A SURVEY OF BUCKLING THEORY AND EXPERIMENT FOR CIRCULAR CONICAL SHELLS OF CONSTANT THICKNESS

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SUMMARY

A survey of the state-of-the-art for the stability of thin-walled conical shells is presented. Known theoretical results are summarized and compared with experiment. The shortcomings of present knowledge and recommended work for the future are discussed.

INTRODUCTION

Only four years ago, in 1958, the state of knowledge of the elastic stability of conical shells was described (ref. 7) as being "quite unsatisfactory at the present time". In the ensuing period, however, a considerable amount of work has been done so that although not all of the desired information is available, conical shells can be designed somewhat more intelligently to withstand many of the loading conditions of interest. The purpose of the present paper is to review, as concisely as possible, the available theoretical and experimental knowledge, to indicate the gaps in the present state-of-the-art, and to suggest additional problems that should be studied.

A reasonably complete bibliography of papers on the stability of circular conical shells is included. The entries consist of those in "Bibliography on Shells and Shell-Like Structures" and the 1954-56 Supplement (both by William A. Nash*), "Structural Mechanics in the U.S.S.R., 1917-1957" (edited by I. M. Rabinovich[+]) and ref. 25, and of other papers that have come to the author's attention.

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* David W. Taylor Model Basin Report 863, Nov. 1954, and Dept. of Eng. Mechanics University of Florida, Report for Office of Ordnance Research (Contract DA-01-009-ORD-404), respectively.

[+] Translation published by Pergamon Press, 1960.

SYMBOLS

D	Bending stiffness of cone wall $\left[\frac{Et^3}{12(1-\nu^2)}\right]$
E	Young's modulus of cone material
L, ℓ	axial and slant length of cone, respectively
M	bending moment
P, P_o	applied and critical axial compressive forces, respectively.
P_{cr}	theoretical critical axial compressive force $\left[\frac{2\pi}{\sqrt{3(1-\nu^2)}} E\,(t \cos \alpha)^2\right]$
P^1	axial compressive collapse force
p, p_o	applied and critical external uniform hydrostatic pressure, respectively; internal uniform hydrostatic pressure
p_{cr}	theoretical critical external uniform hydrostatic pressure
$\bar{p}$	theoretical critical external uniform hydrostatic pressure for "equivalent" cylinder
p^*	internal pressure parameter $\left[\sqrt{12(1-\nu^2)}\ \frac{p}{E}\left(\frac{R_1}{t \cos \alpha}\right)^2\right]$
R_1, R_2	radius of small and large cone cross sections, respectively
T_{cr}	critical torque
t	cone wall thickness
α	semi-vertex angle of cone
ζ	geometry parameter $\left[\sqrt{12(1-\nu^2)}\ \frac{R_1}{t \cos \alpha} \cot^2\alpha\right]$
ν	Poisson's ratio of cone material

ρ_1, ρ_{av} cone radius of curvature at small end and center of cone generator, respectively

$$\left(R_1/\cos\alpha,\ \frac{R_1 + R_2}{2\cos\alpha}\right)$$

σ_b^1/σ_c ratio of net compressive stress due to bending and critical axial compressive stress

$$\left(\sqrt{3(1-\nu^2)}\ \frac{\frac{2M}{R_1} - \pi pR_1^2}{2\pi\ E(t\cos\alpha)^2} = \frac{\sqrt{3(1-\nu^2)}\,M}{\pi\ E\ R_1(t\cos\alpha)^2} - \frac{1}{4}\ p^*\right)$$

τ_{max} maximum critical torsional stress $\left[T_{cr}/(2\pi\ R_1^2 t)\right]$

SMALL DEFLECTION THEORY

The only load conditions for which theoretical small deflection solutions are reasonably well established are some of those for which the stress distribution prior to buckling is independent of position around the circumference of the conical shell. The problem of the buckling of conical shells under axial compression has been studied in refs. 40 and 49. Both investigations are of the axisymmetric form of buckling and indicate that the critical axial compression load for a simply supported conical shell is given approximately by the simple formula:

$$P_{cr} = \frac{2\pi}{\sqrt{3(1-\nu^2)}}\ E\ (t\cos\alpha)^2 \qquad (1)$$

a value which should be relatively good for cones with clamped edges as well. Investigation of the axisymmetric state of buckling is extended to the combined case of axial compression and internal pressure in ref. 45. The results are not given by any simple expression, but can be represented by a series of curves for the variation of an axial load parameter as a function of a pressure parameter for various values of a geometry parameter for the small end of the cone. The results are not very dependent on cone length, but do depend strongly on the boundary conditions at the small end of the conical shell. Values for a particular kind of simply supported edge and a particular kind of clamped edge are shown in Fig. 1.

For other types of loading, more general modes of deformation have to be taken into account. Many sets of equations have been derived for this task, with the rigorous theory of ref. 28 at one end of the spectrum and the Donnell-type theory of refs. 23, 41 and 53 at the other. Success in obtaining reasonably accurate, but simple, solutions appears to depend, however, on the use of a high speed digital computer and a combination of luck and educated guesswork to correlate the resulting mass of data depending on at least three independent parameters. The correlation process has been carried out for simply supported conical frustums under external hydrostatic pressure in ref. 43 where the critical pressure is shown to be given approximately by the expression

$$p_{cr} \approx \bar{p}\, f(1 - {}^{R_1}/R_2) \qquad (2)$$

where $\bar{p}$ is the critical pressure of the "equivalent" cylinder having a length equal to the slant length of the cone, a radius equal to the average radius of curvature of the cone, and the same thickness, and $f(1 - {}^{R_1}/R_2)$ is given by the solid curve of Fig. 2. The pressure $\bar{p}$ can itself be approximated by

$$\bar{p} \approx \frac{0.92\, E}{\left(\frac{\ell}{\rho_{av}}\right)\left(\frac{\rho_{av}}{t}\right)^{5/2}} \qquad (3)$$

The same problem has been studied in numerous other papers (refs. 3, 8, 9, 11, 14, 26, 27, 28, 30, 31, 32, 35, 38, 50, 55, 56 and 59) which give similar results. Some results for external pressure which varies along the generator but is uniform around the circumference are given in ref. 50 and for thermal buckling of conical shells under axisymmetric temperature distributions in ref. 52.

Investigations have also been carried out for simply supported cones under torsion (refs. 29 and 44) and combined external hydrostatic pressure and axial load (refs. 26 and 46). For the latter loading conditions, the interaction curves appear to be a function of the taper ratio $1 - {}^{R_1}/R_2$ and are shown in ref. 46 to lie between the limiting curves shown in Fig. 3. For the former loading condition, an approximate expression for the critical torque is given in ref.. 44 by

$$\frac{T_{cr}}{\pi D} \approx 16.2 \left(\frac{t}{L}\right)^{\frac{1}{2}} \left\{ \left[1 + \left(\frac{1 + {}^{R_2}/R_1}{2}\right)^{\frac{1}{2}} - \left(\frac{2}{1 + {}^{R_2}/R_1}\right)^{\frac{1}{2}} \right] \frac{R_1 \cos\alpha}{t} \right\}^{5/4} \qquad (4)$$

It is obvious that much work remains to be done to complete small deformation investigations of only the loading conditions which yield axisymmetric stress distributions, let alone such asymmetric cases as pure bending. In all of the investigations, membrane theory has been used to define the stress state prior to buckling. For those cases involving internal pressure, it would be desireable to know if consideration of bending effects changes the results to any great extent, since buckling is confined to the immediate vicinity of the small end of the cone where the pre-buckling stress distribution may differ considerably from the membrane state. Large deformation investigations are known to be necessary and have been attempted in refs. 10, 37, and 39. Present knowledge of both small deflection results and test buckle patterns indicate, however, that these analyses are probably quite inaccurate, so that this area of stability theory remains to be explored.

COMPARISON OF THEORY AND EXPERIMENT

Axial Compression and Internal Pressure

As is usual for the stability of thin shells, experimental results are in qualitative, but not quantitative, agreement with theoretical results. In Fig. 4, the available experimental load coefficients (refs. 19 and 47) for conical shells in axial compression are shown as a function of the small radius of curvature-thickness ratio, together with a lower bound curve for cylinders. The agreement between theory and experiment appears to be about the same as for cylinders, possibly a little better. It would appear that the usual empirical cylinder formulas, with the substitution of the small radius of curvature of the cone for the cylinder radius, can be used to design conical shells under axial compression. More test data is needed, however, to establish the effect of cone length and to verify the conjecture that no other parameters are important.

When internal pressure is added to cones under axial compression, the critical compressive loads tend to approach those predicted theoretically. The results of refs. 4, 20, and 47 for clamped cones (see Figs. 5(a) to 5(e)) indicate, however, that discrepancies may exist between theory and experiment at all pressure levels. For low values of internal pressure these discrepancies are most likely due to the decreasing effects of initial imperfections as for cylindrical shells and parameters other than those shown should be investigated. For large values of internal pressure, the discrepancy between theory and experiment is suspected to be due to plastic yielding at the small clamped edge which makes the results fall closer to those for cones with simply supported edges. Thus, more test data is needed to establish design curves for cones of various materials as well as for those of various geometries and end conditions.

External Pressure

Considerable test data is available in refs. 12, 13, 15, 16, 22, 34, 47, 48, 54 and 57 for cones subjected to uniform external hydrostatic pressure. While it is almost gospel that theory and experiment compare favorably for this loading condition, the comparison shown in Fig. 6 indicates that such is actually not the case for either cylinders or cones since the scatter is considerable, with experimental values ranging from 60% to 140% of the values predicted for simply supported ends. Some of the scatter is due to the fact that the end conditions are not the same for all of the test specimens, a good many being clamped rather than simply supported. Since it is known that clamping theoretically increases the critical pressure of a cylinder by 40%, it may be presumed that clamped cones will, on the average, have higher critical pressures than simply supported cones. Initial imperfections are very likely another cause of scatter since they undoubtedly differ from specimen to specimen. Still another cause of scatter is the difficulty of determining the so-called buckling load. Buckling under external pressure is not a collapse phenomenon defined by a maximum load, as is the case for axial compression, but a phenomenon usually defined by the visual perception of large skin deformations and as such, depends on the variable judgement of the observer. It is obvious, therefore, that further investigation of the problem of buckling under external pressure is required.

Axial Compression and External Pressure

For cones under combined external pressure and axial compression, some data is available in ref. 47. If the results are plotted as ratios of applied external pressure to critical external pressure and critical axial compression to critical axial compression in the absence of pressure, as in Fig. 7(a) and 7(b), the various interaction curves are seen to closely agree with the theoretical curves. Such behavior indicates that the ratio of experimental to theoretical critical compressive loads is relatively independent of pressure, a result which differs from that obtained for cylinders under combined axial compression and external pressure. Cones also exhibit an elastic phenomenon which does not appear to be obtainable for cylinders, the non-coincidence of axial buckling and collapse loads for external pressures near the critical value. As indicated in Fig. 7, cones can withstand additional axial load after buckles appear in the shell wall and continue to do so at external pressures considerably larger than the critical value. The behavior seems to depend on the semi-vertex angle of the cone. Since the axial load carrying capacity of a buckled cone can be significant, about 40% of the critical compressive load for a 60° cone buckled at the critical external pressure, this aspect should be investigated further.

Torsion

Experiments on conical shells in torsion are reported in refs. 21 and 47. While the cone of ref. 21 did not differ enough from a cylinder to test the theory of ref. 44, the results of ref. 47 (see Table 1) indicate that the agreement between theory and experiment for cones in torsion is about as good as for cylinders in torsion. On the average the 10 clamped shell specimens buckled at about 95% of the torque predicted by the theory for simply supported conical shells, with individual specimens buckling at torques ranging from 68% to 122% of the theoretical velues. Thus it would appear that eq. (4), multiplied by the same reduction factor as for cylinders, may be used to determine critical torques of cones with the same degree of confidence as for cylinders.

ADDITIONAL EXPERIMENTAL RESULTS

Some additional load conditions, for which no theoretical results are available, have been investigated experimentally in ref. 47. Although the number of tests is small, enough is available to allow some tentative conclusions to be drawn.

Pure Bending

The results for clamped conical shells in pure bending are expressed in terms of a moment coefficient $\dfrac{M}{\pi E R_1 (t \cos \alpha)^2}$ and the ratio of the small radius of curvature to the wall thickness, $\dfrac{\rho_1}{t}$. The moment parameter is a constant times the ratio of the maximum membrane compressive stresses due to bending and to axial compression, the reasoning being that a theoretical solution for the problem would very likely yield the same result as for cylinders, that buckling occurs when the maximum compressive stress due to bending is equal to the critical axial compressive stress. When corresponding values of the parameters are plotted (Fig. 8), the resulting chart is similar to that for cylindrical shells in bending, for which a suggested lower bound curve is also shown.

Despite the success in correlating the data, it is entirely possible that additional parameters, such as semi-vertex angle and length, may be important. In addition, the effect of the type of edge restraint should be significant. The data available is insufficient or lacking, however, to permit any decision to be made concerning these questions.

Bending and Internal Pressure

The addition of internal pressure to conical shells causes the bending moment carrying capacity to increase by a very significant amount, as is the case for cylinders. The data is given in terms of a net membrane compressive stress parameter σ_b/σ_c which is plotted in Fig. 9 as a function of a pressure parameter p^*. Elsewhere in the present compilation+, collapse of pressurized cylindrical shells in bending is explained in terms of collapse of cylindrical membranes and the test results are shown to fall between the limits, expressed in the notation of the present paper,

$$2 + \frac{1}{4} p^* > \frac{\sigma_b^1}{\sigma_c} > \frac{1}{4} p^* \qquad (5)$$

where the lower limit corresponds to the theoretical membrane collapse load and the upper limit to a modified membrane collapse load. The same upper and lower bound lines are plotted in Fig. 9 and are seen to bound the data for cones as well. Thus, at high pressures the bending collapse of conical shells can very likely also be explained in terms of a membrane collapse theory.

For cylinders, a good lower bound to the available data is shown in the cited paper to be given for the entire pressure range, by

$$\frac{\sigma_b^1}{\sigma_c} = \frac{1}{2} + \frac{1}{4} p^* \qquad (6)$$

This does not, however, appear to suffice for conical shells since results for some of the specimens lie below this bound, closer to the membrane collapse moment. For pressurized cones in the intermediate range of pressure values, therefore, additional testing is needed to establish important parameters and values of collapse moments suitable for design purposes.

+ McComb, Harvey, G., Jr, Zender, George W., and Mikulas, Martin M., Jr.: The Membrane Approach to Bending Instability of Pressurized Cylindrical Shells.

Bending, Axial Compression, and Internal Pressure

The final set of data is for a single 30° conical shell subjected to combined bending, axial compression, and internal pressure. The results, are shown in Fig. 10 in the form of combined values of the moment divided by the critical moment for no net axial force at the small end and the net axial force at the small end divided by the critical net axial force for no moment. The data for zero internal pressure and two other values of pressure indicate that there is a single interaction relation between three loads which is very similar in appearance to that obtained theoretically for combined axial compression and uniform external hydrostatic pressure. When the net axial force is compressive, this relation may be approximated by

$$\frac{M}{M_{(P - \pi p R_1^2 = 0)}} + \frac{P - \pi p R_1^2}{(P - \pi p R_1^2)_{M=0}} = 1 \qquad (7)$$

More data is needed, of course, to firmly establish this relationship and to indicate other parameters that may be significant for other cone geometries.

CONCLUDING REMARKS

Although a good deal of territory has been covered in trying to establish design and analysis criteria for the buckling of conical shells, the number of unanswered and bypassed questions is still considerable. Many elastic small deflection problems remain to be investigated as well as large deformation and plastic stability problems which appear to be necessary for our understanding of the experimental results for several loading conditions. The large number of parameters in all of the problems makes it necessary for theory and experiment to proceed together. Thus far the theory has been necessary to provide correlation parameters for the experimental data. In many of the problems that should be considered, however, the theoretician will need experimental data to guide him in making his analyses.

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Table 1. Comparison of Theory and Experiment for Steel Cylinders and Cones in Torsion (Ref. 47).

t (in.)	R_1 (in.)	R_2 (in.)	L (in.)	τ_{max} (experimental) psi	τ_{max} (computed) psi	$\frac{\tau_{exp}}{\tau_{comp}}$
$\alpha = 30^o$						
0.010	4	10	10.39	11650	11100	1.05
0.010	4	10	10.39	11490	11100	1.03
0.020	4	10	10.39	32400	26570	1.22
0.020	4	10	10.39	23400	26570	0.88
$\alpha = 60^o$						
0.010	2	10	4.62	14300	21030	0.68
0.010	2	10	4.62	14300	21030	0.68
0.010	3	10	4.04	10800	12460	0.87
0.010	3	10	4.04	13600	12460	1.09
0.010	5	10	2.89	8390	7850	1.07
0.010	5	10	2.89	6900	7850	0.88

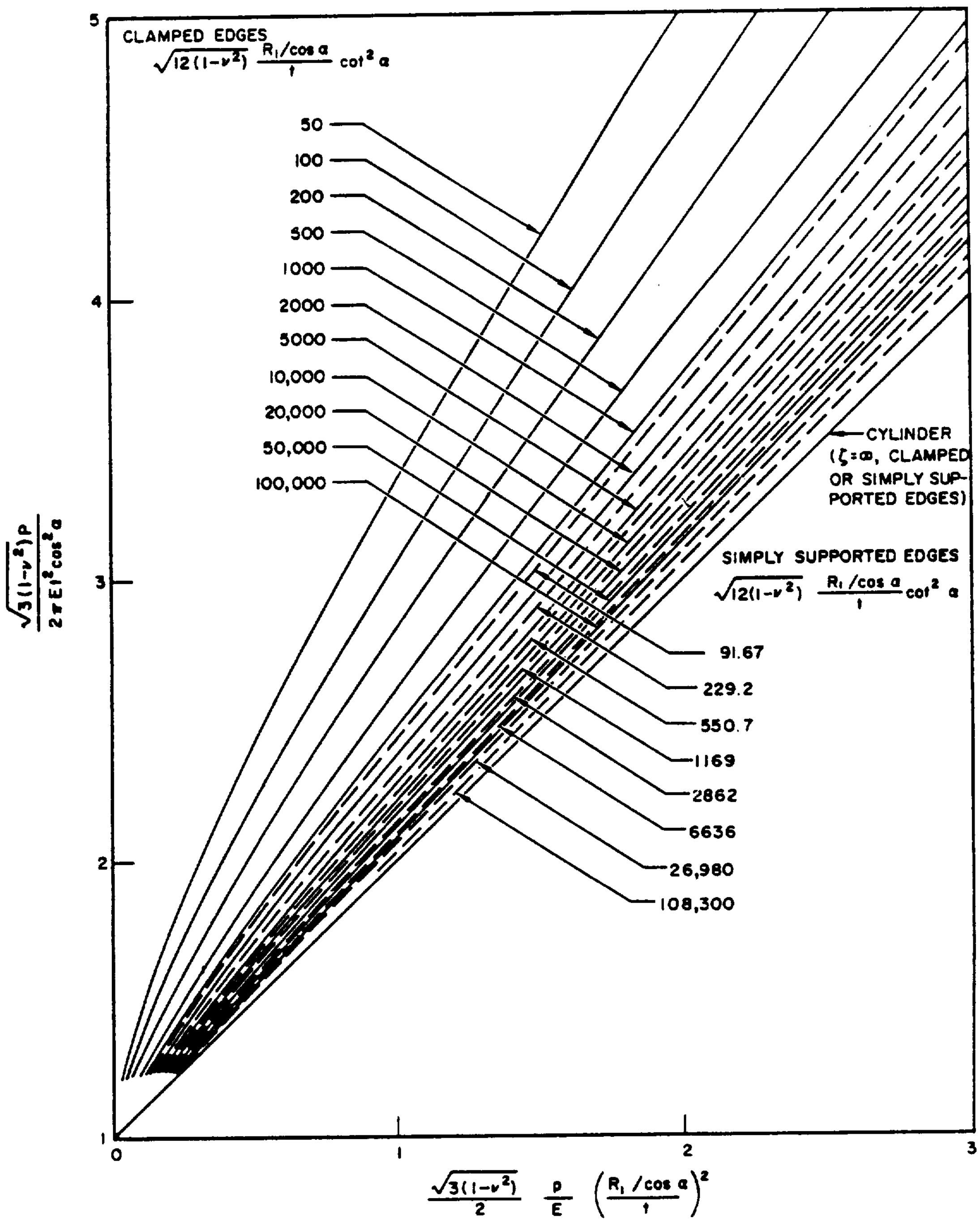


Figure 1.- Theoretical variation of axial load parameter with internal pressure parameter for conical shells with clamped or simply supported edges (ν = 0.3).

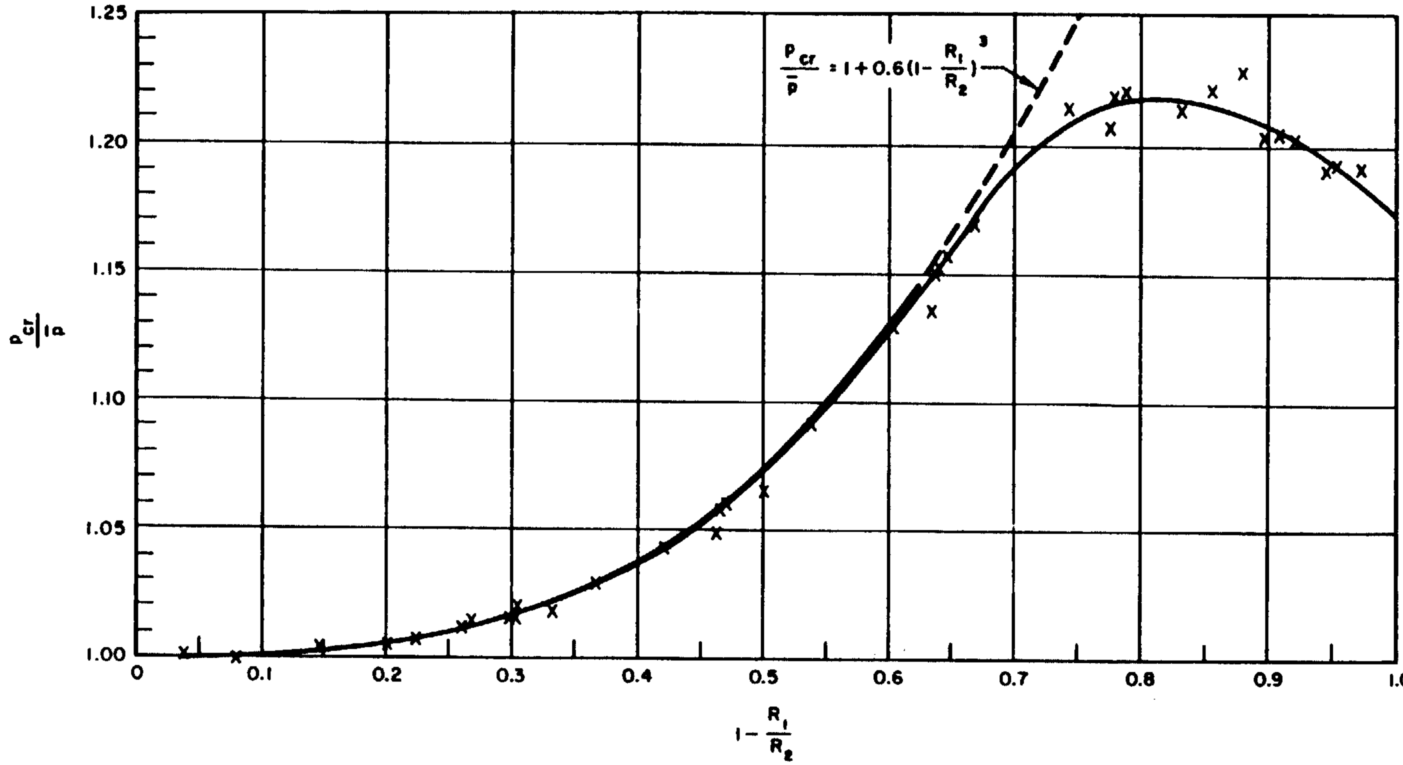


Figure 2.- Theoretical variation of buckling pressure ratio with taper ratio for simply supported cones.

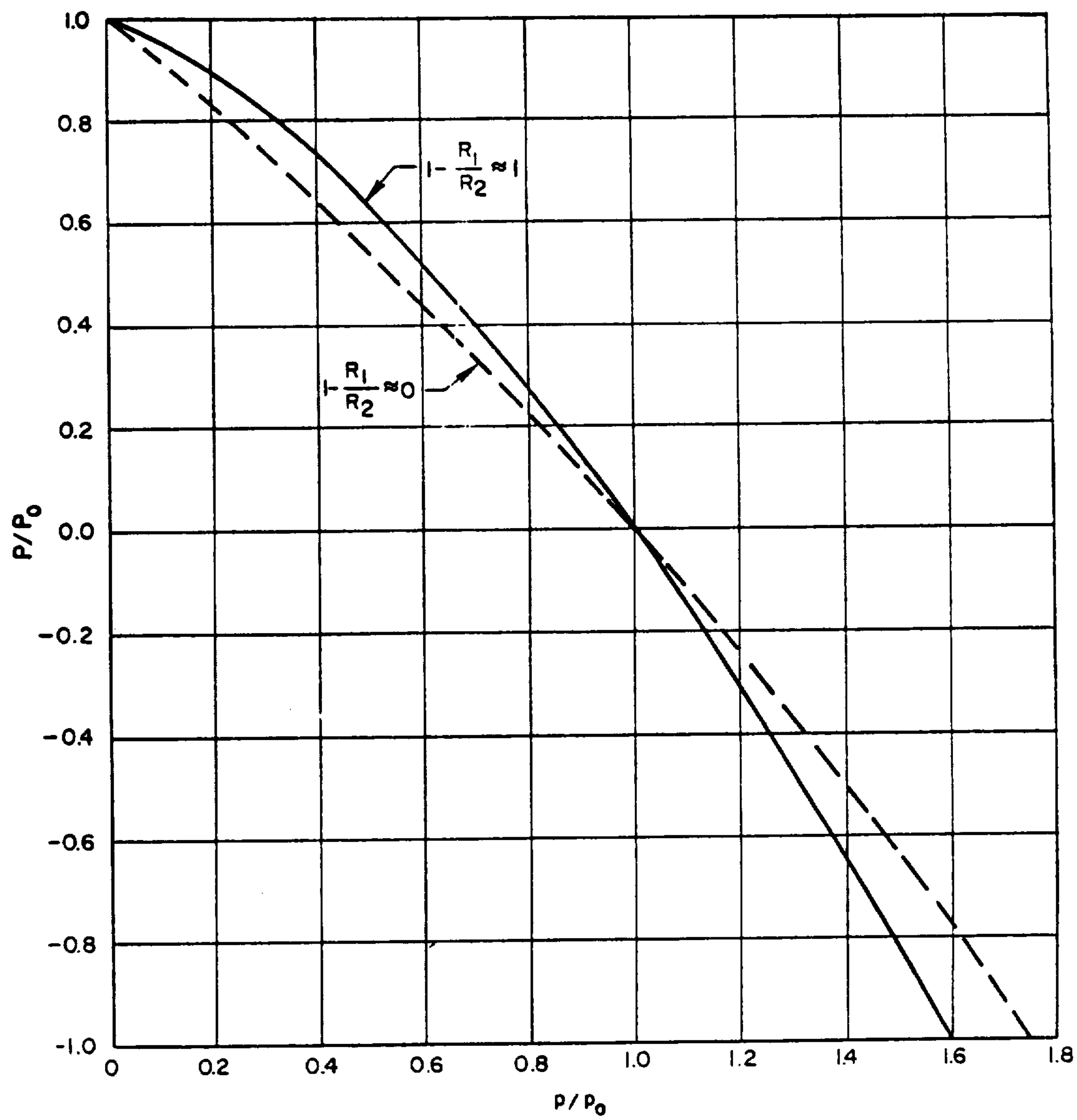


Figure 3.- Theoretical interaction curves for cones under combined axial load and external uniform hydrostatic pressure.

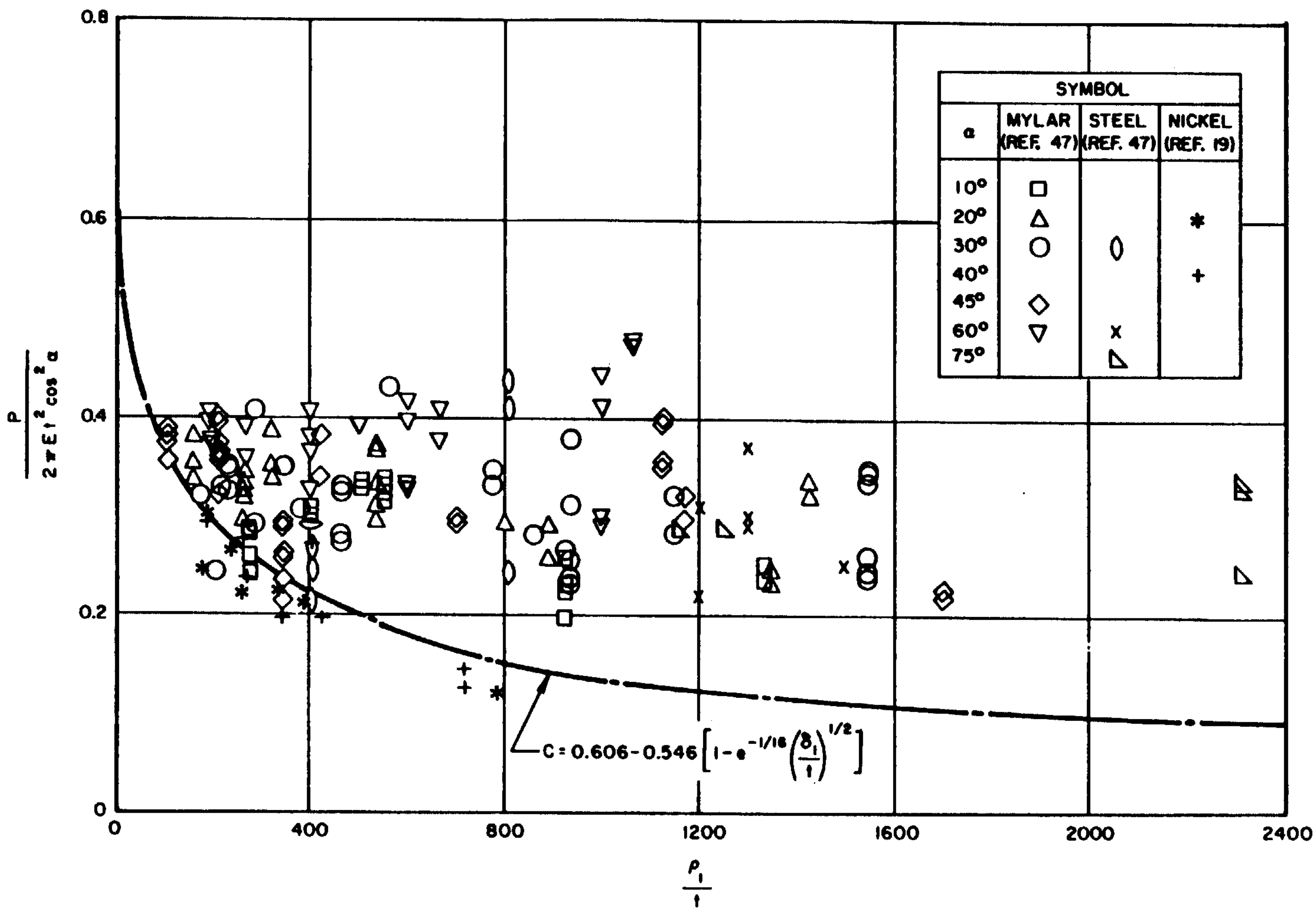


Figure 4.- Comparison of axial compression coefficients for conical shells with lower bound curve for cylinders.

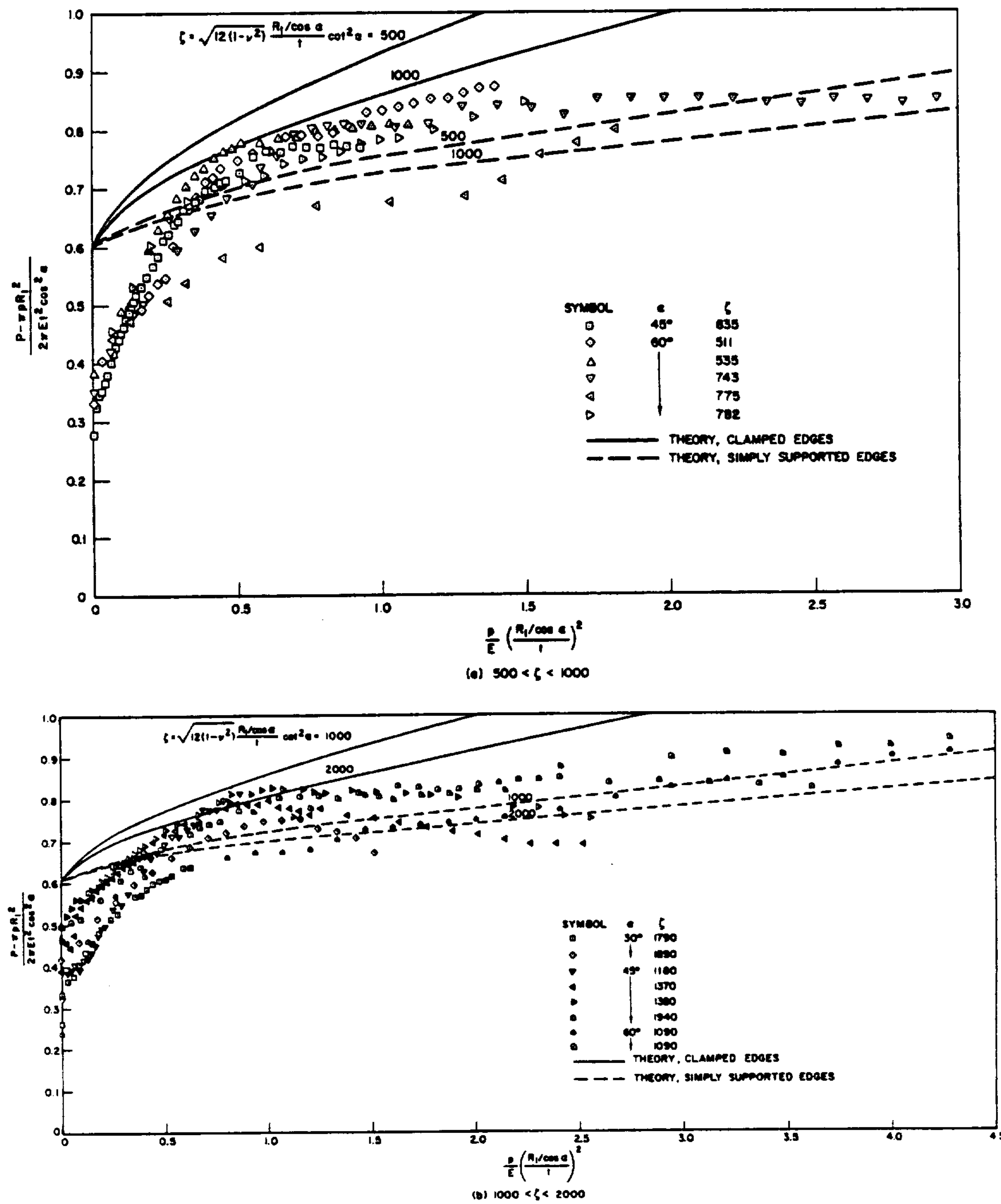
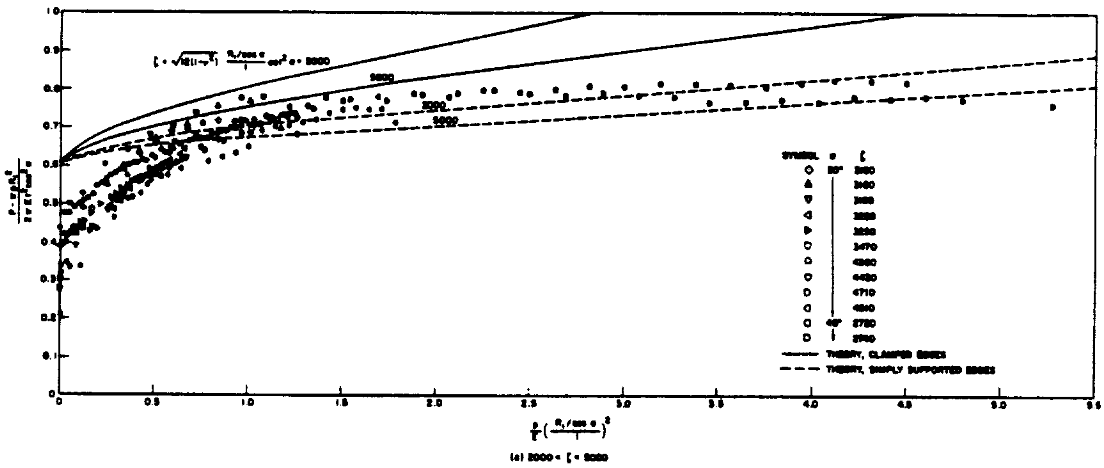


Figure 5.- Comparison of theory and experiment for pressurized conical shells under axial compression.



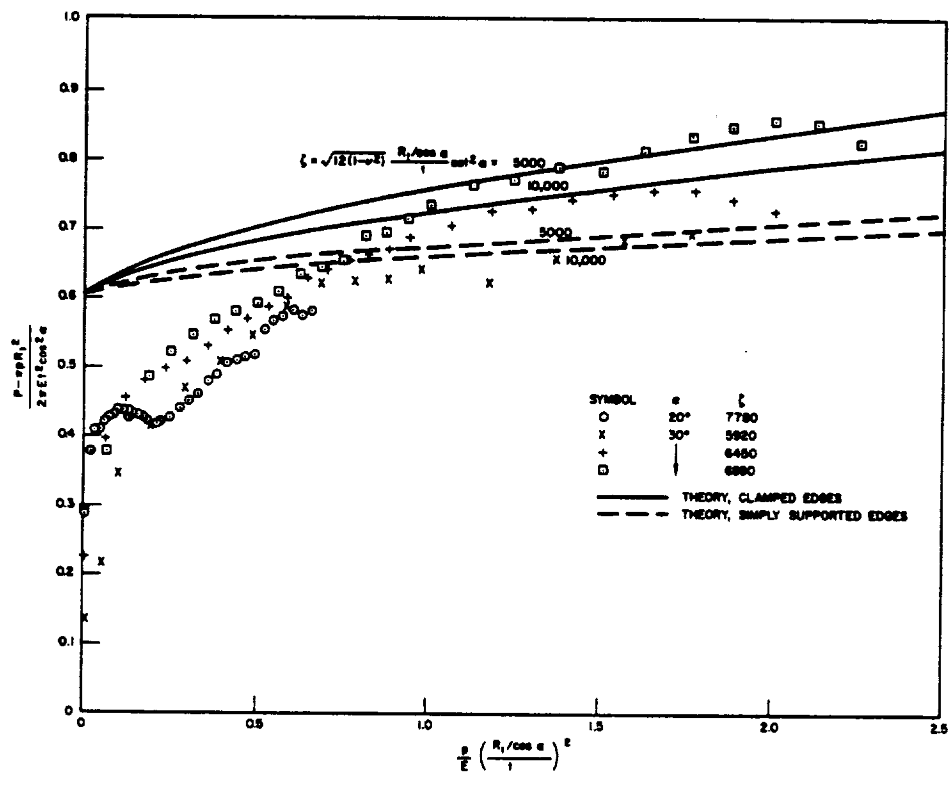
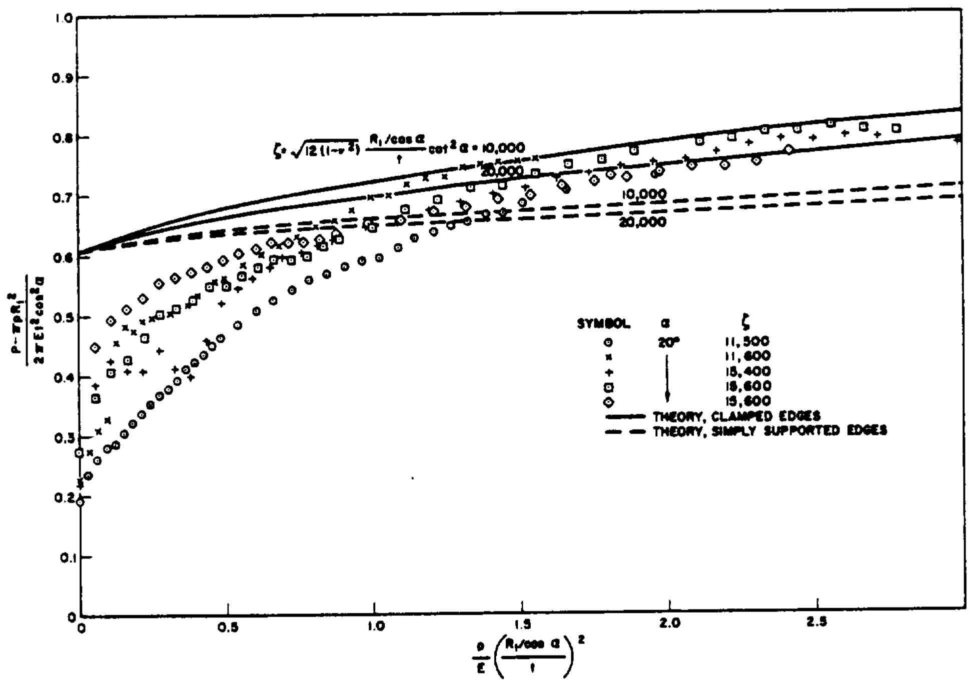


Figure 5.- Continued.



(e) 10,000 < ζ < 20,000

Figure 5.- Concluded.

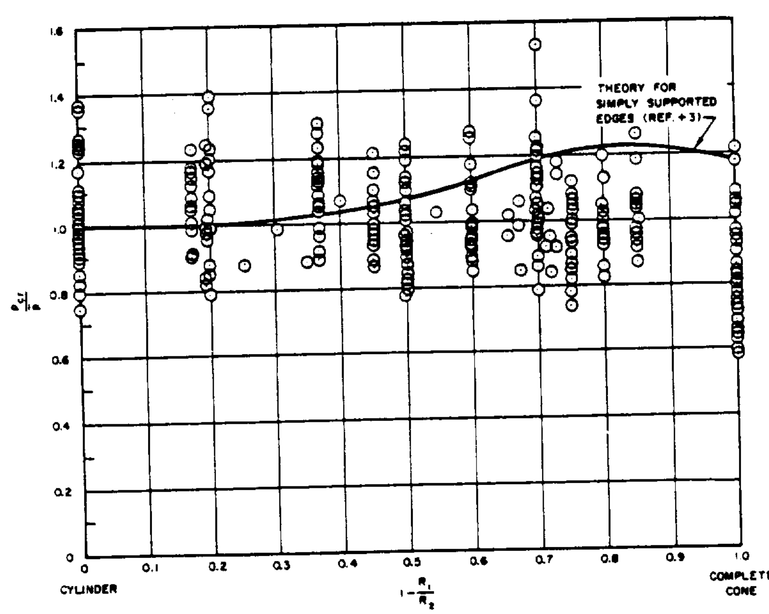
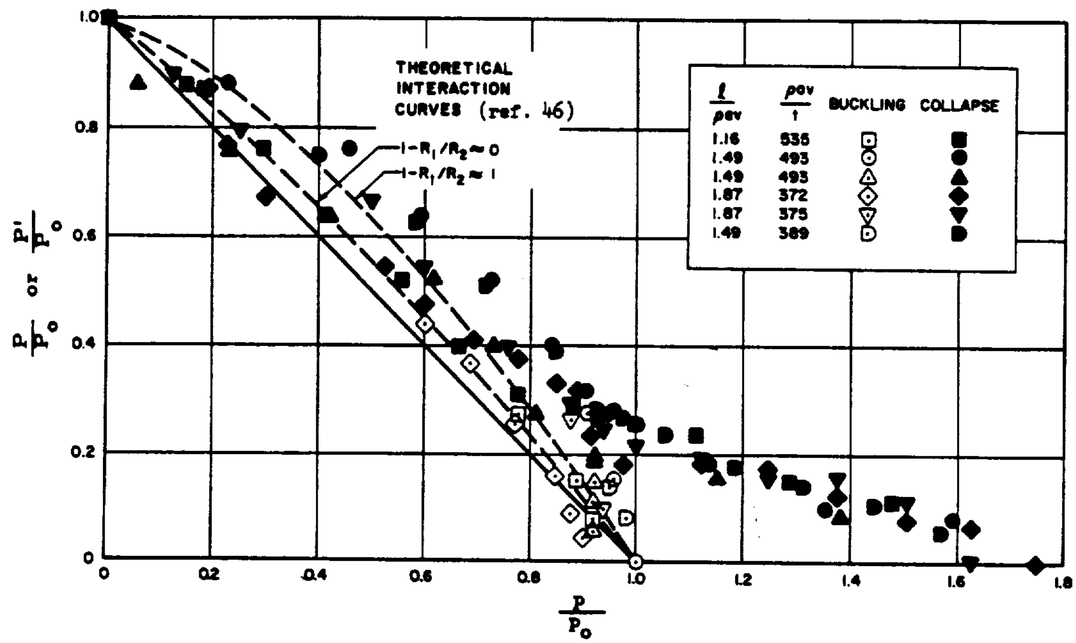
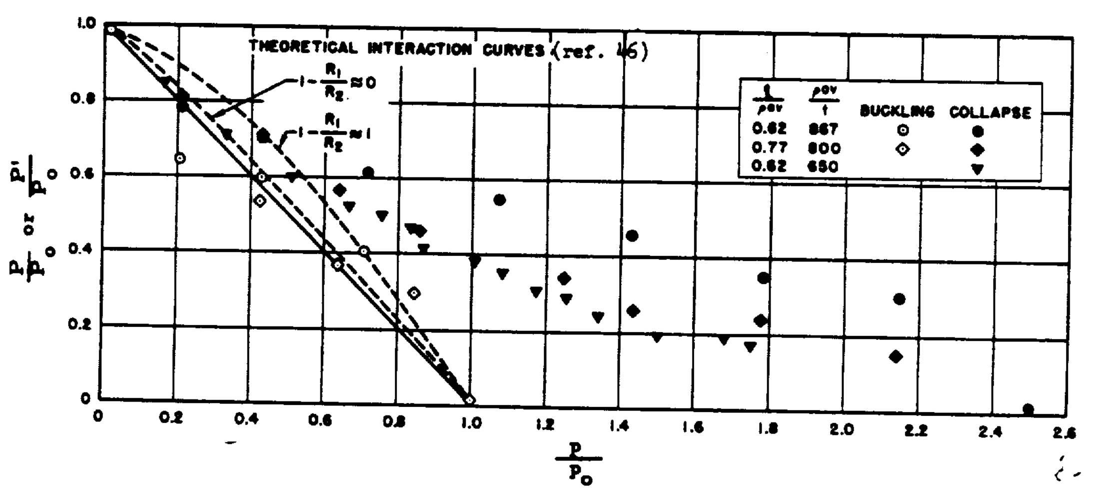


Figure 6.- Comparison of theory and experiment for conical shells under external hydrostatic pressure.



(a) $\alpha = 30^{\circ}$.



(b) $\alpha = 60^{\circ}$.

Figure 7.- Interaction curves for cones under axial compression and external uniform hydrostatic pressure.

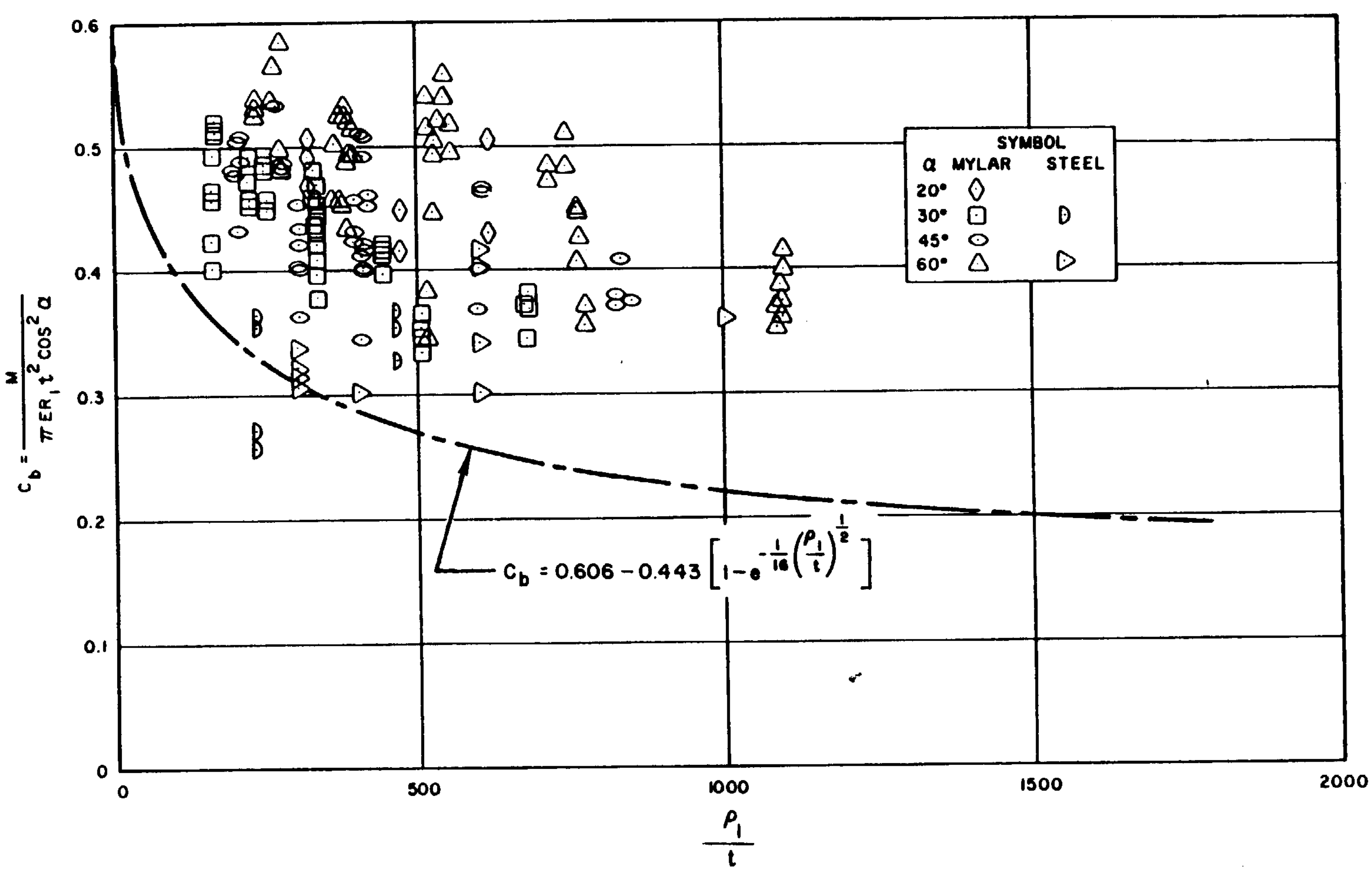


Figure 8.- Comparison of bending moment coefficients for conical shells with lower bound curve for cylinders (data from ref. 47).

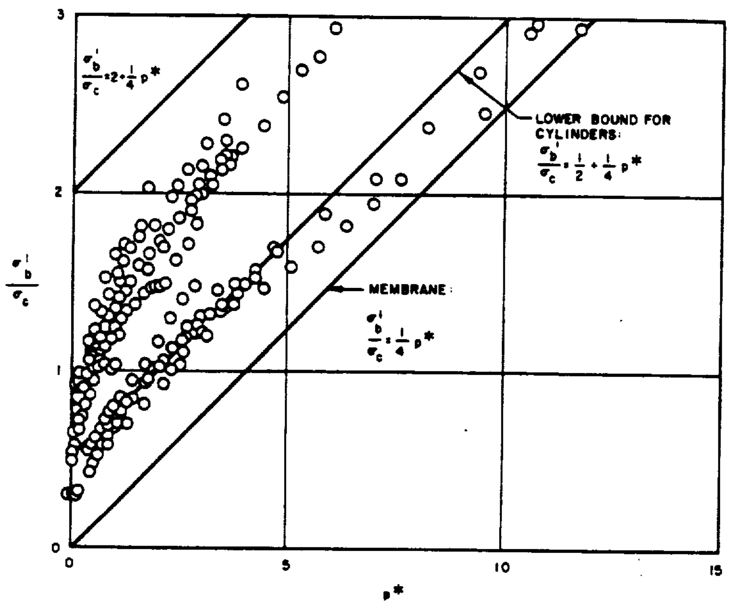


Figure 9.- Variation with internal pressure parameter of net bending stress ratios for 30° and 60° cones under uniform hydrostatic pressure.

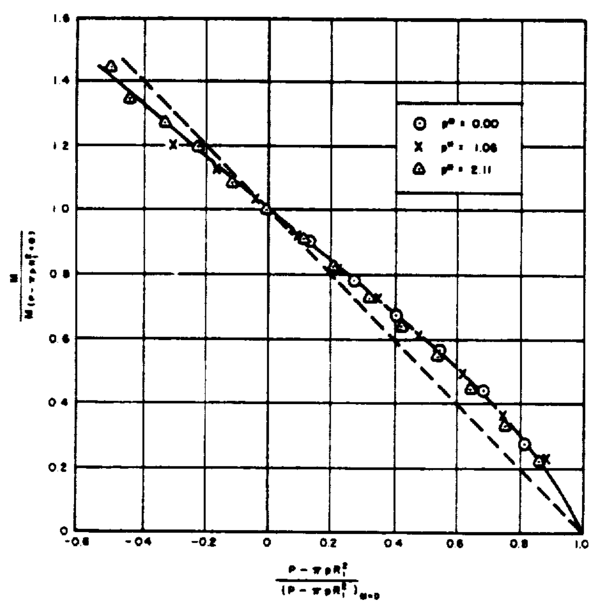


Figure 10.- Interaction curve for a pressurized cone under bending and axial compression $\left(\alpha = 30^\circ, \ \frac{\rho_1}{t} = 670\right)$.