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*N63-12159
code-1*

TECHNICAL NOTE

D-1546

THE IMPEDANCE OF AN ELECTRICALLY SHORT ANTENNA IN THE IONOSPHERE

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON

January 1963

6/20/11

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SUMMARY

Measurements from rocket flights are used to show that the impedance of a short antenna operating above the local plasma frequency can be predicted if the following factors are considered:

1. The formation of an ion sheath around the antenna and the vehicle,
2. The enhancement of this sheath when large RF voltages are applied to the antenna, and
3. The loading that apparently arises from electroacoustic waves excited in the medium by the RF field close to the antenna.

Sufficient design data are included to calculate the effect of the ionosphere on both the resistive and reactive parts of the impedance of a short antenna.

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INTRODUCTION

An investigation on the use of RF impedance probes for measuring ionospheric electron densities has disclosed several important factors affecting the impedance of an antenna immersed in the ionosphere. If the ratio of the operating frequency to the electron gyrofrequency is sufficiently high that magnetic field effects may be neglected, then these factors are the following:

1. The capacitive part of the antenna impedance is affected as though the antenna were immersed in a medium with a dielectric constant given by

$$K = 1 - \frac{\omega_p^2}{\omega^2} \quad (1)$$

(where ω_p and ω are the plasma and operating frequencies), except that there is a sheath region around the antenna where the electron density is usually less than ambient because of the negative potential acquired by the vehicle. If the sheath's dimensions are known, a correction can be made for this effect.

2. Unless the RF voltage applied to the antenna is small, the ionospheric electrons tend to leave the vicinity of the antenna; this creates an enhanced ion sheath, which may be many times the size of that present with no RF field. Computations of the size of the sheath formed in this way agree with experimental observations.

3. The resistive part of the antenna impedance contains at least two terms: The first, arising from the electromagnetic radiation, varies approximately as \sqrt{K} ; while the second, which can be quantitatively explained as arising from an electroacoustic type of radiation, varies approximately as $(1 - K) / \sqrt{K}$.

*Presented at The Institute of Physics and The Physical Society Conference on "The Ionosphere," London, England, July 1962.

†On leave from the University of Auckland, New Zealand, with NASA Post-Doctoral Research Associateship from the National Academy of Science.

EXPERIMENTAL REACTANCE MEASUREMENTS

The most important of the measurements used as a basis for this paper are those made on Naval Research Laboratory (Reference 1) rocket flight NN 3.08 F and on NASA rocket flight 4.07 (Reference 2). The type of data obtained is exemplified in Figure 1, where the apparent electron density was derived from Equation 1 in the form

$$K' = 1 - \frac{Ne^2}{\epsilon_0 m \omega^2}, \quad (2)$$

where

- K' = apparent dielectric constant of the medium,
- N = density in electrons per cubic meter,
- e = the electronic charge,
- m = the electronic mass, and
- $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ farad/meter.

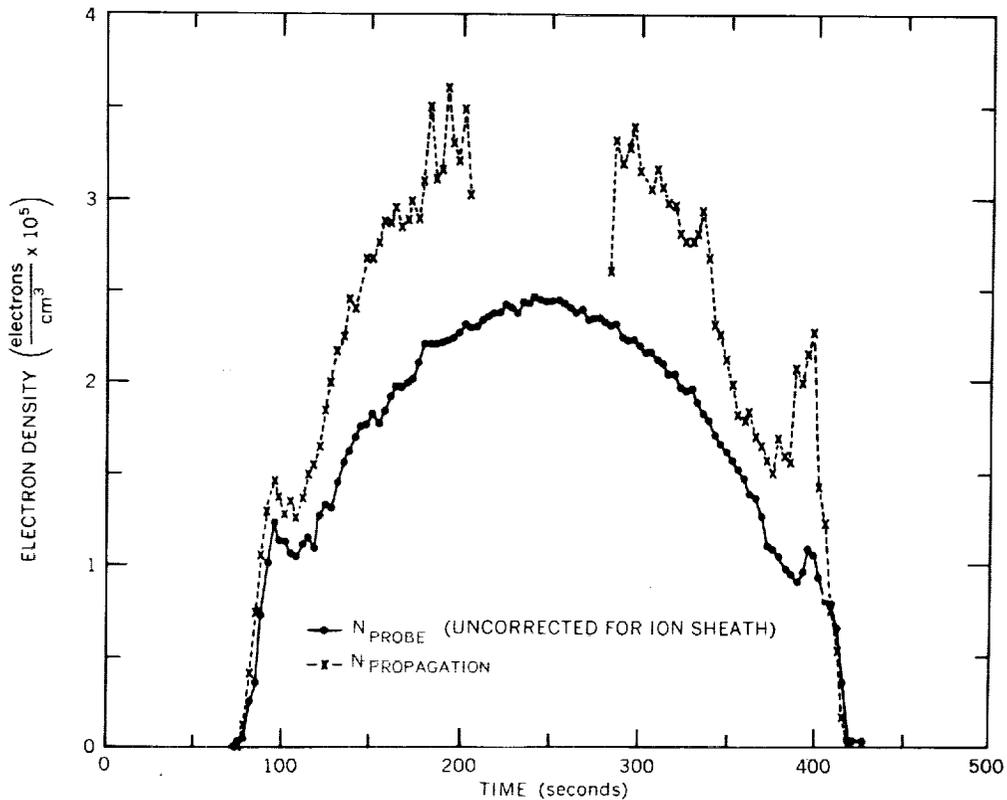


Figure 1—Curves of electron density versus flight time for the RF probe and CW propagation methods (NASA Rocket 4.07, Churchill, Canada, 1128 CST, September 14, 1959)

Since the antennas were electrically short, the value of K' was obtained from

$$K' C_0 = C , \quad (3)$$

where C is the measured antenna capacity in the ionosphere, and C_0 the free space antenna capacity.

A more general treatment indicates that the dielectric constant of the medium in a direction ψ with respect to the earth's magnetic field is given by

$$K = 1 - \frac{X}{1 - Y^2} + \frac{XY^2}{1 - Y^2} \cos^2 \psi , \quad (3a)$$

where

$$X = \frac{\omega_p^2}{\omega^2} ,$$

$$Y = \frac{\omega_H}{\omega} , \text{ and}$$

ω_H = the electron gyrofrequency.

Equation 3a indicates that the effective dielectric constant of the medium, when investigated with a capacitor of any geometry, should appear to be of the form (Reference 2)

$$K = 1 - \frac{X}{1 - Y^2} + \frac{XY^2}{1 - Y^2} G , \quad (3b)$$

where G is a factor which depends on the geometry and orientation of the capacitor and is less than unity. Under the conditions of the NASA 4.07 experiment, the difference in electron density obtained by using Equation 3b instead of the simplified expression, Equation 1, would be less than 4 percent.

The comparison curve showing the true electron density (Figure 1) was obtained by a CW propagation method (Reference 3) which, under the conditions chosen for this experiment, yielded very accurate results for the major portion of the ascending trajectory.

If the ion sheath that formed around the antennas on the NASA 4.07 flight is assumed to have been cylindrical, the two results shown in Figure 1 can be reconciled. It is difficult to assign an exact value to the sheath radius, and a comparison of the results obtained by different methods is shown in Figure 2. Here, the dots are the values of sheath radius (in meters) calculated from the difference between the apparent and true dielectric constant of the ionosphere. The solid line is obtained from a measurement of the vehicle potential (measured as approximately -1 volt with respect to the plasma potential), while the dashed line is derived from Jastrow and Pearse's (Reference 4) theoretical treatment of the problem. Some error in the measured results was inevitable in this particular experiment because of the complicated geometrical configuration near the base of the antenna.

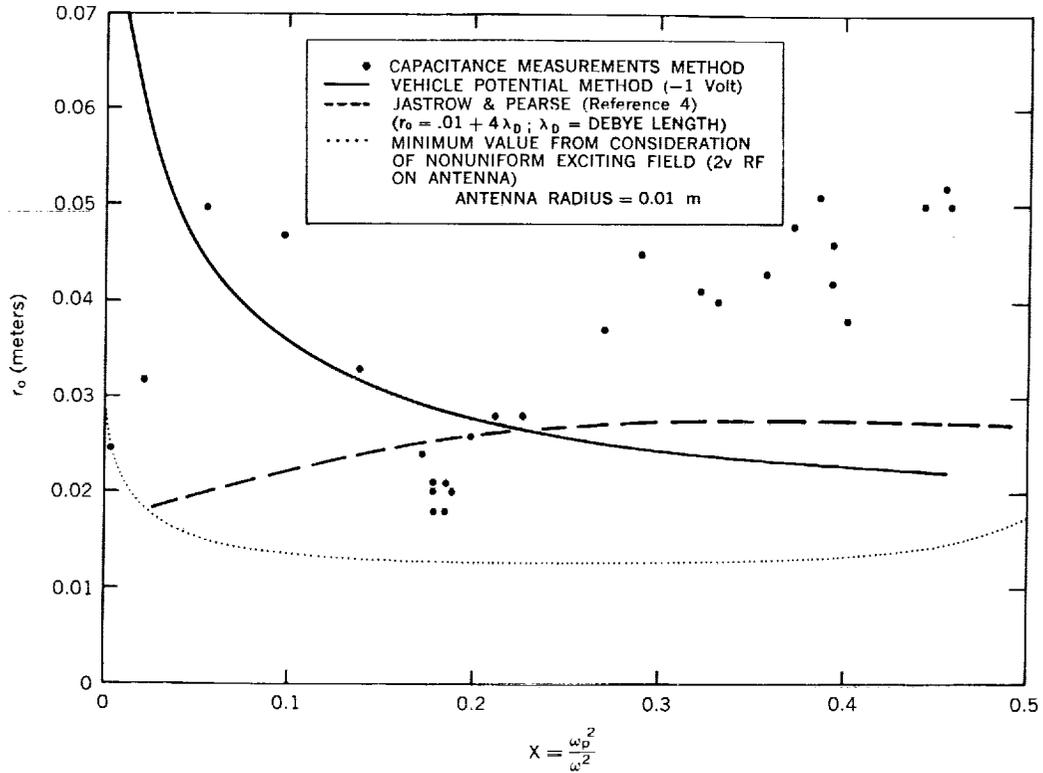


Figure 2—Sheath radius obtained by different methods

It should be noted that, with the 2-volt peak RF voltage used on the antenna in these measurements, the sheath radius was of the order of 2 to 5 centimeters. In two other experiments, when the apparent dielectric constant was obtained with a 200-volt peak on the antenna, much larger sheaths, generally of the order of 15 centimeters, were obtained. The mechanism responsible for these large sheath distances is described in the following section.

SHEATH FORMATION BY LARGE RF VOLTAGES

At distances from the cylindrical antenna that are small compared with the antenna length, the RF is essentially perpendicular to the antenna axis and is inversely proportional to the distance from the axis. The electron motion will be along the field lines but, because of the field gradient, will not be simply sinusoidal. The net effect of this nonlinearity is that the electrons tend to move away from the antenna, and leave an electron-deficient region, or sheath, near the antenna.

The differential equation describing the motion of the electrons near the sheath edge is of the form

$$\ddot{r} = -\frac{\omega_p^2}{2} r + \frac{eE_0}{mr} \cos \omega t, \quad (4)$$

where r is the instantaneous distance of the electron from the antenna axis. Solution of this nonlinear differential equation by numerical methods gives the mean radius of the sheath for various values of ω_p (plasma frequency of the medium) and E_0 (peak radial field strength from the antenna axis). The results of this analysis are plotted in Figure 3, where \bar{r}_0 is the average sheath radius (i.e., time-averaged over one RF cycle) and X is the normalized electron density. The quantity B is given by

$$B = \frac{eE_0}{m\omega^2} \quad (5)$$

and is numerically equivalent to the amplitude of vibration that a single electron would have in an RF field of peak field strength E_0 at a frequency $\omega/2\pi$. For easy computation,

$$B = \frac{E_0}{224.3f^2} \text{ meters,} \quad (6)$$

where f is the operating frequency in Mc.

Comparison of the computed effective sheath radius with experimental measurements is shown in Figure 4, where all the results have been normalized to $B = 10^{-2}$. Of the two sets of results shown,

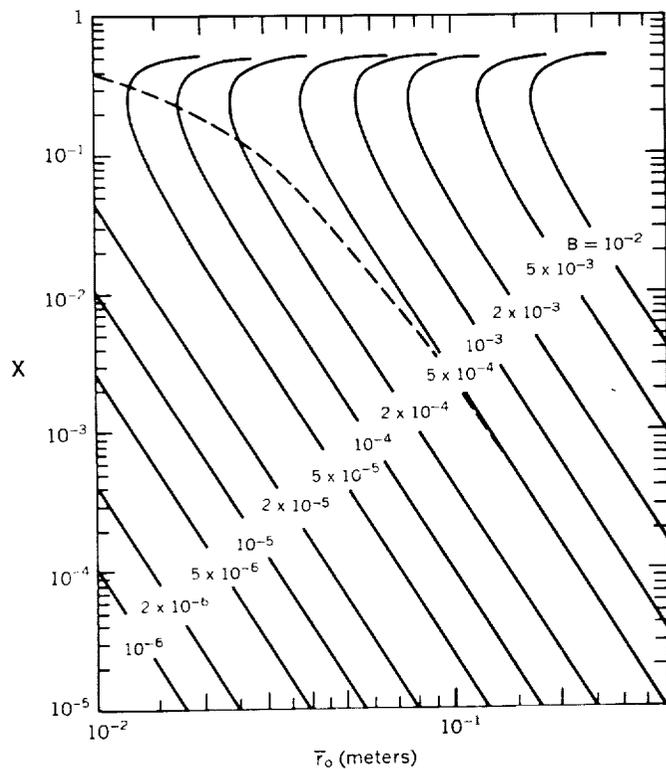


Figure 3—Effective sheath radius \bar{r}_0 as a function of $X = \omega_p^2/\omega^2$ for various values of the parameter B (Dashed line shows the closest distance from the antenna for electrons vibrating about the mean position \bar{r}_0 for $B = 5 \times 10^{-4}$.)

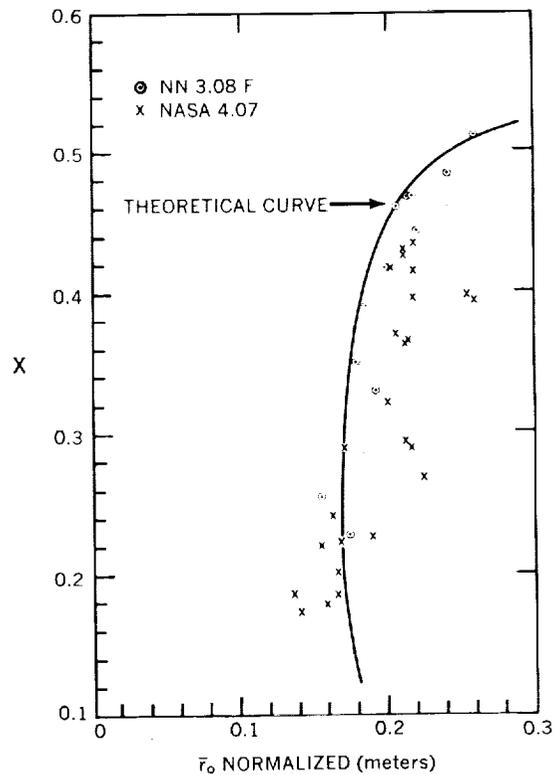


Figure 4—Experimental measurements of sheath radius normalized to $B = 10^{-2}$

the NN 3.08 F data are more accurate, since the method of measurement permitted allowance for the changes in the resistive component of the antenna impedance. In the NASA 4.07 measurements at high RF voltages, the data were obtained from the variation of the antenna voltage (i.e., detuning from resonance) only.

It is to be noted that, since the controlling parameter B is inversely proportional to the frequency squared, the effect of sheath formation by the applied RF voltage is very marked at low frequencies.

RESISTIVE COMPONENT OF ANTENNA IMPEDANCE

The radiation resistance for the electromagnetic radiation of a small dipole of constant dipole moment, when immersed in a dielectric medium of dielectric constant κ , is proportional to $\sqrt{\kappa}$. Under the conditions of the NASA 4.07 experiment, the relative amplitude of the peak obtained as the output circuit is tuned through resonance should then be as shown in the two upper curves in Figure 5. The parameter r_0 , referring to the sheath radius, appears in the upper curves, since it affects the impedance of the antenna and hence the current flowing in it. With the two collinear 3-meter antennas

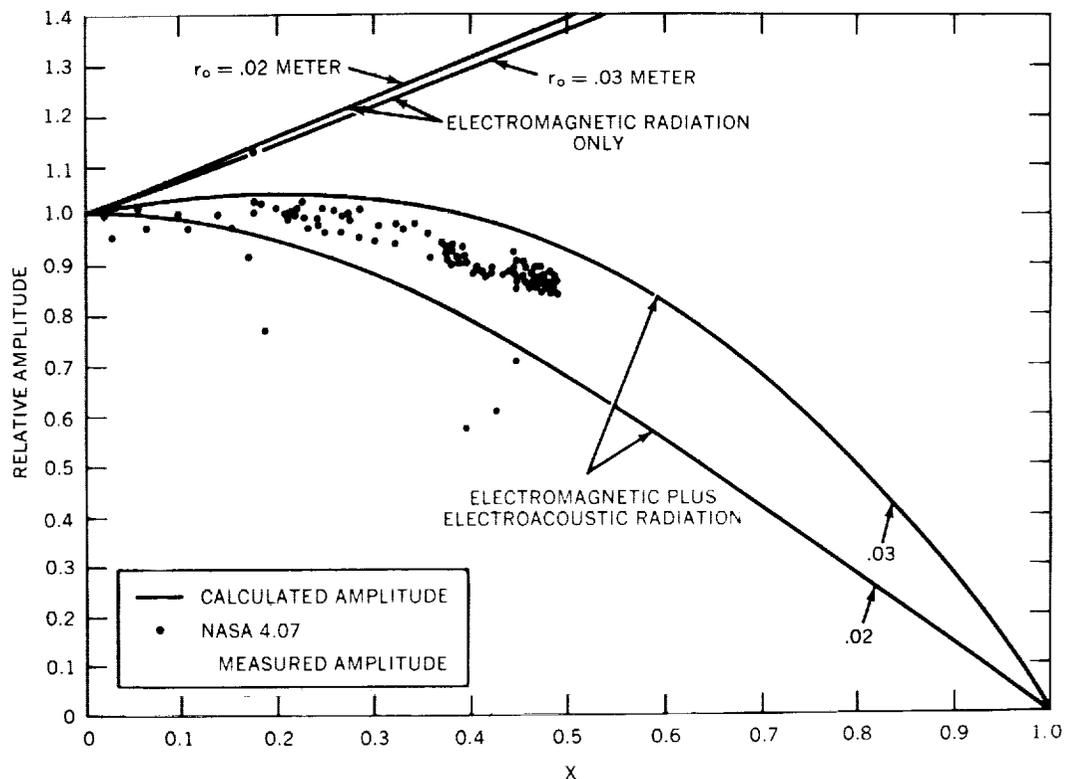


Figure 5—Comparison of the calculated variation of the amplitude of the resonance peak as a function of X with the observed data points. The curves are computed for values of ion sheath radius, 0.02 and 0.03 meter

operating at 7.75 Mc, the power P radiated as electromagnetic radiation is calculated as

$$P = 1.4 \times 10^{-5} K'^2 \sqrt{K} V^2 \text{ watts ,} \quad (7)$$

where

- K' = apparent dielectric constant obtained by the antenna reactance measurement,
- K = ambient dielectric constant of the medium,
- V = 1/2 the peak RF voltage applied between the antenna terminals.

The actual measured relative amplitude of the resonance peak is shown by the data points in Figure 5. There is apparently some mechanism in addition to electromagnetic radiation operating in absorbing power from the antenna. On the assumption that this power is radiated as an electron-pressure (electroacoustic) wave excited in the medium by the RF field close to the antenna, computation gives, for this additional power Q,

$$Q = \frac{(1 - K)v}{4\pi\epsilon_0 r_0 \sqrt{K}} B^2(kr_0) V^2 C^2 \text{ watts,} \quad (8)$$

where

$v = \sqrt{\gamma\kappa T/m}$, a velocity corresponding to the sound wave phase velocity in a neutral gas; γ is the ratio of specific heats, κ the Boltzmann constant, T the electron temperature, and m the electron mass;

$B(kr_0)$ = a factor accounting for the simultaneous excitation of all the electrons in the vicinity of the antenna, with

k = the propagation constant of the longitudinal electron pressure wave and

r_0 = the effective radius of the ion sheath boundary;

C = capacity of the antenna per unit length (farads/meter); and

$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ farad/meter.

The total power radiated (with 1-volt antenna voltage) for the conditions of the NASA 4.07 experiment is then given by

$$P + Q = 1.4 \times 10^{-5} K'^2 \sqrt{K} + 3.6 \times 10^{-6} \frac{1 - K}{\sqrt{K}} \frac{B^2(kr_0)}{r_0} \text{ watts.} \quad (9)$$

By this formula, the calculated relative amplitude of the resonance peaks would then be as shown in the two lower curves in Figure 5. It is apparent that the assumption of an electroacoustic effect, calculated for a sheath radius of 0.025 meter, gives excellent agreement with the actual measurements.

CONCLUSIONS

In considering the impedance of an antenna in the ionosphere, due allowance must be made for the ion sheath that forms around the surface. An estimate of the size of the sheath can be made if the electron density and electron temperature are known or if the vehicle's potential with respect to the ionospheric plasma is known.

When a large RF voltage is applied to the antenna, an enhanced sheath may be formed — an effect particularly noticeable at high RF voltages and low frequencies. Since it arises from a nonlinear process, the reciprocal relations between transmitting and receiving antennas are no longer valid for antennas immersed in the ionosphere.

The resistive component of the antenna impedance contains terms arising from the radiation of power both as electromagnetic waves and as longitudinal electron-pressure (electroacoustic) waves. The two effects (P and Q, Equations 7 and 8) become of comparable magnitude when the dielectric constant of the medium drops to about 0.9; an important consequence of the electroacoustic mechanism is that the antenna should become very heavily damped at the plasma resonance frequency. Thus receiving antennas in the ionosphere may receive energy arising from the electron thermal motions via the electroacoustic wave with an efficiency considerably exceeding that with which they receive electromagnetic waves.

There is little evidence as to how well these electroacoustic waves propagate in the ionosphere. If the attenuation is not too high, they may be a very useful tool for investigating the temperature and structure of an ionized medium.

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