551564
106



# A SURVEY OF RESEARCH ON THE STABILITY OF HYDROSTATICALLY-LOADED SHELL STRUCTURES CONDUCTED AT THE DAVID TAYLOR MODEL BASIN

By Thomas E. Reynolds

David Taylor Model Basin

## SUMMARY

Experimental and theoretical studies have been carried out to investigate the stability of ring-stiffened cylinders and hemispherical shells under hydrostatic pressure. The use of accurately machined models has been found extremely valuable in improving the quality of experimental data, thereby permitting a more careful examination of theory. As a result, the effects of boundary conditions and other factors usually masked by the influence of structural imperfections have been clarified.

So far these studies have caused the investigators to regard classical small-deflection theory with increasing confidence.

## INTRODUCTION

As the major structural element in a submersible, the ring-stiffened cylinder has long been of prime interest to the naval architect. It is likely that future vehicles attaining greater operational depths may also make extensive use of spherical shells for the main pressure hull as well as for terminating closures. This paper is concerned with recent studies of the buckling characteristics exhibited by these two shell types when subjected to hydrostatic pressure.

## RING-STIFFENED CYLINDERS

The basic buckling configuration to be considered is the antisymmetric or lobar mode. It has been convenient to investigate separately two distinct classes: general instability, wherein both rings and shell deflect radially (figure 1), and shell instability, in which the rings do not deflect radially and buckles appear between them (figure 2).

### General Instability

Considerable work has been directed toward an experimental evaluation of Kendrick's small-deflection solution (reference 1, second solution) for the elastic instability of a cylinder with simply-supported ends. Tests of a variety of small, externally-stiffened cylinders machined from high-strength steel tubing have revealed that buckling pressures can be obtained with remarkable accuracy using Southwell's nondestructive technique (references 2 and 3). It has also been established that variations in end restraint, even for cylinders as long as five diameters, can have an appreciable influence on buckling strength (reference 4).

As an example of the investigations being conducted, a recent study by W.F. Blumenberg is cited. The objective was to obtain buckling pressures by Southwell's method as a function of cylinder length with boundary conditions held constant. The test arrangement is shown in figure 3. The length of the central test section was varied by rearranging the pairs of movable discs whose rounded edges were in contact with the inner wall of the cylinder. The outer discs were maintained at one frame space from the inner ones as an approach to isolating the central section from variations in conditions of support resulting from changes in the lengths of the end sections. The maximum test pressure attained averaged about 98 per cent of the Southwell buckling pressure. The maximum measured stress was about 43,000 psi compared with a value of 85,000 psi for the yield strength.

Since it was not expected that this arrangement would closely approximate the condition of simple support, it was not surprising that the experimental pressures were somewhat higher than those given by Kendrick's solution. However, a plot of the results is instructive. In figure 4 the circles represent the experimental pressures and the solid curve is Kendrick's solution for a nominal Young's modulus of $30 \times 10^6$ psi. The abscissa is N, the number of frame spaces separating the inner discs. These results suggest the possibility that, so far as buckling is concerned, a cylinder of length L (N frame spaces) whose ends are arbitrarily restrained behaves as if its length were $L_{eff}$ ($N_{eff}$ frame spaces) and its ends simply-supported, where

$$L_{eff} = kL \qquad (N_{eff} = kN) \tag{1}$$

k is a constant whose value depends on the degree of restraint - being less than unity where the restraint is more restrictive than simple support and greater than unity where the opposite is true. This would mean that for a given cylinder any degree of restraint can be represented on a single plot of buckling pressure versus $L_{eff}$ or $N_{eff}$ and that the transition from one circumferential mode to another must occur at the same pressure regardless of the boundary conditions.

In figure 4 the slight difference between the theoretical and experimental pressures for the transition from 2 lobes to 3 could easily result from a small disparity between the actual and assumed values of Young's modulus. If the corresponding transition values for N are substituted into equation (1) the resulting value for k is 0.725. Using this number the experimental points were replotted with $N_{eff}$ as the new abscissa and with the appropriate adjustment in Young's modulus. These points are the triangles in figure 4.

This approach has been used with data from a variety of tests to obtain values of k ranging from 0.71 to 0.94, and corresponding pressure variations of as much as 65 per cent. The results to date have strengthened the investigators' belief that Kendrick's solution will give reliable predictions when appropriate adjustments are made for the degree of restraint.

## Shell Instability

To summarize very briefly, small-deflection solutions for elastic shell instability have not been entirely satisfactory when applied to shells with closely-spaced stiffeners. Von Mises' solution (reference 5), for example, does not account for the effect of the stiffeners on buckling strength, hence is not strictly applicable. Von Sanden and Tölke (reference 6) considered the stiffeners as they affect the deflections prior to buckling, but neglected their influence on the buckling deformations. Experimental results in many cases have not been particularly illuminating. Because of inadequate yield strengths and fabrication imperfections, elastic shell instability has seldom been observed with closely-spaced stiffeners. In cases where the Von Mises pressure has not been attained "snap-through" buckling has sometimes been offered as the explanation.

A small-deflection solution recently developed by the author accounts for the influence of the stiffeners on deformations occuring both before and during buckling by expressing all deflections as trigonometric series. The solution is obtained using the Ritz procedure and includes the resistance of the stiffeners to deformations in and out of their planes. To evaluate this solution data are available from collapse tests of four machined cylinders where again elastic buckling was achieved through the use of high-strength steel tubing. Two of these tests are reported in reference 7. The results are shown in table 1. It appears that the performances of the cylinders are adequately explained by the new small-deflection solution. This is supported by the fact that the buckling strength in one case was accurately determined using Southwell's method, which is only successful where small-deflection theory applies. It is further indicated, by comparing the Von Mises pressures with the collapse pressures, that the influence of the stiffeners can be appreciable. It is only partially accounted for by the solution of Von Sanden and Tölke.

## SPHERICAL SHELLS

In spite of a long history of investigation it appears that the buckling of spherical shells is not yet properly understood. The elastic buckling pressure given by the classical small-deflection analysis of Zoelly (reference 8) is far in excess of anything that has been observed experimentally, and various attempts to explain these vast discrepancies on the basis of finite deflection theory have been less than satisfactory (reference 9). Furthermore, most work has been devoted to the study of shallow spherical caps whereas the interest of the pressure vessel designer is in deep and complete spherical shells.

Krenzke (reference 10) has recently completed tests of a series of machined hemispherical shells about 1.6 inches in diameter which were designed to study both elastic and inelastic buckling. One group of shells was machined from 6061-T6 aluminum (yield strength of 43,000 psi), another from 7075-T6 aluminum (80,000 psi). Each hemisphere was terminated by a stiffened cylinder which, in all but three cases, was designed so that no bending stresses could develop in the hemisphere prior to buckling. The three exceptions were cases in which the cylinders had to be made somewhat more rigid to provide them with adequate buckling strength. Accurate machining assured nearly perfect sphericity in all cases.

Figure 5 shows a few of the observed failures. The three shells having the more rigid boundaries buckled well within the elastic range. It appeared that the buckling strengths of these shells were not adversely affected by the boundary conditions since, in each case, the portion immediately adjoining the cylinder was undamaged. Their buckling pressures are compared in table 2 with the theoretical values given by the small-deflection solution of Zoelly:

$$p_e = \frac{1.154\,E}{\sqrt{1-\nu^2}}\left(\frac{h}{R}\right)^2 = 1.21\,E\left(\frac{h}{R}\right)^2 \text{ for } \nu = 0.3, \qquad (2)$$

where E is Young's modulus, $\nu$ is Poisson's ratio, h is the shell thickness and R is the mean radius. The ratios of experimental pressures to $p_e$, although less than one, are much larger than have been observed for shells formed from flat plates, indicating the sensitivity of buckling strength to imperfections and residual stresses. The tests also show that the minimum buckling pressures defined by large-deflection theory can be greatly exceeded. From the results of table 2 Krenzke has proposed an empirical buckling formula for the elastic range:

$$p_e' = \frac{0.80\,E}{\sqrt{1-\nu^2}} \left(\frac{h}{R_0}\right)^2 = 0.84\,E \left(\frac{h}{R_0}\right)^2 \text{ for } \nu = 0.3, \qquad (3)$$

where the use of the outer radius, $R_0$, is dictated by simple load equilibrium.

The rest of the shells collapsed in the yield region at pressures ranging from 6 to 46 per cent of $p_e$. For this range Krenzke has suggested the following formula to represent $p_c$, the inelastic collapse pressure:

$$p_c = \sqrt{\frac{E_s E_t}{E^2}}\, p_e' = 0.8 \sqrt{\frac{E_s E_t}{(1-\nu^2)}} \left(\frac{h}{R_0}\right)^2 \qquad (4)$$

$E_s$ and $E_t$ are the secant and tangent moduli which can be determined from the stress-strain diagram for the material under uniaxial loading. The quantity under the radical which multiplies $p_e'$ is a simplified plasticity reduction factor based on theoretical studies by Bijlaard and Gerard (references 11 and 12). In the elastic range equation (4) reduces to equation (3). Figure 6 shows how well this formula fits the experimental data. The abscissa is the ratio of $p_c$ to $p_e$, the elastic buckling pressure according to equation (2), and the ordinate is the ratio of the experimental pressure, $p_{exp}$, to $p_e$.

Despite the consistency of these results it is not necessarily conclusive that they represent the maximum buckling strength attainable. Each of the shells had small deviations in thickness which presumably had some weakening effects. Future studies will include tests of larger machined shells in which such deviations can be further reduced. Other tests are presently underway with spun and pressed hemispheres and with machined spherical shells having central angles greater than as well as less than 180°. The benefits of stiffening are also being studied.

## REFERENCES


1. Kendrick, S.: The Buckling under External Pressure of Circular Cylindrical Shells with Evenly Spaced Equal Strength Circular Ring Frames - Part III. N.C.R.E./R. 244, Sept. 1953.

2. Galletly, G. D., and Reynolds, T. E.: A Simple Extension of Southwell's Method for Determining the Elastic General Instability Pressure of Ring-Stiffened Cylinders Subject to External Hydrostatic Pressure. Proc. SESA, vol. XIII, no. 2, 1956, p. 141.

3. Galletly, G. D., Slankard, R. C., and Wenk, E., Jr.: General Instability of Ring-Stiffened Cylindrical Shells Subject to External Hydrostatic Pressure - A Comparison of Theory and Experiment. Jour. Appl. Mech., vol. 25, Trans. ASME, vol. 80, June 1958, pp. 259-266.

4. Reynolds, T. E., and Blumenberg, W. F.: General Instability of Ring-Stiffened Cylindrical Shells Subject to External Hydrostatic Pressure. DTMB Report 1324, June 1959.

5. Von Mises, R.: The Critical External Pressure of Cylindrical Tubes under Uniform Radial and Axial Load. EMB Report 366, Aug. 1933.

6. Von Sanden, K., and Tölke, F.: On Stability Problems in Thin Cylindrical Shells. DTMB Translation 33, Dec. 1949.

7. Hom, K., and Couch, W. P.: Hydrostatic Tests of Inelastic and Elastic Stability of Ring-Stiffened Cylindrical Shells Machined from Strain-Hardening Steel. DTMB Report 1501, Dec. 1961.

8. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., 1936, p. 491.

9. Fung, Y. C., and Sechler, E. E.: Instability of Thin Elastic Shells. Proc. First Symposium on Naval Structural Mechanics, 1960, p. 115.

10. Kronzke, M. A.: Tests of Machined Deep Spherical Shells under External Hydrostatic Pressure. DTMB Report 1601, May 1962.

11. Bijlaard, P. P.: Theory and Tests on the Plastic Stability of Plates and Shells. Jour. Aero. Sci., vol. 9, no. 10, Aug. 1942.

12. Gerard, G.: Plastic Stability Theory of Thin Shells. Jour. Aero. Sci., vol. 24, no. 4, April 1957.

Table 1 - Results of Elastic Shell Buckling Studies

| $\frac{A_f}{L_f h}$ | Flexibility Parameter $\sqrt[4]{\frac{3(1-\nu^2)}{R^2 h^2}}\, L_f$ | Experimental Buckling Pressure, psi | Theoretical Buckling Pressures, psi | | |
|---|---|---|---|---|---|
| | | | New Solution | von Sanden and Tölke (ref. 6) | von Mises (ref. 5) |
| 0.418 | 4.13 | 803(14)* | 811(11) | 693(16) | 665(15) |
| 0.526 | 4.75 | 725(13) | 744(13) | 665(14) | 654(14) |
| 0.650 | 5.26 | 475(14) | 460(14) | 386(16) | 387(14) |
| 0.228 | 6.80 | 633(11)** | 633(11) | 599(11) | 600(10) |

*Number of circumferential lobes in parentheses
**Southwell method gave 637 psi
$A_f$ = cross-sectional area of stiffener; $L_f$ = stiffener spacing
h = shell thickness; R = mean radius
All cylinders had external rectangular stiffeners

Table 2 - Comparison of Collapse Pressures from the Classical Theory with Experimental Values

| $\frac{h}{R}$ | Buckling Pressures, psi | | $\frac{p_{exp}}{p_e}$ |
|---|---|---|---|
| | Experiment, $p_{exp}$ | Equation (2), $p_e$ | |
| 0.0095 | 800 | 1180 | 0.68 |
| 0.0096 | 830 | 1210 | 0.69 |
| 0.0120 | 1230 | 1875 | 0.66 |

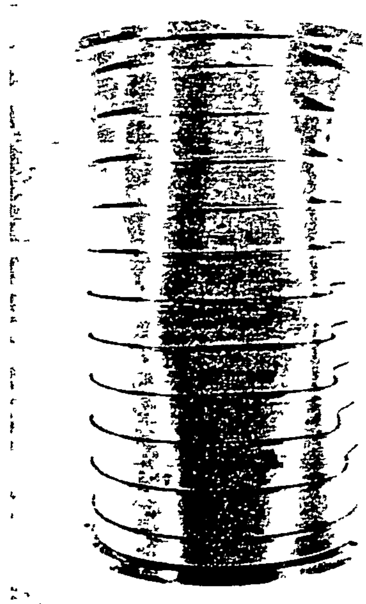

Figure 1.- General instability type of collapse.

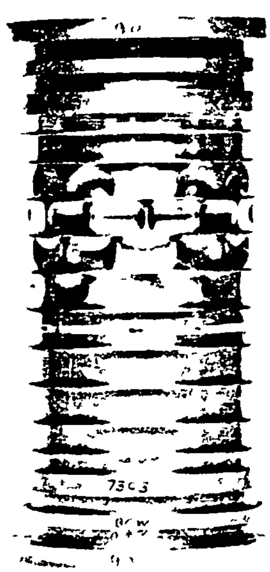

Figure 2.- Shell instability type of collapse.

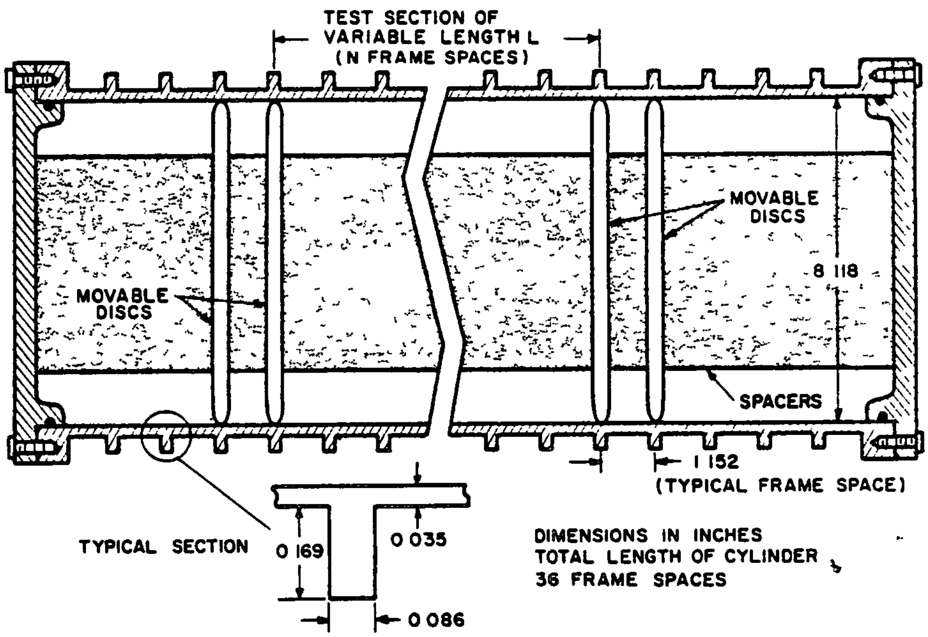


Figure 3.- General instability test arrangement.

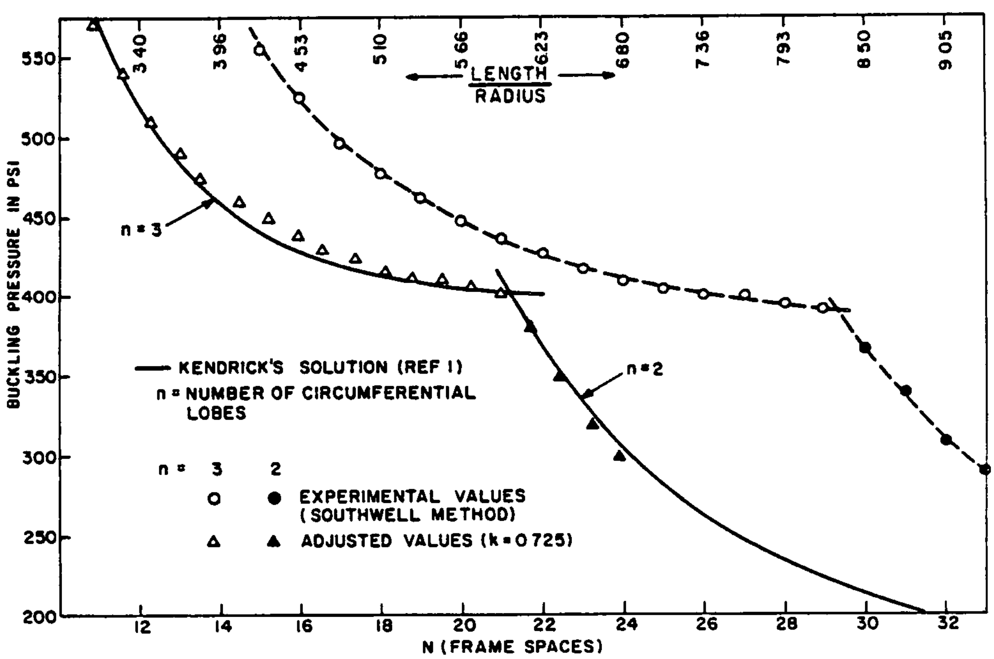


Figure 4.- Stiffened cylinder - elastic general instability results.

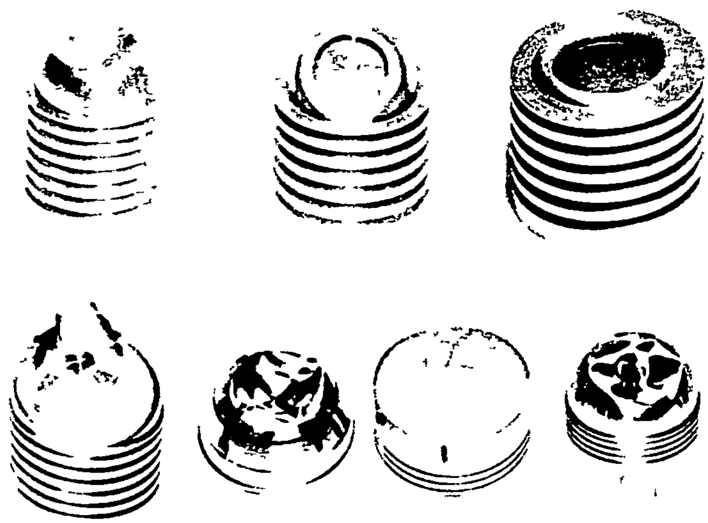

Figure 5.- Examples of collapsed hemispheres.

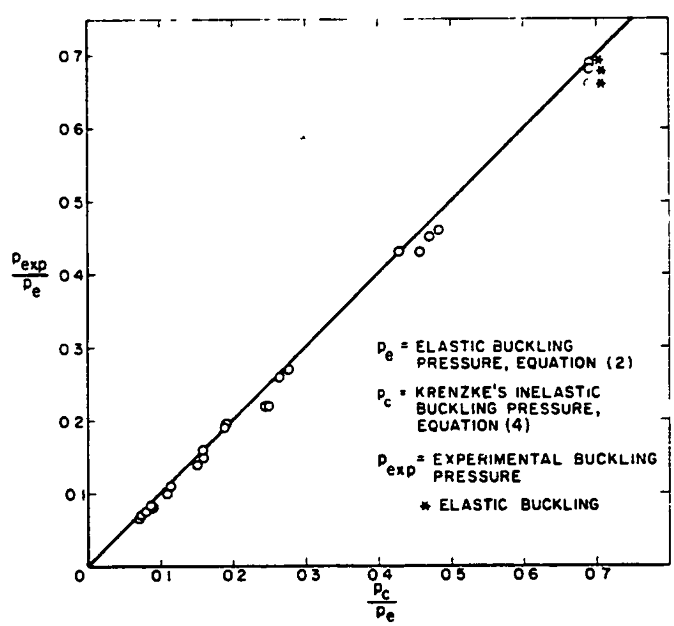


Figure 6.- Buckling data for hemispheres.