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TECHNICAL NOTE

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A STUDY OF THE EFFECT OF A DEADBAND ABOUT A DESIRED PERIGEE ON THE GUIDANCE OF A SPACE VEHICLE APPROACHING THE EARTH

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SUMMARY

An analysis is made of the guidance of a space vehicle which is approaching the earth at supercircular velocities and is attempting to control the trajectory to a desired vacuum perigee point. Perigee altitude was controlled by applying a corrective impulse each time the predicted perigee altitude deviated from the desired perigee altitude by a given amount; this given amount is called the deadband. Random errors were assumed in the measurement of velocity and flight-path angle and in obtaining the desired magnitude of thrust impulse. The error magnitudes and deadband described in NASA Technical Note D-957 were used for the present investigation.

It was found that the method of making a correction each time the deadband was exceeded yielded poorer perigee-altitude control at approximately the same (or slightly less) cost than did the correctional procedure of NASA Technical Note D-957 for which corrections were applied at a series of prescheduled points along the trajectory and for which no deadband was utilized.

INTRODUCTION

In order for a space vehicle approaching the earth at supercircular velocities to intercept the earth's atmosphere at desired entry conditions, it may be necessary for corrective thrust to be applied during midcourse guidance. Considerable research has been directed toward the solution of problems associated with guidance to a specified perigee altitude and the results of some of these studies are presented in references 1 to 6. The basic problem involves the guidance accuracies required to place the vehicle in a position to enter the earth's atmosphere successfully.

Reference 1 gives the results of a study of three methods of scheduling corrective-thrust impulses in the presence of random errors assumed in measuring velocity and flight-path angle and in obtaining the desired thrust impulse. Corrections were applied at preselected correction points along the trajectory. In reference 2, a similar series of scheduled correction points was employed, first, by applying a correction only if the predicted perigee altitude deviated from th desired perigee altitude by a certain amount (called the deadband) and, second, by applying a correction at each scheduled point (no deadband). The width of th deadband was decreased as the vehicle approached the earth, as were the errors i velocity and flight-path angle.

The guidance scheme hypothesized in reference 3 is based on a large number of prescheduled decision points and employs a constant deadband about the assume measured value of the trajectory angular rate. The general method of navigation used in references 5 and 6 is based on the perturbation theory wherein only devi tions in position and velocity from a reference path are utilized. The guidance equations which are dependent upon prescheduled decision points use linear predi tions of the final deviation to obtain the minimum required corrective velocity and were formulated by using optimal filtering of measured data.

It is of interest to note that each of the guidance schemes presented in references 1 to 6 was found capable of controlling the approach trajectory and required a relatively small corrective-velocity increment. One area of agreement among these studies is the fact that the final deviation from the desired perigee point depends on the location of the final correction point. It is also apparent from these results that corrective maneuvers should be executed at the longest feasible range inasmuch as the cost of delaying the corrective action becomes sub stantial as the range decreases.

In the present study the effect of a deadband in the guidance scheme upon control accuracy and total-corrective-velocity requirements is further investigated. The same formalization of a deadband as defined in reference 2 is employe and a correction is made whenever the predicted perigee altitude exceeds a boundary of the deadband. This method differs from reference 2 in that the corrective action is not made at preselected points along the approach trajectory. The magnitude of each correction was calculated on the basis of assumed measurements of position and velocity. According to a conclusion of reference 7, the scheduled correction points of reference 2 occur at near optimal frequency along the trajec tory. The purpose of this paper is to compare the results of the present study with the results of reference 2.

SYMBOLS

The English system of units is used in this study. In case conversion to metric units is desired, the following relationships apply: 1 foot = 0.3048 mete and 1 statute mile = 5,280 feet = 1,609.344 meters.

g acceleration due to gravity, 32.2 ft/sec^2

R radius of earth, 3,960 international statute miles

r radial distance from center of earth to vehicle, ft, except when used in determining the standard deviations of errors in V and γ and then in international statute miles ɔ radial distance from center of earth to perigee point of flight path, ft

ɔ,a radial distance from center of earth to perigee point of flight path after final correction, ft

p,d desired perigee radial distance, ft

velocity of space vehicle, ft/sec

d velocity at a given radial distance for the desired trajectory, ft/sec

T magnitude of corrective-velocity vector, ft/sec

V increment of velocity used to establish deadband, ft/sec

flight-path angle, deg

d flight-path angle at a given radial distance for the desired trajectory, deg

γ increment of flight-path angle used to establish deadband, deg

angle between a line from center of earth to space vehicle and perigee radius vector, deg

f magnitude of θ at final correction point, deg

θ change in θ, deg

standard deviation of normal distribution

$_{V}$ standard deviation of error in V, ft/sec

$_{V_T}$ standard deviation of error in V_T, percent V_T, ft/sec

$_{\gamma}$ standard deviation of error in γ, deg

Subscript:

) conditions that define initial trajectory

METHOD OF ANALYSIS

Approach Conditions and Assumptions

In all cases investigated in the present study, the space vehicle is approaching the earth on an elliptical path with an eccentricity of almost 1. A any point along the approach trajectory, the desired trajectory is that part of an ellipse which passes through the point and has a semimajor axis of 100,000 international statute miles and a perigee radius of $R + 250{,}000$ feet. This stu is concerned with the portion of the approach trajectory beginning at $\theta_o = 160^\circ$ and ending at the vacuum perigee radius, as shown in the diagram of figure 1.

The following assumptions, the same as those of references 1 and 2, are mad in this study:

(1) The earth is spherical.

(2) Motion is considered only in the plane of the orbit for a nonrotating earth.

(3) The space vehicle is close enough to the earth for the gravitation fiel of all other bodies to be neglected (a two-body problem).

Correction Technique

The present study is based on the application of a thrust impulse to contro the perigee altitude. The basic technique for controlling perigee altitude is t apply corrections throughout the course of the approach trajectory each time the predicted perigee altitude exceeds a boundary of a specified deadband about the desired perigee altitude. At any point along the approach trajectory the measur values of V and γ (obtained by adding random errors to the true values) are used to calculate the orbital characteristics. The calculated (predicted) perig radius is compared with a limit (the boundaries of the deadband) of the perigee radius to determine if a corrective impulse is needed. If a correction is indicated, calculations are then made to determine the optimum direction and magnitu of corrective velocity required to correct the trajectory to the center of the deadband. An assumed error in corrective velocity is added and the correction i applied in the optimum direction. The procedure given in reference 2 to determi the direction in which to apply corrective thrust was used in the present study.

The standard equations of an ellipse were used (ref. 2) to calculate the orbital characteristics of a space vehicle approaching the earth on an elliptical path. The following expression for the perigee radius in terms of the trajectory variables r, V, and γ (eq. (1) of ref. 2) was used to calculate the deadband within which the space vehicle was to be controlled:

$$r_p = \frac{gR^2\left[1 - \sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(\frac{2gR^2}{r} - V^2\right)}{\left(gR^2\right)^2}}\right]}{\frac{2gR^2}{r} - V^2}$$

where $V = V_d + \Delta V$ and $\gamma = \gamma_d - \Delta\gamma$ are used to determine r_p on one side of the deadband and $V = V_d - \Delta V$ and $\gamma = \gamma_d + \Delta\gamma$ are used to determine r_p on the other side.

Range of Initial Conditions

Two sets of assumed errors, the same as those investigated in reference 2, were used in the present study. These errors, which are errors in measuring velocity and flight-path angle and in applying corrective thrust, were assumed to have a normal distribution. The standard deviations of the errors investigated were:

First	Second
$\sigma_V = \frac{r}{10,000}$ ft/sec	$\sigma_V = \frac{3r}{10,000}$ ft/sec
$\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees	$\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees
$\sigma_{V_T} = 0.013V_T$ ft/sec	$\sigma_{V_T} = 0.039V_T$ ft/sec

Initial conditions were assumed so that without corrections perigee radii of 0.75R, 0.99R, 1.01R, 1.25R, 1.5R, and 2.0R would be obtained. The initial range was that associated with $\theta_o = 160^\circ$.

The deadband width studied for each set of instrumentation errors was that obtained when $\Delta V = \sigma_V$ and $\Delta\gamma = \sigma_\gamma$ and is the same as the "σ deadband" in reference 2. The r term in σ_V and σ_γ causes the width of the deadband to decrease as the vehicle approaches the earth.

Method of Control

For perigee-altitude control, the present investigation employed a corrective impulse at every point along the approach trajectory where the predicted perigee altitude exceeded a boundary of the deadband. In order to simulate the continuous predicted perigee altitude needed in the present study, the digital-computer program for the angular method of perigee-altitude control for the study reported in reference 2 was modified in the following manner. Small angular increments were used to schedule observation points along the approach trajectory. At these preselected points, the predicted perigee altitude was determined and compared with the deadband. If the predicted perigee altitude was not in the deadband, a straight-line approximation was made to determine where the predicted perigee altitude exceeded the deadband between the present and previous observation points. At the point where the predicted perigee altitude exceeded the deadband, the correctional maneuver was made and the new approach trajectory was determined.

In order to assess the effect of the straight-line approximation used to determine the point where the predicted perigee altitude exceeded the deadband, angular increments from 2° to 30° were investigated. The perigee altitude and total-corrective-velocity probability curves were the same for all angular increments investigated. Therefore angular increments of 30° were used for the study.

RESULTS AND DISCUSSION

General Discussion

The results of this study, presented as solid curves in figures 2 to 7, are shown as probability distribution curves. The results are shown for the two sets of errors and in the case of the smaller errors for two values of θ_f. The perigee-altitude probability curves, where each curve is based on 1,000 runs, are presented as the probability of the perigee-altitude error being less than a given value. Likewise, the total-corrective-velocity probability curves, where each curve is based on 100 runs, are presented as the probability of the total corrective velocity being less than a given value. For comparison with the present results, data obtained under the investigation reported in reference 2 for methods with and without a deadband are included in figures 2 to 7. Although all results of the present study and those of reference 2 are presented for a desired perigee altitude of 250,000 feet, these results are applicable to any desired perigee altitude.

Results of Correcting at Edge of Deadband

A number of values of $r_{p,o}$ were used in the analysis to determine the effect of applying a correction each time the predicted perigee altitude exceeded the boundary of the deadband about the desired perigee altitude. The results presented in figure 2, where the set of measurement errors is represented by $\sigma_V = \frac{r}{10,000}$ ft/sec, $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees, and $\sigma_{V_T} = 0.013V_T$ ft/sec, and where $\theta_f = 10^{\circ}$, show a perigee-altitude control within about $\pm 3,000$ feet. For the same set of measurement errors, but where no corrective action was taken beyond $\theta = 40^\circ$, the results presented in figure 4 show a perigee-altitude control within about ±10,000 feet. The total-corrective-velocity requirements for the perigee-altitude control shown in figures 2 and 4 are presented in figures 3 and 5.

Figures 6 and 7 show the perigee-altitude control and total-corrective-velocity requirements for the assumed measurement errors of $\sigma_V = \frac{3r}{10,000}$ ft/sec, $\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees, and $\sigma_{V_T} = 0.039V_T$ ft/sec, and for $\theta_f = 10^\circ$. These results show a perigee-altitude control within about ±10,000 feet.

Comparison of Results

In reference 2 - the results of which are here compared with those of the present study - a method of scheduling corrections at different values of the angle between perigee and the vehicle's position vector was investigated with and without a deadband. Cases for which errors, initial conditions, deadband, and the final observation point were the same as those of the present study were selected from reference 2 and are included here for comparison with the results of the present study. Attention is called to the step or abrupt change in slope of the total-corrective-velocity probability-distribution curves of some of the data taken from reference 2. As pointed out in reference 2, this step simply means that a certain percent of the runs required the same or approximately the same value of V_T.

Figures 2, 4, and 6 show that the perigee-altitude-error band for the correction procedure of reference 2 where no deadband was utilized was much smaller (approximately 50 percent for most cases) than the band for either of the correction procedures utilizing a deadband.

Figures 3, 5, and 7 show that the probability-distribution curves of total corrective velocity are approximately the same for the correction procedure of this study and the correction procedure without a deadband. The differences between the two methods are not too significant. However, it is indicated that for the runs with small errors and small values of θ_f (fig. 3) the V_T required was always slightly lower for the present procedure.

A comparison of the three control procedures leads to the conclusion that either the method without a deadband or the method of the present study would be the best regarding efficiency. However, if achieving a perigee altitude nearest to $r_{p,d}$ is required, then the method which uses no deadband is the best. In just about all cases investigated, the deadband procedure of reference 2 was poor in comparison with the other two methods either for efficient use of V_T or for close control of $r_{p,d}$.

CONCLUDING REMARKS

A study of the effects of employing a deadband about a desired perigee altitude on the guidance of a space vehicle approaching the earth was made. A corrective maneuver was made at any point along the approach trajectory where the predicted perigee altitude did not fall within the deadband. A comparison was made of these results and the results of the two procedures of perigee altitude control of NASA Technical Note D-957. These two procedures were as follows: (1) Corrective maneuvers were made at scheduled observation points along the trajectory if the predicted perigee altitude fell outside a deadband. (2) Corrective maneuvers were made at all scheduled observation points (no deadband).

By using a correction procedure which omitted the deadband (no deadband of NASA Technical Note D-957), the perigee-altitude control about the desired perigee value was better under all initial conditions, instrumentation inaccuracies, and location of the final correction point than either of the two correction procedures which included a deadband. The total-corrective-velocity requirements for the procedure which omitted the deadband and for the procedure in which a correction was made when the predicted perigee altitude exceeded a boundary of the deadband were, in general, approximately the same. The deadband procedure of NASA Technical Note D-957 was poor, in comparison with other procedures, either for efficient use of corrective velocity or for close control of perigee altitude.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., November 14, 1962.

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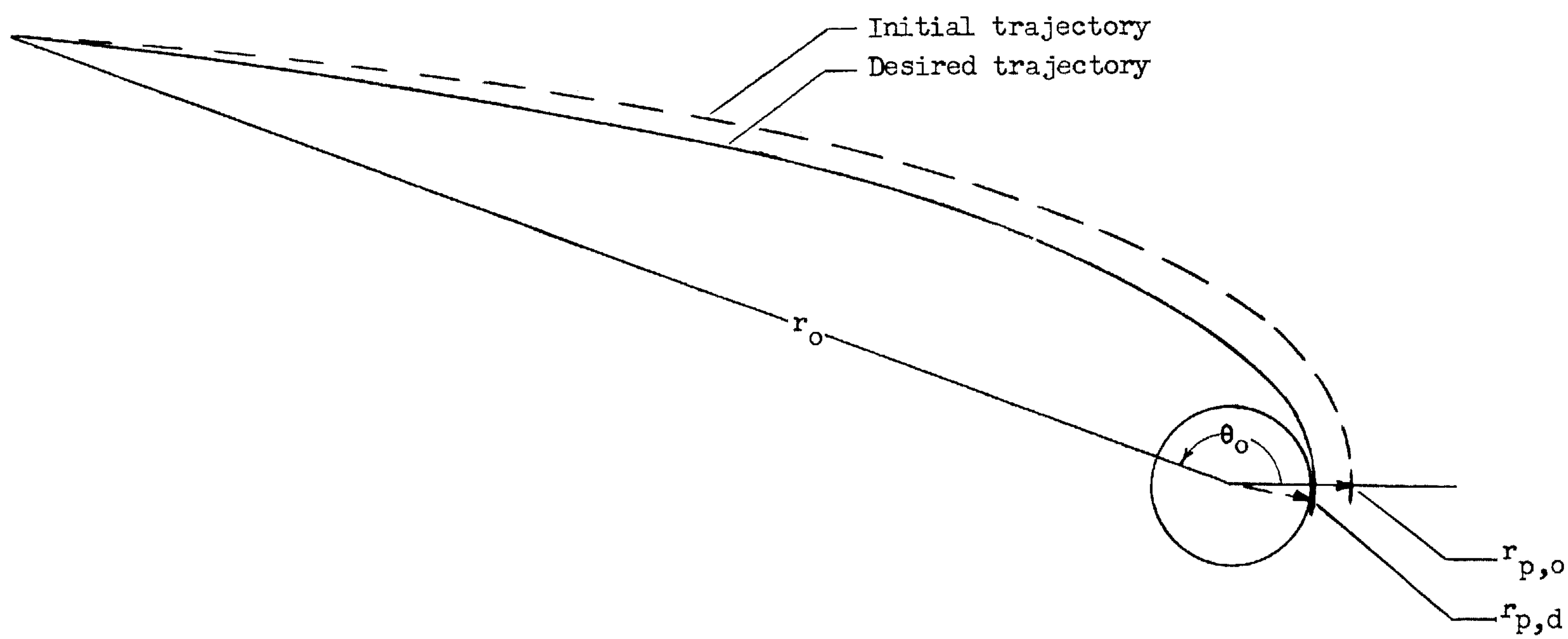


Figure 1.- Diagram of initial and desired trajectories. $\theta_o = 160^\circ$.

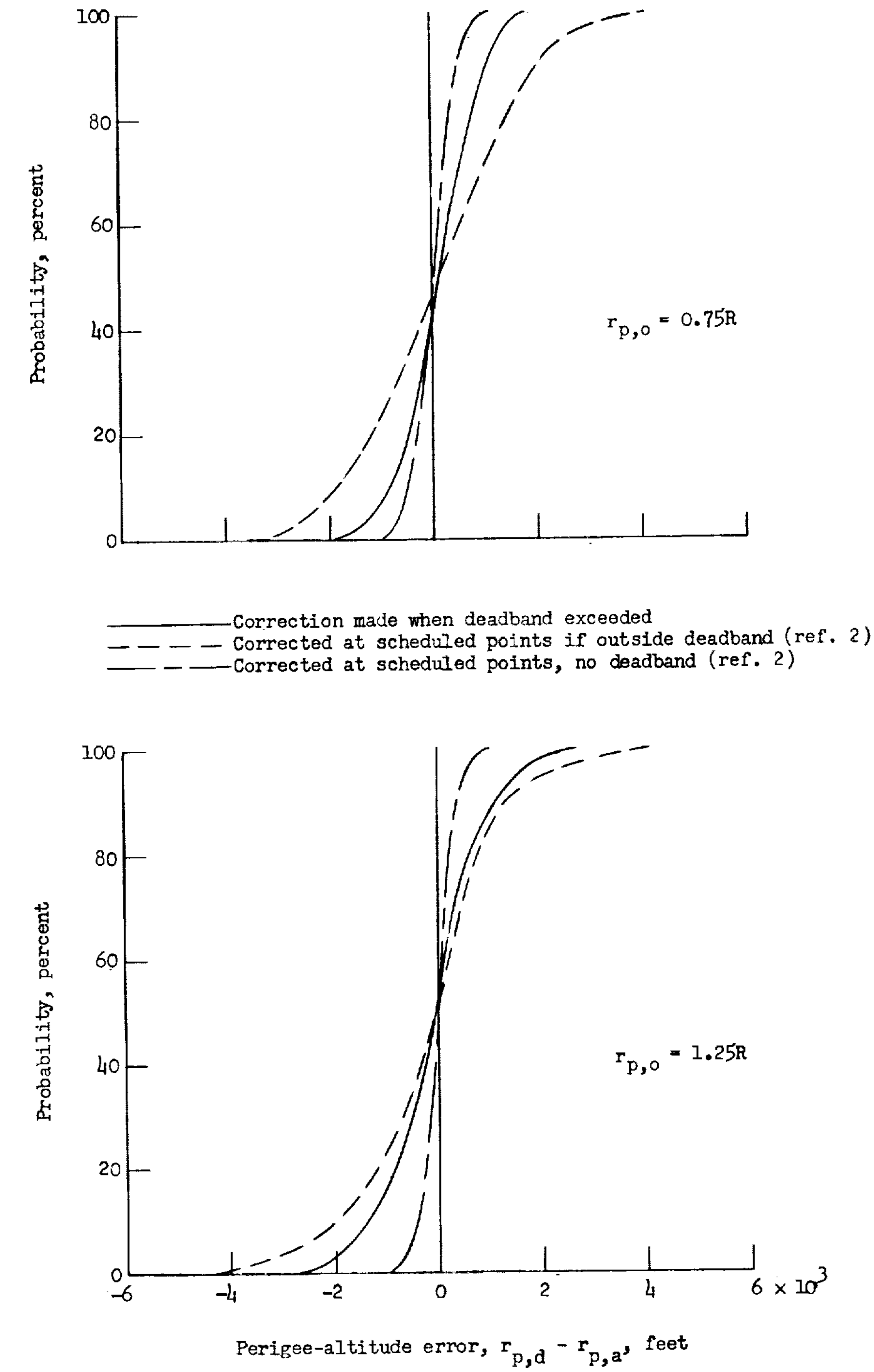


Figure 2.- Probability-distribution curves of perigee-altitude error. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 0.013V_T$ ft/sec; $\theta_f = 10^\circ$.

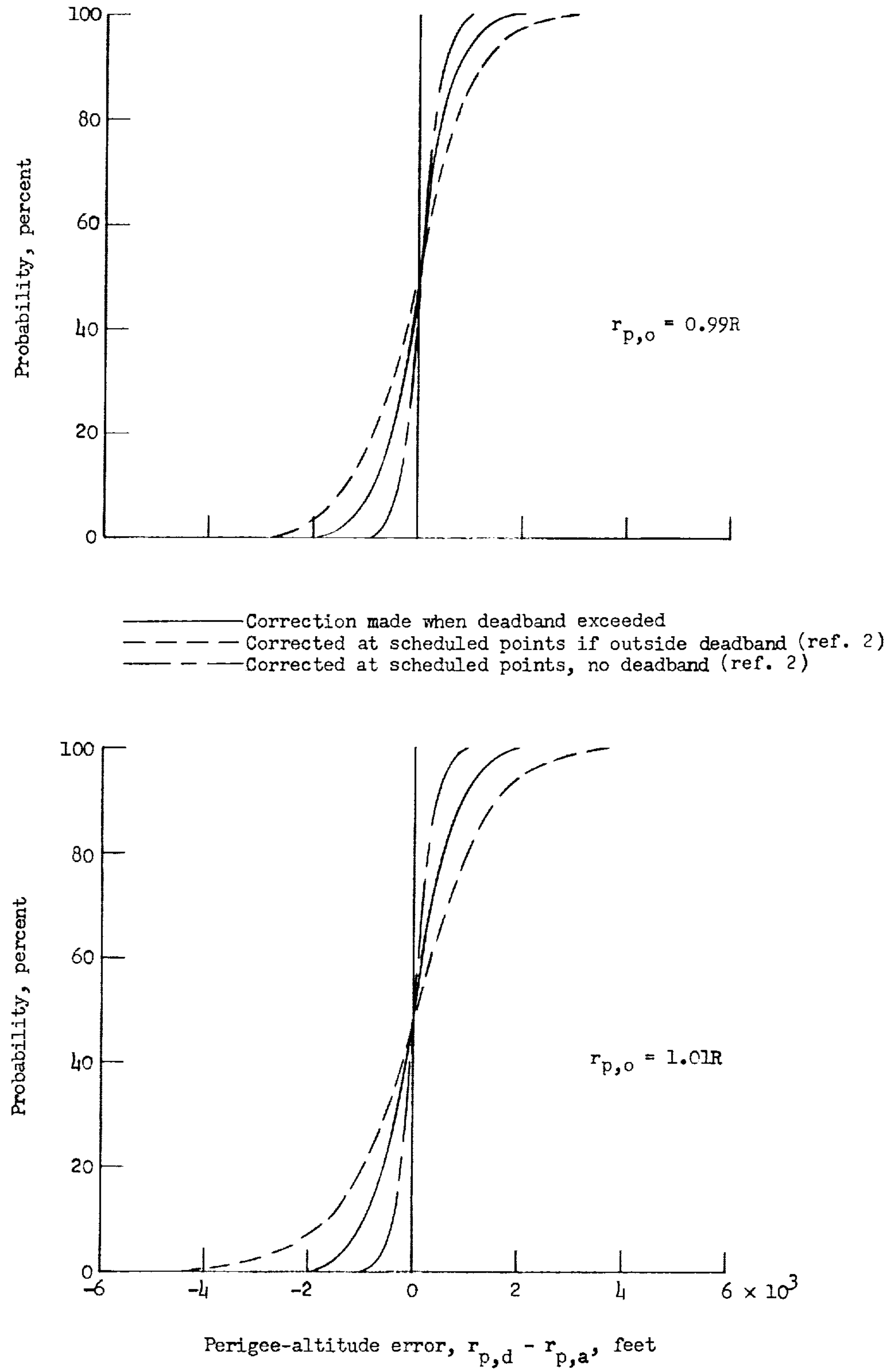
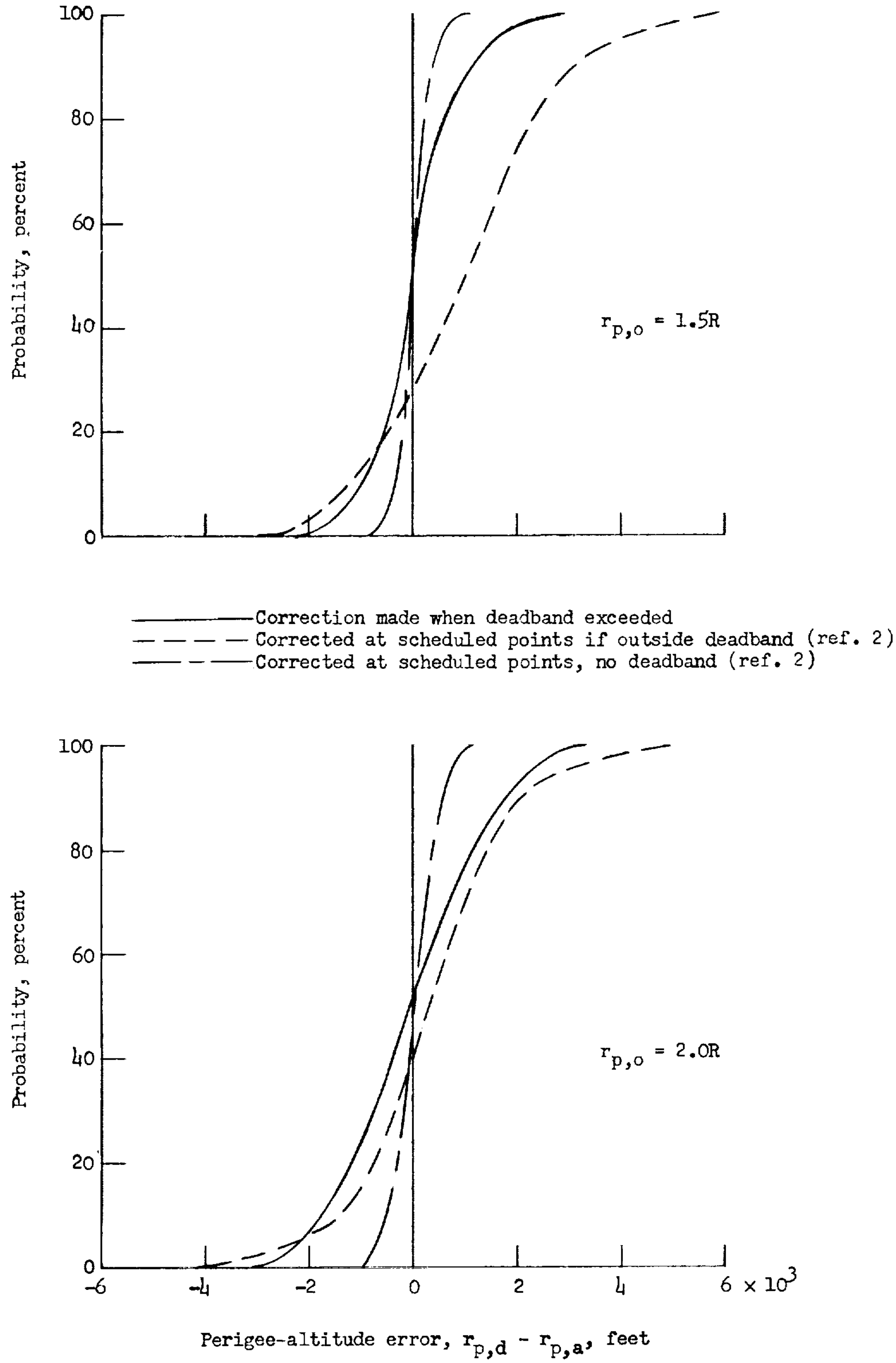


Figure 2.- Continued.



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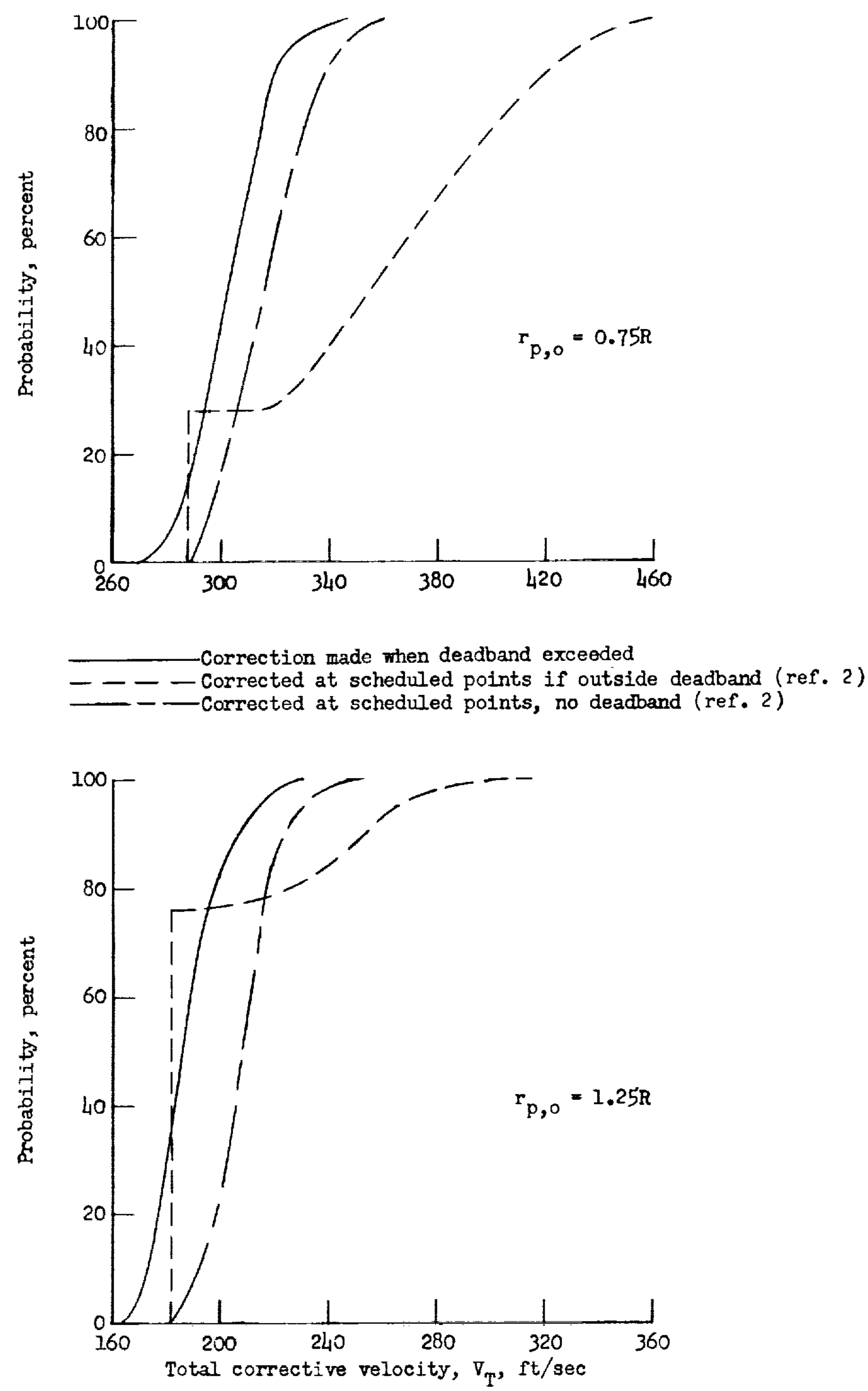
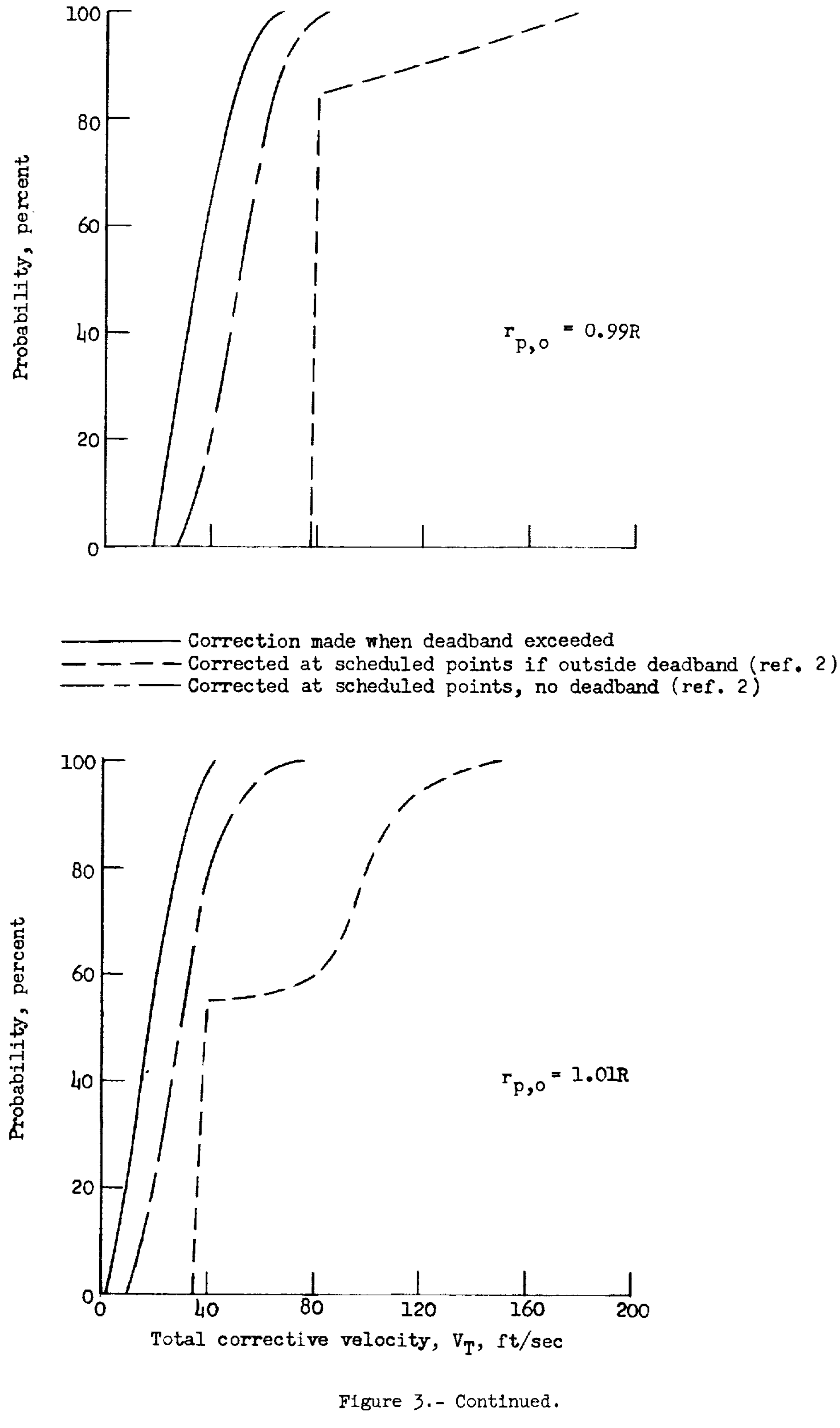


Figure 3.- Probability-distribution curves of total corrective velocity. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 0.013V_T$ ft/sec; $\theta_f = 10^\circ$.



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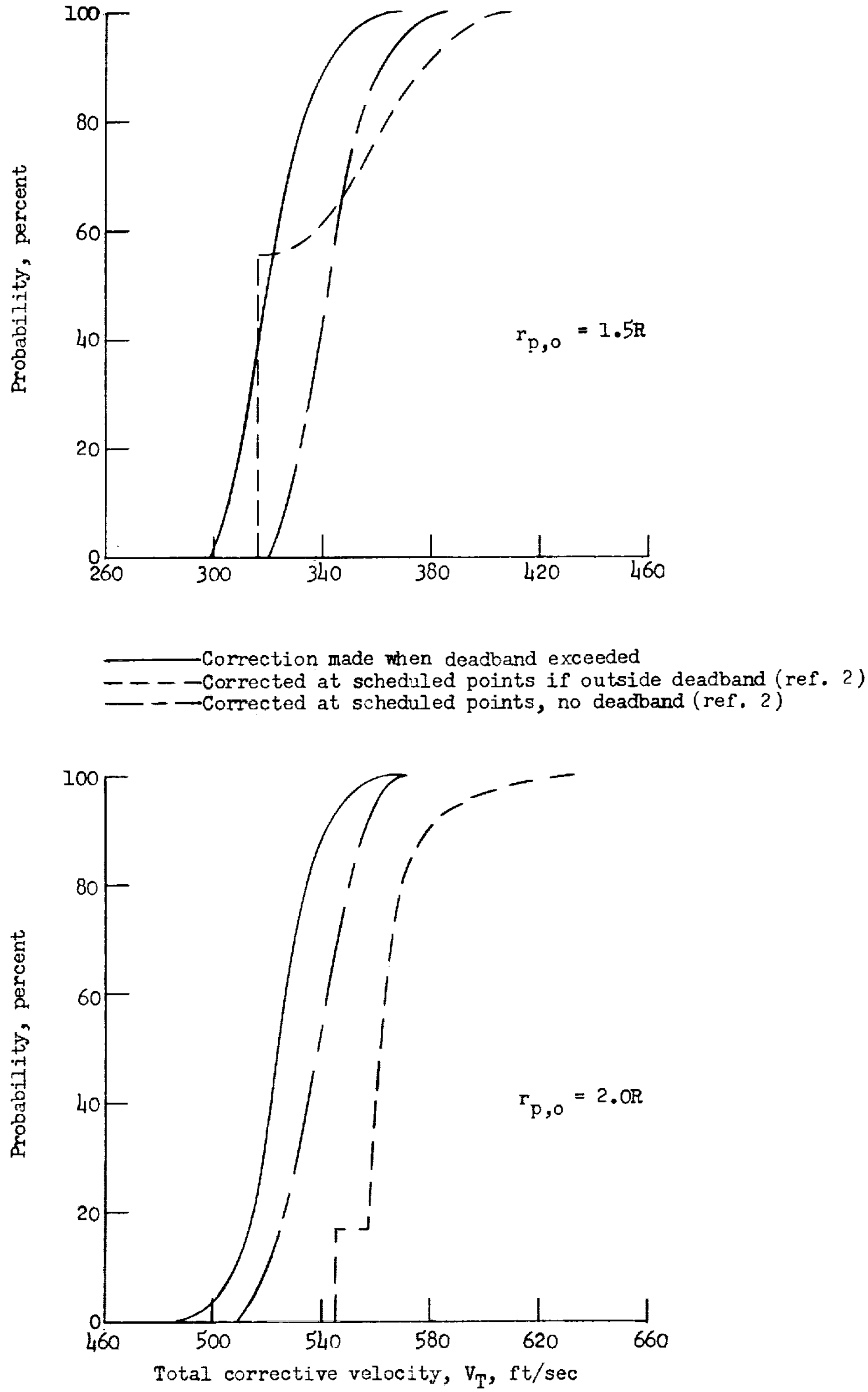

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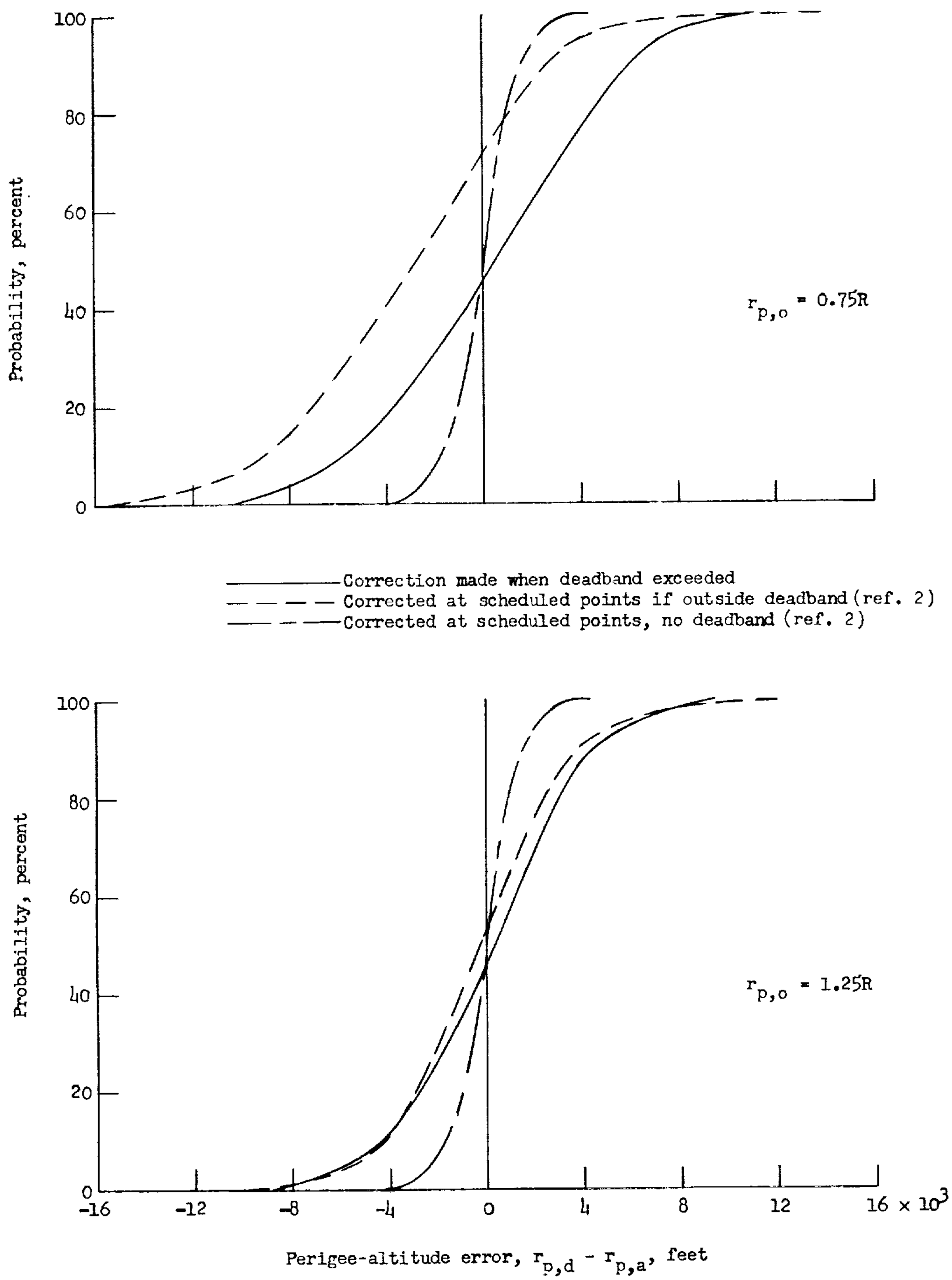
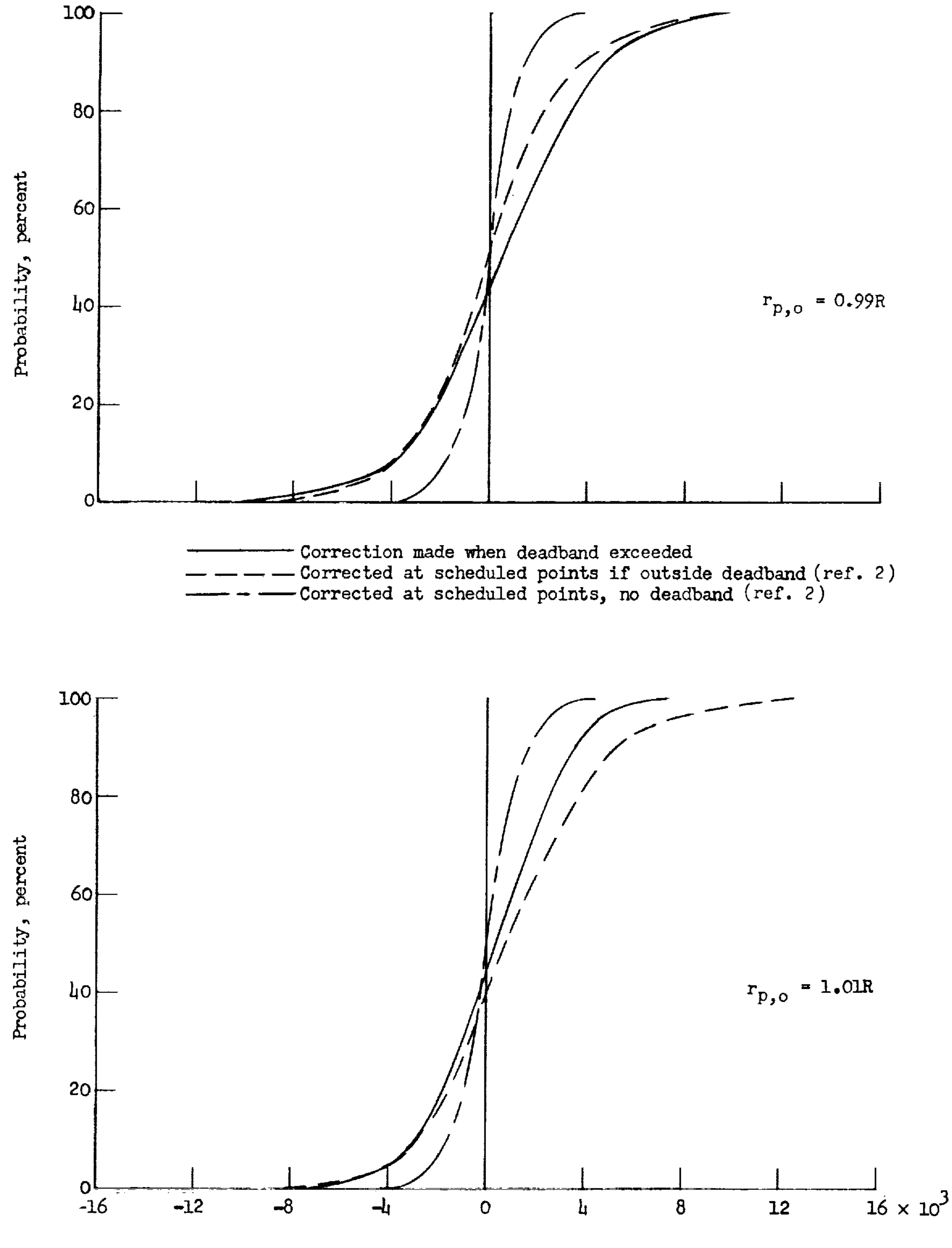


Figure 4.- Probability-distribution curves of perigee-altitude error. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 0.013V_T$ ft/sec; $\theta_f = 40^\circ$.



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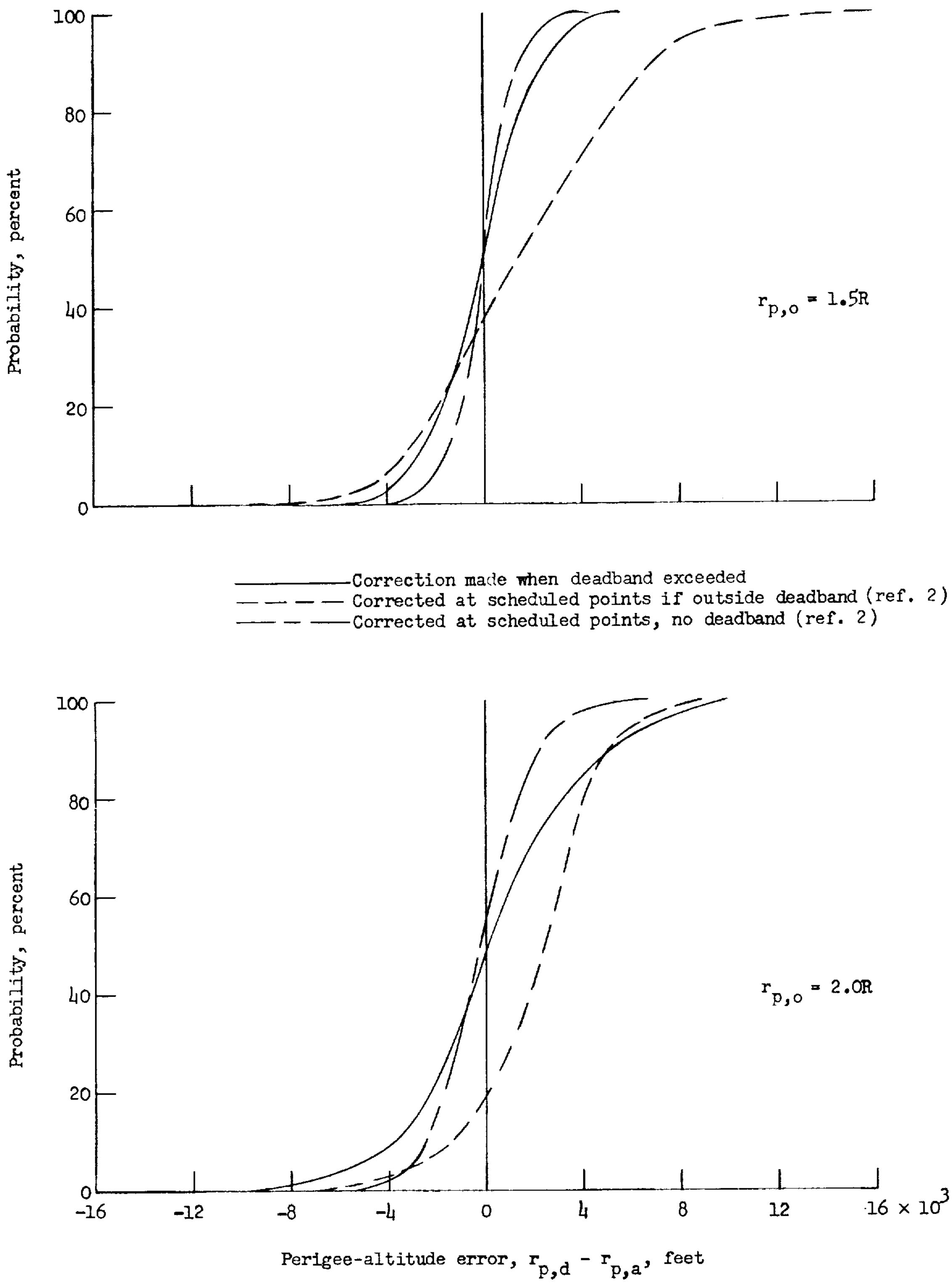

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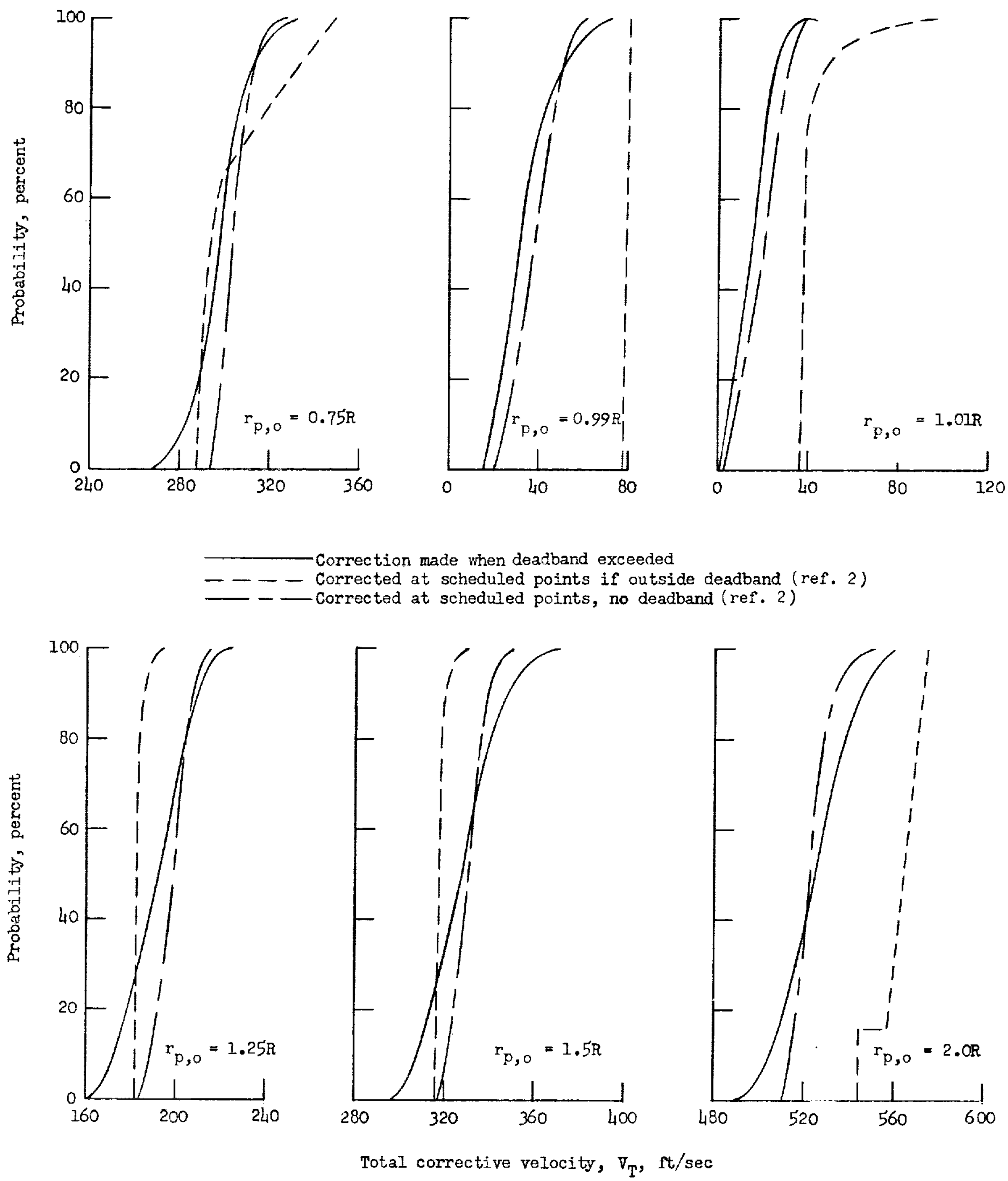


Figure 5.- Probability-distribution curves of total corrective velocity. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 0.013V_T$ ft/sec; $\theta_f = 40^\circ$.

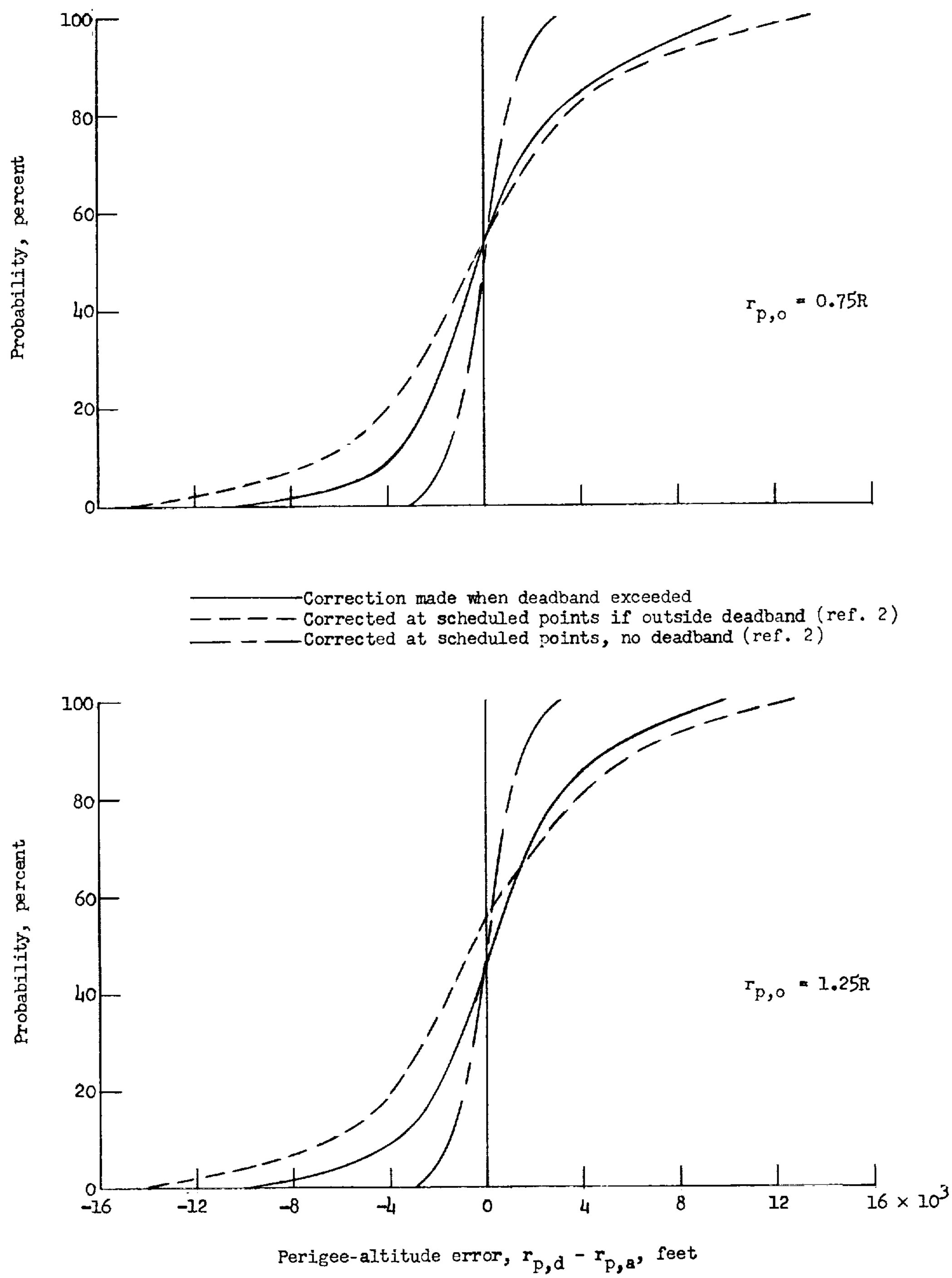
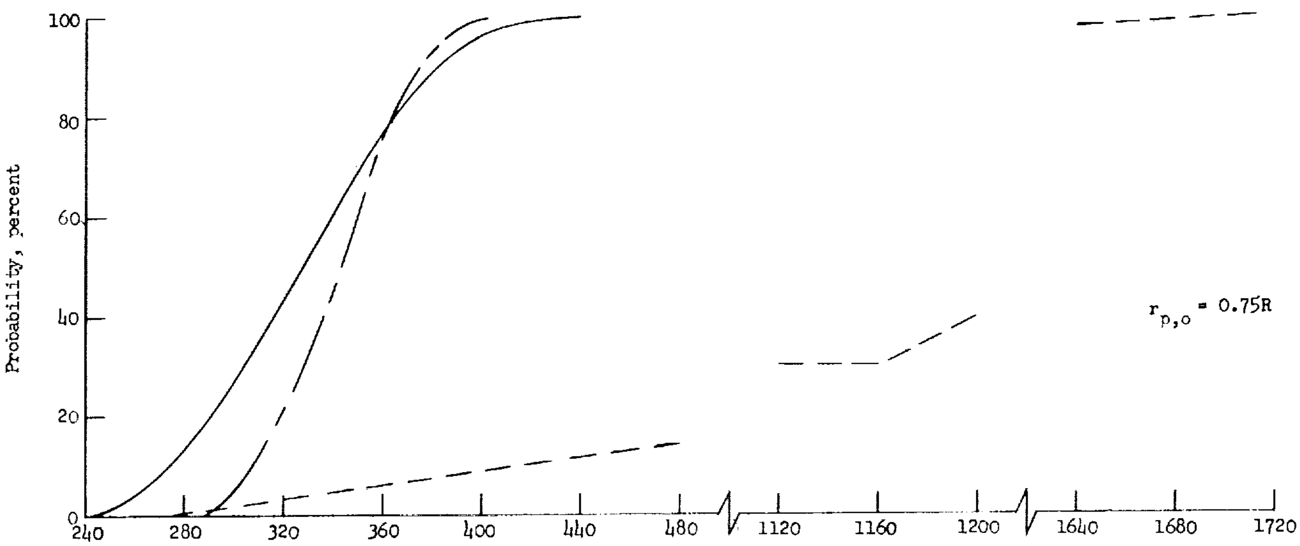


Figure 6.- Probability-distribution curves of perigee-altitude error. $\sigma_V = \frac{3r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees; $\sigma_{V_T} = 0.039V_T$ ft/sec; $\theta_f = 10^\circ$.



———— Correction made when deadband exceeded
— — — Corrected at scheduled points if outside deadband (ref. 2)
—— — — Corrected at scheduled points, no deadband (ref. 2)

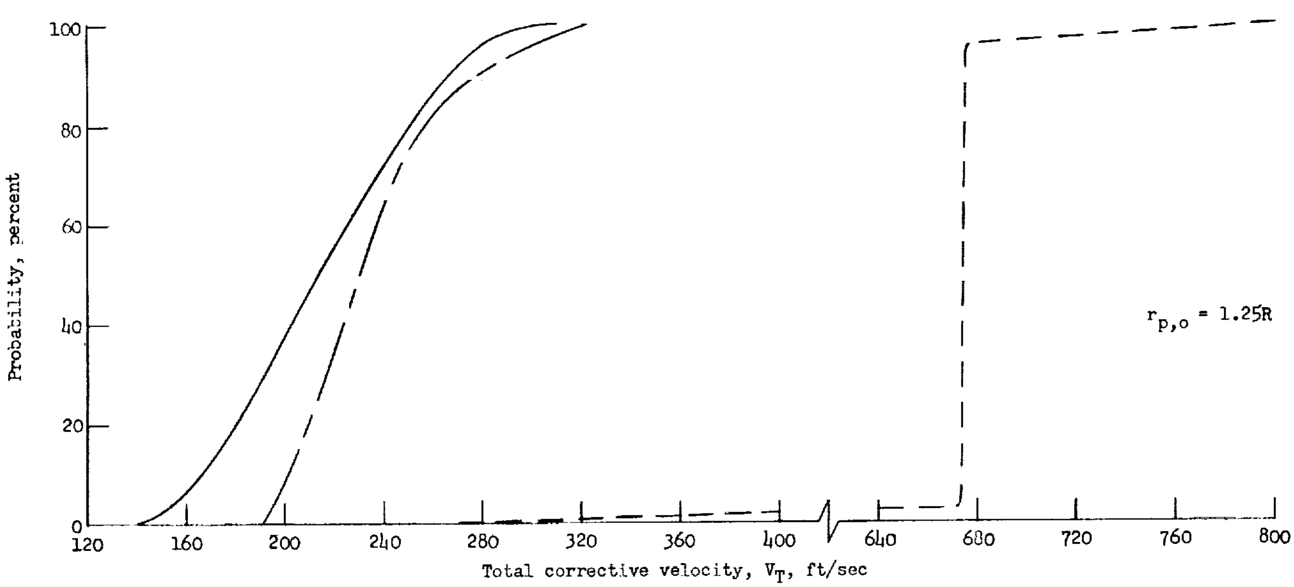


Figure 7.- Probability-distribution curves of total corrective velocity. $\sigma_V = \frac{3r}{10,000}$ ft/sec;

$\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees; $\sigma_{V_T} = 0.039V_T$ ft/sec; $\theta_f = 10^\circ$.

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