) the predicted buckling stress according to the various theories. The results are shown in Figures 8, 9, and 10. The predicted buckling stresses and the actual failing stresses are given in Table II. The predicted buckling stresses from small deflection theory and from wrinkling theory were calculated for $G_c$ = 20,000 psi and $G_c$ = 40,000 psi and showed only a very slight effect of a variation of core shear modulus in the rigid core region. A conventional stress-strain curve (Figure 11) was obtained from coupons cut from the sheets used to fabricate the cylinders.

The predicted buckling loads from the small deflection general instability equation and the wrinkling equation (assuming no initial waviness of the faces) are in close agreement with each other. One can not expect exact agreement with test values as some uncertainty is introduced in the inelastic range. Also, the stress-strain curve used is for tension rather than compression as it was not feasible to conduct compression tests on the .010" thick coupons cut from the facing sheets. Note that one of the compression failing stresses (78,800 psi) actually exceeds the ultimate strength in tension (77,000).

Various wrinkling equations corresponding to symmetrical and unsymmetrical failure modes were examined. The equation for symmetrical buckling predicted the minimum critical loads and was used for theoretical predictions (Equation 9). The thin specimen actually failed by unsymmetrical wrinkling, probably due to some slight imperfection.

In summary, the thin 0.010" faces were stabilized well into the inelastic region and failures occurred at stresses much higher than predicted by the large deflection theory.

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TABLE I

| Specimen Number | Core Thickness Inches | Buckling Load Pounds | Face Stress Psi | Type of Failure |
|---|---|---|---|---|
| 1 | 0.125 | 137,600 | 61,000 | Wrinkle About 6" from End |
| 2 | 0.188 | 156,000 | 69,000 | Wrinkle At Center of Cylinder |
| 3 | 0.188 | 141,400 | 62,500 | Wrinkle at Center of Cylinder |
| 4 | 0.400 | 168,500 | 74,000 | Wrinkle at Center of Cylinder |
| 5 | 0.400 | 178,000 | 78,800 | Wrinkle at Edge of Doubler |

TABLE II

| Specimen Number | Predicted Buckling Stresses (psi) | | | | Actual Failing Stress (Psi) |
|---|---|---|---|---|---|
| | Wrinkling | Monocell | General Instability | | |
| | | | Small Deflection | Large Deflection | |
| 1 | 74,800 | 70,000 | 57,000 | 31,000 | 61,000 |
| 2 | 74,800 | 70,000 | 64,800 | 45,400 | 69,000 |
| 3 | 74,800 | 70,000 | 64,800 | 45,400 | 62,500 |
| 4 | 74,800 | 70,000 | 69,000 | 64,000 | 74,000 |
| 5 | 74,800 | 70,000 | 69,000 | 64,000 | 78,800 |

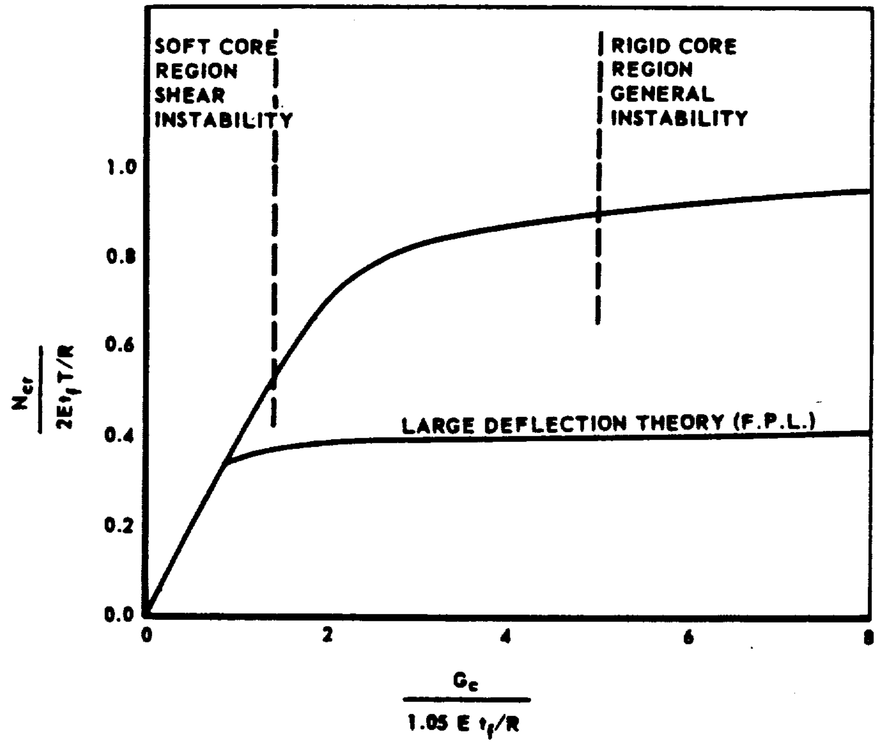



FIGURE 1. – EFFECT OF SHEAR RIGIDITY ON CRITICAL BUCKLING OF CYLINDER

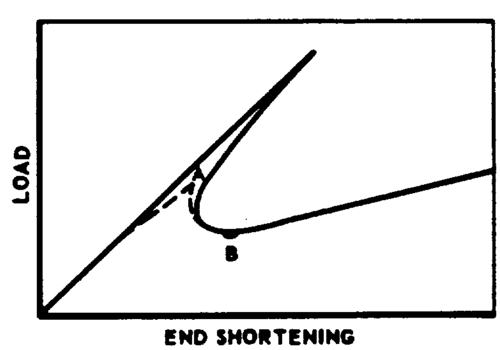



FIGURE 2. – LOAD CARRYING CAPACITY BY LARGE DEFLECTION THEORY

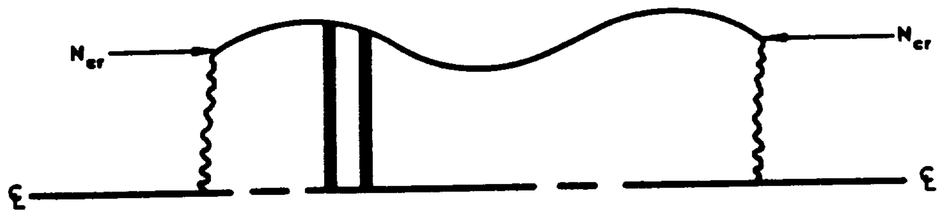



Figure 3.- Wrinkling of sandwich construction.

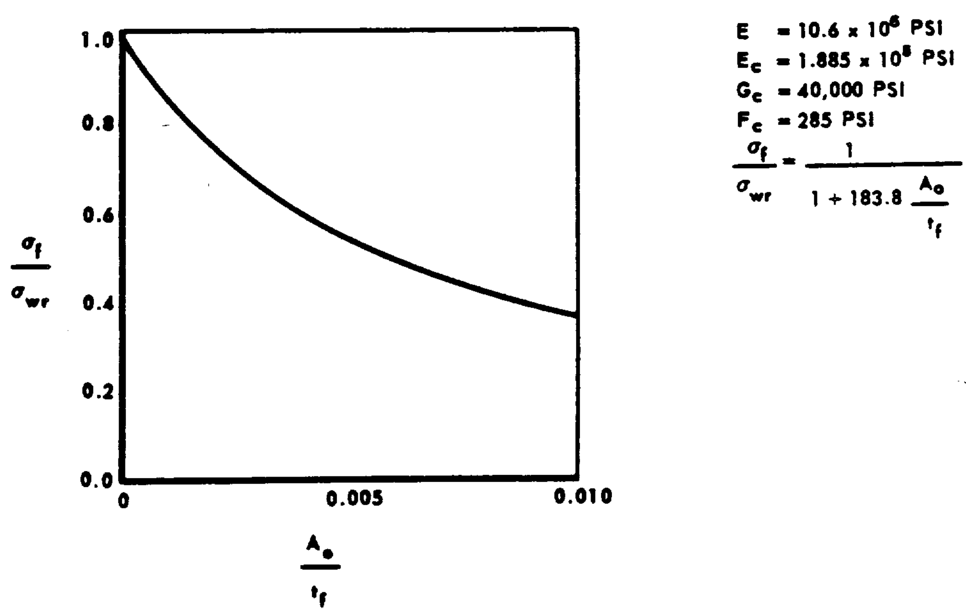
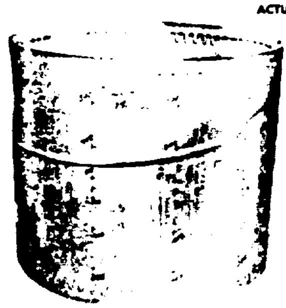



Figure 4.- Reduction of buckling strength due to imperfections.

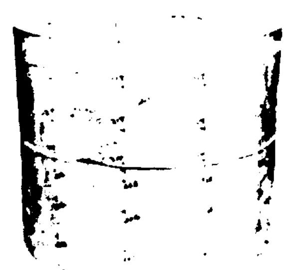
| CORE THICKNESS | |
|---|---|
| THEORETICAL FAILING STRESS, PSI | |
| LARGE DEFLECTION | 31,000 |
| SMALL DEFLECTION | 57,000 |
| SYMM. WRINKLING | 74,800 |
| MONOCELL | 70,000 |

SPECIMEN 1
ACTUAL FAILING STRESS 61,000 PSI



SPECIMEN 2
ACTUAL FAILING STRESS 69,000 PSI

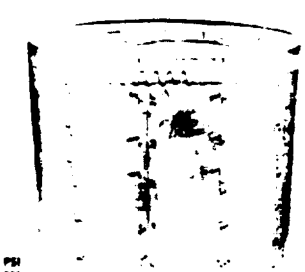
| CORE THICKNESS 0.100" | |
|---|---|
| THEORETICAL FAILING STRESS, PSI | |
| LARGE DEFLECTION | 45,400 |
| SMALL DEFLECTION | 64,800 |
| SYMM. WRINKLING | 74,800 |
| MONOCELL | 70,000 |



SPECIMEN 3
ACTUAL FAILING STRESS 62,500 PSI

SPECIMEN 4
ACTUAL FAILING STRESS 74,000 PSI

| CORE THICKNESS 0.400" | |
|---|---|
| THEORETICAL FAILING STRESS, PSI | |
| LARGE DEFLECTION | 64,000 |
| SMALL DEFLECTION | 69,000 |
| SYMM. WRINKLING | 74,800 |
| MONOCELL | 70,000 |



SPECIMEN 5
ACTUAL FAILING STRESS 70,000 PSI

**FIGURE 5**

FIGURE 6. - PREMATURE FAILURE AT UNSCALLOPED DOUBLER

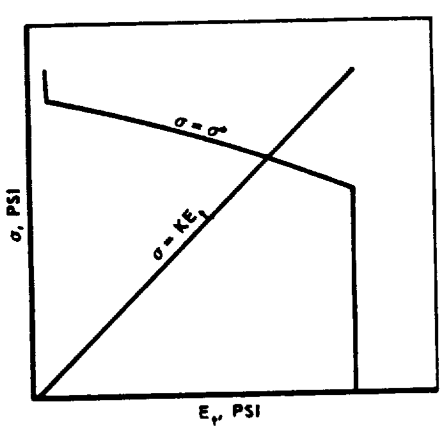



FIGURE 7. - INSTABILITY CRITERION

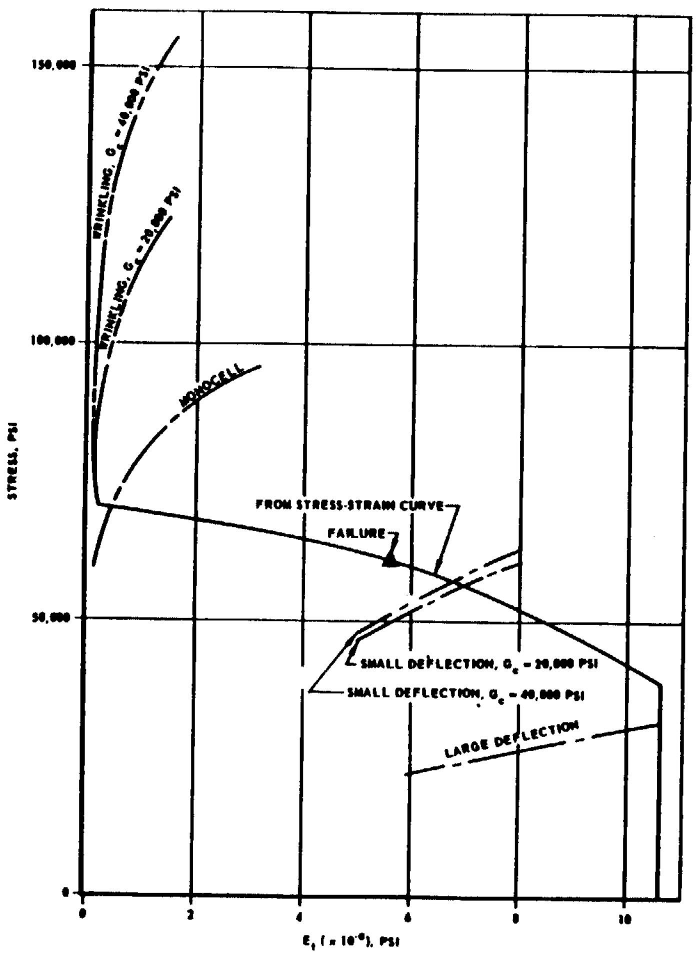



FIGURE 8. - INSTABILITY CRITERIA WITH CORE THICKNESS OF 0.125 INCH

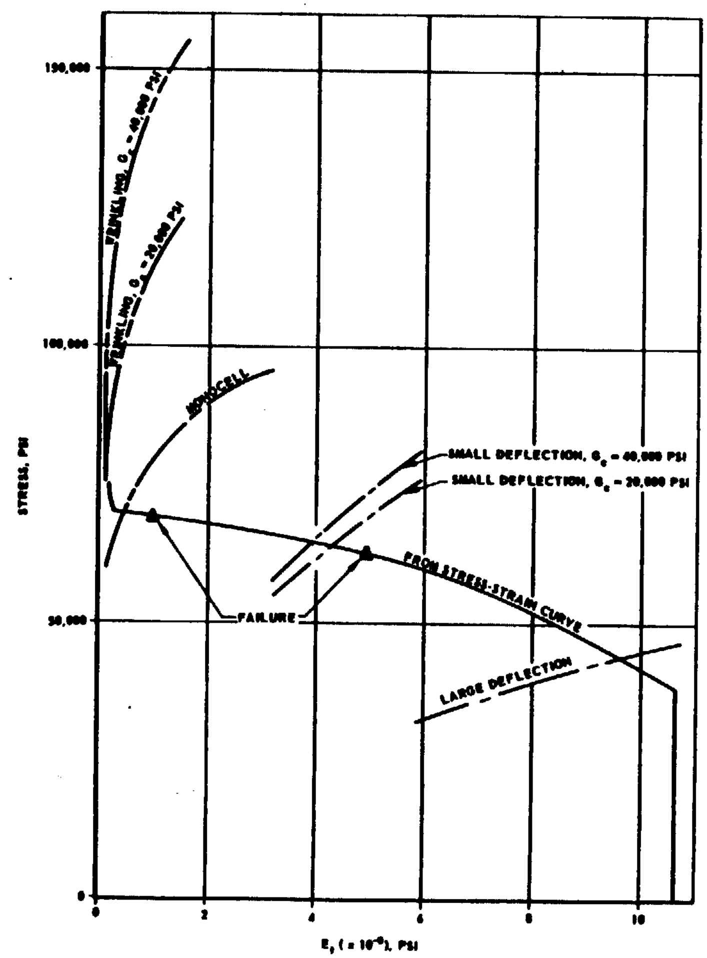



FIGURE 9. - INSTABILITY CRITERIA WITH CORE THICKNESS OF .188 INCH

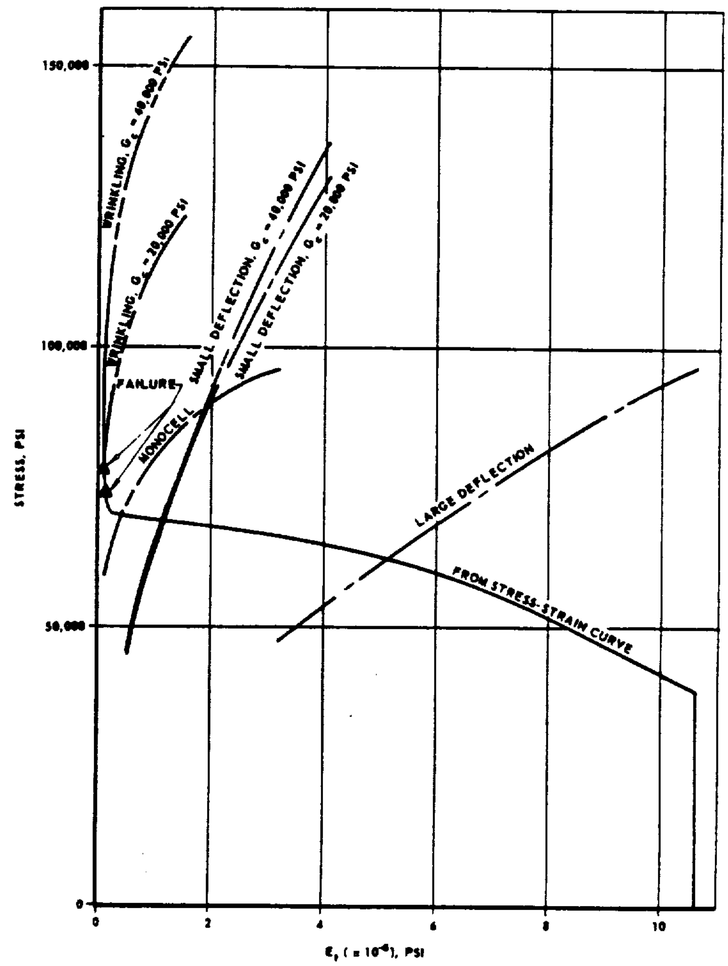



FIGURE 10 INSTABILITY CRITERIA WITH CORE THICKNESS OF 0.400 INCH

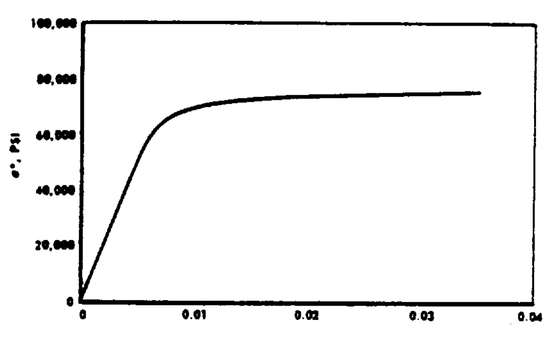



FIGURE 11 STRESS-STRAIN CURVE OF 7075-T6 ALUMINUM