18p

N 63 10868

NASA TN D-1445

code 1

NASA TN D-1445

TECHNICAL NOTE

D-1445

DIFFERENTIAL CORRECTION FOR VINTI'S ACCURATE INTERMEDIARY ORBIT

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WASHINGTON December 1962

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by

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SUMMARY

This report presents an orbit improvement method in which mean values of certain parameters characteristic of Vinti's kinetic equations are obtained for a series of observations in a predetermined time interval and are used to determine the corresponding set of Izsak elements which exactly factor Vinti's two quartic polynomials. As a consequence, predicted positions and velocities for an earth satellite at any time can be obtained. This report also includes a section on the partial derivatives resulting from the first order Taylor expansion of the conditional equations.

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INTRODUCTION

Vinti's solution (Reference 1) for a gravitational potential

$$V(\rho, \eta) = \frac{-\mu\rho}{\rho^2 + c^2\eta^2}$$

in oblate spheroidal coordinates that simultaneously satisfies Laplace's equation and separates the Hamilton-Jacobi equation provides three adjustable constants for which this solution can be made to agree with the potential of an axially symmetric earth, expressed by means of an expansion in spherical harmonics. The agreement is exact for the zeroth and second zonal harmonics and, as a consequence of this system, through more than half of the latest accepted value of the earth's fourth harmonic. This solution by means of the canonical transformation for which both the momenta and coordinates are constants of the motion α_i and β_i, differs from others in that the oblateness potential is included in the solution of the equations of motion. In terms of specific initial conditions, both sets of constants can be evaluated and used to factor the two quartic polynomials $F(\rho)$ and $G(\eta)$; thus, Vinti's kinetic equations can produce coordinates of a satellite as functions of time in the form of an orbit generator (References 2 and 3).

To obtain good predictions outside of an arc of observations it is necessary to determine, by an iterated least-square fitting of the solution to many revolutions in the orbit, the Izsak elements which exactly factor these quartic polynomials.

First partials for the normal equations are derived, and hence by inserting coordinates obtained from initial conditions for each specified time of observation into these equations, the results of the iterated least-square procedure will produce the Izsak elements and consequently coordinate predictions.

STATEMENT OF THE PROBLEM

If L_0 and M_0 denote the values of the observed direction cosines of a satellite for a given time of observation, then the corresponding computed values of these functions are given by

$$L_c = \frac{x_m}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{1}{2}}}$$

and

$$M_c = \frac{y_m}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{1}{2}}} ,$$

where x_m, y_m, and z_m are the corresponding local coordinates. These are given by the relation

$$\begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix} = (A_I)^{-1} \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} \right\} ,$$

where x, y, and z define the satellite's coordinates with respect to an earth centered, right-handed, orthogonal inertial system; x_T, y_T, and z_T are the inertial coordinates of the observation point; and the matrix (A_I) rotates the local $x_m y_m z_m$ frame parallel to the inertial xyz. The computed functions can now be written

$$L_c = \frac{\left[(A_I)^{-1}(\sigma - T)\right]_1}{\left\{\left[(A_I)^{-1}(\sigma - T)\right]_1^2 + \left[(A_I)^{-1}(\sigma - T)\right]_2^2 + \left[(A_I)^{-1}(\sigma - T)\right]_3^2\right\}^{\frac{1}{2}}} ,$$

and

$$M_c = \frac{\left[(A_I)^{-1}(\sigma - T)\right]_2}{\left\{\left[(A_I)^{-1}(\sigma - T)\right]_1^2 + \left[(A_I)^{-1}(\sigma - T)\right]_2^2 + \left[(A_I)^{-1}(\sigma - T)\right]_3^2\right\}^{\frac{1}{2}}} ,$$

where

$$\sigma = \begin{pmatrix} x \\ y \\ z \end{pmatrix} ,$$

and

$$T = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} .$$

The subscripts 1, 2, and 3 denote the rows of the product matrix

$$(A_I)^{-1} (\sigma - T) .$$

Thus, the functions are obtained for a specified time of observation by inserting into the matrix σ those values of the earth satellite coordinates computed by the Vinti orbit generator (Reference 3).

It is necessary to develop the first partials for the Taylor expansion in order to complete the first order approximation for the conditional equations, and to obtain the Izsak elements which can factor Vinti's two quartic polynomials $F(\rho)$, and $G(\eta)$ (Reference 2) exactly, and hence to be able to obtain accurate predictions for position and velocity.

THE PARTIAL DERIVATIVES

If in the matrix σ we make the following substitutions (page 22, Reference 3)

$$x = \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \cos\phi ,$$

$$y = \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \sin\phi ,$$

and

$$z = \rho\eta ,$$

where

$$\rho = a(1 - e \cos E) ,$$

and

$$\eta = \eta_0 \sin\psi ,$$

then we can express the computed direction cosines L_c and M_c as functions of the six elements a, e, η_0, E, ψ, and ϕ. Here a and e are taken to be the semimajor axis and eccentricity, respectively, while η_0 is related to the orbit inclination by the expression $\eta_0 = \sin I$. The elements E and ψ are uniformizing variables defined by Vinti (Reference 2) as the eccentric anomaly and a variable analogous to the argument of latitude, respectively. The element ϕ is the geocentric right ascension.

In the expression

$$L_c = \frac{x_m}{\sqrt{x_m^2 + y_m^2 + z_m^2}} ,$$

x_m, y_m, and z_m are functions of the six elements a, e, η_0, E, ψ, and ϕ. Since the local coordinates are themselves functions of the computed inertial coordinates, and if we denote any one of the elements by q_i, then

$$\frac{\partial L_c}{\partial q_i} = \frac{\partial L_c}{\partial x_m}\frac{\partial x_m}{\partial q_i} + \frac{\partial L_c}{\partial y_m}\frac{\partial y_m}{\partial q_i} + \frac{\partial L_c}{\partial z_m}\frac{\partial z_m}{\partial q_i} , \tag{1}$$

with

$$\frac{\partial L_c}{\partial x_m} = \frac{1}{\sqrt{x_m^2 + y_m^2 + z_m^2}} - \frac{x_m^2}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{3}{2}}} = K_{00} , \tag{2}$$

$$\frac{\partial L_c}{\partial y_m} = \frac{-x_m y_m}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{3}{2}}} = K_{01} , \tag{3}$$

and

$$\frac{\partial L_c}{\partial z_m} = \frac{-x_m z_m}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{3}{2}}} = K_{02} . \tag{4}$$

Since,

$$\begin{Bmatrix} x_m \\ y_m \\ z_m \end{Bmatrix} = (A_I)^{-1} \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} \right\}$$

$$= (A_I)^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - (A_I)^{-1} \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} ,$$

we have

$$\begin{Bmatrix} \frac{\partial x_m}{\partial q_i} \\ \frac{\partial y_m}{\partial q_i} \\ \frac{\partial z_m}{\partial q_i} \end{Bmatrix} = (A_I)^{-1} \begin{Bmatrix} \frac{\partial x}{\partial q_i} \\ \frac{\partial y}{\partial q_i} \\ \frac{\partial z}{\partial q_i} \end{Bmatrix} . \tag{5}$$

Here, x_T, y_T, and z_T are independent of the q_i. Substituting

$$\rho = a(1 - e \cos E)$$

and

$$\eta = \eta_0 \sin\psi$$

into

$$x = \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \cos\phi ,$$

$$y = \sqrt{(\rho^2 + c^2)(1 - \eta^2)} \sin\phi ,$$

and

$$z = \rho\eta ,$$

we have the following relations

$$x = \sqrt{\left[a^2(1 - e\cos E)^2 + c^2\right]\left(1 - \eta_0^2 \sin^2\psi\right)} \cos\phi , \tag{6}$$

$$y = \sqrt{\left[a^2(1 - e\cos E)^2 + c^2\right]\left(1 - \eta_0^2 \sin^2\psi\right)} \sin\phi , \tag{7}$$

and

$$z = a(1 - e\cos E)\, \eta_0 \sin\psi . \tag{8}$$

Differentiating Equations 6, 7, and 8 with respect to each of the q_i in turn (that is, a, e,

η_0, E, ψ, and ϕ) and substituting into Equation 5 yields

$$\frac{\partial x_m}{\partial q_i}, \quad \frac{\partial y_m}{\partial q_i}, \text{ and } \frac{\partial z_m}{\partial q_i} .$$

If Equations 2, 3, 4, and 5 are now substituted into Equation 1, we obtain the six first partial derivatives of the 'L' equation of condition:

$$\frac{\partial L_c}{\partial a} = K_{00}K_{20} + K_{01}K_{21} + K_{02}K_{22} ,$$

$$\frac{\partial L_c}{\partial e} = K_{00}K_{23} + K_{01}K_{24} + K_{02}K_{25} ,$$

$$\frac{\partial L_c}{\partial \eta_0} = K_{00}K_{26} + K_{01}K_{27} + K_{02}K_{28} ,$$

$$\frac{\partial L_c}{\partial E} = K_{00}K_{29} + K_{01}K_{30} + K_{02}K_{31} ,$$

$$\frac{\partial L_c}{\partial \psi} = K_{00}K_{32} + K_{01}K_{33} + K_{02}K_{34} ,$$

and

$$\frac{\partial L_c}{\partial \phi} = K_{00}K_{35} + K_{01}K_{36} + K_{02}K_{37} ,$$

in which

$$K_{20} = K_1 \cos\psi_A \cos\phi + K_1 \sin\psi_A \sin\phi ,$$

$$K_{21} = -K_1 \sin\psi_A \sin\theta_D \cos\phi + K_1 \cos\psi_A \sin\theta_D \sin\phi + K_8\eta_0 \sin\psi_A \cos\theta_D ,$$

$$K_{22} = K_1 \sin\psi_A \cos\theta_D \cos\phi - K_1 \cos\psi_A \cos\theta_D \sin\phi + K_8\eta_0 \sin\psi_A \sin\theta_D ,$$

$$K_{23} = K_2 \cos\psi_A \cos\phi + K_2 \sin\psi_A \sin\phi ,$$

$$K_{24} = -K_2 \sin\psi_A \sin\theta_D \cos\phi + K_2 \cos\psi_A \sin\theta_D \sin\phi + K_9\eta_0 \sin\psi_A \cos\theta_D ,$$

$$K_{25} = K_2 \sin\psi_A \cos\theta_D \cos\phi - K_2 \cos\psi_A \cos\theta_D \sin\phi + K_9\eta_0 \sin\psi_A \sin\theta_D ,$$

$$K_{26} = K_3 \cos\psi_A \cos\phi + K_3 \sin\psi_A \sin\phi ,$$

$$K_{27} = -K_3 \sin\psi_A \sin\theta_D \cos\phi + K_3 \cos\psi_A \sin\theta_D \sin\phi + K_{10} \sin\psi_A \cos\theta_D ,$$

$$K_{28} = K_3 \sin\psi_A \cos\theta_D \cos\phi - K_3 \cos\psi_A \cos\theta_D \sin\phi + K_{10} \sin\psi_A \sin\theta_D ,$$

$$K_{29} = K_4 \cos\psi_A \cos\phi + K_4 \sin\psi_A \sin\phi ,$$

$$K_{30} = -K_4 \sin\psi_A \sin\theta_D \cos\phi + K_4 \cos\psi_A \sin\theta_D \sin\phi + K_{11}\eta_0 \sin\psi_A \cos\theta_D ,$$

$$K_{31} = K_4 \sin\psi_A \cos\theta_D \cos\phi - K_4 \cos\psi_A \cos\theta_D \sin\phi + K_{11}\eta_0 \sin\psi_A \sin\theta_D ,$$

$$K_{32} = K_5 \cos\psi_A \cos\phi + K_5 \sin\psi_A \sin\phi ,$$

$$K_{33} = -K_5 \sin\psi_A \sin\theta_D \cos\phi + K_5 \cos\psi_A \sin\theta_D \sin\phi + K_{10}\eta_0 \cos\psi_A \cos\theta_D ,$$

$$K_{34} = K_5 \sin\psi_A \cos\theta_D \cos\phi - K_5 \cos\psi_A \cos\theta_D \sin\phi + K_{10}\eta_0 \cos\psi_A \sin\theta_D ,$$

$$K_{35} = K_6 \cos\psi_A \sin\phi + K_6 \sin\psi_A \cos\phi ,$$

$$K_{36} = K_6 \sin\psi_A \sin\theta_D \sin\phi + K_6 \cos\psi_A \sin\theta_D \cos\phi ,$$

and

$$K_{37} = K_6 \sin\psi_A \cos\theta_D \sin\phi - K_6 \cos\psi_A \cos\theta_D \cos\phi ,$$

where angles with the subscripts D and A refer to the A_I matrix, and the factors K_1, K_2, $\cdots$, K_{11} are

$$K_1 = \frac{a(1 - e\cos E)^2 \sqrt{1 - \eta_0^2 \sin^2\psi}}{\sqrt{a^2(1 - e\cos E)^2 + c^2}} ,$$

$$K_2 = \frac{-a^2 \cos E(1 - e\cos E) \sqrt{1 - \eta_0^2 \sin^2\psi}}{\sqrt{a^2(1 - e\cos E)^2 + c^2}} ,$$

$$K_3 = \frac{-\eta_0 \sin^2\psi \sqrt{a^2(1 - e\cos E)^2 + c^2}}{\sqrt{1 - \eta_0^2 \sin^2\psi}} ,$$

$$K_4 = \frac{a^2(1 - e\cos E)\ e\sin E\ \sqrt{1 - \eta_0^2 \sin^2\psi}}{\sqrt{a^2(1 - e\cos E)^2 + c^2}} ,$$

$$K_5 = \frac{-\eta_0^2 \sin\psi\cos\psi\ \sqrt{a^2(1 - e\cos E)^2 + c^2}}{\sqrt{1 - \eta_0^2 \sin^2\psi}} ,$$

$$K_6 = - \sqrt{a^2(1 - e\cos E)^2 + c^2}\ \sqrt{1 - \eta_0^2 \sin^2\psi} ,$$

$$K_8 = (1 - e\cos E) ,$$

$$K_9 = -a\cos E ,$$

$$K_{10} = a(1 - e\cos E) ,$$

and

$$K_{11} = a\, e\sin E .$$

Similarly,

$$M_c = \frac{y_m}{\sqrt{x_m^2 + y_m^2 + z_m^2}} ,$$

and

$$\frac{\partial M_c}{\partial q_i} = \frac{\partial M_c}{\partial x_m}\frac{\partial x_m}{\partial q_i} + \frac{\partial M_c}{\partial y_m}\frac{\partial y_m}{\partial q_i} + \frac{\partial M_c}{\partial z_m}\frac{\partial z_m}{\partial q_i} . \tag{9}$$

Then

$$\frac{\partial M_c}{\partial x_m} = \frac{-x_m y_m}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{3}{2}}} = K_{01} , \tag{10}$$

$$\frac{\partial M_c}{\partial y_m} = \frac{1}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{1}{2}}} - \frac{y_m^2}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{3}{2}}} = K_{00} , \tag{11}$$

and

$$\frac{\partial M_c}{\partial z_m} = \frac{-y_m z_m}{\left(x_m^2 + y_m^2 + z_m^2\right)^{\frac{3}{2}}} = K_{03} \ . \tag{12}$$

Proceeding as above, and substituting Equations 10, 11, and 12 into Equation 9, we obtain the six first partial derivatives of the 'M' equation of condition:

$$\frac{\partial M_c}{\partial a} = K_{01}K_{20} + K_{00}K_{21} + K_{03}K_{22} \ ,$$

$$\frac{\partial M_c}{\partial e} = K_{01}K_{23} + K_{00}K_{24} + K_{03}K_{25} \ ,$$

$$\frac{\partial M_c}{\partial \eta_0} = K_{01}K_{26} + K_{00}K_{27} + K_{03}K_{28} \ ,$$

$$\frac{\partial M_c}{\partial E} = K_{01}K_{29} + K_{00}K_{30} + K_{03}K_{31} \ ,$$

$$\frac{\partial M_c}{\partial \psi} = K_{01}K_{32} + K_{00}K_{33} + K_{03}K_{34} \ ,$$

and

$$\frac{\partial M_c}{\partial \phi} = K_{01}K_{35} + K_{00}K_{36} + K_{03}K_{37} \ .$$

PRIME CONSTANTS

Once the corrected set of these six elements has been obtained, the epoch values of Izsak elements are computed by the following (Reference 2):

$$p = a\left(1 - e^2\right) \ ,$$

$$D = \left(ap - c^2\right)\left(ap - c^2\eta_0^2\right) + 4a^2c^2\eta_0^2 \ ,$$

$$D' = D + 4a^2c^2\left(1 - \eta_0^2\right),$$

$$A = -2ac^2D^{-1}\left(1 - \eta_0^2\right)\left(ap - c^2\eta_0^2\right),$$

$$B = c^2\eta_0^2 D^{-1}D',$$

$$b_1 = -\frac{1}{2}A,$$

$$b_2 = B^{\frac{1}{2}},$$

$$-2\alpha_1 = \mu\left(a + b_1\right)^{-1} = Q_1,$$

$$\frac{-\alpha_2^2}{2\alpha_1} = -c^2\left(1 - \eta_0^2\right) + apD^{-1}D' = Q_2,$$

$$\alpha_2 = Q_1^{-\frac{1}{2}} Q_2^{\frac{1}{2}},$$

$$\cos I = \left(1 - \eta_0^2\right)^{\frac{1}{2}},$$

$$\alpha_3 = \alpha_2\left(1 - \frac{c^2\eta_0^2}{Q_1}\right)^{\frac{1}{2}} \cos I,$$

$$\eta_2^{-2} = \frac{c^2D}{a\,p\,D'},$$

$$\eta_2^2,$$

$$q^2 = \left(\frac{\eta_0}{\eta_2}\right)^2,$$

$$q^4,$$

$$K = \frac{c^2}{p^2}.$$

Note: If

$$90^\circ < I \leq 180^\circ - I_c \quad ,$$

where

$$I_c = 1^\circ 54' \ ,$$

use

$$\cos I = -\left(1 - \eta_0^2\right)^{+\frac{1}{2}} \quad .$$

MUTUAL CONSTANTS

Compute:

$$\left(1 - e^2\right)^{\frac{1}{2}} \ ,$$

$$\left(\frac{b_1}{b_2}\right) \ ,$$

$$A_1 = \left(1 - e^2\right)^{\frac{1}{2}} p \sum_{n=2}^{\infty} \left(\frac{b_2}{p}\right)^n P_n\left(\frac{b_1}{b_2}\right) P_{n-2}\left[\left(1 - e^2\right)^{\frac{1}{2}}\right] \ ,$$

$$A_2 = \left(1 - e^2\right)^{\frac{1}{2}} p^{-1} \sum_{n=0}^{\infty} \left(\frac{b_2}{p}\right)^n P_n\left(\frac{b_1}{b_2}\right) R_n\left[\left(1 - e^2\right)^{\frac{1}{2}}\right] \ ,$$

where $P_n(b_1/b_2)$ is the Legendre polynomial of degree n, $R_n(x_s) = x_s^n P_n(x_s^{-1})$ is a polynomial of degree $[n/2]$ in x_s^2, and $x_s = (1 - e^2)^{\frac{1}{2}}$.

If m is an even integer, compute

$$D_m = D_{2i} = \sum_{n=0}^{i} (-1)^{i-n} \left(\frac{c}{p}\right)^{2i-2n} \left(\frac{b_2}{p}\right)^{2n} P_{2n}\left(\frac{b_1}{b_2}\right) .$$

If m is an odd integer, compute

$$D_m = D_{2i+1} = \sum_{n=0}^{i} (-1)^{i-n} \left(\frac{c}{p}\right)^{2i-2n} \left(\frac{b_2}{p}\right)^{2n+1} P_{2n+1}\left(\frac{b_1}{b_2}\right) \ .$$

$$A_3 = \left(1 - e^2\right)^{\frac{1}{2}} p^{-3} \sum_{m=0}^{\infty} D_m R_{m+2} \left[\left(1 - e^2\right)^{\frac{1}{2}}\right] ,$$

$$B_1 = \frac{1}{2} + \frac{3}{16} q^2 + \frac{15}{128} q^4 ,$$

$$B_2 = 1 + \frac{q^2}{4} + \frac{9}{64} q^4 ,$$

$$\gamma_m = \frac{(2m)!}{2^{2m}(m!)^2} \sum_{n=1}^{m-1} \frac{(2n)!\, \eta_0^{2n}}{2^{2n}(n!)^2} ,$$

$$B_3 = 1 - \left(1 - \eta_2^{-2}\right)^{-\frac{1}{2}} - \sum_{m=2}^{\infty} \gamma_m \eta_2^{-2m} ,$$

$$A_{11} = \frac{3}{4} \left(1 - e^2\right)^{\frac{1}{2}} p^{-3}\, e\left(-2b_1 b_2^2 p + b_2^4\right) ,$$

$$A_{12} = \frac{3}{32} \left(1 - e^2\right)^{\frac{1}{2}} b_2^4 e^2 p^{-3} ,$$

$$A_{21} = \left(1 - e^2\right)^{\frac{1}{2}} p^{-1}\, e\left[b_1 p^{-1} + \left(3b_1^2 - b_2^2\right) p^{-2} - \frac{9}{2} b_1 b_2^2 \left(1 + \frac{e^2}{4}\right) p^{-3} + \frac{3}{8} b_2^4 \left(4 + 3e^2\right) p^{-4}\right] ,$$

$$A_{22} = \left(1 - e^2\right)^{\frac{1}{2}} p^{-1} \left[\frac{e^2}{8} \left(3b_1^2 - b_2^2\right) p^{-2} - \frac{9}{8} e^2 b_1 b_2^2 p^{-3} + \frac{3}{32} b_2^4 \left(6e^2 + e^4\right) p^{-4}\right] ,$$

$$A_{23} = \left(1 - e^2\right)^{\frac{1}{2}} p^{-1} \frac{e^3}{8} \left(-b_1 b_2^2 p^{-3} + b_2^4 p^{-4}\right) ,$$

$$A_{24} = \frac{3}{256} \left(1 - e^2\right)^{\frac{1}{2}} p^{-5} b_2^4 e^4 ,$$

$$A_{31} = \left(1 - e^2\right)^{\frac{1}{2}} p^{-3}\, e \left[2 + b_1 p^{-1} \left(3 + \frac{3}{4} e^2\right) - p^{-2} \left(\frac{1}{2} b_2^2 + c^2\right)\left(4 + 3e^2\right)\right] ,$$

$$A_{32} = \left(1 - e^2\right)^{\frac{1}{2}} p^{-3} \left[\frac{e^2}{4} + \frac{3}{4} b_1 p^{-1} e^2 - p^{-2}\left(\frac{b_2^2}{2} + c^2\right)\left(\frac{3}{2} e^2 + \frac{e^4}{4}\right)\right] ,$$

$$A_{33} = \left(1 - e^2\right)^{\frac{1}{2}} e^3 \left[\frac{b_1}{12p} - \frac{p^{-2}}{3}\left(\frac{b_2^2}{2} + c^2\right)\right] p^{-3} ,$$

$$A_{34} = -\frac{1}{32}\left(1 - e^2\right)^{\frac{1}{2}} p^{-5} e^4 \left(\frac{1}{2} b_2^2 + c^2\right) ,$$

$$2\pi\gamma_1 = \left(-2\alpha_1\right)^{\frac{1}{2}} \left(a + b_1 + A_1 + c^2\eta_0^2 A_2 B_1 B_2^{-1}\right)^{-1} ,$$

$$2\pi\gamma_2 = \left(\alpha_2^2 - \alpha_3^2\right)^{\frac{1}{2}} \eta_0^{-1} A_2 B_2^{-1} \left(a + b_1 + A_1 + c^2\eta_0^2 A_2 B_1 B_2^{-1}\right)^{-1} ,$$

$$e' = ae\left(a + b_1\right)^{-1} ,$$

where the parameter e' is always less than e.

IZSAK ELEMENTS

From the anomaly connections

$$\cos v = \frac{\cos E - e}{1 - e\cos E}$$

and

$$\sin v = \frac{\left(1 - e^2\right)^{\frac{1}{2}} \sin E}{1 - e\cos E} ,$$

the value of v is determined within the limits $0 \leq v < 2\pi$.

The set of mean Izsak elements a, e, η_0, β_1, β_2, and β_3 are obtained by substituting the above results into the equations

$$\beta_1 = (-2\alpha_1)^{\frac{1}{2}}\left[b_1 E + a(E - e\sin E) + A_1 v + A_{11}\sin v + A_{12}\sin 2v\right]$$

$$+ c^2\left(\alpha_2^2 - \alpha_3^2\right)^{-\frac{1}{2}} \eta_0^3\left[B_1\psi - \frac{1}{8}\left(2 + q^2\right)\sin 2\psi + \frac{q^2}{64}\sin 4\psi\right] - t_{epoch} ,$$

$$\beta_2 = -\alpha_2\left(-2\alpha_1\right)^{-\frac{1}{2}}\left[A_2 v + A_{21}\sin v + A_{22}\sin 2v + A_{23}\sin 3v + A_{24}\sin 4v\right]$$

$$+ \left(\alpha_2^2 - \alpha_3^2\right)^{-\frac{1}{2}}\eta_0\alpha_2\left[B_2\psi - \frac{q^2}{32}\left(4 + 3q^2\right)\sin 2\psi + \frac{3q^4}{256}\sin 4\psi\right],$$

and

$$\beta_3 = \phi - \alpha_3\left(\alpha_2^2 - \alpha_3^2\right)^{-\frac{1}{2}}\eta_0\left[\left(1 - \eta_0^2\right)^{-\frac{1}{2}}\left(1 - \eta_2^{-2}\right)^{-\frac{1}{2}}\chi + B_3\psi + \frac{3}{32}\eta_0^2\,\eta_2^{-4}\sin 2\psi\right]$$

$$+ c^2\alpha_3\left(-2\alpha_1\right)^{-\frac{1}{2}}\left[A_3 v + \sum_{n=1}^{4} A_{3n}\sin n v\right],$$

where χ is determined, within the limits $0 \le \chi < 2\pi$, by the equations

$$\sin\chi = \frac{\sqrt{1 - \eta_0^2}\,\sin\psi}{\sqrt{1 - \eta_0^2\sin^2\psi}}$$

and

$$\cos\chi = \frac{\cos\psi}{\sqrt{1 - \eta_0^2\sin^2\psi}} .$$

Predicted positions and velocities of the satellite can now be obtained by using these results in the Vinti-orbit generator as given, beginning on page 18, in Reference 3.

REMARKS

A refined version of this satellite orbit computation program is to contain corrections for atmospheric drag. Furthermore, it is intended to incorporate the effects of the residual oblateness potential unaccounted for in the existing version of this program.

Preliminary tests for the Explorer XI (1961 ν), on the IBM 7090 electronic digital computer, indicate that the orbit generator (Reference 3), feeding into, and predicting from the orbit improvement program, can compute approximately 1600 minute points (time; x, y, and z; $\dot{x}$, $\dot{y}$, and $\dot{z}$ each minute for 1600 minutes) in one minute of computer operation, while simultaneously producing BCD tape.

Extension of the results of Vinti's theory (References 1 and 2), to the cases of parabolic and hyperbolic orbits is also being investigated.

ACKNOWLEDGMENTS

The author is especially grateful to Dr. John P. Vinti of the National Bureau of Standards for his valuable advice in preparing this report, and to Messrs. Imre G. Izsak of the Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts, and R. V. Borchers and W. D. Kahn, Goddard Space Flight Center, for several technical discussions on the subject matter of this report.

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