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# TECHNICAL NOTE

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## PERIODIC ORBITS FOR MOON PROBES

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## SUMMARY

A general discussion is presented on the broadening of scope, purpose, and — consequently — technique of celestial mechanics as a result of the space age. The discussion is focused on the time scale of the objects being studied. In order to differentiate from the more rigorous part of classical celestial mechanics, the name *Space Mechanics* is suggested to cover this field of study, which is empirical in purpose and numerical in approach.

In the sense of *Space Mechanics,* some periodic orbits that enclose both the two finite bodies have been investigated within the framework of the restricted three-body problem. This represents only the first step in looking for orbits that will permit a moon-probing vehicle to make periodic encounters with the moon on its other side. Two families of periodic orbits have been found — one stable, one unstable — in the orbital plane of the hypothetical moon. Such periodic orbits also have been sought outside the orbital plane. Although a periodic orbit has been obtained numerically, it does not enclose the moon.

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## TIME SCALES AND METHODS OF APPROACH

Since the coming of the space age, the purpose of celestial mechanics and, consequently, the technique and concepts involved have been drastically broadened. This broadening of the scope results from the time scale of objects under study. Before man introduced artificial satellites and space-probing vehicles, the objects of study in celestial mechanics were confined to celestial bodies that are naturally present in the solar system. These bodies have been in existence, according to most astrophysicists, cosmic chemists, and geophysicists, for a time scale of about 4.5 billion years. With a background of such a long time scale, it would be absurd to talk about an orbit that lasts less than, say, a few thousand years. Indeed, a periodic orbit in celestial mechanics generally was expected to be a mathematically rigorous solution of the equations of motion. Since the problems are so difficult that only men of considerable mathematical talent can make contributions, celestial mechanics naturally becomes a branch of mathematics. In the past centuries many great mathematicians have left their marks in this field.

Our interest in space exploration by means of probing vehicles has modified this situation. We can ask, "What is the time scale of rockets that man on the earth has sent, or will send, out to space?" Without doubt, many of them will last as long as the solar system itself. However, we are not interested in their *entire* life span. For the exploration of space, the upper limit of *useful* rocket life will perhaps be of the order of 100 years. In most cases, the useful life span of a space-probing rocket will be much shorter than this limit.

One hundred years is chosen as a critical time for two reasons. First, it is of the same order of magnitude as the life span of an individual human being. We should remember that the space vehicle is used for experiments whose purpose is to understand the physical nature of an astronomical universe. Whether in physics, chemistry, or biology, an experiment is expected in general to be performed in a length of time shorter than the life span of the investigator who designs the experiment in the earthbound laboratory. This attitude probably will not change drastically for experiments in the spacebound laboratory. Thus a time scale of the order of 100 years for performing an experiment may be regarded as a limit in most cases of space exploration. Secondly, a round trip in a free

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orbit from the earth to Pluto, the outermost planet in the solar system, would also be of the order of magnitude of 100 years. Now this does not mean that space travel should be limited forever to the solar system, but the step from interplanetary travel to a visit to our stellar neighbors is wider than the step from Columbus' voyage across the Atlantic Ocean to the astronaut's trip to the moon. Thus considering interstellar travel at the present time is thinking way into the future. For these two reasons, we should be concerned in the next decade or two mainly with orbits of space-probing vehicles having a lifetime of the order of magnitude of 100 years or less.

As a result, the method for attacking the problems in celestial mechanics broadens correspondingly. While it would be most ridiculous to suggest that a numerical solution be attempted for the entire solar system in the next billion years, it is within reason to use the electronic computer for solving many problems connected with rocket trajectories in the solar system during a time interval of a few hundred years or less. Broadening the scope of celestial mechanics as thus understood may be regarded by many investigators, with justification, as its "degeneration." Therefore the name *Space Mechanics* is proposed to cover that part of celestial mechanics used to meet the conditions required by certain kinds of experiments in space research. Thus studies of the trajectories of all kinds of probing vehicles belong to the domain of space mechanics. In this way, *celestial* mechanics will maintain its traditionally high level of mathematical requirements while the numerical results of *space* mechanics will satisfy scientists who design the spacebound experiments.

## A PROCEDURE FOR DERIVING PERIODIC ORBITS

With space mechanics in mind, we will talk about periodic orbits for the moon probe. The periodic orbits are supposed to enclose both the earth and the moon and, for practical reasons, to pass around the moon at short distances. Obviously this is a very difficult problem and perhaps has no solution in the rigorous sense. Because of the presence of the sun and the eccentricity of the moon's orbit, it is not certain that we can find such required orbits — which will last for the time interval of a few years. The present paper serves only as a preliminary probing toward this end. It is because we have studied solely the orbits of a test particle with negligible mass in a hypothetical circumstance that the moon is assumed to be revolving in a circular orbit around the earth in the sun's absence. This problem is known as the restricted three-body problem in celestial mechanics (recently reviewed by Szebehely, Reference 1). The periodic orbits thus found (Reference 2) encourage us to search for the desired orbits in the actual earth-moon-sun system, although they do not insure that periodic orbits lasting one or more years in the actual system will necessarily be found.

We will undertake our search of periodic orbits in the restricted three-body problem by successive approximation. Thus we first neglect the mass of the moon. Then, if an orbit meets the condition of periodic encounters with the moon, it must satisfy the following equation:

$$\frac{p}{p_0} = \frac{n}{m}, \tag{1}$$

where $p_0$ and $p$ are respectively the periods of the moon and the third body and where both $n$ and $m$ are integers. Equation 1 determines the semimajor axis $a$ of the orbit of the third body; that is,

$$a = \left(\frac{n}{m}\right)^{2/3}$$

if the radius of the hypothetical moon's orbit is taken as the unit of length. Because of the requirement that the third body, which will be a moon-probing vehicle, must pass the other side of the moon at a comparatively short distance from it, the semimajor axis $a$, or equivalently the ratio of integers $n/m$, must be limited to a certain range of values. Since any two successive close encounters between the moon and the third body take place in a time interval of $np_o$, a small value of $n$ is preferred in order to have frequent encounters. It is because of these two conditions that there are only a few desirable choices for the value of $n/m$. The value $n/m = 2/3$ proved suitable for our purpose, while Message (Reference 3) and Newton (Reference 4) have separately studied the case of $n/m = 1/2$.

With the ratio $n/m$ (or the semimajor axis $a$) fixed, we still have a wide choice of possible orbits because of the freedom in assigning values to the orbital eccentricity. Also, the orbital motion of the third body can be in the same direction (*direct* motion) as, or in the opposite direction (*retrograde* motion) to, the motion of the moon. Therefore we have two families of desired orbits if we consider only close encounters at the apogee of the third body's orbit. In the case where $n/m > 1$, another two families of desired orbits may be obtained for encounters at the perigee of the third body's orbit. However encounters at the perigee are not of interest for the present purpose. Thus we shall confine our present study to the two families of orbits resulting from close encounters at the apogee.

The above considerations are based on the assumption that the moon has a negligible mass. We now reason that these two families of periodic orbits may exist even when the mass of the moon, though small, is not negligible. By direct computation two families of periodic orbits have indeed been found for a mass ratio of the two finite bodies corresponding to the earth and the moon.

No analytical proof of the existence of these two families of periodic orbits has been attempted, since the problem is treated as an empirical one and the desired results are derived by numerical experiments. However the procedure by which these orbits are derived may provide some *intuitive* ground to believe in the existence of the periodic solutions in the mathematical sense.

Let us first introduce a rotating coordinate system with the origin at the entire system's center of mass and with the $x$ axis joining the two finite bodies. Let us assume that the third body has the initial conditions given by

$$x = x_o, \quad y = 0, \quad \dot{x} = 0, \quad \dot{y} = \dot{y}_o.$$

We can then define the period $p_n$ of the $n^{th}$ cycle of a nearly periodic orbit by the time interval between the $n+1$ and $n^{th}$ crossings of the $x$ axis by the third body at about the initial value $x_o$. Thus $p_n$ can be obtained by interpolation from the results of numerical integration. For a true periodic orbit, it must necessarily be true that

$$p_1 = p_2 = \cdots p_n = \cdots$$

and the difference between two successive values of $p$'s; that is,

$$\Delta_{n+1,n} = p_{n+1} - p_n$$

measures the deviation from a periodic orbit.

Now we can describe our procedure: First, a value of $x_0$ is arbitrarily chosen and a few trial values of $\dot{y}_0$ are surmised (from the case when the moon's mass is neglected). We then integrate the equations of motion for the restricted three-body problem and obtain, by interpolation, $\Delta_{2,1}$ for each trial value $\dot{y}_0$. Table 1 illustrates the results for two orbits: one direct, and the other retrograde. We assume the initial condition that yields

$$\Delta_{2,1} = 0$$

to be the one that leads to the desired orbit. Thus, the correct initial value of $\dot{y}_0$ should be somewhere between those given in the second and third lines of the table.

Table 1
Deviation from Periodicity ($x_0$ = -.39215)

| Direct Orbit | | Retrograde Orbit | |
|---|---|---|---|
| $\dot{y}_0$ | $\Delta_{2,1}$ | $\dot{y}_0$ | $\Delta_{2,1}$ |
| -1.6102479 | -.001294 | 2.3516409 | +.000026 |
| -1.6102480 | -.000457 | 2.3516410 | +.000009 |
| -1.6102481 | +.000380 | 2.3516411 | -.000011 |
| -1.6102482 | +.001216 | 2.3516412 | -.000030 |

## A STUDY OF STABILITY

Next we examine the stability of the obtained orbits by investigating the variation in $\Delta_{n+1,n}$ with n. We immediately find that, in the case of *direct* orbits, $\Delta_{n+1,n}$ oscillates with an ever increasing magnitude roughly as the exponential function of n. This clearly indicates instability of the periodicity. For *retrograde* orbits, on the other hand, the variation in $\Delta_{n+1,n}$ with n for each given $\dot{y}_0$ slightly different from the correct $\dot{y}_0$ of the periodic orbit is simply oscillatory without any increase in magnitude, as Figure 1 shows. Moreover, the amplitude of variation in $\Delta_{n+1,n}$ decreases with the decrease in the deviation of $\dot{y}_0$ from the correct value that corresponds to the periodic orbit. This shows most clearly that the periodic orbit is stable under a small change in the initial conditions.

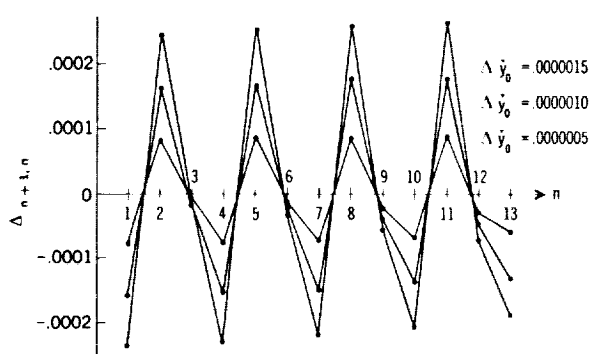


Figure 1 - Stability of the retrograde orbit showing the variation of $\Delta_{n+1,n}$ with n. The amplitude of variation appears to be proportional to the deviation $\Delta\dot{y}_0$ of the initial ejection velocity from the correct value. This figure provides a heuristic argument for the existence of periodic orbits in a mathematical sense.

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