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TECHNICAL NOTE

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THERMAL RADIATION INCIDENT ON

AN EARTH SATELLITE

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SUMMARY

Equations have been derived, using vector analysis, which predict the incident energy at any given point on an earth satellite. The derivation is based on a non-spinning satellite and extended to include spinning vehicles. The satellite was assumed to be oriented in any one of three ways - toward the sun, toward the earth, and tangent to the flight path at perigee. The amount of energy received depends upon the intensity of the radiation and the view factor between the object and source. The view factor equations are applicable to any mathematically describable shape. To determine the length of exposure to the sun, simple relations have been developed which predict the ingress and egress points of the earth's shadow. The solution to the equations was programmed in a general manner on an IBM 7090. The input data required consisted of parameters to specify the orbit and an equation to describe the surface of the satellite. As an illustration a cylindrical satellite with hemispherical ends was chosen. The view factor equations were derived and numerically integrated, and the incident energy was calculated for an arbitrarily selected orbit. The results indicated that spinning reduces the peak heat fluxes in all cases, and the minimum incident energy occurred when the vehicle was oriented toward the sun.

The methods outlined in this report are applicable to satellites of other celestial bodies also. The only additional information required is a knowledge of the position of the sun and the orbital plane relative to the body.

INTRODUCTION

One of the major problems encountered in the operation of manned and unmanned space vehicles is heating by thermal radiation. The net heat received by the satellite may be controlled by many methods, both active and passive. For example, the vehicle may be oriented to obtain the minimum incident energy, or it may be shielded and insulated to reduce the heating, or a combination of these methods may be used. This report will consider the thermal advantages to be gained by minimizing the incident energy through vehicle orientation. For a spinning satellite, the problem is simplified somewhat since the incident energy is uniformly distributed about the spin axis. This fact will be used to determine the irradiation of a spinning satellite from the analyses of the non-spinning vehicles.

An object in orbit about the earth receives significant amounts of radiant energy from three sources - direct solar radiation, earth radiation, and albedo or earth-reflected solar energy. As long as the satellite is near the earth it will always receive appreciable amounts of terrestrial radiation. However, it may or may not be receiving heat from the remaining two sources depending on the relative position of the earth, the sun, and the satellite. The amount of heat received from each source depends on the intensity of the radiation and the view factor between the object and the source. Equations for the view factors will be derived for any mathematically describable shape traveling in a circular or elliptical orbit. These equations will include three possible vehicle orientations - earth-oriented, sun-oriented, and tangent to flight path at perigee. To determine the length of exposure to the solar heat source, equations will be derived which give the points of ingress and egress of the earth's shadow. The use of these methods will be illustrated by sample calculations for a cylindrical satellite with hemispherical ends and a verification obtained for certain simple cases.

DEFINITION OF SYMBOLS

a	=	semi-major axis of orbit ellipse
a_s	=	semi-major axis of earth's shadow ellipse
A_1	=	total surface area of radiant heat source
A_2	=	total surface area of body receiving radiant energy
A_{2i}	=	finite area increments of satellite surface
$\overline{A}$	=	average albedo of a planet
b_s	=	semi-minor axis of earth's shadow ellipse
D	=	number of days after vernal equinox
d	=	diameter of a cylindrical satellite
dA_1	=	differential surface area on the radiant heat source
dA_2	=	differential surface area on the body receiving radiant energy
dA_{2i}	=	differential area increment of irradiated surface
$\overrightarrow{dq_{1\rightarrow 2}}$	=	differential radiant heat flux from dA_1 to dA_2
e	=	eccentricity of orbit ellipse
$F_E = F_{12}$	=	radiation view factor between the earth and the satellite area, A_{2i}
F_H	=	radiation view factor for a flat plate parallel to a planet's local horizon
F_R	=	radiation view factor between the satellite area A_{2i} and earth-reflected solar energy

DEFINITION OF SYMBOLS (Continued)

F_s = radiation view factor between the satellite area A_{2_i} and the sun

F_v = radiation view factor for a flat plate vertical to the local planet horizon

g = general mathematical function defined as $g(X_4, X_5, X_6) = 0$

h = altitude of a satellite above a planet's surface

i = inclination of orbital plane to earth's equatorial plane

I_R = intensity of earth-reflected solar energy

I_p = intensity of planetary radiation

I_t = total energy radiated per unit time and area by the earth

$\vec{I}_{12}$ = intensity of radiation between the earth and the satellite

$\vec{i}, \vec{j}, \vec{k}$ = unit vectors along the coordinate axes of a rectangular cartesian coordinate system

K_2 = 2.206×10^4 $(KM)^2$ (constant)

K_4 = 1.580570×10^9 $(KM)^4$ (constant)

L = length of cylindrical part of satellite

L_θ = right ascension of sun in plane of ecliptic

LST = local sidereal time

N = North

$\vec{N}_1$ = unit vector normal to surface of heat source

$\vec{N}_2$ = unit vector normal to satellite surface

DEFINITION OF SYMBOLS (Continued)

P	=	period of orbit
$\overline{p}$	=	semi-latus rectum of orbit ellipse
$\vec{Q}_{1\rightarrow 2}$	=	incident radiant energy from body 1 to body 2
$\vec{Q}_{1\rightarrow 2i}$	=	incident radiant energy from earth to satellite area A_{2_i}
Q_p	=	incident energy from a planet
$\vec{Q}_R$	=	earth-reflected solar energy incident on area A_2
$\vec{Q}_S$	=	solar energy incident on area A_2
R	=	radius from center of earth to the satellite
R_E	=	radius of the earth = 6378.150 ± 0.070 (km)
R_p	=	radius of perigee
R_s	=	radius from center of earth to any point on the earth's shadow ellipse
$\vec{r}_{12}$	=	radius vector from earth surface element to satellite surface element
S	=	solar constant 443 Btu/ft^2 hr $\pm$ 2%
$\vec{S}$	=	unit vector from center of earth directed toward sun location
S_1, S_2, S_3	=	scalar components of solar vector along the X_1, X_2, X_3 coordinate axis, respectively
T_1	=	black body equilibrium temperature of the earth
W	=	West
X	=	a coordinate of a rectangular cartesian system
x, y, z	=	rectangular cartesian coordinate system

DEFINITION OF SYMBOLS (Continued)

$\vec{\nabla}$ = vector operator "del"

α = right ascension of sun in plane of equator

β_L = spherical angle in launch triangle (FIG 6)

β_P = spherical angle in perigee triangle (FIG 6)

Γ = angle in the plane of orbit, measured clockwise from perigee to the projection of the sun into the orbital plane (FIG 9)

γ = declination of the sun

γ_1 = angle between unit surface normal to radiating surface and vector, $\vec{r}_{12}$ (FIG 2)

γ_2 = angle between the unit normal to the surface of the object receiving radiation and the vector, $\vec{r}_{12}$ (FIG 2)

γ_{1s} = angles between unit normals to the earth and sun

γ_{2s} = angle between unit normals to the satellite and the sun (FIG 4)

δ_L = latitude of launch site

δ = latitude of initial perigee point

ϵ = obliquity of the ecliptic - 23° 27' 8.2"

ζ = angular coordinate on cylindrical satellite (FIG 15)

η_1 = geocentric angular coordinate for position description on the earth's surface (FIG 2)

η_2 = geocentric angular coordinate for satellite location (FIG 3)

DEFINITION OF SYMBOLS (Continued)

θ = angular measurement in the plane of orbit (FIG 12)

θ_1 = geocentric angular coordinate for position description on the earth's surface (FIG 2)

θ_2 = geocentric angular coordinate for satellite location (FIG 3)

λ_L = angle subtending a side of launch spherical triangle (FIG 6)

λ = angle subtending a side of launch perigee spherical triangle (FIG 6)

μ = spherical coordinate (FIG 17)

ν = true anomaly (FIG 8)

ξ = angle between the normal to the plane of orbit and the sun vector (FIG 9)

π = constant 3.1416

ρ = spherical coordinate (FIG 17)

ρ_P = radius of planet

σ = Stefan-Boltzman constant -.174 $\times 10^{-8}$ Btu/hr ft^2 $(R°)^4$

ϕ = spherical coordinate (FIG 16)

ψ = spherical coordinate (FIG 16)

Ω = right ascension of ascending node

$\dot{\Omega}$ = regression rate of ascending node

ω_L = side of launch spherical triangle (FIG 6)

DEFINITION OF SYMBOLS (Concluded)

ω	=	argument of perigee
$\dot{\omega}$	=	advance of perigee

Coordinate Subscripts

1, 2, 3	=	earth centered triad (FIG 2)
4, 5, 6	=	satellite coordinate system (FIG 5)
7, 8, 9	=	system given by first rotation in equatorial plane
10, 11, 12	=	orbital plane triad
13, 14, 15	=	earth-oriented triad
16, 17, 18	=	system given by first rotation to align with the sun (FIG 10)
19, 20, 21	=	sun-oriented system
22, 23, 24	=	positive hemisphere system
E. O.	=	earth-oriented system
p	=	perigee system
S. O.	=	sun-oriented system

Acknowledgement

Dr. H. G. L. Krause reviewed the orbital part of this report and Mr. Tom Yarbrough programmed the equations. The following people checked the equations, drew the figures and proof-read the many revisions: Messrs. Carroll Boatwright, Warren Jensen, David E. Price III, and Christopher Wright.

RADIATION HEAT TRANSFER

In this section, equations will be developed which predict the thermal energy incident on the satellite. The analysis will be made for a non-spinning vehicle and then extended to include the spinning satellite. The irradiation will be assumed to be independent of wavelength and angle of incidence. It is convenient to use the celestial sphere concept which assumes that the earth is stationary and the heavenly bodies revolve about it. Two rectangular cartesian coordinate systems will be employed: one moving with the satellite, but fixed relative to it, and a geocentric equatorial system. By calculating the maximum heating that could be received by an earth satellite, it will be shown that only the earth and sun contribute significant amounts of thermal radiation.

Consider two diffusely radiating surface elements dA_1 and dA_2, as shown in FIG 1, which are separated by a non-absorbing medium. The radiation emitted from dA_1 and striking dA_2 is

$$\vec{dq}_{1\rightarrow 2} = \vec{I}_{12} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \, dA_2 \tag{1}$$

where $\vec{I}_{12}$ is the radiation intensity. The angles γ_1 and γ_2 are the angles between the respective normals and a line connecting the two elements. The total radiation received per unit time by area A_2 from A_1 is

$$\vec{Q}_{1\rightarrow 2} = \int_{A_1} \int_{A_2} \vec{I}_{12} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \, dA_2 \tag{2}$$

Or assuming that A_1 is a black body at temperature T_1, then

$$\vec{Q}_{1\rightarrow 2} = \frac{\sigma T_1^4}{\pi} \int_{A_1} \int_{A_2} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \, dA_2 \tag{3}$$

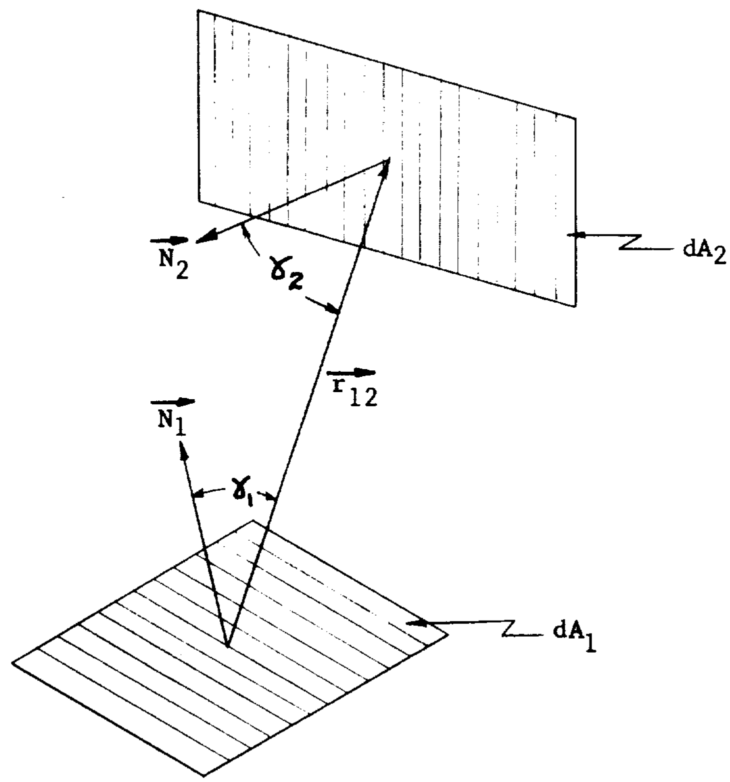

Figure 1. Radiation Heat Transfer Between Two Surfaces

and the limits of integration are $\cos \gamma_1 \geq 0, \quad \cos \gamma_2 \geq 0$ (4)

A. EARTH PLANETARY RADIATION

If area dA_1 represents an elemental area on the surface of the earth and dA_2 is the satellite area as shown in FIG 2, then

$$\cos \gamma_1 = \frac{\vec{r}_{12} \cdot \vec{N}_1}{|\vec{r}_{12}|} \tag{5}$$

and

$$\cos \gamma_2 = \frac{-\vec{r}_{12} \cdot \vec{N}_2}{|\vec{r}_{12}|} \tag{6}$$

where the unit normal to the earth's surface is

$$\vec{N}_1 = \sin \eta_1 \cos \theta_1 \, (\vec{i}_1) + \sin \eta_1 \sin \theta_1 \, (\vec{i}_2) + \cos \eta_1 \, (\vec{i}_3) \tag{7}$$

and the elemental area is given by

$$dA_1 = R_E^{\,2} \sin \eta_1 \, d\eta_1 \, d\theta_1 \tag{8}$$

The position of the satellite relative to the earth is given by the vector $\vec{R}$, as shown in FIG 3, and is defined in terms of the geocentric angles η_2 and θ_2. If the magnitude of $\vec{R}$ is denoted as R then the vector $\vec{r}_{12}$ from the earth's surface to the satellite is

$$\begin{aligned} \vec{r}_{12} = \; & (R \sin \eta_2 \cos \theta_2 - R_E \sin \eta_1 \cos \theta_1)\,(\vec{i}_1) \; + \\ & (R \sin \eta_2 \sin \theta_2 - R_E \sin \eta_1 \sin \theta_1)\,(\vec{i}_2) \; + \\ & (R \cos \eta_2 - R_E \cos \eta_1)\,(\vec{i}_3) \end{aligned} \tag{9}$$

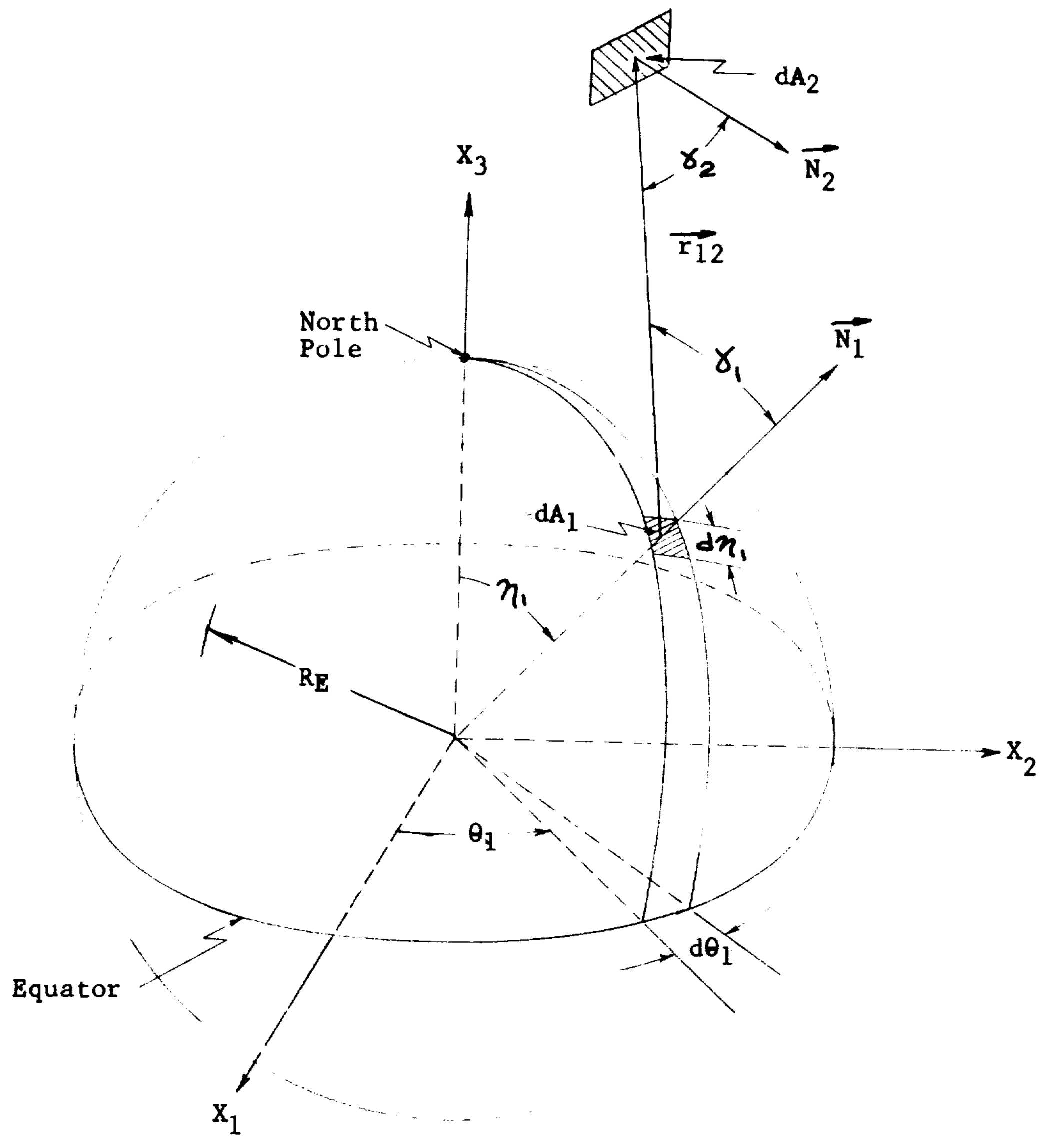

Figure 2. Radiation Heat Transfer Between the Earth and Satellite

$$\cos \theta_2 = \frac{X_1}{R \sin \eta_2}$$

$$\sin \theta_2 = \frac{X_2}{R \sin \eta_2}$$

$$\cos \eta_2 = \frac{X_3}{R}$$

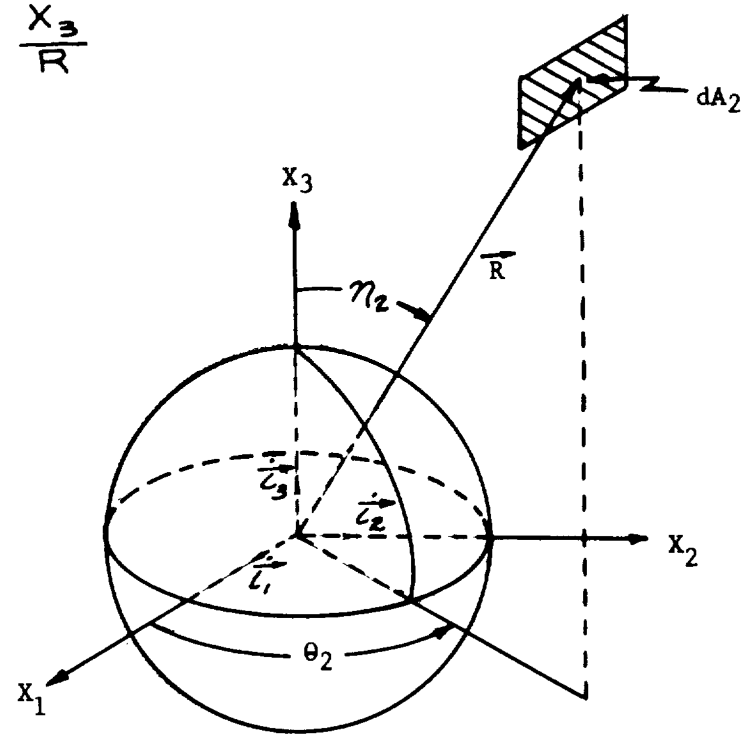

Figure 3. **Satellite Position Co-ordinates**

The area dA_2 on the satellite will depend on the particular shape of the vehicle and will be defined later.

The only vector as yet undetermined is the satellite unit normal. Let X_4, X_5, X_6 represent the coordinate axes of a rectangular cartesian coordinate system which moves with the satellite. This coordinate system is fixed relative to the satellite and chosen to give the simplest mathematical description of the satellite surfaces. The general equation of the satellite surface would be

$$g(X_4, X_5, X_6) = 0 \tag{10}$$

and its unit normal at any point is

$$\vec{N}_2 = \frac{\vec{\nabla} g}{|\vec{\nabla} g|} \tag{11}$$

with the sign chosen so that $\vec{N}_2$ is always directed outward from the surface.

The integration of equation (3) may be simplified by studying the functional dependence of the vectors on area A_2. $\vec{r}_{12}$ is not a function of position on the satellite surface, since even for a 100-foot satellite in a low orbit (100 miles), the differences in $\vec{r}_{12}$ at extreme points on the satellite are negligible. Also by subdividing the satellite surface into increments (A_{2i}) over which the direction of the unit normal is constant, the cos γ_2 is made piecewise independent of A_2. The equation for the energy received by A_{2i} from the earth is

$$\vec{Q}_{1 \to 2i} = \frac{\sigma T_1^4}{\pi} \int_{A_{2i}} dA_{2i} \int_{A_1} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \tag{12}$$

or

$$\vec{Q}_{1 \to 2i} = \sigma T_1^4 \; A_{2i} \; F_{21} \tag{13}$$

where F_{21} is the view factor defined as

$$F_{21} = F_E = \frac{1}{\pi} \int \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \tag{14}$$

Additional information concerning the definition of the view factor is given in Appendix A.

To apply equation (13) the surface temperature of the earth must be determined. For a given point on the earth the temperature varies with the time of day, season and atmospherical conditions, thus making it impossible to use the Stefan-Boltzman Law. However, it is possible to compute the total energy radiated by the earth by means of an energy balance, independent of these variables, and obtain a mean effective temperature. The thermal energy incident on the earth consists almost entirely of solar radiation. Since the sun is so far from the earth, it can be considered as a point source emitting radiation which impinges on the earth in parallel lines. The intensity of this solar energy is called the solar constant, S, and is based on the irradiation of a flat plate normal to the sun's rays at the earth's mean distance from the sun. The net energy absorbed by the earth is the difference between the incident solar radiation and the fraction that is reflected as defined by the albedo. Since the mean temperature of the earth and its atmosphere does not vary appreciably over extended periods, it may be concluded that this absorbed energy is in turn reradiated. Both the solar constant and average albedo, $\overline{A}$, are reasonably well known for the earth so that this reradiated energy can be readily evaluated. If I_t is the total energy radiated per unit area and time, the energy balance is

$$(1 - \overline{A}) S \pi R_E^2 = 4 \pi R_E^2 I_t \tag{15}$$

This assumes that all the surface of the earth is radiating with the same intensity as a black body; therefore

$$I_t = \sigma T_1^4 = \frac{(1 - \overline{A})}{4} S \tag{16}$$

The most accurate value for the solar constant is that given by Johnson (Reference 1), as

$$S = 443 \pm 9 \frac{\text{Btu}}{\text{ft}^2 \text{ hr}}$$

and the average albedo is 0.40 (Reference 2). Substituting these values into equation (16) gives a mean effective temperature for the earth of 443.5°R, and an intensity of 66.36 Btu/hr-ft^2.

B. SOLAR RADIATION

Since the irradiation of a flat plate normal to the sun's rays is known (solar constant), it is easy to find the incident energy on any flat surface. The amount of heat received will be proportional to the projected area in the direction of the sun. For example, if the unit normal to the area A_{2i} makes an angle γ_{2s} with the sun's rays, as shown in FIG 4, then the radiation received from the sun would be

$$Q_{s_i} = S \cos \gamma_{2s} A_{2i} \tag{17}$$

From the form of the above equation, and the definition of a view factor, as explained in Appendix A, it can be seen that the solar view factor is

$$F_s = \cos \gamma_{2s} = \vec{S} \cdot \vec{N}_2 \tag{18}$$

where

$$\vec{S} = S_1 (\vec{i_1}) + S_2 (\vec{i_2}) + S_3 (\vec{i_3}) \tag{19}$$

C. ALBEDO RADIATION

The third source of energy irradiating the satellite is earth-reflected solar energy. Neglecting the atmosphere, the amount of solar energy incident on the earth per unit time and area would be

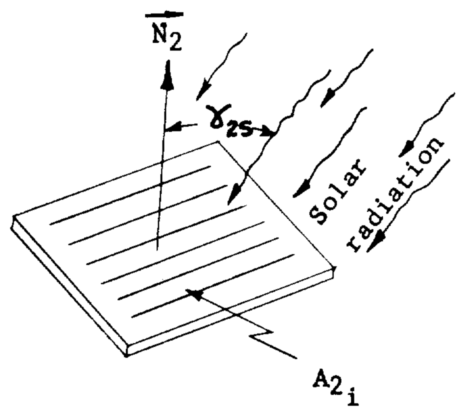

Figure 4. Solar Irradiation

$$S \cos \gamma_{1s} \tag{20}$$

where

$$\cos \gamma_{1s} = \vec{S} \cdot \vec{N}_1 \tag{21}$$

However, all of this energy is not absorbed; some of it is reflected back to space. The average fraction reflected is defined as the albedo and accounts for the actual reflected radiation regardless of the cause of its reflection. Therefore, the amount of the energy returned to space, per unit time and area, by the earth would be

$$\bar{A} S \cos \gamma_{1s} \tag{22}$$

If the energy is reflected diffusely according to Lambert's Cosine Law then the intensity in any direction is

$$I_R = \frac{\bar{A} S \cos \gamma_{1s}}{\pi} \tag{23}$$

The amount of this energy received by the satellite can now be computed in a manner similar to direct earth radiation. The only difference is the intensity of the radiation. Such an approach gives

$$Q_{R_i} = S \bar{A} A_{2i} F_{R_i} \tag{24}$$

where

$$F_{R_i} = \frac{1}{\pi} \int \frac{\cos \gamma_{1s} \cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \tag{25}$$

and the limits of integration are

$$\cos \gamma_2 \geq 0 \qquad \cos \gamma_1 \geq 0 \qquad \cos \gamma_{1s} \geq 0 \tag{26}$$

The limits of integration given by equations (4) and (26) determine whether or not a surface is receiving radiation. Specifically, the limit $\cos \gamma_1 \geq 0$ means that a given element of area on the earth is in a position to emit energy to a particular satellite location. If the $\cos \gamma_2$ is positive, this means that the element of area on the satellite is in a position to receive the radiation. The limit on $\cos \gamma_{1s}$ insures that the earth surface area element is irradiated by the sun and therefore should be considered when summing the reflected solar energy.

D. OTHER CELESTIAL SOURCES

To determine whether or not the radiation from a nearby celestial body should be considered, the maximum possible heating loads were calculated. At any given altitude above a planet this maximum incident energy would occur on a flat plate which is parallel to the local planet horizon. The view factor between the plate and the planet is determined by Smolak's equation (Reference 3) and found to be

$$F_H = \left[\frac{1}{1 + \frac{h}{\rho_P}} \right]^2 \tag{27}$$

The intensity of the planetary radiation is determined in a manner similar to that previously described on Page 16. The albedo values are taken from Reference 2 and listed with the calculated intensitis in Table 1. The incident thermal energy per unit area on a flat plate parallel to the planet's local horizon would be

$$Q_P = I_P \; F_H \tag{28}$$

Table 2 gives the amount of incident energy for various altitudes above the moon and the neighboring planets. It can be seen that planetary radiation is negligible beyond 30,000 miles from the surface of the bodies considered. Therefore, the only sources which contribute significant amounts of thermal radiation to an earth satellite are the sun, the earth, and albedo or earth-reflected solar energy.

TABLE 1

Planetary Constants

Celestial Body	Earth	Moon	Venus	Mars
I_t Btu/hr-ft^2	66.36	102.86	50.78	40.48
$\bar{A}$	0.40	0.07	0.76	0.15

TABLE 2

Planetary Radiation to a Flat Plate Parallel to the Planet's Local Horizon

h (Statute Miles)	q_p (Btu/hr-ft^2)			
	Earth	Moon	Venus	Mars
100	63.0	86.4	48.3	36.7
500	52.0	48.5	39.7	26.2
1000	42.1	28.0	32.0	18.8
5000	13.0	3.17	9.48	3.48
10000	5.33	1.00	3.87	1.19
20000	1.88	0.274	1.30	0.357
30000	0.906		0.636	
40000	0.538		0.386	

E. SPINNING SATELLITE

Spinning the satellite causes the heating loads to be uniformly distributed about the spin axis. To calculate the incident energy the satellite is divided into segments which are bounded by planes perpendicular to the spin axis. The total irradiation of each segment is then calculated for the non-spinning case and this value divided by the exposed surface area of the segment. This solution assumes a constant spin rate during any one revolution, which will usually be the case. However, variable rates can also be accounted for by weighting the averaging process.

ORBITAL PARAMETERS

The parameters which describe the satellite's orbit and motion will be given in this section. Spherical trigonometry will be used to relate the launching conditions to the resulting orbit, and the movement of the satellite in the orbital plane will be described in polar coordinates. Because the earth is not a perfect sphere, orbital parameters will not remain fixed but will experience perturbations, which will be accounted for by Krause's equations (Reference 4).

As the vehicle travels in orbit it is assumed to be oriented in any one of three ways: toward the earth, toward the sun, or tangent to the flight path at perigee. In each case the vehicle will be assumed to be capable of maintaining its orientation and not be influenced by drag or electromagnetic forces. The primary interest here is in the angular relations and not in expressing the coordinates of a point in terms of the different coordinate systems; therefore, the transposition of the origins will be neglected. The equations relating various unit vectors of the different coordinate systems are given in detail in Appendix B, based on the analysis of this section.

The position of the sun relative to the geocentric coordinate system will be derived and the attitude of the earth's shadow.

cylinder[1] will be determined. Since the intersection of the orbital plane and the shadow-cylinder describes an ellipse, the ingress and egress points will be found by simultaneously solving the equations for the orbital and shadow ellipses. Such an approach assumes that the satellite is a point, which is a reasonable assumption for the purposes of angular location.

The orbital plane is defined by the angle, i, which it makes with the equatorial plane of the earth and the right ascension of the ascending node, Ω, as shown in FIG 5. The position of the orbit in its plane is given by the argument of perigee, ω, measured from the ascending node to the perigee point.

The initial track of the satellite, on the earth's surface, is illustrated in FIG 6, based on the assumption of a spherical earth and a launch trajectory confined to the orbital plane.

To correlate the orbital and launch parameters, spherical trigonometry is used. This method requires the solution of two similar right spherical triangles. These triangles are formed by the earth's equator and the track of the satellite, with the third side being the meridian of longitude through either the launch site or perigee projection. The launch triangle is solved first since two of its parts are known. However, the known quantities here are the inclination of the orbit, i, and the side opposite which is the latitude of the launch site. This case is ambiguous, giving two possible solutions; thus, a sketch should be drawn to interpret the results properly. The spherical angle relations are

$$\sin \delta_L = \sin i \sin \omega_L \tag{29}$$

$$\tan \lambda_L = \cos i \tan \omega_L \tag{30}$$

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[1] Actually the earth's shadow is a cone with an included angle of 0.53° and an apex located 746,800 nautical miles from the earth's center (Reference 5). However, the cylindrical assumption is practical since even at a distance of 10,000 nautical miles from the earth the diameter of the cylinder would only be 80 nautical miles more than the cone, or one percent of the earth's diameter.

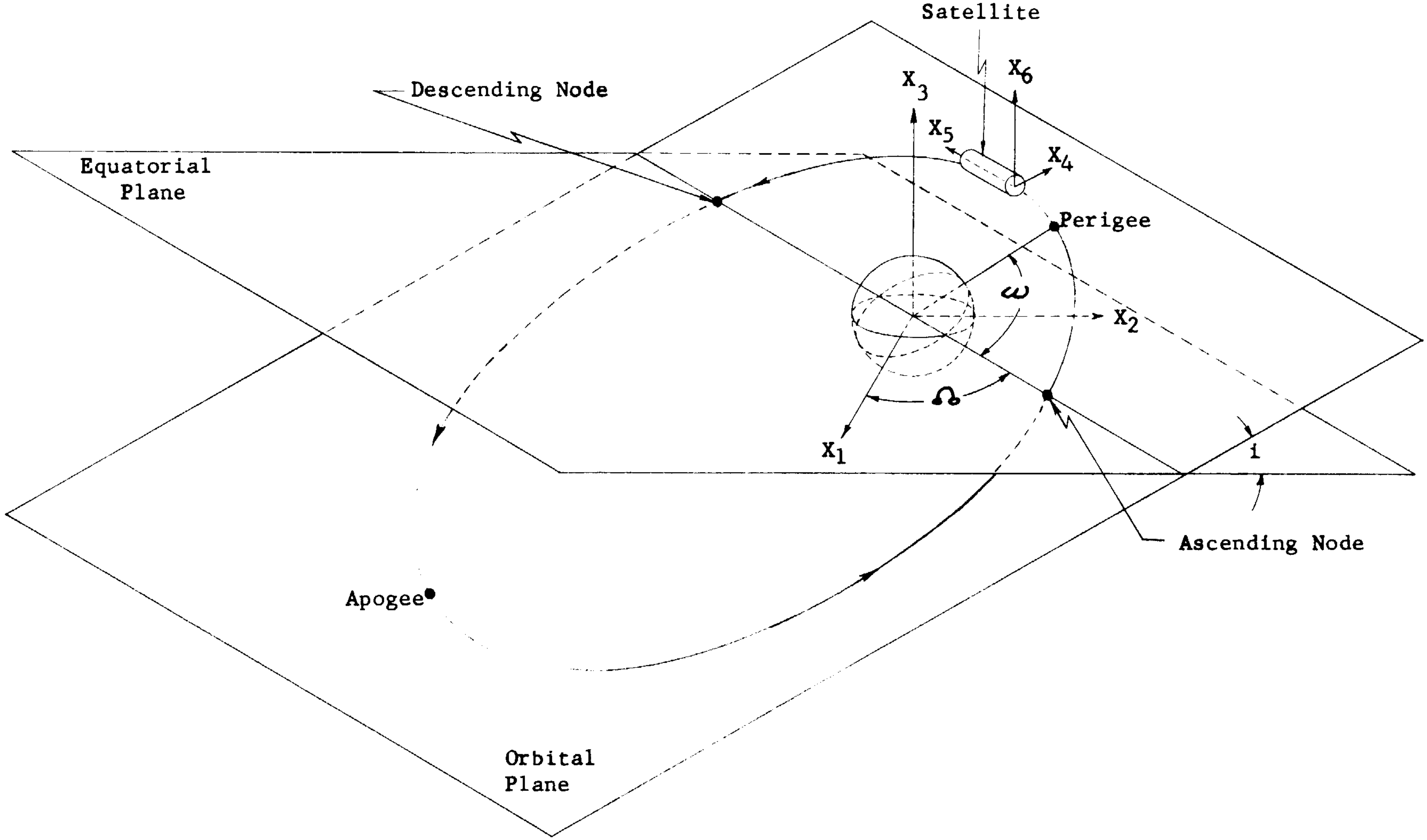

Figure 5. Illustration of the Orbit Orientation Relative to the Earth

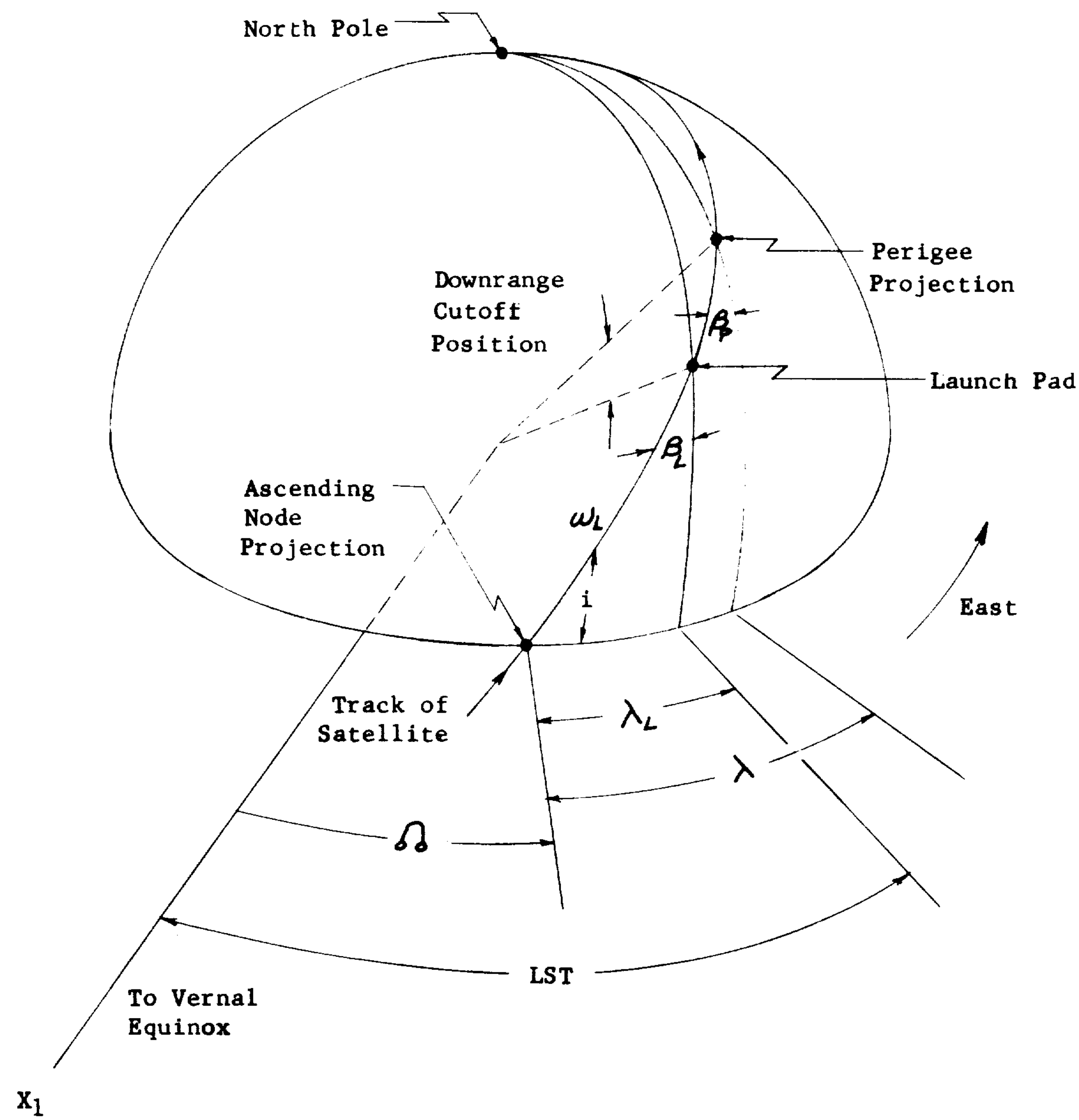

Figure 6. **Path of Satellite Projected onto the Earth's Surface**

and

$$\sin \beta_L = \sin \lambda_L \csc \omega_L \tag{31}$$

Using the above results, the parts of the spherical triangle with the meridian of perigee as a side may now be found. The known quantities are the angle of inclination, i, and the argument of perigee, ω, which is given by

$$\omega = \omega_L + \text{downrange cutoff} \tag{32}$$

The remaining relations are calculated by the following equations which are similar to equations (29) through (31) above

$$\sin \delta = \sin i \sin \omega \tag{33}$$

$$\tan \lambda = \cos i \tan \omega \tag{34}$$

and

$$\sin \beta_p = \sin \lambda \csc \omega \tag{35}$$

The date and time of launch appear in the determination of Ω, since

$$\Omega = LST - \lambda_L \tag{36}$$

as can be seen from FIG 6. The angle, λ_L, is found from the solution of the launch triangle, and Reference 6 gives the local sidereal time, LST.

The polar form of the equation of an ellipse which is

$$R = \frac{a(1 - e^2)}{1 + e \cos \nu} \tag{37}$$

will be used to describe the position of the satellite in the plane of orbit. The angle, ν, is called the true anomaly and is measured counterclockwise from perigee. The radius is measured from the perifocal point to the satellite. In the case of an earth satellite the perifocal point is chosen at the center of the earth and the radius, R, as shown in FIG 3. The eccentricity of the ellipse is e, and a is the semi-major axis length.

Because of the oblateness of the earth, the perigee and nodal locations do not remain fixed but vary according to the equations given below (Reference 4). The location of the ascending node will change with time according to

$$\Omega = \Omega\Big)\begin{matrix}\text{initial passage}\\ \text{of the satellite}\end{matrix} + \left(\begin{matrix}\text{Number of days since}\\ \text{initial passage of the}\\ \text{satellite}\end{matrix}\right)\dot{\Omega} \tag{38}$$

where

$$\dot{\Omega} = -\frac{2\pi}{P}(24)\cos i\left[3\frac{K_2}{(\bar{p})^2} + 10\frac{K_4}{(\bar{p})^4}\left(1 + \frac{3}{2}e^2\right)\left(1 - \frac{7}{4}\sin^2 i\right)\right]\frac{\text{radians}}{\text{days}} \tag{39}$$

and

$$K_2 = 2.206 \times 10^4 \quad (\text{KM})^2 \tag{40}$$

$$K_4 = 1.580570 \times 10^9 \quad (\text{KM})^4 \tag{41}$$

$$\bar{p} = R_p(1 + e) \tag{42}$$

$$P = (2.764 \times 10^{-6})\left[\frac{R_p\,(\text{KM})}{1 - e}\right]^{3/2} \quad \text{hours} \tag{43}$$

The perturbation of the argument of perigee is

$$\dot{\omega} = \frac{2\pi}{P} (24) \left[3 \frac{K_2}{(\bar{p})^2} \left(1 - \frac{3}{2} \sin^2 i\right) + 10 \frac{K_4}{(\bar{p})^4} \left(1 + \frac{3}{4} e^2\right) \left(1 - 5 \sin^2 i + \frac{35}{8} \sin^4 i\right)\right] - (\cos i) (\dot{\Omega}) \quad \frac{\text{radians}}{\text{day}} \tag{44}$$

and the position of perigee will be

$$\omega = \omega \Bigg)\begin{matrix}\text{initial passage} \\ \text{of the satellite}\end{matrix} + \left(\begin{matrix}\text{Number of days} \\ \text{since initial passage} \\ \text{of the satellite}\end{matrix}\right) \dot{\omega} \tag{45}$$

The relationship between the satellite coordinate system (X_4, X_5, X_6) and the earth-centered system (X_1, X_2, X_3) depends on the attitude of the vehicle. The derivation of the transformation equations between these two systems will be explained by a series of rotations. (FIG 5 is an aid to visualize the physical significance of these rotations.) First, rotate the X_1, X_2, X_3 coordinate system about the X_3 axis through the angle, Ω. This rotation places the X_7 axis on the line of nodes.

The matrix relating the new triad to the earth-centered system is given below in terms of the direction cosines.

$$\begin{bmatrix} \cos \Omega & \cos (\pi/2 - \Omega) & \cos \pi/2 \\ \cos (\pi/2 + \Omega) & \cos \Omega & \cos \pi/2 \\ \cos \pi/2 & \cos \pi/2 & \cos 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} X_7 \\ X_8 \\ X_9 \end{bmatrix} \tag{46}$$

or simplifying

$$\begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} X_7 \\ X_8 \\ X_9 \end{bmatrix} \quad (47)$$

To place the base of the triad in the plane of the orbit, the X_7, X_8, X_9 system is rotated about the X_7 axis through the orbital inclination angle, i (FIG 7)

The matrix relating this system to the previous one is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \begin{bmatrix} X_7 \\ X_8 \\ X_9 \end{bmatrix} = \begin{bmatrix} X_{10} \\ X_{11} \\ X_{12} \end{bmatrix} \quad (48)$$

Further orientation in the plane of orbit is accounted for by a rotation about the X_{12} axis. When the coordinate system is rotated through the angle, ω, the X_{10} axis will be pointing toward perigee and the X_{11} axis will be tangent to the flight path of the satellite at the perigee point. This coincides with one of the desired satellite orientations; and if the axis of the orbiting triad, which is tangent to the flight path, is X_5, then the matrix relating the satellite axis system to the X_{10}, X_{11}, X_{12} triad is

$$\begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{10} \\ X_{11} \\ X_{12} \end{bmatrix} = \begin{bmatrix} X_4 \\ X_5 \\ X_6 \end{bmatrix} \quad (49)$$

Now the satellite vectors may be expressed in terms of the geocentric coordinates by successive application of the transformation equations.

To orient the satellite so that the X_5 axis is always pointing toward the center of the earth requires only a turning of the satellite

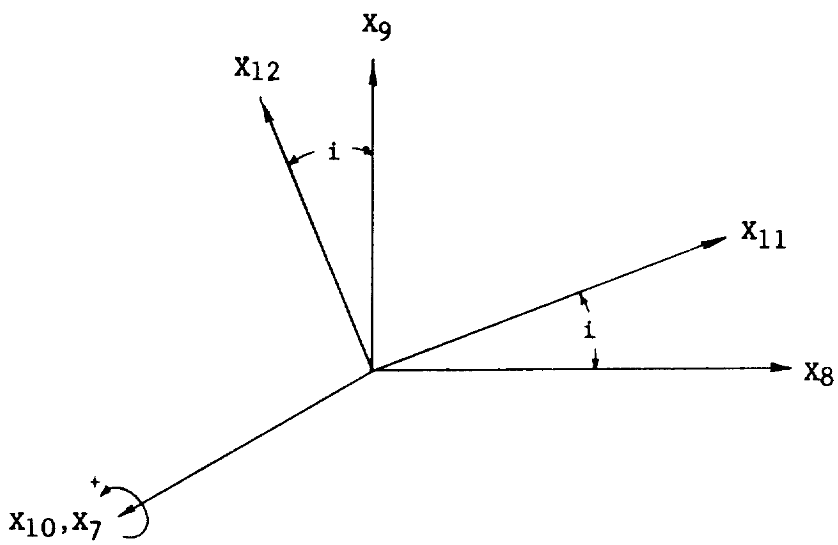

Figure 7. Rotation of Co-ordinate System About X_7 Axis

in the plane of its orbit. The angle through which the satellite must be rotated is measured relative to the perigee attitude of the vehicle and, as can be seen from FIG 8, is simply $\frac{\pi}{2} + \nu$.

If the earth-oriented triad is called X_{13}, X_{14}, X_{15} as shown in FIG 8, then the relationship between it and the perigee system is given by the matrix

$$\begin{bmatrix} -\sin\nu & \cos\nu & 0 \\ -\cos\nu & -\sin\nu & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} X_{13} \\ X_{14} \\ X_{15} \end{bmatrix} \quad (50)$$

The sun orientation is more difficult to express mathematically since the satellite must not only be turned in the plane of its orbit but must also be elevated out of the orbital plane. These rotations will be expressed in terms of the angles which the sun's unit vector makes with the perigee system as defined in FIG 9. The scalar components of the sun vector in the perigee system are

$$\sin\xi \cos\Gamma = \vec{S} \cdot \vec{i}_4 \quad (51)$$

$$\cos\xi = \vec{S} \cdot \vec{i}_6 \quad (52)$$

and

$$-\sin\xi \sin\Gamma = \vec{S} \cdot \vec{i}_5 \quad (53)$$

Since the vectors in the above equations are known in the geocentric system (equation 19 and Appendix B), the indicated vector operations may be completed and a scalar expression obtained for the right hand side of each equation. Using these results and the fact that $0 \leq \xi \leq \pi$, equations (51-53) may be solved for ξ and Γ.

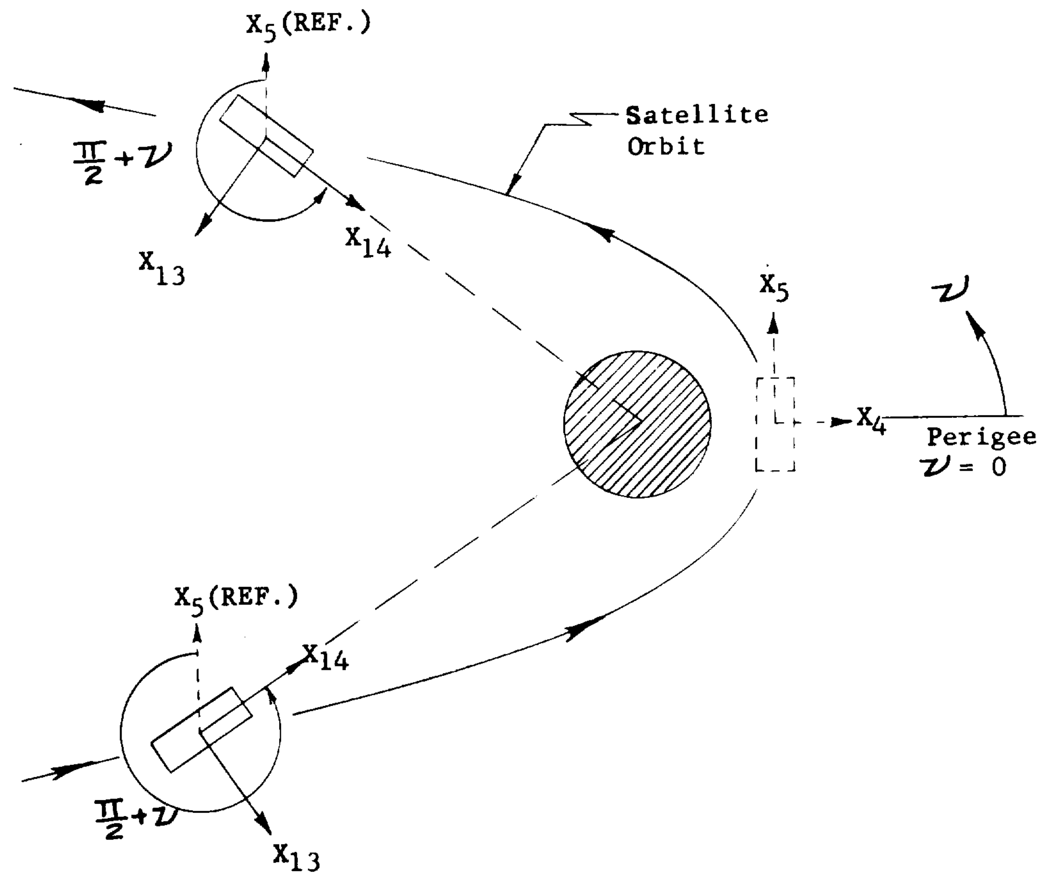

Figure 8. Earth-Oriented Satellite

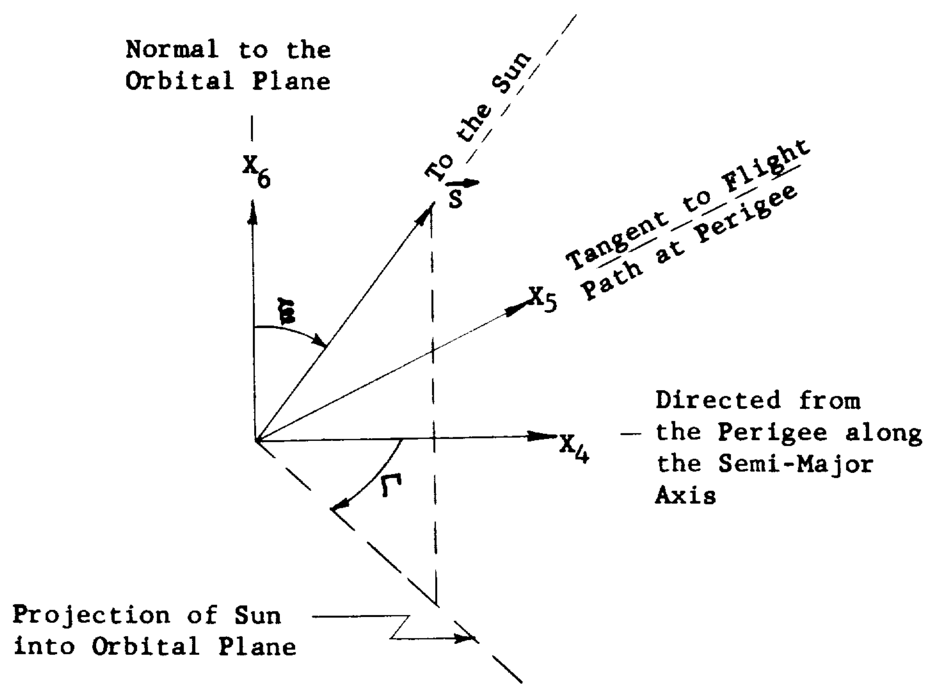

Figure 9. Position of Sun Relative to Satellite Co-ordinate System

By rotating the perigee system through the angle $\frac{\pi}{2} + \Gamma$, as shown in FIG 10, the X_5 axis is aligned with the sun's projection in the orbital plane (FIG 9). The relationship between the new triad (X_{16}, X_{17}, X_{18}) and the perigee system is

$$\begin{bmatrix} -\sin\Gamma & -\cos\Gamma & 0 \\ \cos\Gamma & -\sin\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} X_{16} \\ X_{17} \\ X_{18} \end{bmatrix} \tag{54}$$

To complete the alignment with the sun, the satellite must now be elevated about the X_{16} axis through the angle, $\frac{\pi}{2} - \xi$, so that the X_{17} axis now points directly at the sun. The axes of the triad resulting from the last rotation are labeled X_{19}, X_{20}, X_{21}. The transformation between this system and the previous one is given by the following matrix array:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\xi & \cos\xi \\ 0 & -\cos\xi & \sin\xi \end{bmatrix} \begin{bmatrix} X_{16} \\ X_{17} \\ X_{18} \end{bmatrix} = \begin{bmatrix} X_{19} \\ X_{20} \\ X_{21} \end{bmatrix} \tag{55}$$

To aid in remembering the orientation associated with each arithmetical subscript, the important coordinates will hereafter be referred to by more informative subscripts as listed below.

(perigee system) $X_4, X_5, X_6 = x_p, y_p, z_p$ (56)

(earth-oriented) $X_{13}, X_{14}, X_{15} = x_{E.O.}, y_{E.O.}, z_{E.O.}$ (57)

and

(sun-oriented) $X_{19}, X_{20}, X_{21} = x_{S.O.}, y_{S.O.}, z_{S.O.}$ (58)

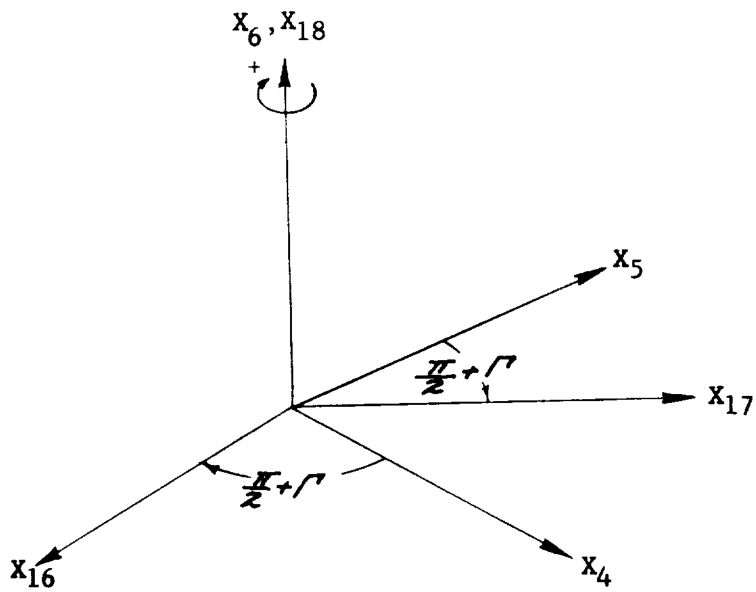

Figure 10. Rotation of Co-ordinate System About X_6 Axis

The location of the sun may be found by using the celestial sphere concept and spherical trigonometry as shown in Reference 7. The projection of the path of the sun on the celestial sphere is called the ecliptic and makes an angle, ϵ, with the celestial equator of 23.45° or 23° 27' 8.2". Using the vernal equinox as a reference point the right spherical triangle to be solved is formed by the ecliptic, the celestial equator, and the meridian circle passing through the sun's position. If the right ascension in the plane of the ecliptic, $L_\odot$, was known, then the equations could be solved. The relations from spherical trigonometry would be

$$\tan \alpha = \tan L_\odot \cos \epsilon \tag{59}$$

and

$$\sin \gamma = \sin \epsilon \sin L_\odot \tag{60}$$

where α is the right ascension of the sun in the plane of the equator and γ is the declination of the sun. The expression for $L_\odot$ will be found by use of the equation relating true anomaly to mean anomaly, since the mean motion of the sun is well known. This equation is (Reference 4)

$$\nu = M + 2 e \sin M + \frac{5}{4} e^2 \sin 2M + \ldots \tag{61}$$

where the eccentricity of the earth's orbit is 0.0167 (Reference 5). Since the sun completes an orbit every 365.25 days, the mean daily motion would be

$$\frac{360}{365.25} = 0.9856 \frac{\text{deg}}{\text{day}} \qquad (62$$

and the mean anomaly expressed relative to the vernal equinox would be

$$M = 0.9856\,(77 + \text{Number of days after v.e.}) \tag{63}$$

The number 77 appears because the passage of the vernal equinox occurs 77 days after perihelion of the earth. However, basing the position of the sun on the calendar introduces an error in the calculations. The calendar neglects the one-quarter day and compensates by adding a full day every four years. To locate the true position of the sun in any given year a correction factor is added to account for this discrepancy. The right ascension in the plane of the ecliptic would then be

$$\begin{aligned} L_{\odot} = {} & 0.9856\,(D + 77) + 1.9481 \sin\left[0.9856\,(D + 77)\right] \\ & + 0.0207 \sin\left[2(0.9856)(D + 77)\right] - \\ & 77.617° + \frac{\text{Number of years since leap year}}{4} \end{aligned} \tag{64}$$

where 77.617° is the true anomaly of vernal equinox; and the number of years since leap year will be 0, 1, 2, or 3. A new leap year is not counted as four years since the last one, but as zero years. Now $L_{\odot}$ can be calculated, and equations 59 and 60 solved for α and γ. The relationship between the coordinates α and γ, and the earth triad, is shown in FIG 11 along with the conversion equations.

The intersection of the earth's shadow cylinder with the orbital plane is illustrated in FIG 12 and will be described in terms of previously defined parameters. The angle of intersection is measured by ξ, the angle between the normal to the orbital plane and the sun vector. The plane view shows elliptical shape of the locus of intersection and its relation to the X_{16}, X_{17}, X_{18} triad system. The equation of the earth's shadow ellipse is

$$\left(\frac{X_{17}}{a_s}\right)^2 + \left(\frac{X_{16}}{b_s}\right)^2 = 1 \tag{65}$$

$$\vec{S} = S_1(\vec{i_1}) + S_2(\vec{i_2}) + S_3(\vec{i_3})$$

$$S_1 = \cos\alpha \cos\gamma$$
$$S_2 = \sin\alpha \cos\gamma$$
$$S_3 = \sin\gamma$$

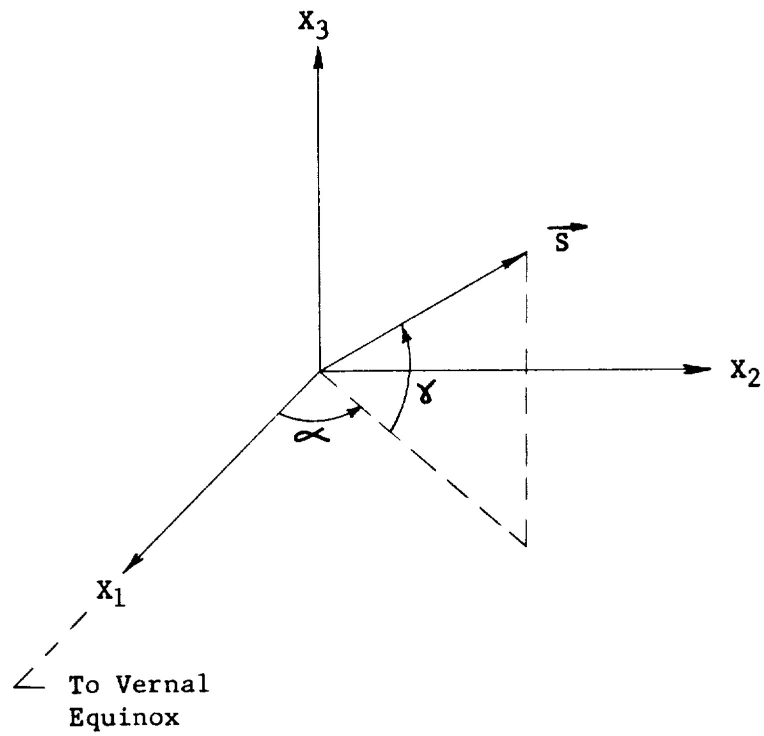

Figure 11. Position of Sun Relative to Earth Co-ordinate System

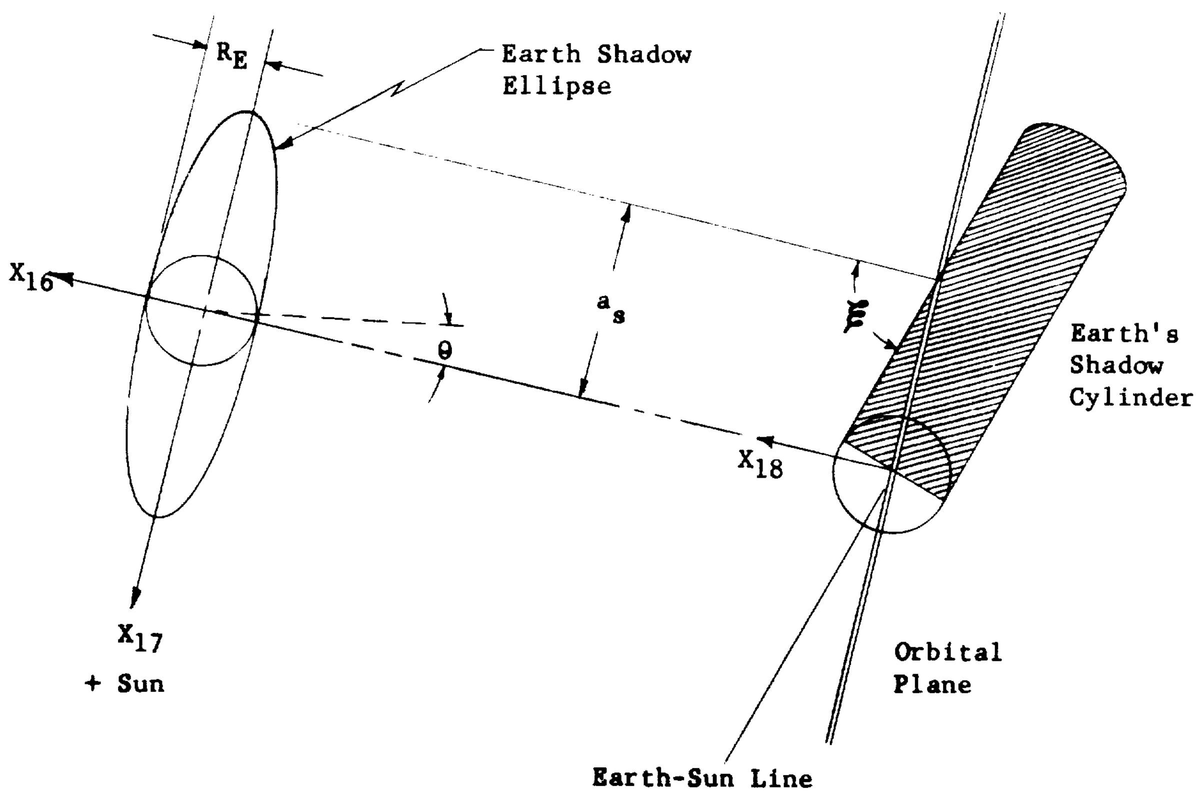

Figure 12. Earth's Shadow

where $a_s = \frac{R_E}{\cos \xi}$ and $b_s = R_E$ (FIG 12). Or, using the angle θ and the radius R_s, measured from the center of the earth to any point on the shadow ellipse, the equation may be expressed as

$$R_s = \frac{R_E^2}{\cos^2 \theta (1 - \cos^2 \xi) + \cos^2 \xi} \qquad (66)$$

The position of the shadow ellipse in the orbital plane is defined relative to perigee by the angle Γ.

The equations of the two ellipses in the orbital plane (the shadow ellipse and the satellite orbital ellipse) are now known. Positions on satellite orbit are specified by the true anomaly, ν, while the angle θ locates points on the shadow ellipse. From FIG 13 it can be seen that these angular coordinates are related by the following equation

$$\nu = \theta + \pi/2 - \Gamma \qquad (67)$$

The ingress and egress points are two of the four possible intersections of these ellipses, and may be found by simultaneously solving their equations. This has been accomplished on an electronic computer by simultaneously computing the two radii and comparing them. By limiting the comparison to the values of ν corresponding to a θ range of $0 \leq \theta \leq \pi$, the proper intersections are obtained (FIG 14).

EXAMPLE PROBLEM

As an example, the incident energy on a cylindrical satellite with hemispherical bulkheads will be calculated for all three vehicle orientations. The thermal heating depends on the intensity of the radiation and the view factor between the object and the source. The intensities are known and the view factor equations may be evaluated by double integration. The input data needed to perform the integration consists of an expression for the satellite surface normal and specification of the orbital parameters.

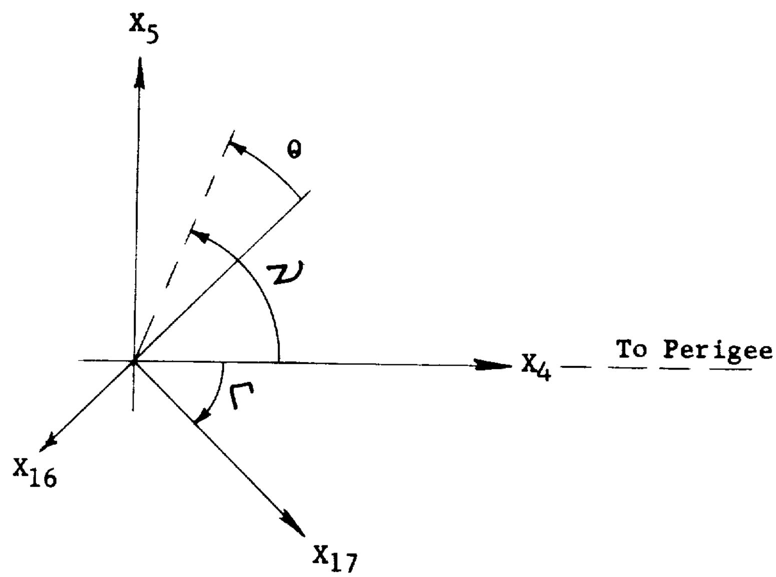

Figure 13. Angular Relations in the Orbital Plane

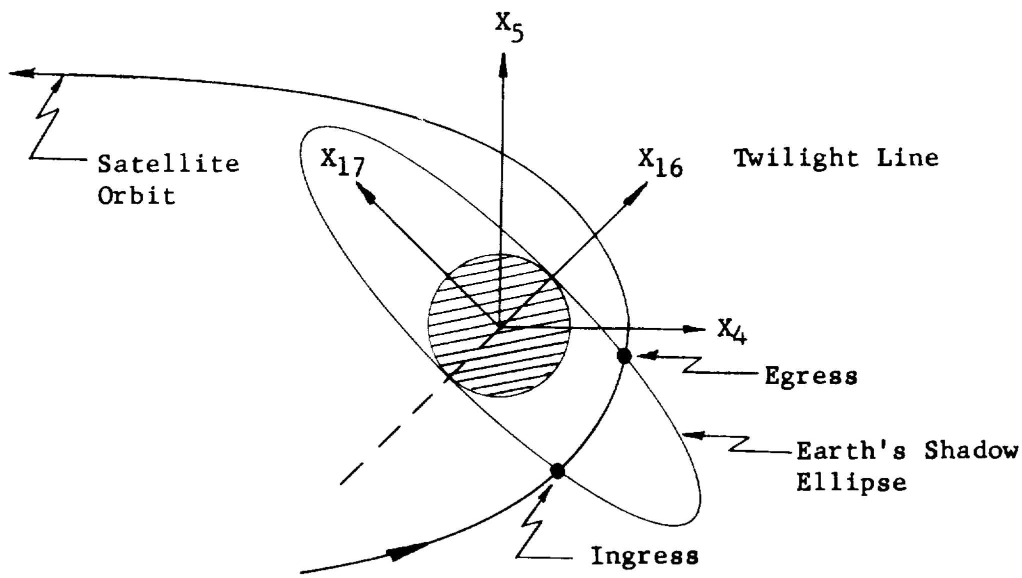

Figure 14. Ingress and Egress of the Earth's Shadow

The surface normal equations will be derived in detail for the satellite oriented tangent to the flight path at perigee. The other derivations are similar, and the results are given in Appendix C.

Consider the cylinder oriented as shown in FIG 15. The equation for its surface would be

$$(x_p)^2 + (z_p)^2 = \frac{d^2}{4} \tag{68}$$

and the unit normal would be (see equation 11)

$$\vec{N_2} = \frac{(\vec{i_p})(x_p) + (\vec{k_p})(z_p)}{d/2} \tag{69}$$

or using the coordinate transformation

$$x_p = d/2 \cos \zeta \tag{70}$$

$$z_p = d/2 \sin \zeta$$

The surface normal then becomes

$$\vec{N_2} = (\vec{i_p}) \cos \zeta + (\vec{k_p}) \sin \zeta \tag{72}$$

The relationship between the satellite coordinate system and the earth-centered triad is given by equations (B4-B6) of Appendix B. Substituting those expressions into equation 72 gives

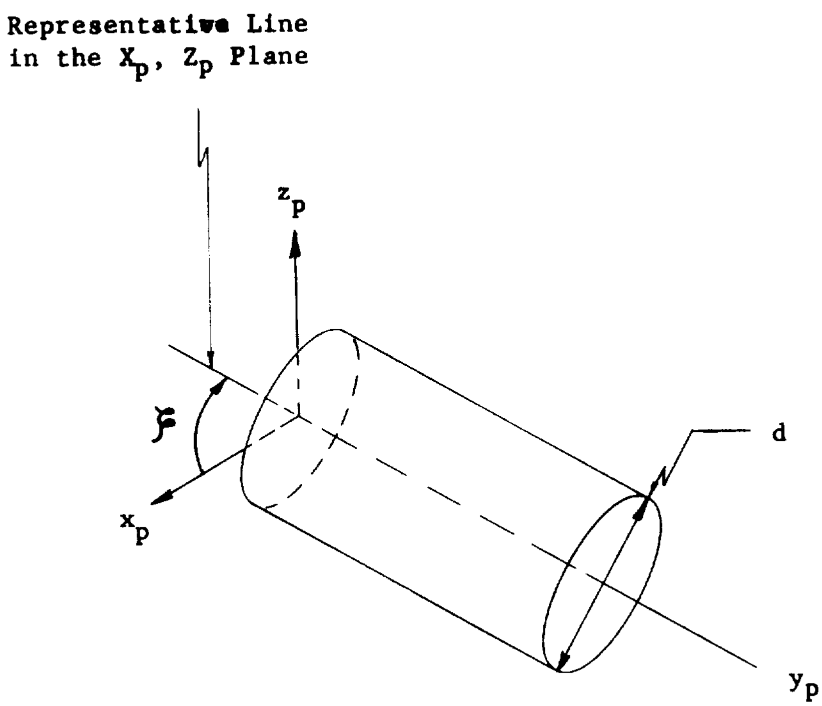

Figure 15. Co-ordinate System for Description of Cylindrical Satellite Surface

$$\vec{N_2} = (\vec{i_1})\left\{\cos\zeta\left[\cos\omega\cos\Omega - \sin\omega\cos i\sin\Omega\right] + \sin\zeta\left[\sin i\sin\Omega\right]\right\} + (\vec{i_2})\left\{\cos\zeta\left[\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega\right] + - \sin\zeta\left[\sin i\cos\Omega\right]\right\} + (\vec{i_3})\left\{\cos\zeta\left[\sin\omega\sin i\right] + \sin\zeta\left[\cos i\right]\right\} \tag{73}$$

The hemispherical tank ends will be described by placing limits on the equation of a sphere. The end nearest the origin of the satellite coordinate system (negative hemisphere) would be part of a sphere of radius $\frac{d}{2}$ whose equation is

$$(x_p)^2 + (y_p)^2 + (z_p)^2 = d^2/4 \tag{74}$$

and the unit normal would be given as (see equation 11)

$$\vec{N_2} = \frac{(\vec{i_p})(x_p) + (\vec{j_p})(y_p) + (\vec{k_p})(z_p)}{d/2} \tag{75}$$

To restrict the surface to a hemisphere, it is convenient to switch to spherical polar coordinates as shown in FIG 16. Then the unit normal is

$$\vec{N_2} = \sin\psi\sin\phi\,(\vec{i_p}) + \cos\phi\,(\vec{j_p}) + \sin\phi\cos\psi\,(\vec{k_p}) \tag{76}$$

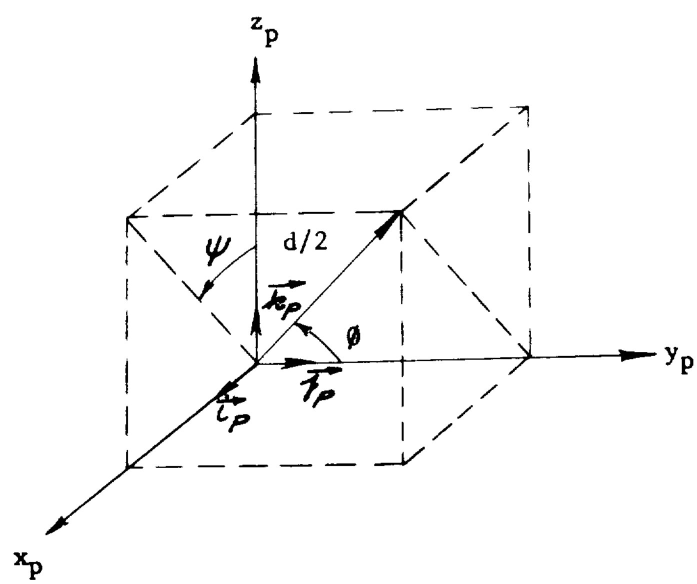

Figure 16. Co-ordinate System for Description of Negative Hemispherical Bulkhead

and the limits which define the hemisphere are

$$0 \leq \psi \leq 2\pi \tag{77}$$

$$\pi/2 \leq \phi \leq \pi \tag{78}$$

To describe the other hemispherical end (positive hemisphere) the origin of the coordinate axes is shifted to $x_p = 0$, $y_p = L$, $z_p = 0$, where L is the length of the cylindrical portion of the satellite. Denoting this new system as shown in FIG 17, the unit normal here is

$$\vec{N_2} = \sin\mu \sin\rho\,(\vec{i_{22}}) + \cos\rho\,(\vec{i_{23}}) + \sin\rho\cos\mu\,(\vec{i_{24}}) \tag{79}$$

and

$$\vec{i_p} = \vec{i_{22}} \qquad \vec{j_p} = \vec{i_{23}} \qquad \vec{k_p} = \vec{i_{24}} \tag{80}$$

The angular limits are

$$0 \leq \mu \leq 2\pi \tag{81}$$

$$0 \leq \rho \leq \pi/2 \tag{82}$$

The equations for the hemisphere unit normals are expressed in terms of the earth-centered coordinate system just as the unit normal to the cylinder was giving

$$\begin{aligned}\vec{N_2} = (\vec{i_1}) \Big\{ &\sin\psi \sin\phi \left[\cos\omega\cos\Omega - \sin\omega\cos i \sin\Omega\right] + \\ &\cos\phi \left[-\sin\omega\cos\Omega - \cos\omega\cos i\sin\Omega\right] + \\ &\cos\psi\sin\phi\left[\sin i \sin\Omega\right]\Big\} +\end{aligned}$$

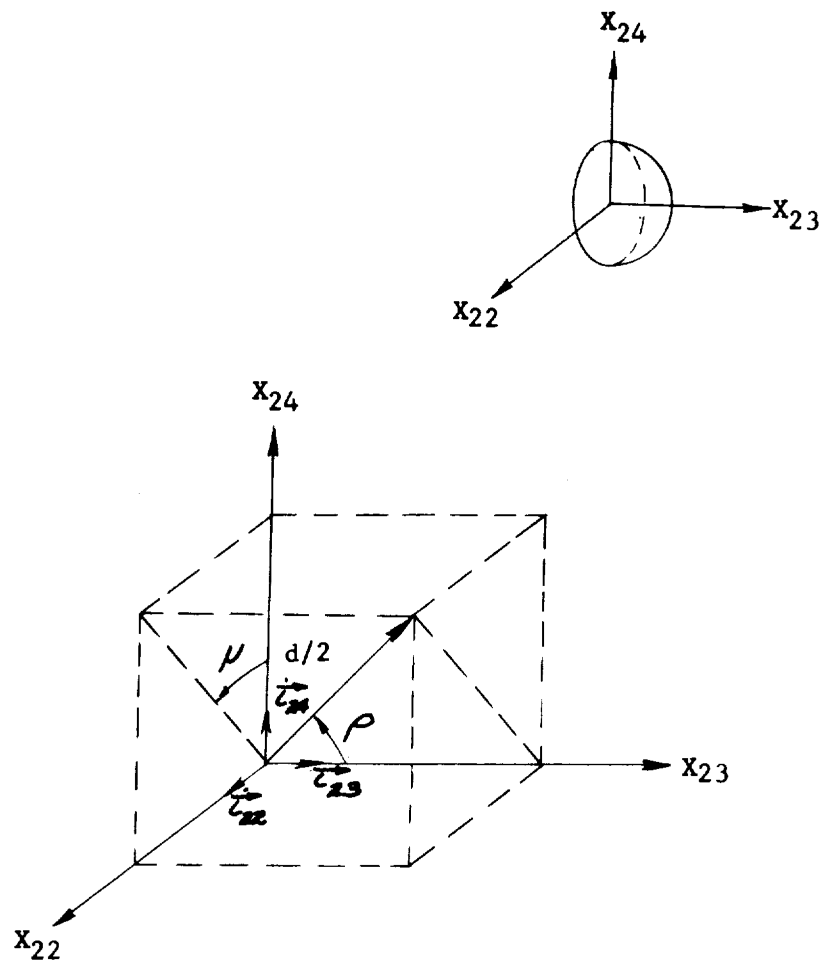

Figure 17. Co-ordinate System for Description of Positive Hemispherical Bulkhead

$$
\begin{aligned}
(\vec{i_2}) \; \Big\{ & \sin\psi \;\sin\phi \;\big[\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega\big] \; + \\
& \cos\phi \;\big[\cos\omega\cos i\cos\Omega - \sin\omega\sin\Omega\big] \\
& \cos\psi \;\sin\phi \;\big[-\sin i\cos\Omega\big]\Big\} \; + \qquad (83) \\
(\vec{i_3}) \; \Big\{ & \sin\psi \;\sin\phi \;\big[\sin\omega\sin i\big] \; + \; \cos\phi \;\big[\cos\omega\sin i\big] \; + \\
& \cos\psi \;\sin\phi \;\big[\cos i\big]\Big\}
\end{aligned}
$$

and

$$
\begin{aligned}
\vec{N_2} = (\vec{i_1}) \; \Big\{ & \sin\mu \;\sin\rho \;\big[\cos\omega\cos\Omega - \sin\omega\cos i\sin\Omega\big] \; + \\
& \cos\rho \;\big[-\sin\omega\cos\Omega - \cos\omega\cos i\sin\Omega\big] \; + \\
& \cos\mu \;\sin\rho \;\big[\sin i\sin\Omega\big]\Big\} \; + \\
(\vec{i_2}) \; \Big\{ & \sin\mu \;\sin\rho \;\big[\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega\big] \; + \\
& \cos\rho \;\big[\cos\omega\cos i\cos\Omega - \sin\omega\sin\Omega\big] \; + \qquad (84) \\
& \cos\mu \;\sin\rho \;\big[-\sin i\cos\Omega\big]\Big\} \; + \\
(\vec{i_3}) \; \Big\{ & \sin\mu \;\sin\rho \;\big[\sin\omega\sin i\big] \; + \; \cos\rho \;\big[\cos\omega\sin i\big] \; + \\
& \cos\mu \;\sin\rho \;\big[\cos i\big]\Big\}
\end{aligned}
$$

The orbit selected has the following pertinent characteristics

$$
i = 28^\circ\, 45' \qquad a = 4118.7 \text{ miles} \qquad e = 0 \qquad (85)
$$

Launching will be assumed to be made from Cape Canaveral on May 11, 1962, in the south-east direction. The geographic co-ordinates of the Cape are

latitude, 28° 45' N

longitude, 80° 34' W

From this information, the "launch triangle" may be solved using equations 29 through 31; the results are

$$\omega_L = \pi/2 \qquad \lambda_L = \pi/2 \qquad \beta_L = \pi/2 \tag{87}$$

Assuming a downrange cutoff of 15° gives a value for the argument of perigee of

$$\omega = 105°$$

from equation 32. The other parts of the perigee triangle are found from equations 33 through 35 and are

$$\delta = 27° \; 41' \tag{89}$$

$$\lambda = 107° \tag{90}$$

$$\beta = 81° \; 55' \tag{91}$$

The local sidereal time at midnight on May 11, 1962, is given in Reference 5 as

$$LST = 228° \; 31' \; 22'' \tag{92}$$

and the right ascension of the ascending node is then

$$\Omega = 228° \, 31' \, 22'' - 90° = 138° \, 31' \, 22'' \tag{93}$$

from equation 36.

Since this is a circular orbit, equation 37 reduces to

$$R = a \tag{94}$$

The perturbations of the orbital parameters are calculated from equations 39 through 44; these give the position of the ascending node as

$$\Omega = 138° \, 31' \, 22'' + (-7.6725) \text{ (Numbers of days since initial passage of the satellite)} \tag{95}$$

and the argument of perigee varies according to

$$\omega = 105° + (12.423) \text{(Number of days since initial passage of the satellite)} \tag{96}$$

The ingress and egress points of the earth's shadow are found to be at $\nu = 270.2°$ and $57.7°$, respectively, for the initial orbit.

The position of the satellite in terms of earth-centered coordinates, θ_2 and η_2, is needed to compute the vector $\vec{r}_{12}$ (equation 9). The equations for these angles were given with FIG 3 and are

$$\cos \eta_2 = \frac{X_3}{R} \tag{97}$$

$$\sin \theta_2 = \frac{X_2}{R \sin \eta_2} \tag{98}$$

$$\cos\ \theta_2 = \frac{X_1}{R\ \sin\ \eta_2} \tag{99}$$

The relationship between the earth-centered coordinates, X_1, X_2, and X_3, and the perigee system may be found from equations 47 through 49. Substituting this information along with the fact that

$$\left.\begin{aligned} x_p &= R \cos \nu \\ y_p &= R \sin \nu \\ z_p &= 0 \end{aligned}\right\} \quad \text{in the plane of orbit} \tag{100}$$

into (97), (98), and (99) gives

$$\cos\ \eta_2 = (\sin i \sin \omega) \cos \nu + (\sin i \cos \omega) \sin \nu \tag{101}$$

$$\begin{aligned} \sin\ \theta_2 = & \left[\sin \Omega \cos \omega + \cos \Omega \cos i \sin \omega\right] \frac{\cos \nu}{\sin \eta_2} \\ & + \left[\cos \Omega \cos i \cos \omega - \sin \Omega \sin \omega\right] \frac{\sin \nu}{\sin \eta_2} \end{aligned} \tag{102}$$

and

$$\begin{aligned} \cos\ \theta_2 = & \left[\cos \Omega \cos \omega - \sin \Omega \cos i \sin \omega\right] \frac{\cos \nu}{\sin \eta_2} \\ & - \left[\cos \Omega \sin \omega + \sin \Omega \cos i \cos \omega\right] \frac{\sin \nu}{\sin \eta_2} \end{aligned} \tag{103}$$

The sine of η_2 is given by

$$\sin\ \eta_2 = \sqrt{1 - (\cos\ \eta_2)^2}$$

since η_2 always lies between zero and π.

The components of the other vectors needed in the calculations of the view factors may be determined from the above equations and data. To integrate the view factors the areas A_{2i} must be chosen. This selection is easy for the cylindrical part of the satellite since the direction of $\vec{N}_2$ does not change with axial distance (see equation 73) but only with angular position, ζ. Therefore, if the cylinder is approximated by a circumscribed polygon, $\vec{N}_2$ will have a constant direction over the surfaces of any side. The hemispherical ends present more of a problem since the unit normal varies with both angular coordinates. By approximating the hemispherical surface with tangent planes the surface normal is made constant over the areas. Using methods such as these, the surfaces of a satellite may be approximated to any degree of accuracy desired by choice of the sizes of area, A_{2i}. The resulting integrals have been programmed in a general manner on the IBM 7090, using the numerical method of Gauss. Typical curves for the planetary, albedo and solar radiation view factors are shown in FIG 18 through 20 for each orientation.

A detailed description of the irradiation may now be determined, using equations 13, 17, and 24. A typical curve showing the earth planetary radiation incident on the cylindrical part of the satellite, at $\zeta = 0$, is presented in FIG 21. The solid line gives the heat flux for a non-spinning vehicle and the dotted line for a spinning satellite. The total incident energy on the cylinder is listed in Table 3 according to the source of the radiation and the orientation of the satellite. The same information is given in Tables 4 and 5 for the positive and negative hemispheres, respectively.

RESULTS AND DISCUSSION

A check on the view factor equations, as derived in this report, may be obtained for certain simple cases. Smolak in Reference 3 gives the view factor equations for planetary radiation to a flat plate parallel or perpendicular to the local horizon. If the plate is perpendicular to the local horizon this equation is

$$F_v = \frac{1}{\pi}\left[\tan^{-1}\left(\frac{\rho p/h}{\sqrt{1 + 2\rho p/h}}\right) - \frac{\left(\rho p/h\right)\sqrt{1 + 2\rho p/h}}{(1 + \rho p/h)^2}\right] \tag{105}$$

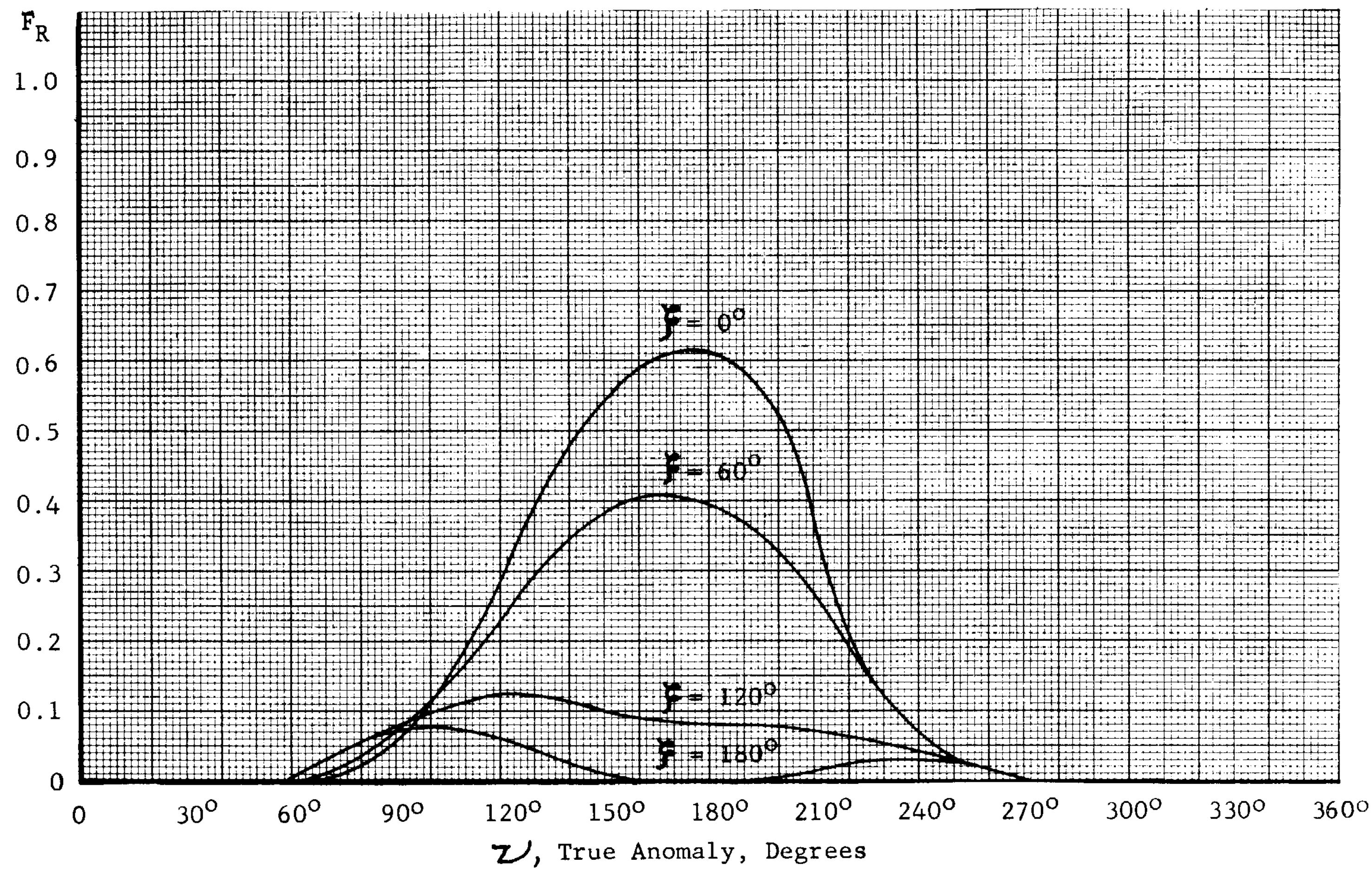

Figure 18. Typical View Factor Curves for Albedo Radiation to Cylinder Oriented Tangent to the Flight Path at Perigee

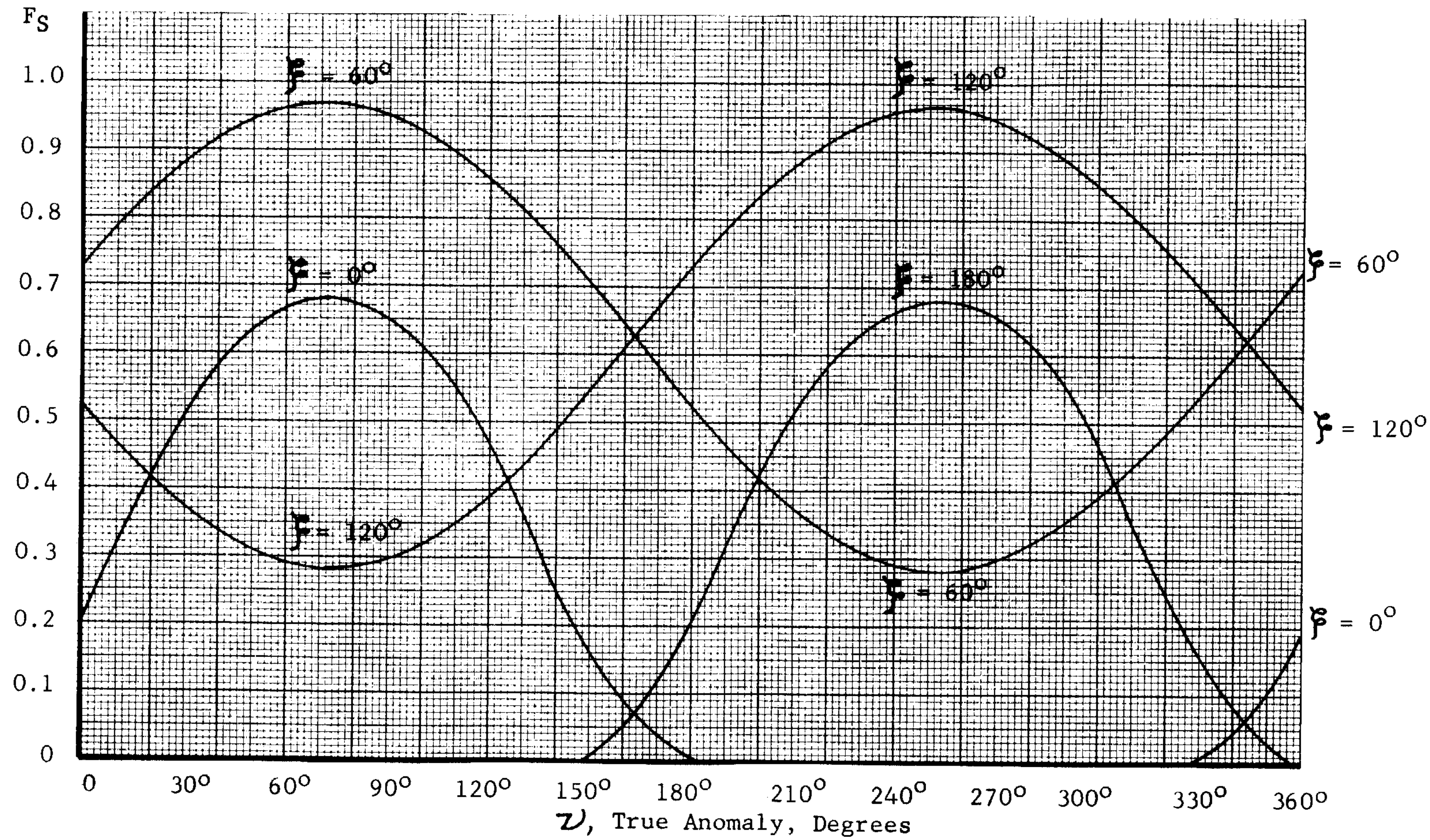

Figure 19. Typical View Factor Curves for Solar Radiation to Cylinder Oriented Toward the Earth

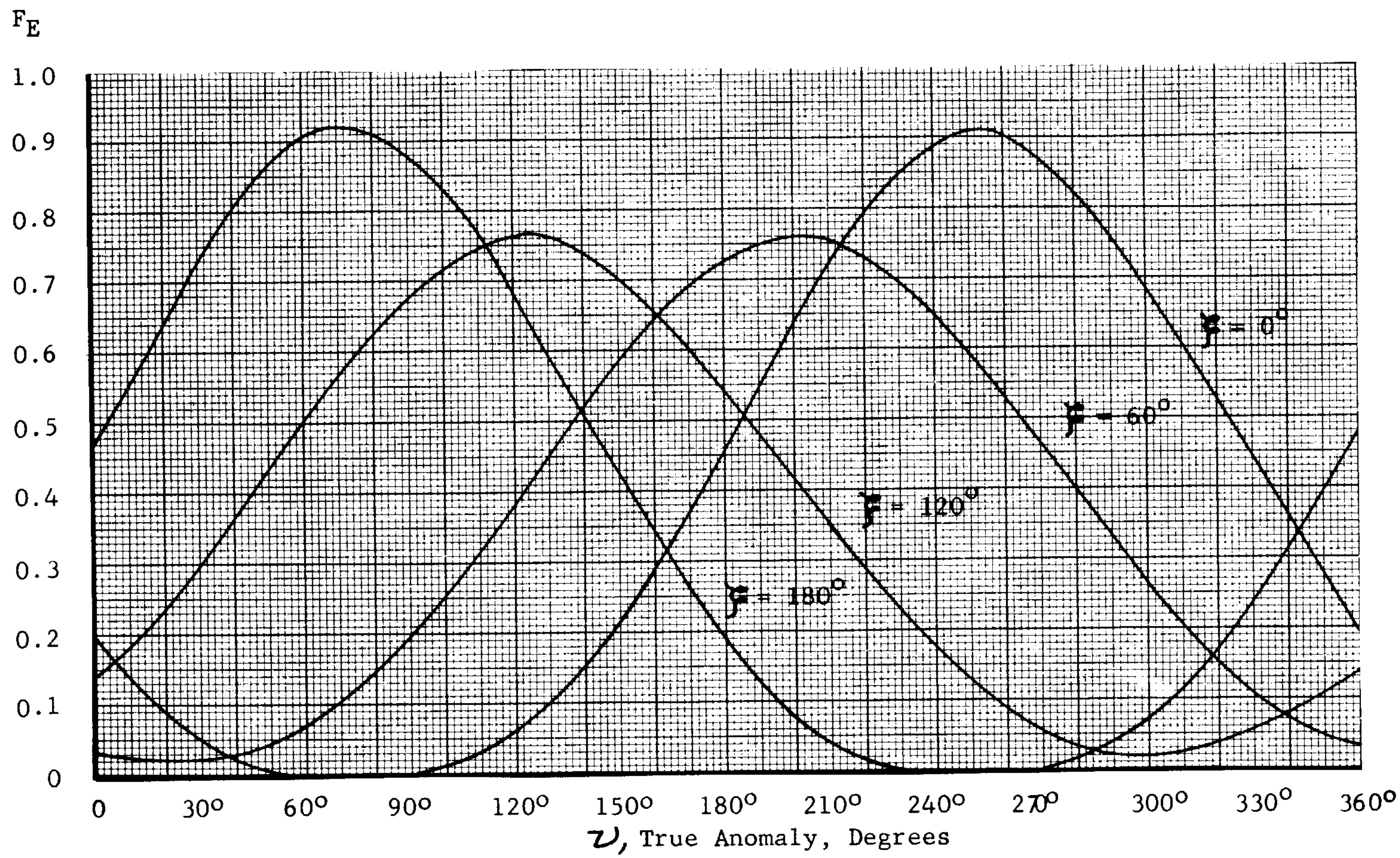

Figure 20. Typical View Factor Curves for Earth Radiation to Cylinder Oriented toward the Sun

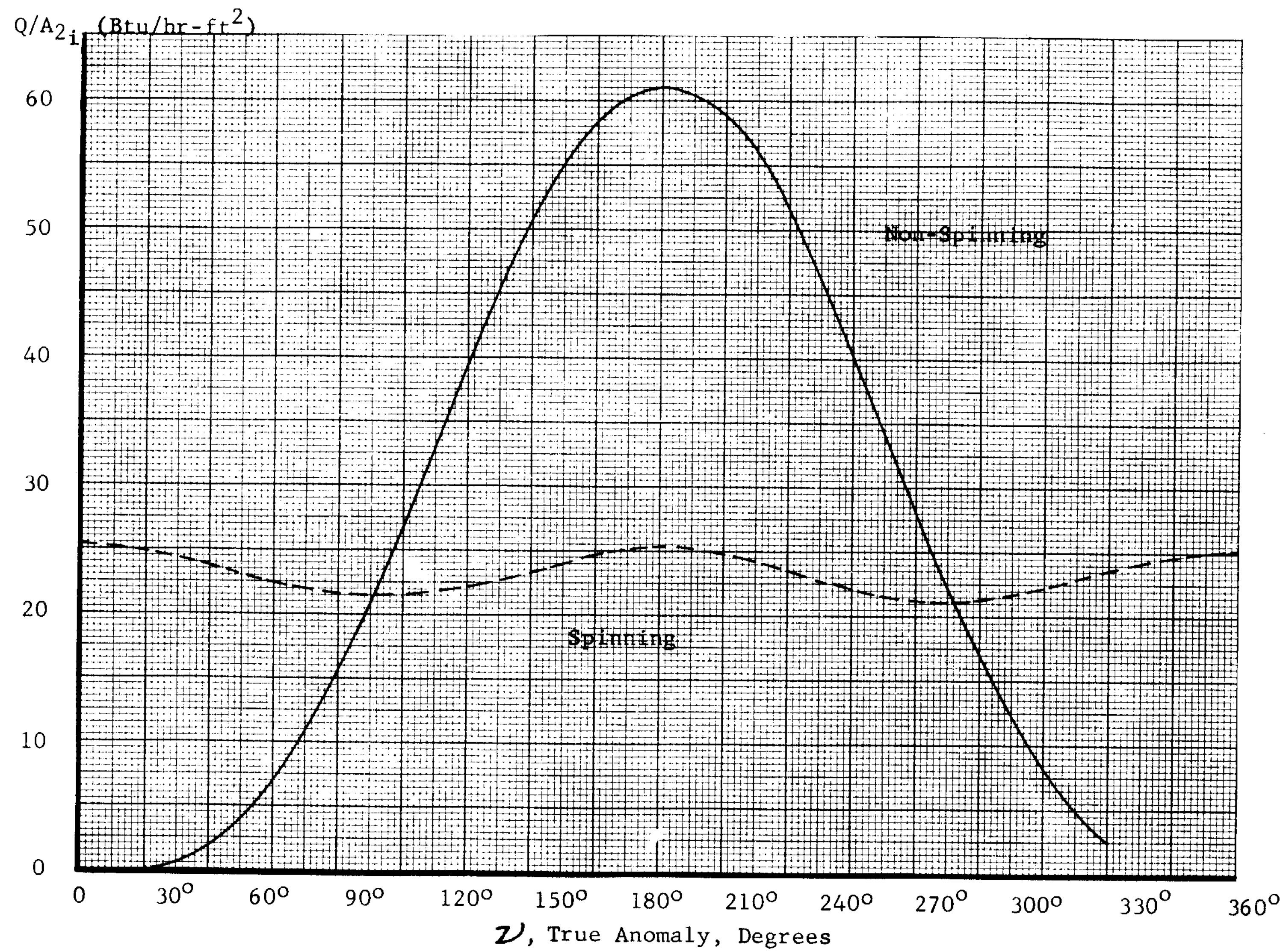

Figure 21. Incident Earth Radiation at $\zeta = 0$ of a Cylinder Oriented Tangent to Flight Path at Perigee both with and without Spin

TABLE 3

Cylinder

Source of Radiation	Incident Thermal Energy for One Orbit $\frac{Q}{\pi DL}$ $\frac{Btu}{ft^2}$		
	Satellite Orientation		
	Tangent Flight Path at Perigee	Toward Earth	Toward Sun
Earth	35	32	36
Sun	123	111	0
Albedo	21	19	22
Total	179	162	58

TABLE 4

Positive Hemisphere

Source of Radiation	Incident Thermal Energy for One Orbit $\frac{Q}{2\pi R^2}$ $\frac{Btu}{ft^2}$		
	Satellite Orientation		
	Tangent Flight Path at Perigee	Toward Earth	Toward Sun
Earth	36	58	36
Sun	112	64	191
Albedo	20	35	16
Total	168	157	243

TABLE 5

Negative Hemisphere

Source of Radiation	Incident Thermal Energy for One Orbit $\frac{Q}{2\pi R^2}$ $\frac{Btu}{ft^2}$		
	Satellite Orientation		
	Tangent Flight Path at Perigee	Toward Earth	Toward Sun
Earth	24	14	36
Sun	86	131	0
Albedo	24	8	27
Total	134	153	63

(The horizontal view factor has previously been given as equation 27.) In FIG 22 the results of the numerical integration of the earth view factor are shown for a flat plate oriented tangent to the flight path at perigee. The values given by the Smolak's formulas are shown as circle symbols in FIG 22 and agree exactly with the predictions of this report.

To determine the intensity of the earth's planetary radiation, a uniform black body temperature is assumed for the entire surface. This assumption is necessary since the local temperature is a function of many variables and is not predictable. In Reference 8, Francis made a more detailed analysis of the earth's radiosity. Using the meterological data of other researchers, a mean environment was obtained for each locale, at various times of the year, and the resulting irradiation of a plane surface was studied. It was found that the local variations from the uniform temperature results were quite high for polar orbits (-17% to +33%) but agreed within 5% for equatorial orbits and average conditions on the earth.

The benefits to be gained from controlling the incident energy through vehicle orientation may be determined by studying the results of the example problem. By orienting the satellite toward the sun, the total heat flux to the cylinder is reduced by 120 Btu/ft^2 each orbit. However, the incident energy to the positive hemisphere is increased by 76 Btu/ft^2 and the heat load to the negative hemisphere is reduced by 50%. Consequently, it is very advantageous from a thermal viewpoint to orient the satellite toward the sun. Orienting the vehicle toward the earth does not offer much, since the proximity of the earth reduces the effectiveness of the satellite as a shield.

The methods outlined in this report may be extended easily to predict the incident energy on satellites of other celestial bodies. The major problems would be locating the orbital plane and sun relative to the body. However, the extension to a lunar satellite is very simple since the sun and the vernal equinox have almost exactly the same direction at the moon as at the earth.

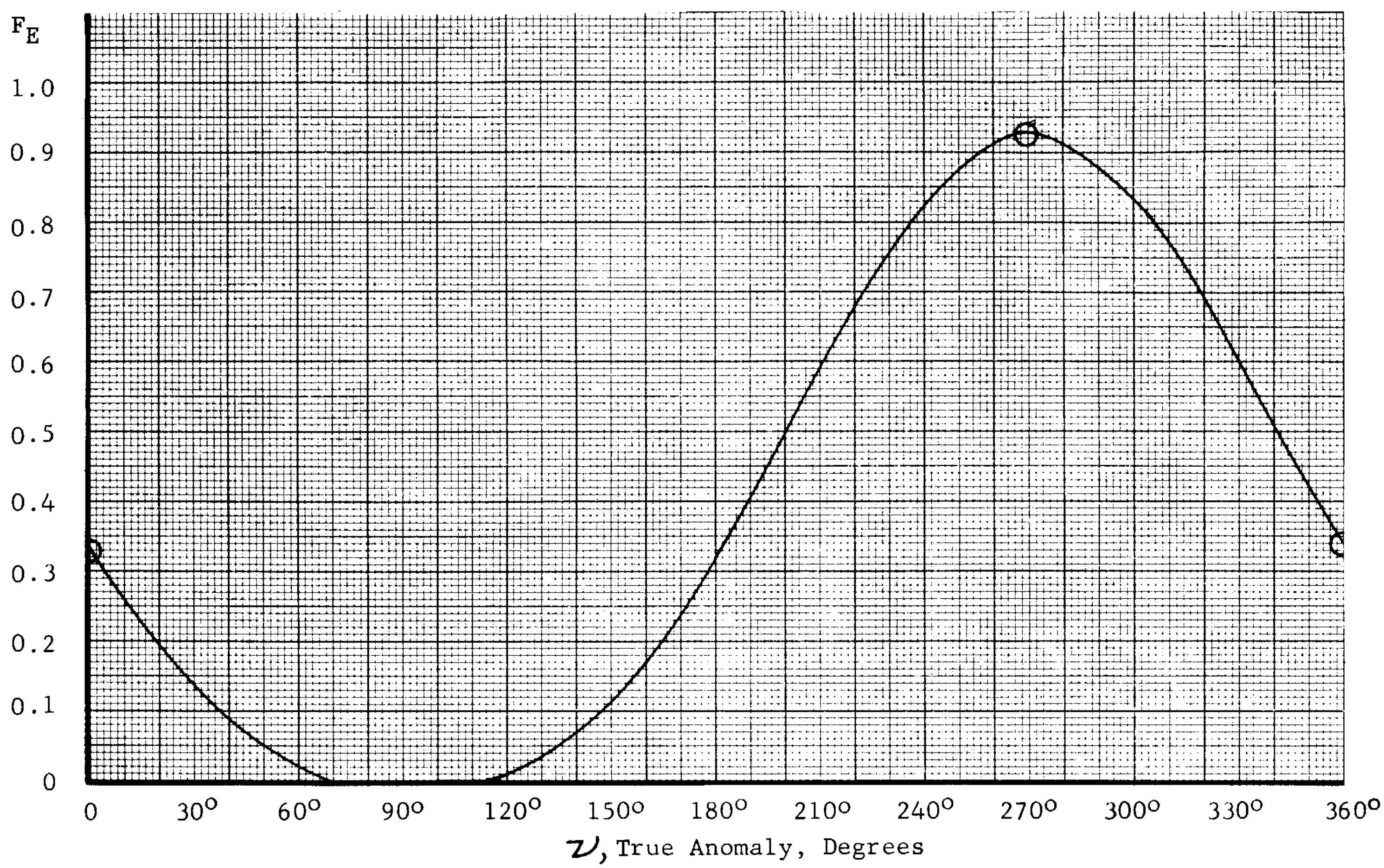

Figure 22. Comparison of View Factors to Flat Plate Oriented Normal to Flight Path at Perigee

APPENDIX A

VIEW FACTORS

The standard equation for the radiation emitted by black body A_1 which falls on the body A_2 is

$$\vec{Q}_{1\rightarrow 2} = \frac{\sigma T_1^4}{\pi} \int_{A_1} \int_{A_2} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} \, dA_1 \, dA_2 \qquad (1A)$$

The integral in this equation is usually combined with either the area of the receiver, or the emitter, and is referred to as the view factor. If the area of the emitter is used, the physical meaning of the view factor is more obvious. Multiplying the numerator and denominator of the R.H.S. of the equation by A_1 gives

$$\vec{Q}_{1\rightarrow 2} = \sigma T_1^4 \; A_1 \; F_{12} \qquad (2A)$$

where

$$F_{12} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} \, dA_1 \, dA_2 \qquad (3A)$$

The term $\sigma T_1^4 \; A_1$ is the total energy radiated by area A_1 and, therefore, the view factor represents the fraction that is intercepted by A_2. However, it is equally correct to base the view factor on the area of the receiver, if the numerator and denominator of the R.H.S. of equation 1A are multiplyed by A_2. Then

$$\vec{Q}_{1\rightarrow 2} = \sigma T_1^4 \; A_2 \; F_{21} \qquad (4A)$$

where

$$F_{21} = \frac{1}{\pi A_2} \int_{A_1} \int_{A_2} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \, dA_2 \qquad (5A)$$

and the order of the subscripts on the view factor is reversed to indicate that the reference used in the definition is the area A_2. A comparison of equations 5A and 3A shows the following relationship to exist:

$$A_2 F_{21} = A_1 F_{12} \qquad (6A)$$

This is known as the reciprocity theorem.

In this report the area of the receiver has been used to define the view factor and it is assumed that the integration is piecewise independent of the area A_2. That assumption simplifies equation 5A to

$$F_{21} = \frac{1}{\pi} \int_{A_1} \frac{\cos \gamma_1 \cos \gamma_2}{|\vec{r}_{12}|^2} dA_1 \qquad (7A)$$

The equation for the thermal radiation received is unchanged, retaining the form of equation 4A.

Another identifying property of the view factor is its numerical value which ranges from zero to plus one. It should also be recognized from studying the form of the radiation heat transfer equation, since the product of the intensity of the source and the view factor gives the incident radiant energy per unit area and time.

APPENDIX B

COORDINATE TRANSFORMATION

Using the transformation equations 47 through 49, the unit vectors of the satellite triad may be expressed in terms of the geocentric coordinate system. The method consists of successively applying these equations until the desired result is obtained. Consider the $\vec{i_p}$ vector.

$$\vec{i_p} = \cos\omega\,(\vec{i_{10}}) + \sin\omega\,(\vec{i_{11}}) + (0)\,(\vec{i_{12}}) \tag{1B}$$

or substituting for $\vec{i_{10}}$, and $\vec{i_{11}}$, then

$$\begin{aligned}\vec{i_p} = {} & \cos\omega\left[(1)\,(\vec{i_7}) + (0)\,(\vec{i_8}) + (0)\,(\vec{i_9})\right] + \\ & \sin\omega\left[(0)\,(\vec{i_7}) + \cos i\,(\vec{i_8}) + \sin i\,(\vec{i_9})\right]\end{aligned} \tag{2B}$$

Expressing $\vec{i_7}$, $\vec{i_8}$, $\vec{i_9}$ in terms of earth triad gives

$$\begin{aligned}\vec{i_p} = {} & \cos\omega\left[\cos\Omega\,(\vec{i_1}) + \sin\Omega\,(\vec{i_2}) + (0)\,(\vec{i_3})\right] + \\ & \sin\omega\cos i\left[-\sin\Omega\,(\vec{i_1}) + \cos\Omega\,(\vec{i_2}) + (0)\,(\vec{i_3})\right] + \\ & \sin\omega\sin i\left[(0)\,(\vec{i_1}) + (0)\,(\vec{i_2}) + (1)\,(\vec{i_3})\right]\end{aligned} \tag{3B}$$

which simplifies to

$$\begin{aligned}\vec{i_p} = {} & \left[\cos\omega\cos\Omega - \sin\Omega\sin\omega\cos i\right]\,(\vec{i_1}) + \\ & \left[\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega\right]\,(\vec{i_2}) + \\ & \left[\sin\omega\sin i\right]\,(\vec{i_3})\end{aligned} \tag{4B}$$

The remaining unit vectors are derived similarly and are obtained as

$$\vec{j_p} = \left[- \sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega\right] (\vec{i_1}) + \left[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega\right] (\vec{i_2}) + \left[\cos \omega \sin i\right] (\vec{i_3}) \tag{5B}$$

and

$$\vec{k_p} = \left[\sin i \sin \Omega\right] (\vec{i_1}) + \left[- \sin i \cos \Omega\right] (\vec{i_2}) + \left[\cos i\right] (\vec{i_3}) \tag{6B}$$

The earth-oriented triad may be expressed in terms of the perigee system by similar methods. The results are

$$\vec{i}_{E.O.} = - \sin \nu (\vec{i_p}) + \cos \nu (\vec{j_p}) \tag{7B}$$

$$\vec{j}_{E.O.} = - \cos \nu (\vec{i_p}) - \sin \nu (\vec{j_p}) \tag{8B}$$

$$\vec{k}_{E.O.} = \vec{k_p} \tag{9B}$$

The equations for the sun-oriented triad are

$$\vec{i}_{S.O.} = - \sin \Gamma (\vec{i_p}) - \cos \Gamma (\vec{j_p}) \tag{10B}$$

$$j_{S.O.} = \sin \xi \cos \Gamma (i_p) - \sin \xi \sin \Gamma (j_p) + \cos \xi (k_p) \tag{11B}$$

and

$$\vec{k}_{S.O.} = - \cos \xi \cos \Gamma (\vec{i_p}) + \cos \xi \sin \Gamma (\vec{j_p}) + \sin \xi (\vec{k_p}) \tag{12B}$$

APPENDIX C

SATELLITE SURFACE UNIT NORMALS

The surface unit normals for a cylindrical satellite with hemispherical ends, oriented toward the earth, are given below

Cylinder:

$$
\begin{aligned}
\vec{N_2} = & - \sin \nu \left\{ \cos \zeta \left[\cos \omega \cos \Omega - \sin \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right. \\
& \cos \zeta \left[\cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega \right] (\vec{i_2}) + \\
& \left. \cos \zeta \left[\sin \omega \sin i \right] (\vec{i_3}) \right\} + \\
& \cos \nu \left\{ \cos \zeta \left[- \sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right. \\
& \cos \zeta \left[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \right] (\vec{i_2}) + \\
& \left. \cos \zeta \left[\cos \omega \sin i \right] (\vec{i_3}) \right\} + \\
& \sin \zeta \left[\sin i \sin \Omega \right] (\vec{i_1}) + \sin \xi \left[- \sin i \cos \Omega \right] (\vec{i_2}) + \\
& \left. \sin \zeta \left[\cos i \right] (\vec{i_3}) \right\}
\end{aligned}
\tag{1C}
$$

Positive Hemisphere:

$$
\begin{aligned}
\vec{N_2} = & \cos \nu \sin \mu \sin \rho \left\{ \left[- \sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right. \\
& \left[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \right] (\vec{i_2}) + \\
& \left. \left[\cos \omega \sin i \right] (\vec{i_3}) \right\} +
\end{aligned}
$$

$$- \sin\nu \sin\mu \sin\rho \left\{ \left[\cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega \right] (\vec{i_2}) +$$

$$\left. \left[\sin\omega \sin i \right] (\vec{i_3}) \right\} +$$

$$- \cos\nu \cos\rho \left\{ \left[\cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega \right] (\vec{i_2}) +$$

$$\left. \left[\sin\omega \sin i \right] (\vec{i_3}) \right\} + \qquad (2C)$$

$$- \sin\nu \cos\rho \left\{ \left[- \sin\omega \cos\Omega - \cos\omega \cos i \sin\Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos\omega \cos i \cos\Omega - \sin\omega \sin\Omega \right] (\vec{i_2}) +$$

$$\left. \left[\cos\omega \sin i \right] (\vec{i_3}) \right\} +$$

$$\cos\mu \sin\rho \left\{ \left[\sin i \sin\Omega \right] (\vec{i_1}) + \left[- \sin i \cos\Omega \right] (\vec{i_2}) + \right.$$

$$\left. \left[\cos i \right] (\vec{i_3}) \right\}$$

Negative Hemisphere:

$$\vec{N_2} = \cos\nu \sin\psi \sin\phi \left\{ \left[- \sin\omega \cos\Omega - \cos\omega \cos i \sin\Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos\omega \cos i \cos\Omega - \sin\omega \sin\Omega \right] (\vec{i_2}) +$$

$$\left. \left[\cos\omega \sin i \right] (\vec{i_3}) \right\} +$$

$$- \sin\nu \sin\psi \sin\phi \left\{ \left[\cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega \right] (\vec{i_2}) +$$

$$\left. \left[\sin\omega \sin i \right] (\vec{i_3}) \right\} +$$

$$- \cos \nu \cos \phi \left\{ \left[\cos \omega \cos \Omega - \sin \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega \right] (\vec{i_2}) +$$

$$\left. \left[\sin \omega \sin i \right] (\vec{i_3}) \right\} + \qquad (3C)$$

$$- \sin \nu \cos \phi \left\{ \left[-\sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \right] (\vec{i_2}) +$$

$$\left. \left[\cos \omega \sin i \right] (\vec{i_3}) \right\} +$$

$$\cos \psi \sin \phi \left\{ \left[\sin i \sin \Omega \right] (\vec{i_1}) + \left[- \sin i \cos \Omega \right] (\vec{i_2}) + \right.$$

$$\left. \left[\cos i \right] (\vec{i_3}) \right\}$$

The equations for the sun-oriented vehicle are given below.

Cylinder:

$$\vec{N_2} = - \sin \Gamma \cos \zeta \left\{ \left[\cos \omega \cos \Omega - \sin \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega \right] (\vec{i_2}) +$$

$$\left. \left[\sin \omega \sin i \right] (\vec{i_3}) \right\} +$$

$$- \cos \Gamma \cos \zeta \left\{ \left[- \sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \right] (\vec{i_1}) + \right.$$

$$\left[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \right] (\vec{i_2}) +$$

$$\left. \left[\cos \omega \sin i \right] (\vec{i_3}) \right\} +$$

$$- \sin\zeta \cos\xi \cos\Gamma \left\{ \left[\cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega\right] (\vec{i_1}) + \left[\cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega\right] (\vec{i_2}) + \left[\sin\omega \sin i\right] (\vec{i_3}) \right\} + \tag{4C}$$

$$\sin\zeta \cos\xi \sin\Gamma \left\{ \left[- \sin\omega \cos\Omega - \cos\omega \cos i \sin\Omega\right] (\vec{i_1}) + \left[\cos\omega \cos i \cos\Omega - \sin\omega \sin\Omega\right] (\vec{i_2}) + \left[\cos\omega \sin i\right] (\vec{i_3}) \right\} +$$

$$\sin\xi \sin\zeta \left\{ \left[\sin i \sin\Omega\right] (\vec{i_1}) + \left[- \sin i \cos\Omega\right] (\vec{i_2}) + \left[\cos i\right] (\vec{i_3}) \right\} +$$

Positive Hemisphere:

$$\vec{N_2} = - \sin\Gamma \sin\mu \sin\rho \left\{ \left[\cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega\right] (\vec{i_1}) + \left[\cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega\right] (\vec{i_2}) + \left[\sin\omega \sin i\right] (\vec{i_3}) \right\} +$$

$$- \cos\Gamma \sin\mu \sin\rho \left\{ \left[- \sin\omega \cos\Omega - \cos\omega \cos i \sin\Omega\right] (\vec{i_1}) + \left[\cos\omega \cos i \cos\Omega - \sin\omega \sin\Omega\right] (\vec{i_2}) + \left[\cos\omega \sin i\right] (\vec{i_3}) \right\} +$$

$$\sin\xi \cos\Gamma \cos\rho \left\{ \left[\cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega\right] (\vec{i_1}) + \left[\cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega\right] (\vec{i_2}) + \left[\sin\omega \sin i\right] (\vec{i_3}) \right\} +$$

$$\cos\xi \cos\rho \left\{ \left[\sin i \sin\Omega\right](\vec{i_1}) + \left[-\sin i \cos\Omega\right](\vec{i_2}) + \right.$$

$$\left. \left[\cos i\right](\vec{i_3}) \right\} + \tag{5C}$$

$$-\sin\xi \sin\Gamma \cos\rho \left\{ \left[-\sin\omega\cos\Omega - \cos\omega\cos i \sin\Omega\right](\vec{i_1}) + \right.$$

$$\left[\cos\omega\cos i\cos\Omega - \sin\omega\sin\Omega\right](\vec{i_2}) +$$

$$\left. \left[\cos\omega \sin i\right](\vec{i_3}) \right\} +$$

$$-\cos\xi \cos\Gamma \cos\mu \sin\rho \left\{ \left[\cos\omega\cos\Omega - \sin\omega\cos i \sin\Omega\right](\vec{i_1}) + \right.$$

$$\left[\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega\right](\vec{i_2}) +$$

$$\left. \left[\sin\omega\sin i\right](\vec{i_3}) \right\} +$$

$$\cos\xi \sin\Gamma \cos\mu \sin\rho \left\{ \left[-\sin\omega\cos\Omega - \cos\omega\cos i \sin\Omega\right](\vec{i_1}) + \right.$$

$$\left[\cos\omega\cos i\cos\Omega - \sin\omega\sin\Omega\right](\vec{i_2}) +$$

$$\left. \left[\cos\omega \sin i\right](\vec{i_3}) \right\} +$$

$$\sin\xi \cos\mu \sin\rho \left\{ \left[\sin i \sin\Omega\right](\vec{i_1}) + \left[-\sin i \cos\Omega\right](\vec{i_2}) + \right.$$

$$\left. \left[\cos i\right](\vec{i_3}) \right\} +$$

Negative Hemisphere:

$$\vec{N_2} = -\sin\Gamma \sin\psi \sin\phi \left\{ \left[\cos\omega\cos\Omega - \sin\omega\cos i \sin\Omega\right](\vec{i_1}) + \right.$$

$$\left[\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega\right](\vec{i_2}) +$$

$$\left. \left[\sin\omega\sin i\right](\vec{i_3}) \right\} +$$

$$- \cos \Gamma \sin \psi \sin \phi \Big\{ \Big[-\sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \Big] (\vec{i_1}) +$$

$$\Big[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \Big] (\vec{i_2}) +$$

$$\Big[\cos \omega \sin i \Big] (\vec{i_3}) \Big\} +$$

$$\sin \xi \cos \Gamma \cos \phi \Big\{ \Big[\cos \omega \cos \Omega - \sin \omega \cos i \sin \Omega \Big] (\vec{i_1}) +$$

$$\Big[\cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega \Big] (\vec{i_2}) +$$

$$\Big[\sin \omega \sin i \Big] (\vec{i_3}) \Big\} +$$

$$\cos \xi \cos \phi \Big\{ \Big[\sin i \sin \Omega \Big] (\vec{i_1}) + \Big[-\sin i \cos \Omega \Big] (\vec{i_2}) +$$

$$\Big[\cos i \Big] (\vec{i_3}) \Big\} + \qquad (6C)$$

$$- \sin \xi \sin \Gamma \cos \phi \Big\{ \Big[-\sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \Big] (\vec{i_1}) +$$

$$\Big[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \Big] (\vec{i_2}) +$$

$$\Big[\cos \omega \sin i \Big] (\vec{i_3}) \Big\} +$$

$$- \cos \xi \cos \Gamma \cos \psi \sin \phi \Big\{ \Big[\cos \omega \cos \Omega - \sin \omega \cos i \sin \Omega \Big] (\vec{i_1}) +$$

$$\Big[\cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega \Big] (\vec{i_2}) +$$

$$\Big[\sin \omega \sin i \Big] (\vec{i_3}) \Big\} +$$

$$\cos \xi \sin \Gamma \cos \psi \sin \phi \Big\{ \Big[-\sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega \Big] (\vec{i_1}) +$$

$$\Big[\cos \omega \cos i \cos \Omega - \sin \omega \sin \Omega \Big] (\vec{i_2}) +$$

$$\Big[\cos \omega \sin i \Big] (\vec{i_3}) \Big\} +$$

$$\sin \xi \ \cos \psi \ \sin \phi \left\{ \left[\sin i \sin \Omega \right] (\vec{i_1}) + \left[- \sin i \cos \Omega \right] (\vec{i_2}) + \left[\cos i \right] (\vec{i_3}) \right\}$$

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