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POWER INPUT TO A SMALL FLAT PLATE FROM A DIFFUSELY RADIATING SPHERE WITH APPLICATION TO EARTH SATELLITES: THE SPINNING PLATE

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SUMMARY

A general derivation is presented for the radiation from a uniformly radiating sphere incident on a small flat plate, spinning about an axis not coincident with its normal. The results are presented as a function of the separation of the bodies, the orientation of the plate, and the orientation of the spin axis. In addition, a series of curves is given which represents the power input from earth radiation to one side of a spinning flat plate (averaged over a spin period) for a range of altitude from 200 to 32000 km. These curves are based upon the assumption that the earth is a uniform diffuse emitter radiating as a blackbody at a temperature of 250°K.

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INTRODUCTION

In a previous paper* (Part I) the power input to a stationary flat plate was calculated. The present paper will apply these results to the problem of the spinning plate in order to determine the instantaneous earth-radiated power incident upon the solar cell paddles associated with the various satellites. This analysis does not consider any shielding of the paddles by other paddles or by the body of the satellite itself.

In the previous paper we defined three cases for which the limits of integration were easily determinable. These were dependent upon the value of the angle between the normal to the plate and the earth-satellite line. For a spinning plate, however, this simplification is not possible since a spinning plate whose axis of spin is not coincident with the normal to the plate defines a varying angle between the normal and the earth-satellite line.

In general, a free rigid body whose axis of spin does not coincide with a principal axis exhibits, to an external observer, a precession of the spin axis about the angular momentum vector and a precession of the axis of symmetry (in this case the normal) about the spin axis, so that both the axis of symmetry and the axis of spin appear to precess about the angular momentum vector. However, since we are considering plates attached to larger satellite bodies which usually spin about their axis of symmetry (also a principal axis), the spin axis coincides with the angular momentum vector and the net result is simply a precession of the normal about the spin axis. The main assumption here is that the satellite's spin axis deviates no more than slightly from its axis of symmetry and hence, possesses a negligible amount of precessional motion.

*Cunningham, F. G., "Power Input to a Small Flat Plate from a Diffusely Radiating Sphere with Application to Earth Satellites", NASA Technical Note D-710 (Corrected copy), August 1961.

From the aforementioned paper, we can write the general expression for the instantaneous power input to a stationary flat plate as

$$P = \frac{2\Lambda\epsilon\alpha A}{\pi}\int_{\phi}\int_{\psi} (\cos\lambda\cos\phi + \sin\lambda\sin\phi\cos\psi)\sin\phi \, d\phi \, d\psi \tag{1}$$

where:

Λ = the generalized emittance of the surface (or σT^4 for thermal radiation);

ϵ = the emissivity of the surface of the radiating sphere;

α = the absorptivity of the plate;

$\mathbf{A}$ = the elemental vector area of the plate;

ϕ = the angle defining the position from the earth-satellite line of the radiating element $\mathbf{ds}$ on the disk replacing the sphere;

ψ = the azimuthal designation of the element $\mathbf{ds}$; and

λ = the angle between the normal to the plate and the earth-satellite line.

The upper limit of the ϕ integration is given by $\phi_m = \sin^{-1}(1/H)$ where $\mathbf{H}$ is the earth-satellite vector, and H is given in units of mean earth radii. The upper limit of the ψ integration is determined by the conditions governing the three previously mentioned cases:

(1) $0 \leq \lambda \leq (\pi/2 - \phi_m)$

(2) $(\pi/2 - \phi_m) < \lambda \leq \pi/2$

(3) $\pi/2 < \lambda \leq (\pi/2 + \phi_m)$.

In the first case the upper limit of ψ is π for all ϕ, and in the second case the upper limit is π for $0 \leq \phi \leq (\pi/2 - \lambda)$. For the remaining portion of the second case and for all of the third case the upper limit of ψ is given by

$$\cos\psi_m = \frac{-\cos\lambda\cos\phi}{\sin\lambda\sin\phi} \, . \tag{2}$$

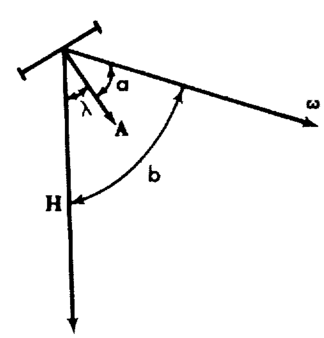

Figure 1—Geometry showing the orientation of the spin axis and normal to the plate

EXTENSION OF THE METHOD TO A SPINNING PLATE

The flat plate whose spin axis ω passes through the center of mass but is not coincident with the normal $\mathbf{A}$ is shown in Figure 1, where

$\mathbf{H}$ = the earth-satellite vector;

a = the angle between $\mathbf{A}$ and ω;

b = the angle between $\mathbf{H}$ and ω.

We now define a new angle ζ as the azimuthal angle of spin of the plate about ω; i.e., $\omega = d\zeta/dt$. Figure 1 shows the zero value of the spin angle ζ for the values of a and b shown. In general the zero value of ζ is defined when $\mathbf{A}$ lies in the $\mathbf{H},\omega$ plane and λ is a minimum. Because of symmetry we need not consider ζ outside the range $0 \leq \zeta \leq \pi$. In addition, ω is assumed to be constant and of sufficient magnitude so that we need not consider a time weighted average over the angle ζ to determine the average power input over the spin period, which is our ultimate goal.

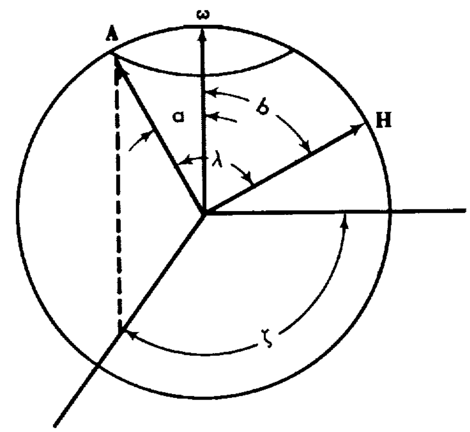

Figure 2—Unit sphere from which the angle λ can be calculated

We shall consider the vectors ω, $\mathbf{H}$ and $\mathbf{A}$ as unit vectors. Figure 2 shows the general orientation of the spinning plate, where the ends of the vectors ω, $\mathbf{H}$ and $\mathbf{A}$ lie on the surface of a unit sphere. With the above definitions, we can employ spherical trigonometry to determine λ. Clearly,

$$\cos\lambda = \cos a \cos b + \sin a \sin b \cos\zeta . \tag{3}$$

By inserting Equation 3 into Equation 1, we have

$$P = \frac{2\Lambda\epsilon\alpha A}{\pi}\int_\phi\int_\psi (\cos a \cos b + \sin a \sin b \cos\zeta)\sin\phi\cos\phi \, d\phi \, d\psi$$

$$+ \frac{2\Lambda\epsilon\alpha A}{\pi}\int_\phi\int_\psi \left[1 - (\cos a\cos b + \sin a \sin b\cos\zeta)^2\right]^{1/2}\cos\psi\sin^2\phi \, d\phi\, d\psi . \tag{4}$$

The upper limit of the ψ integration as given by Equation 2 becomes

$$\cos\psi_m = \frac{-(\cos a\cos b + \sin a\sin b\cos\zeta)\cos\phi}{\left[1 - (\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2}\sin\phi} \tag{5}$$

and

$$\sin\psi_m = \left\{\frac{\sin^2\phi - (\cos a\cos b + \sin a\sin b\cos\zeta)^2}{\left[1 - (\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]\sin^2\phi}\right\}^{1/2} . \tag{6}$$

However, as pointed out in the earlier paper, there are values of ϕ for which the argument of Equation 5 is greater than unity although λ lies within the proper bounds; for this situation $\psi_m = \pi$. The values of ϕ in question are listed on page 2. A complete discussion of the values of ϕ and λ for which Equations 5 and 6 are applicable will be presented subsequently.

Performing the ψ integration of Equation 4 we have:

$$P = \frac{2\Lambda\epsilon\alpha A}{\pi}\int_0^{\phi_m} (\cos a \cos b + \sin a \sin b \cos \zeta)\, \psi_m \sin\phi \cos\phi \, d\phi$$

$$+ \frac{2\Lambda\epsilon\alpha A}{\pi}\int_0^{\phi_m} \left[1 - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2} \sin\psi_m \sin^2\phi \, d\phi \tag{7}$$

In the regions where Equations 5 and 6 apply we have,

$$P = \frac{2\Lambda\epsilon\alpha A}{\pi}\int_0^{\phi_m} (\cos a \cos b + \sin a \sin b \cos \zeta) \cos^{-1}\left\{\frac{-(\cos a \cos b + \sin a \sin b \cos \zeta)\cos\phi}{\left[1 - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}\sin\phi}\right\} \sin\phi \cos\phi \, d\phi$$

$$+ \frac{2\Lambda\epsilon\alpha A}{\pi}\int_0^{\phi_m} \left[\sin^2\phi - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2} \sin\phi \, d\phi \tag{8}$$

The values of λ which define the upper bounds of the regions of interest are $\lambda = \pi/2 - \phi_m$, $\pi/2$, $\pi/2 + \phi_m$, for which the corresponding ζ (for a given a and b) can be determined. These ζ are defined as follows:

$$\cos\zeta_1 = \frac{\sin\phi_m - \cos a \cos b}{\sin a \sin b}, \quad \text{for } \lambda = \pi/2 - \phi_m ; \tag{9a}$$

$$\cos\zeta' = \frac{-\cos a \cos b}{\sin a \sin b}, \quad \text{for } \lambda = \pi/2 ; \tag{9b}$$

$$\cos\zeta_2 = \frac{-(\sin\phi_m + \cos a \cos b)}{\sin a \sin b}, \quad \text{for } \lambda = \pi/2 + \phi_m . \tag{9c}$$

Equation 8 gives the instantaneous power on a spinning flat plate as a function of the altitude (from the upper limit of the ϕ integration), the angle between the instantaneous spin axis and the normal to the plate a, the angle between the instantaneous spin axis and the earth-satellite line b, and the azimuthal angle of spin ζ about the spin axis ω. In the equation the arc cosine factor in the first term and the square root factor in the second term arise from the ψ integration; the first corresponds to ψ_m and the second to $\sin\psi_m$. The arguments of these two factors contain functions of ϕ, and functions of a, b, and ζ. These latter factors arise from the value of $\cos\lambda$ given by Equation 3. In the previous paper the value of λ was assumed to be constant and calculations were made in the appropriate range for various values of λ. In that case it was shown that for certain values of λ the range of integration over ϕ had to be divided into two subdivisions, for the first $\psi_m = \pi$ and the absolute value of the argument of the arc cosine factor is greater than unity, and for the second, ψ_m is less than π and the correct value is given by the arc cosine factor. In fact, since λ was held constant, it was possible to define the limiting value of the ϕ integration which depended upon λ, and hence, to break the integral into its parts — only one of which contained the arc cosine factor as an upper limit. However, in the present discussion we are considering the plate as it spins about an arbitrarily oriented axis. Consequently, it follows from Equation 3 that λ is no longer constant in time but varies with ζ for the constant angles a and b. Then, in order to define the limiting values of the ϕ integration we must make use of the dependence of λ upon ζ where the corresponding limiting values of ζ are given by Equation 9.

As mentioned previously, a plate spinning about an axis not coincident with its normal (which is a principal axis) exhibits a wobbly motion as the axis of symmetry (the normal) precesses about the axis of spin. Before, we were able to hold λ within the range for which the power is incident upon the side of the plate in question. The upper limit of this range is given by Equation 9c. When the combination of the angles a, b, and ζ is such that λ, as given by Equation 3, is greater than $\pi/2 + \phi_m$ then the side of the plate under consideration is not visible from the earth, and the incident power is zero.

Clearly we can define three distinct combinations of the angles a and b:

$$(1) \quad \lambda \leq (\pi/2 - \phi_m) \text{ for } 0 \leq \zeta \leq \pi \; ; \tag{10a}$$

$$(2) \quad 0 \leq \lambda \leq (\pi/2 + \phi_m) \text{ for } 0 \leq \zeta \leq \zeta_2 \; ; \tag{10b}$$

$$(3) \quad \lambda > (\pi/2 + \phi_m) \text{ for } 0 \leq \zeta \leq \pi \; . \tag{10c}$$

Equation 10c defines the situation with no radiation incident on the side of the plate in question; and is of no further interest here. The condition defined in Equation 10a is the least complicated expression, integrable over ζ, for the power because the upper limit of the ψ integration is π for all values of ϕ in the range $0 \leq \phi \leq \phi_m$. The general case is represented by Equation 10b where we intend to imply that, over the range of ζ from 0 to π, the value of λ might be less than $\pi/2 - \phi_m$ for certain values of ζ, greater than $\pi/2 - \phi_m$ but less than $\pi/2 + \phi_m$ for other values of ζ, and greater than $\pi/2 + \phi_m$ for still other values of ζ, or any other variation on the theme.

Equation 8 is the expression which must be integrated over ϕ. It is identical to the equation in Part I except that the corresponding expressions involving a, b, and ζ for $\cos\lambda$ and $\sin\lambda$ are included. Before attempting to calculate a given problem we must first examine the range of values of λ (given by Equation 3) to determine the ranges of ζ which are of interest. After the ϕ integration it then becomes necessary to average over ζ, being careful to keep the appropriate range of ζ coupled to the proper expression.

For the case where Equation 10a applies, Equation 8 integrates to

$$P = 2\Lambda\epsilon\alpha A \int_0^{\sin^{-1}(1/H)} (\cos a \cos b + \sin a \sin b \cos\zeta) \sin\phi \cos\phi \, d\phi \; , \tag{11}$$

which becomes

$$P = \frac{\Lambda\epsilon\alpha A}{H^2}(\cos a \cos b + \sin a \sin b \cos \zeta) \ . \tag{12}$$

Equation 12 is identical to the expression given as P_1 in Part I. We indicate the average over ζ by $\langle P \rangle_\zeta$ which is

$$\langle P \rangle_\zeta = \frac{\Lambda\epsilon\alpha A}{H^2}\left(\cos a \cos b + \frac{\sin a \sin b}{\pi}\int_0^\pi \cos \zeta \, d\zeta\right) \tag{13}$$

$$= \frac{\Lambda\epsilon\alpha A}{H^2}\cos a \cos b \ . \tag{14}$$

The most general integral expression for the case of Equation 10b is:

$$\begin{aligned}
P = {} & 2\Lambda\epsilon\alpha A \int_0^{\sin^{-1}(1/H)} (\cos a \cos b + \sin a \sin b \cos \zeta) \sin\phi \cos\phi \, d\phi \\
& + 2\Lambda\epsilon\alpha A \int_0^{\pi/2-\cos^{-1}(\cos a \cos b + \sin a \sin b \cos \zeta)} (\cos a \cos b + \sin a \sin b \cos \zeta) \sin\phi \cos\phi \, d\phi \qquad \left\{0 \le \zeta \le \cos^{-1}\left(\frac{\frac{1}{H} - \cos a \cos b}{\sin a \sin b}\right)\right\} \\
& + \frac{2\Lambda\epsilon\alpha A}{\pi}\int_{\pi/2-\left[\cos^{-1}(\cos a \cos b + \sin a \sin b \cos \zeta)\right]}^{\sin^{-1}(1/H)} (\cos a \cos b + \sin a \sin b \cos \zeta)\cos^{-1}\left\{\frac{-(\cos a \cos b + \sin a \sin b \cos \zeta)\cos\phi}{\left[1 - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}\sin\phi}\right\}\sin\phi\cos\phi \, d\phi \\
& + \frac{2\Lambda\epsilon\alpha A}{\pi}\int_{\pi/2-\left[\cos^{-1}(\cos a \cos b + \sin a \sin b \cos \zeta)\right]}^{\sin^{-1}(1/H)} \left[\sin^2\phi - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}\sin\phi \, d\phi \qquad \left\{\cos^{-1}\left(\frac{-\cos a \cos b + \frac{1}{H}}{\sin a \sin b}\right) < \zeta \le \cos^{-1}\left(\frac{-\cos a \cos b}{\sin a \sin b}\right)\right\} \\
& + \frac{2\Lambda\epsilon\alpha A}{\pi}\int_{\cos^{-1}(\cos a \cos b + \sin a \sin b \cos \zeta)-\pi/2}^{\sin^{-1}(1/H)} (\cos a \cos b + \sin a \sin b \cos \zeta)\cos^{-1}\left\{\frac{-(\cos a \cos b + \sin a \sin b \cos \zeta)\cos\phi}{\left[1 - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}\sin\phi}\right\}\sin\phi\cos\phi \, d\phi \\
& + \frac{2\Lambda\epsilon\alpha A}{\pi}\int_{\cos^{-1}(\cos a \cos b + \sin a \sin b \cos \zeta)-\pi/2}^{\sin^{-1}(1/H)} \left[\sin^2\phi - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}\sin\phi \, d\phi \ . \qquad \left\{\cos^{-1}\left(\frac{-\cos a \cos b}{\sin a \sin b}\right) < \zeta \le \cos^{-1}\left[\frac{-\cos a \cos b - \frac{1}{H}}{\sin a \sin b}\right]\right\}
\end{aligned} \tag{15}$$

Equation 15 integrates out exactly as Equations 21 and 35 of Part I except that we have replaced the cosine λ and sine λ factors by their equivalents in terms of a, b, and ζ.

We now have:

$$
\begin{aligned}
P = {} & \frac{\Lambda\epsilon\alpha A}{H^2}(\cos a\cos b + \sin a\sin b\cos\zeta) && \left\{0 \le \zeta \le \zeta_1\right\} \\
& + \frac{\Lambda\epsilon\alpha A}{\pi}\Bigg[\frac{\pi}{2} - \sin^{-1}\left\{\frac{(H^2-1)^{1/2}}{H\left[1-(\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2}}\right\} \\
& \qquad + \frac{1}{H^2}\Bigg((\cos a\cos b + \sin a\sin b\cos\zeta)\cos^{-1}\left\{\frac{-(H^2-1)^{1/2}(\cos a\cos b + \sin a\sin b\cos\zeta)}{\left[1-(\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2}}\right\} \\
& \qquad - (H^2-1)^{1/2}\left[1 - H^2(\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2}\Bigg)\Bigg] && \left\{\zeta_1 < \zeta \le \zeta'\right\} \\
& + \frac{\Lambda\epsilon\alpha A}{\pi}\Bigg[\frac{\pi}{2} - \sin^{-1}\left\{\frac{(H^2-1)^{1/2}}{H\left[1-(\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2}}\right\} + \frac{1}{H^2}\Bigg(-(H^2-1)^{1/2}\left[1 - H^2(\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2} \\
& \qquad + (\cos a\cos b + \sin a\sin b\cos\zeta)\cos^{-1}\left\{\frac{-(H^2-1)^{1/2}(\cos a\cos b + \sin a\sin b\cos\zeta)}{\left[1-(\cos a\cos b + \sin a\sin b\cos\zeta)^2\right]^{1/2}}\right\}\Bigg)\Bigg], && \left\{\zeta' < \zeta \le \zeta_2\right\}
\end{aligned}
\tag{16}
$$

where the expression in parentheses following certain terms in Equations 15 and 16 indicates the range of ζ over which the preceding terms are applicable. Because of the complexity of some of the terms in Equation 16 the average over ζ cannot be easily calculated in closed form so that this portion of the problem will be left to the digital computer.

Equation 16 applies in its entirety only to the situation for which λ does, indeed, have values within all three regions defined by $0 \le \lambda \le (\pi/2 - \phi_m)$, $(\pi/2 - \phi_m) < \lambda \le \pi/2$, and $\pi/2 < \lambda \le (\pi/2 + \phi_m)$ each of which corresponds to a particular range of values of ζ as ζ goes from 0 to π. If λ does not take on values in all the regions for the entire range of ζ certain modifications of Equation 16 must be made which are pointed out subsequently. To determine which parts of Equation 16 to use and the corresponding limits for any case, we must examine the range of λ for that case by checking its extremes. Since $\mathbf{A}$ precesses about $\boldsymbol{\omega}$ at a constant angle a, the extremes can be determined by considering the cases when $\mathbf{A}$ lies in the $\mathbf{H}$, $\boldsymbol{\omega}$ plane; i.e., when $\zeta = 0, \pi$. The only additional requirement is that $\lambda \le \pi$. In defining values of a and b it suffices that if we let b lie in the range $0 \le b \le \pi$, we need only consider a in the range $0 \le a \le \pi/2$. For example, the situation represented by $a = 100°$, $b = 10°$ is identically represented by the situation $b = 170°$, $a = 80°$. The direction of $\boldsymbol{\omega}$, and consequently the value of b, is always determined by the right hand rule. It also follows that interchanging the values of a and b does not alter the problem. In addition, if we wish to determine the input to the other side of the plate (called side β) after having determined it for one side (called side α), by the methods outlined above, the extremes of the range of λ for side β can be easily calculated by subtracting the limits for side α from 180 degrees. Then, once the extremes of λ_β are known, it is quite easy to construct the corresponding values of a and b which, used in conjunction with Equation 3, will allow for the determination of all the parameters of the new problem. For example, if $a = 80°$, $b = 170°$, for side α, we have that $90^\circ \leq \lambda \leq 110^\circ$; whereas $70^\circ \leq \lambda \leq 90^\circ$ for side β. The latter set corresponds to $b = 10^\circ$, $a = 80^\circ$ or $a = 10^\circ$, $b = 80^\circ$.

We now outline the various possibilities of the general case represented by Equation 10b. But first we shall make the following definitions for simplicity:

$$A = \frac{\Lambda \epsilon \alpha A}{H^2}\left[\cos a \cos b + \sin a \sin b \cos \zeta\right] , \tag{17a}$$

and

$$B = \frac{\Lambda \epsilon \alpha A}{\pi}\left[\frac{\pi}{2} - \sin^{-1}\left\{\frac{\left(H^2 - 1\right)^{1/2}}{H\left[1 - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}}\right\} + \frac{1}{H^2}\left((\cos a \cos b + \sin a \sin b \cos \zeta) \cos^{-1}\left\{\frac{-\left(H^2 - 1\right)^{1/2}(\cos a \cos b + \sin a \sin b \cos \zeta)}{\left[1 - (\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}}\right\} - \left(H^2 - 1\right)^{1/2}\left[1 - H^2(\cos a \cos b + \sin a \sin b \cos \zeta)^2\right]^{1/2}\right)\right] . \tag{17b}$$

Equation 16 can now be written in symbolic form:

$$P = A\left(0 \leq \zeta \leq \zeta_1\right) + B\left(\zeta_1 < \zeta \leq \zeta'\right) + B\left(\zeta' < \zeta \leq \zeta_2\right) . \tag{18}$$

The first step in any problem is to determine the range of λ. We shall define three cases:

$\lambda \leq \pi/2$ for all ζ; (19a)

$\lambda \gtrless \pi/2$ for suitable values of ζ in the range $0 \leq \zeta \leq \pi$; (19b)

$\lambda > \pi/2$ for all ζ. (19c)

In 19a the only value of ζ of concern is that for which $\lambda = \pi/2 - \phi_m$ or $\zeta = \zeta_1$ as given by Equation 9a. If for a given value of H the argument of the arc cosine in 9a is greater than +1 then $\lambda > \left(\pi/2 - \phi_m\right)$ for all ζ and Equation 16 reduces to

$$P = B(0 \leq \zeta \leq \pi) . \tag{20}$$

If, for a given value of H, the argument of the arc cosine in Equation 9a is greater than or equal to -1 but less than or equal to $+1$, so that Equation 9a defines the value of ζ at which $\lambda = \left(\pi/2 - \phi_m\right)$, then Equation 16 becomes

$$P = A\left(0 \leq \zeta \leq \zeta_1\right) + B\left(\zeta_1 < \zeta \leq \pi\right) . \tag{21}$$

However, if for a given value of H the argument of the arc cosine term in 9a is less than -1, then $\lambda < \left(\pi/2 - \phi_m\right)$ for all ζ and Equation 16 becomes

$$P = A(0 \leq \zeta \leq \pi) , \tag{22}$$

which, when averaged over ζ, yields Equation 14 once again.

From 19b it immediately follows that λ can never be less than $(\pi/2 - \phi_m)$ or greater than $(\pi/2 + \phi_m)$ for all ζ. Equations 9a and 9c define the limits of ζ in question. In the preceding statement, the fact that the value of the argument of the arc cosine in 9a can never be less than −1 and that the value of the argument of the arc cosine in 9c can never be greater than +1 can easily be shown for any problem satisfying the conditions of 19b. If the argument of the arc cosine in 9a is greater than or equal to −1 but less than or equal to +1 and the argument of the arc cosine in 9c is less than −1, Equation 16 reduces to

$$P = A(0 \leq \zeta \leq \zeta_1) + B(\zeta_1 \leq \zeta \leq \pi) . \tag{23}$$

If both the arguments of the arc cosines in 9a and 9c lie between +1 and −1 we then have Equation 16. If the argument of the arc cosine in 9a is greater than +1 while the argument of the arc cosine of 9c lies between +1 and −1, Equation 16 becomes

$$P = B(0 \leq \zeta \leq \zeta_2) \tag{24}$$

Finally, if the argument of the arc cosine in 9a is greater than +1 while the argument of the arc cosine in 9c is less than −1, Equation 16 reduces to

$$P = B(0 \leq \zeta \leq \pi) . \tag{25}$$

In 19c the only value of ζ of concern is that for which $\lambda = \pi/2 + \phi_m$ or $\zeta = \zeta_2$ as given by Equation 9c. If the argument of the arc cosine in 9c is less than −1, $\lambda < (\pi/2 + \phi_m)$ for all ζ and Equation 16 becomes

$$P = B(0 \leq \zeta \leq \pi) . \tag{26}$$

If the argument of the arc cosine of 9c lies between +1 and −1, the value of ζ_2 is defined and Equation 16 is

$$P = B(0 \leq \zeta \leq \zeta_2) . \tag{27}$$

If the argument of the arc cosine of 9c is greater than +1 then $\lambda > (\pi/2 + \phi_m)$ for all ζ, and the corresponding power input is zero.

DISCUSSION

Equation 16 is general and applies to any sort of diffusely radiating sphere. However, in the following calculations we are considering the diffusely radiating sphere to be the earth which radiates as a blackbody at a temperature approximately equal to 250°K. Thus

$$\Lambda\epsilon = \sigma T_0^4 , \tag{28}$$

where ϵ equals unity because of the equivalent blackbody approximation and α is taken equal to unity in order to give the energy incident upon the plate. In addition, Equation 16 can be modified to give directly the incident power to the plate as a function of the orbital position of the satellite by giving H as a function of the orbital parameters and the azimuthal position of the satellite in orbit. This can be done by utilizing the polar equation of an ellipse,

$$H = \frac{a(1 - e^2)}{1 + e \cos \eta} , \tag{29}$$

where η is the angle giving the azimuthal position of the satellite from perigee, a is the semi-major axis and e is the eccentricity. However, in the subsequent evaluations of Equation 16 this additional information will not be included because the presence of the arc cosine factor in the equation renders it almost impossible to write, in terms of the orbital parameters, the equation in closed form (after averaging over ζ) so that not a great deal is to be gained unless we wish the computer to print out the value for each orbital position.

So far we have only considered the case where the spin axis passed through the center of the plate. However, it is easy to see that the dependence of λ on ζ remains identical to that given by Equation 3 for the case where a and b are defined as usual but the axis of spin is displaced from the center of the plate. As long as this lateral displacement (usually half the width of a satellite) is small in comparison to the distance of the plate from the surface of the sphere (which always obtains for earth satellites) the previous analysis remains applicable.

RESULTS

In the graphs that follow, the average incident power to the flat plate per spin period is plotted as a function of altitude above the surface of the earth for various values of the angles a and b. The situation occurring when b equals 0 or π results in $\lambda = a$ for all ζ so that these values are not considered, since the incident power for $\lambda =$ constant is readily available in Part I. This situation also applies for $a = 0$ for which $\lambda = b =$ constant. The first 16 graphs apply for H in the range $200 \leq H \leq 3200$ km (Appendix A), while the remaining graphs apply to the entire range $200 \leq H \leq 32000$ km (Appendix B).

ACKNOWLEDGMENT

The author wishes to thank Mr. E. Monasterski, Goddard Space Flight Center, for the IBM 7090 computations.

Appendix A

The Average Incident Power to a Flat Plate per Spin Period for Various Angles a and b as a Function of Altitude for the Range 200 $\leq$ H $\leq$ 3200 km

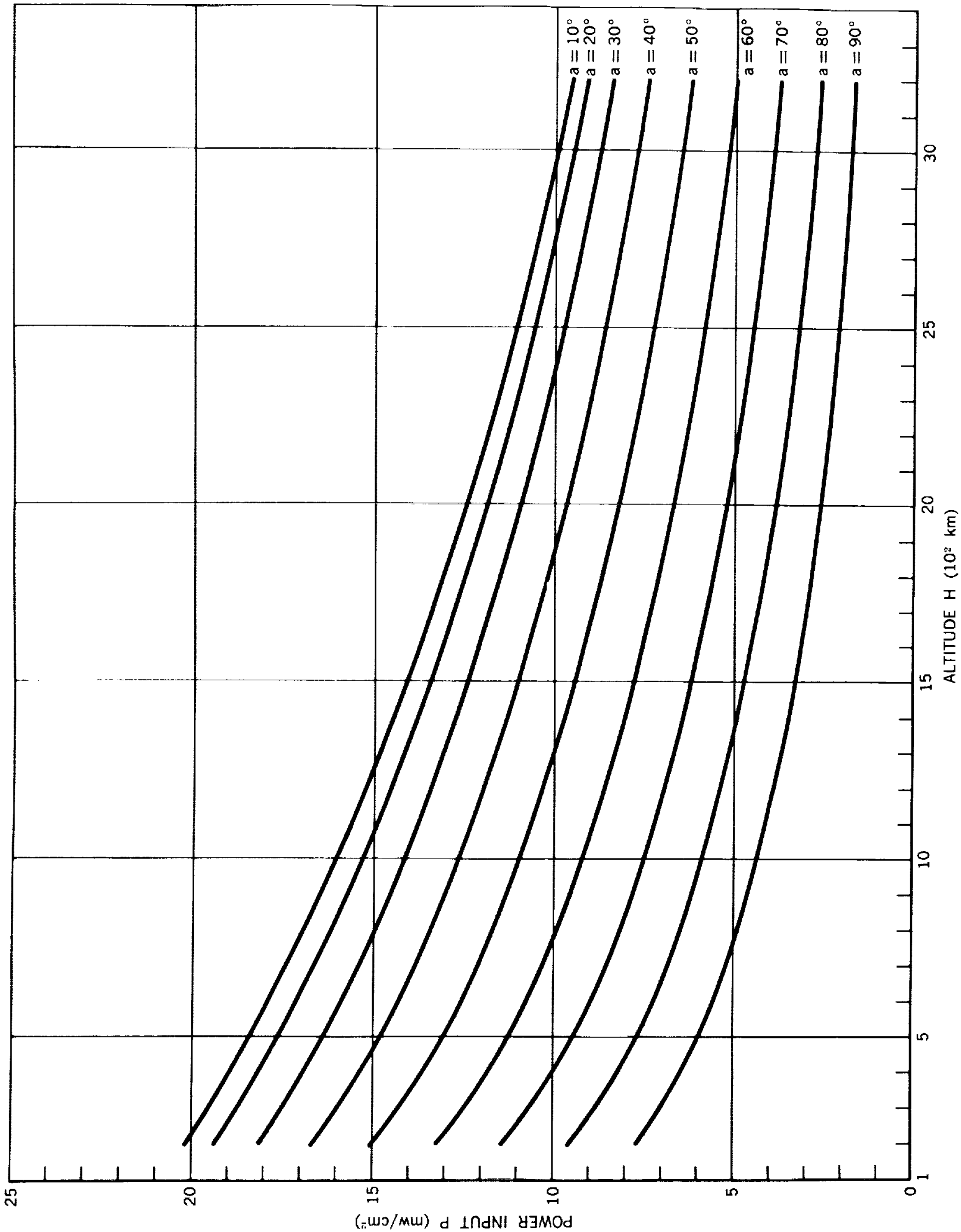

Figure A1—b = 10°

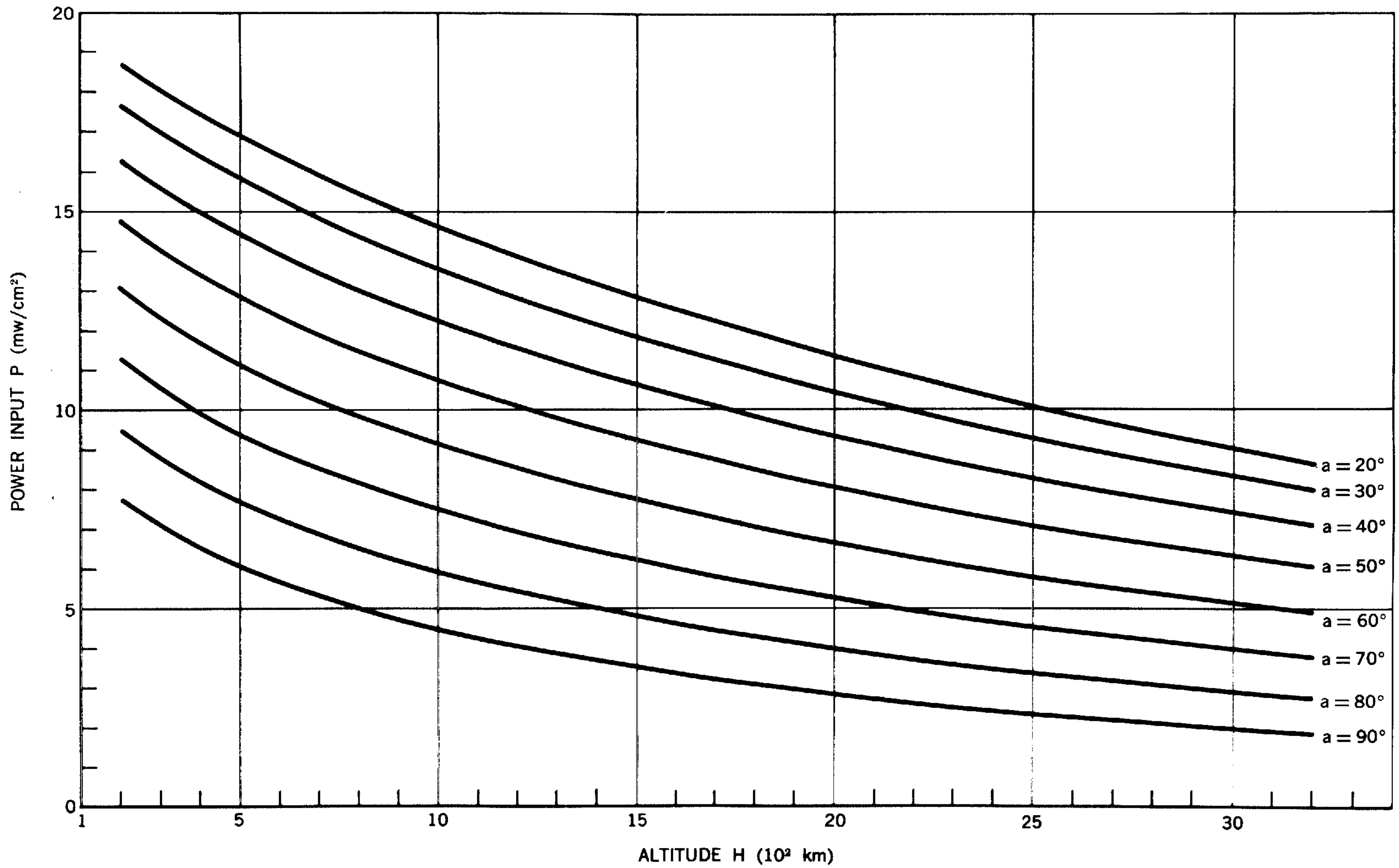

Figure A2—b = 20°

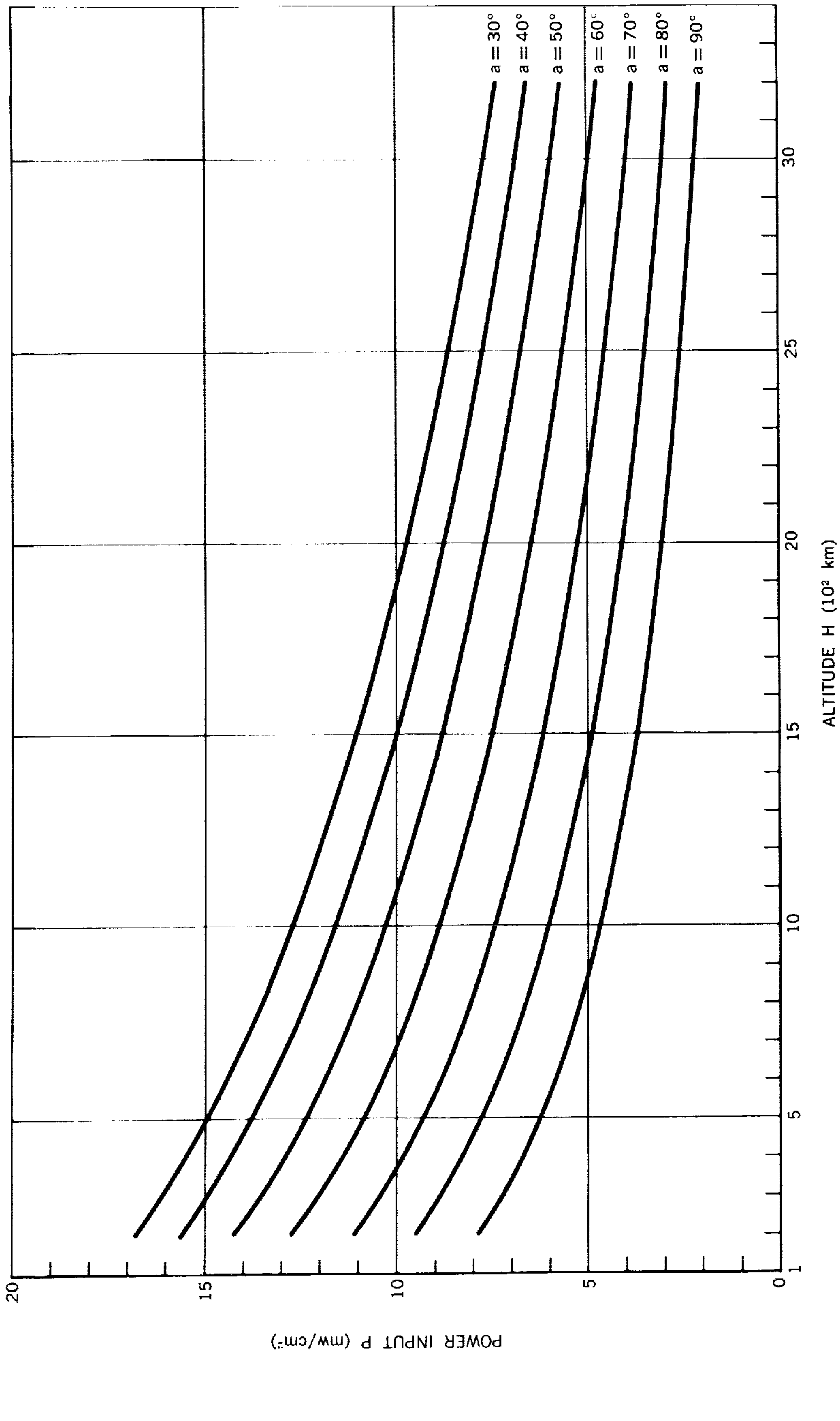

Figure A3—b = 30°

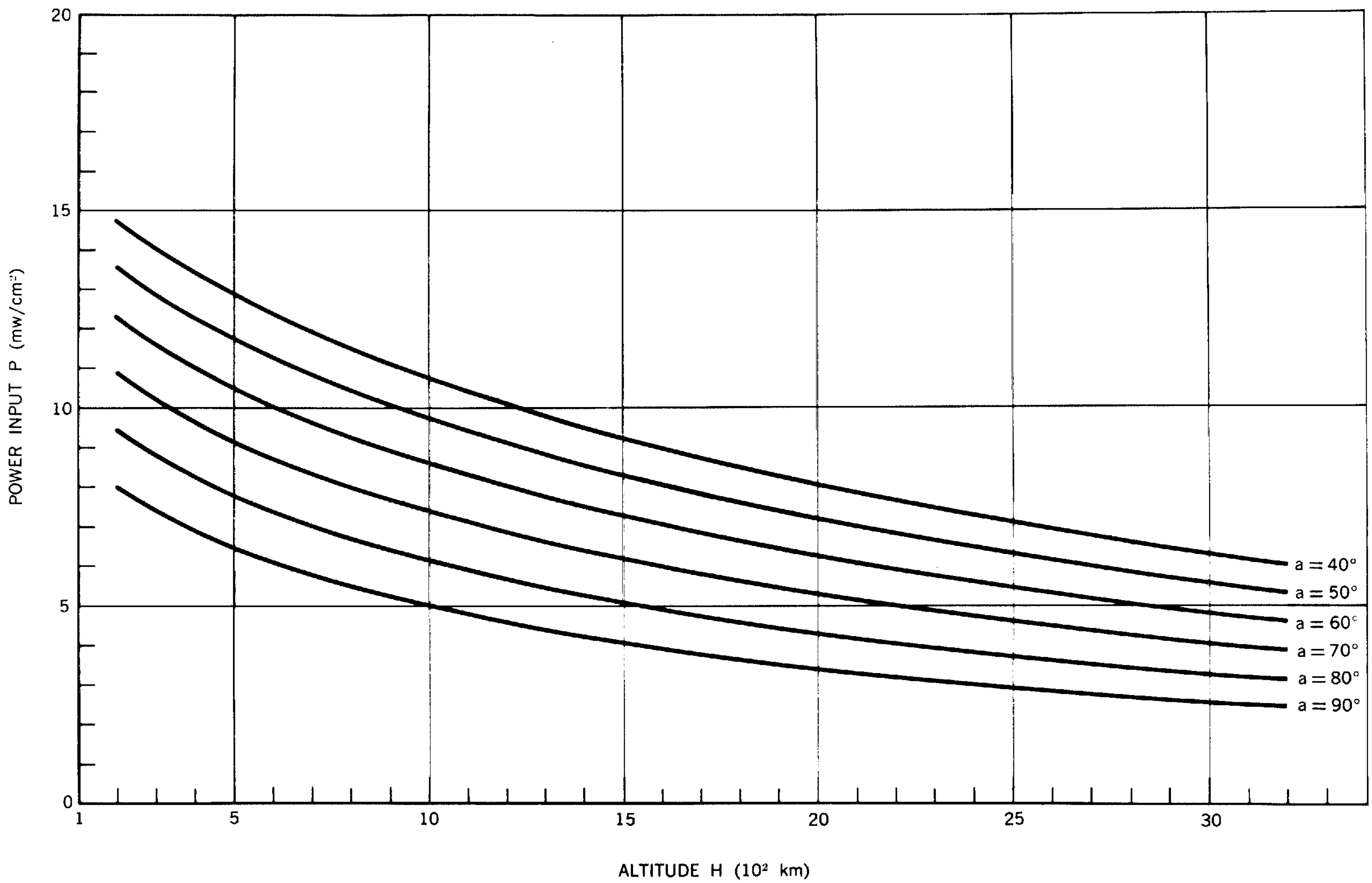

Figure A4—b = 40°

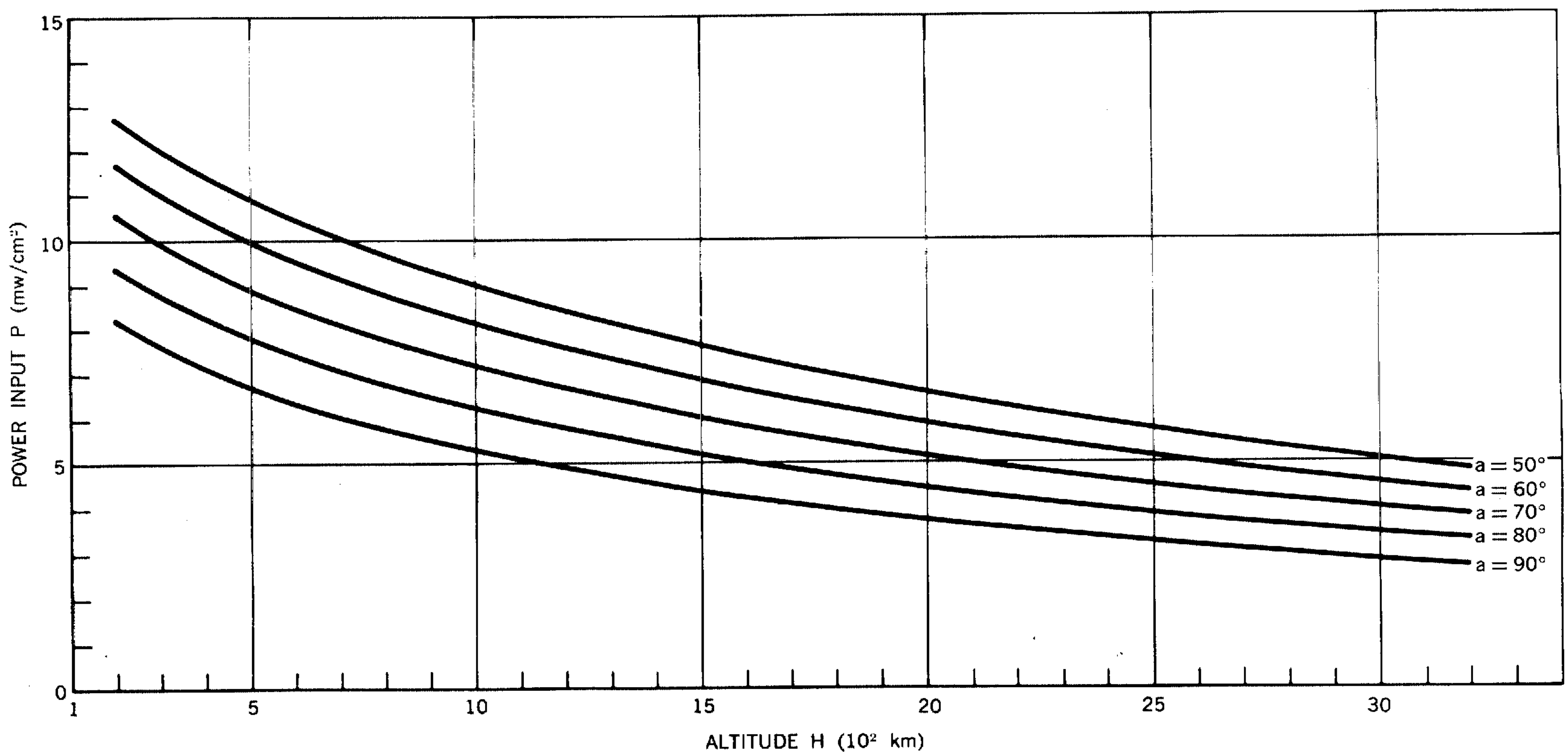

Figure A5—b = 50°

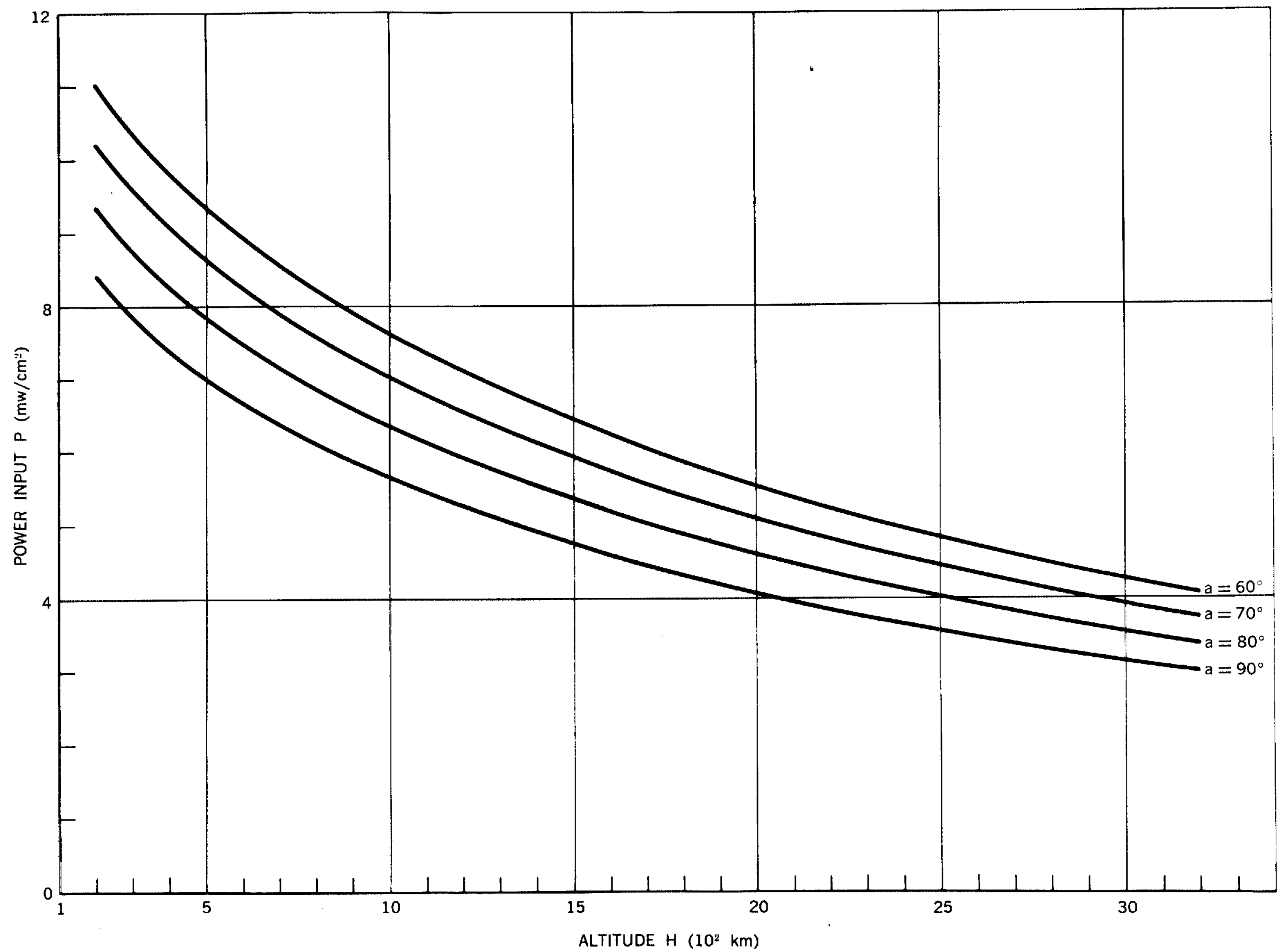

Figure A6—b = 60°

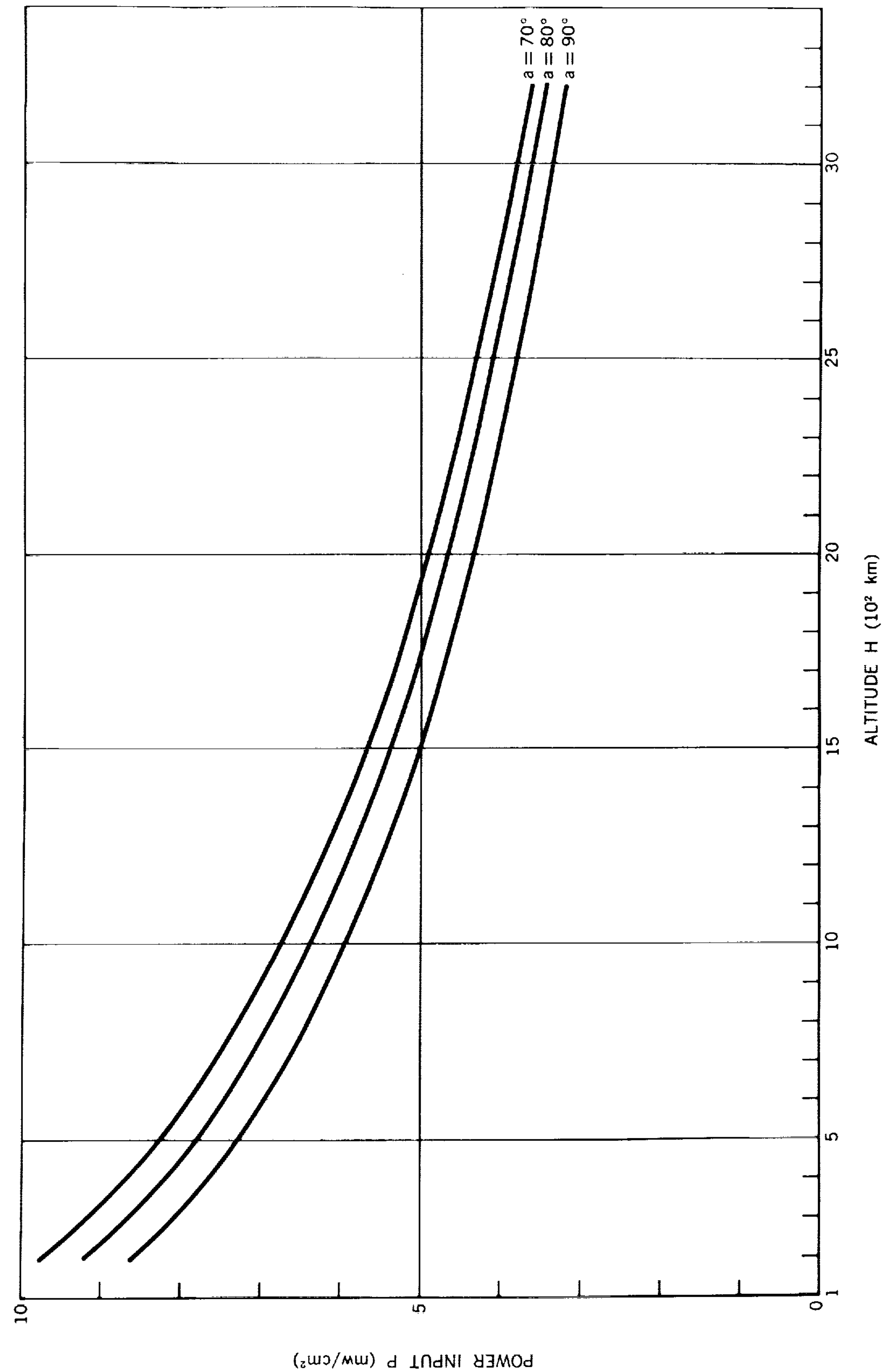

Figure A7—b = 70°

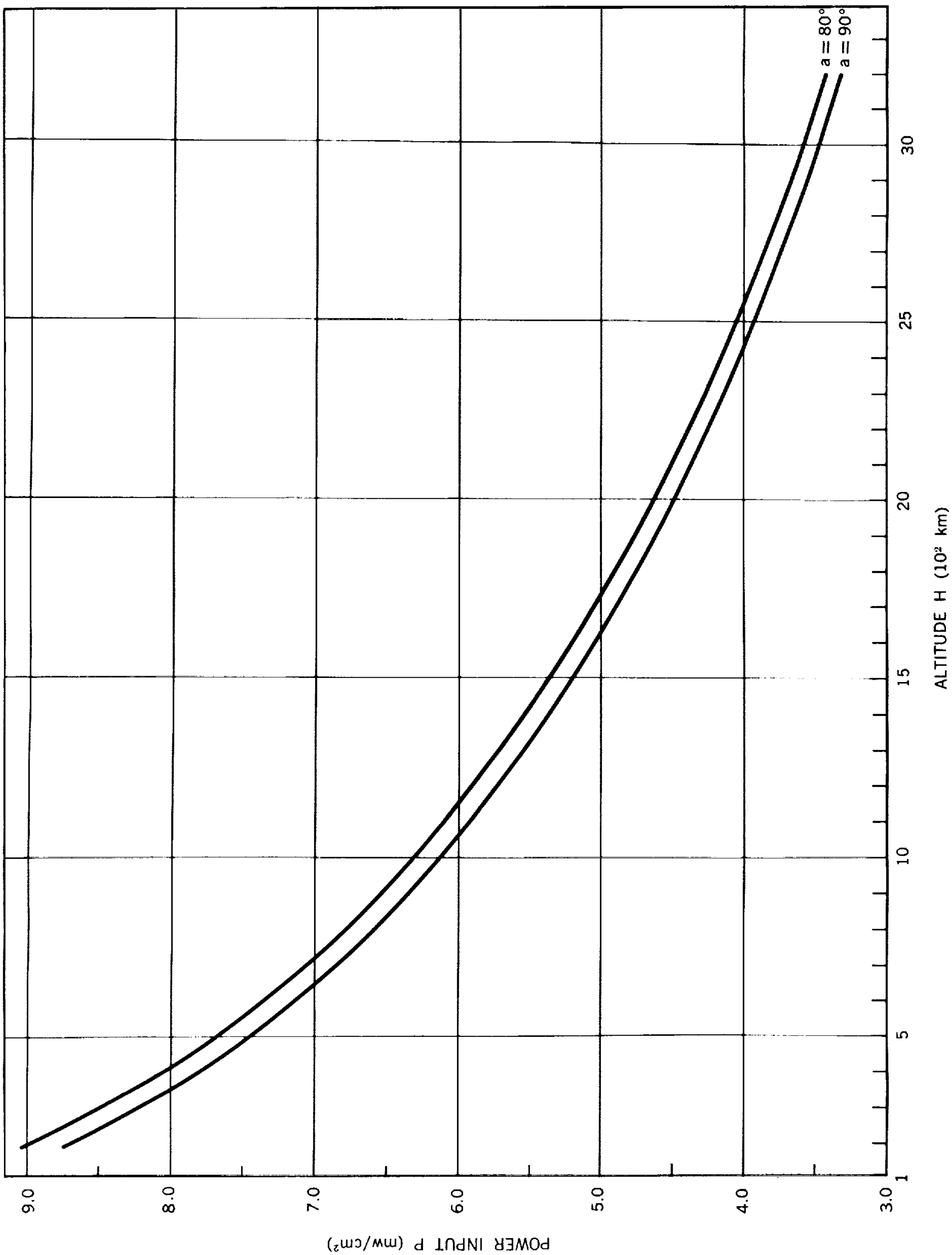

Figure A8—b = 80°

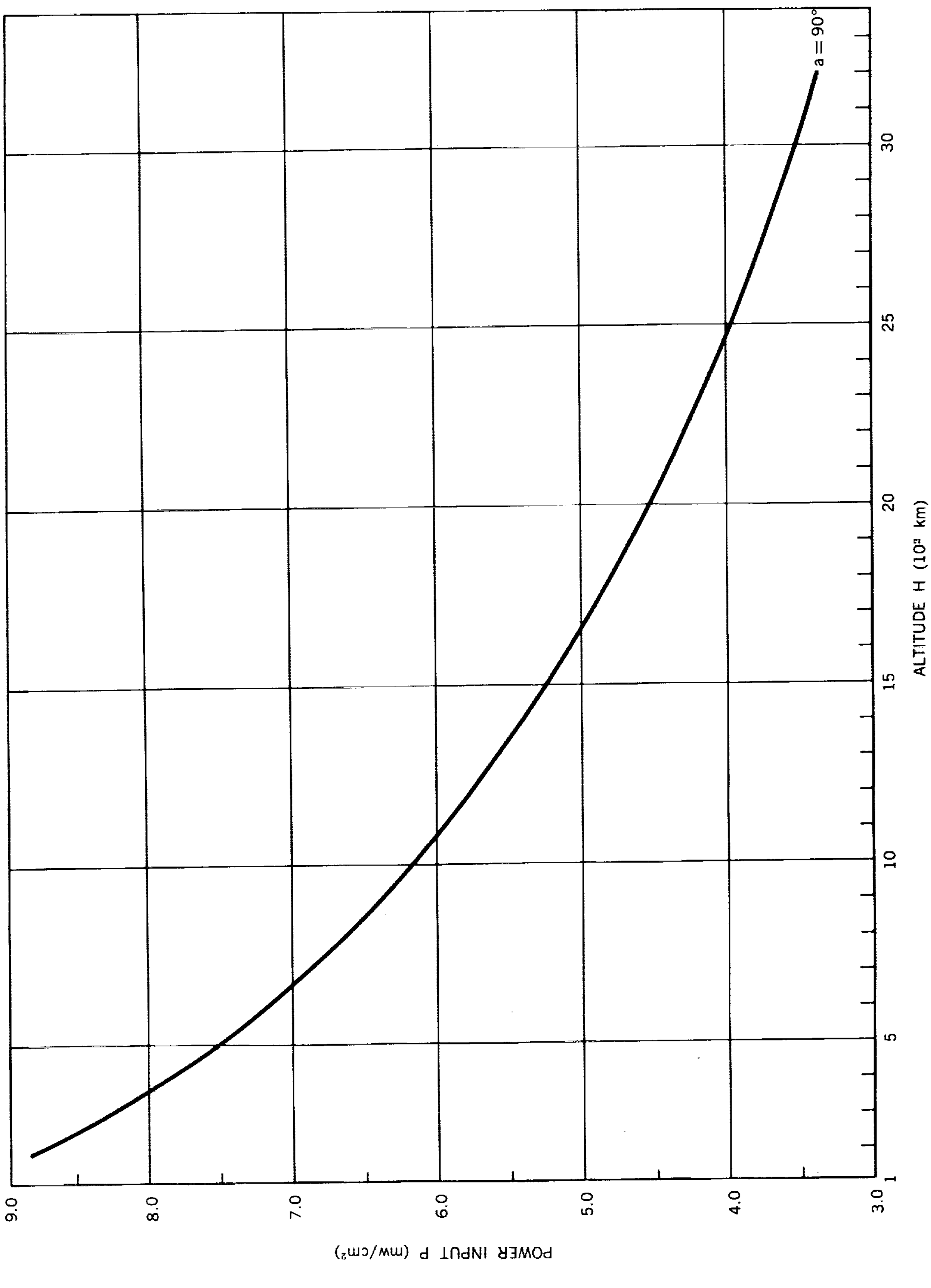

Figure A9—b = 90°

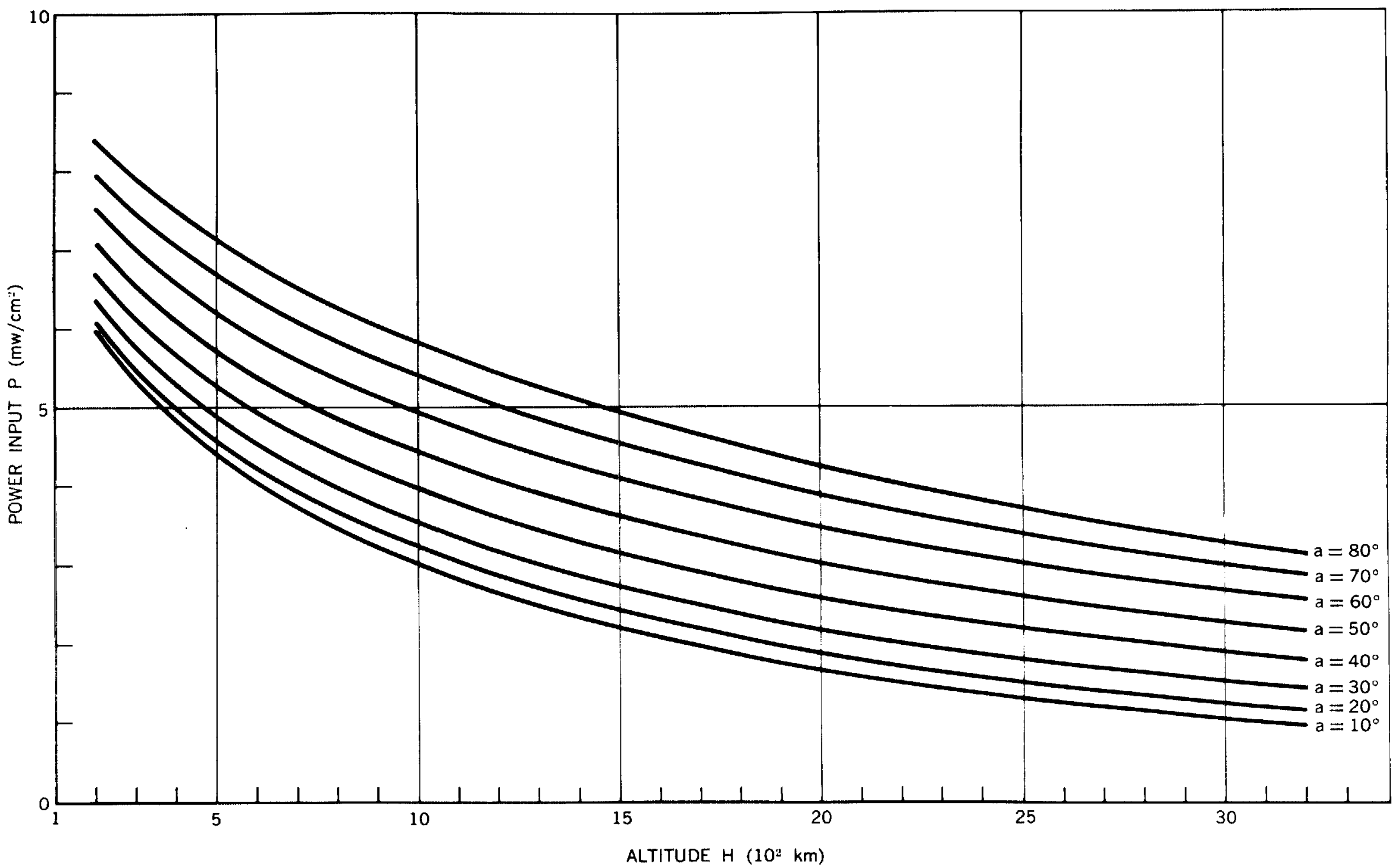

Figure A10—b = 100°

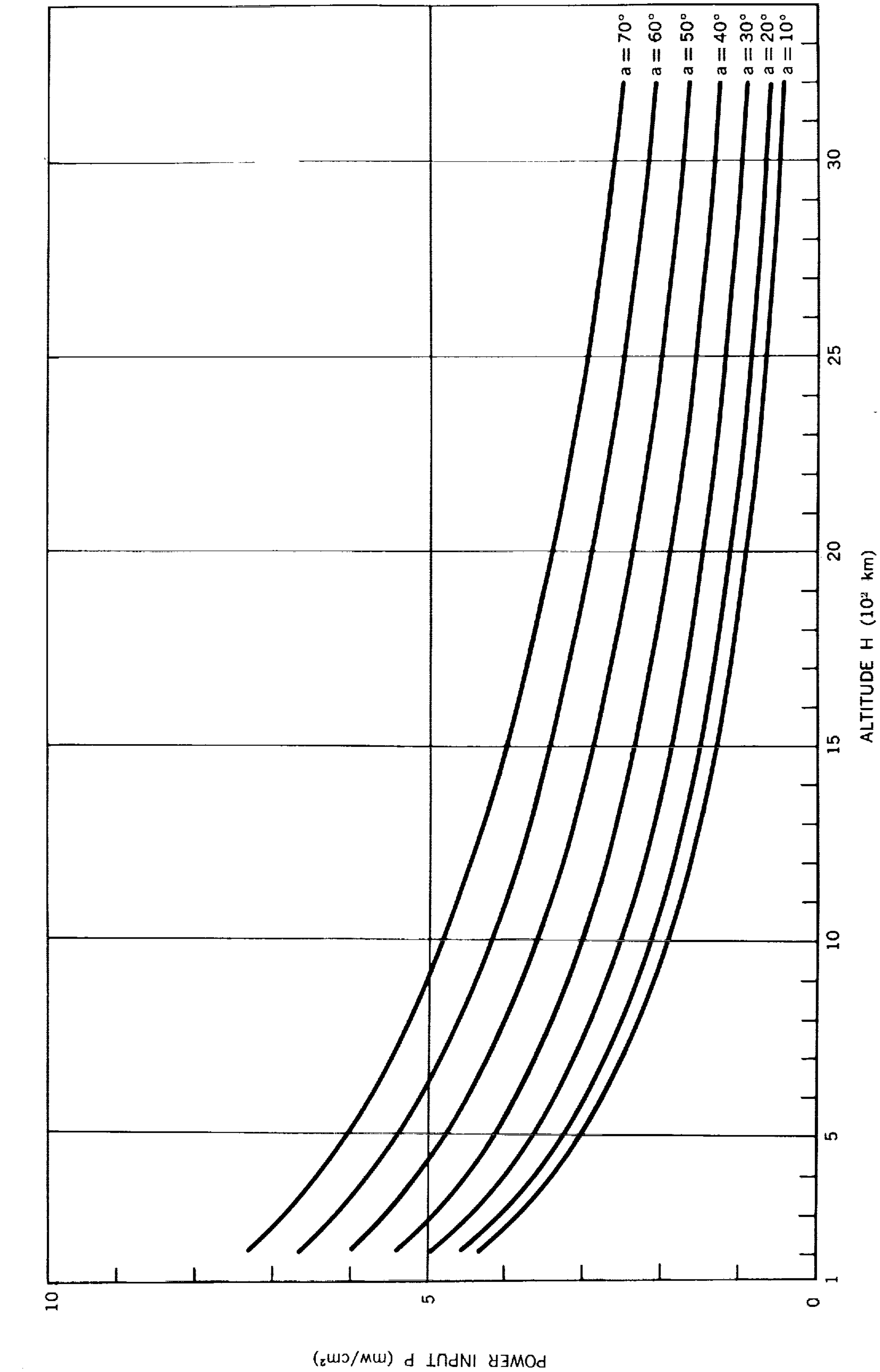

Figure A11—b = 110°

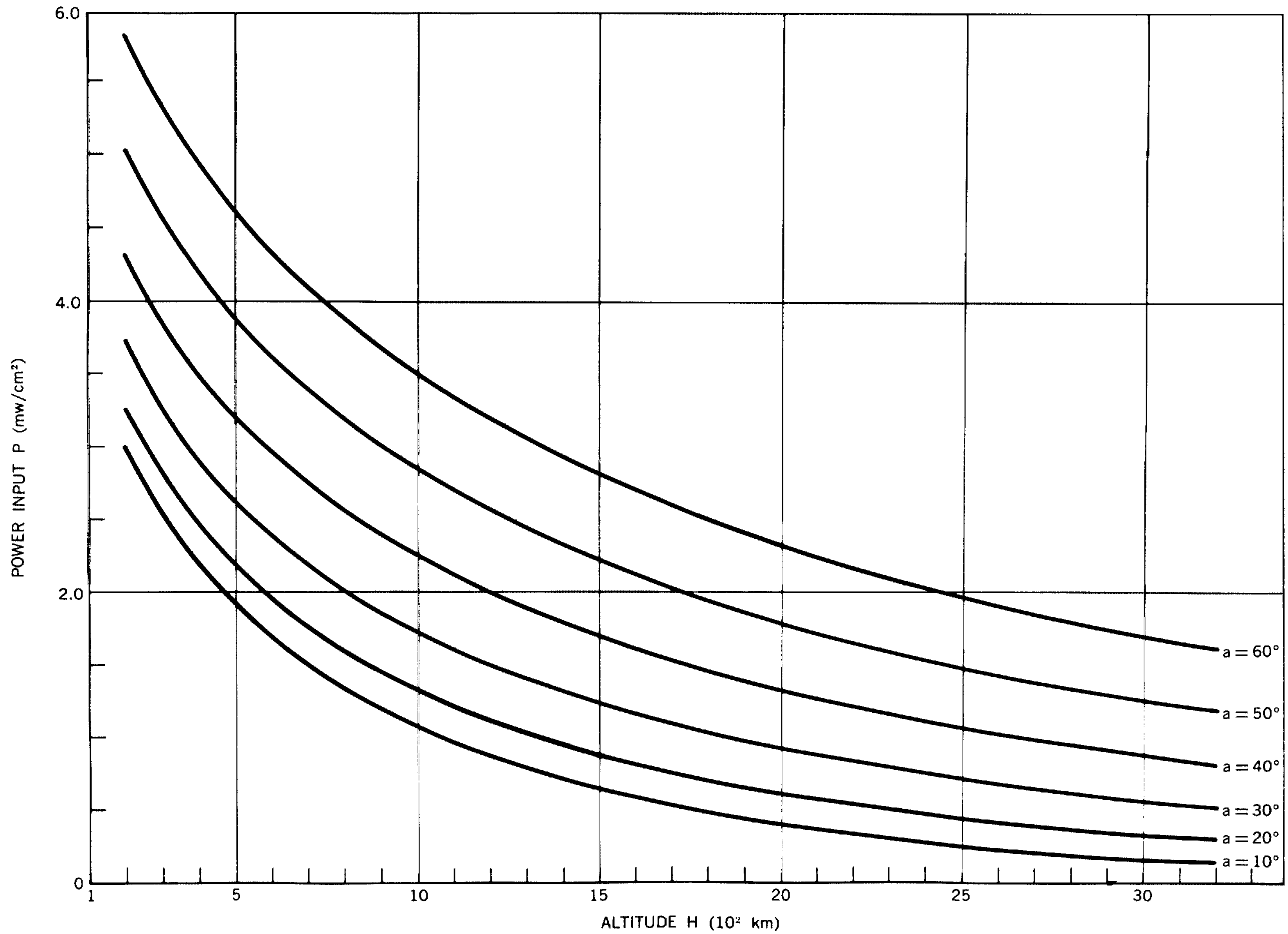

Figure A12—b = 120°

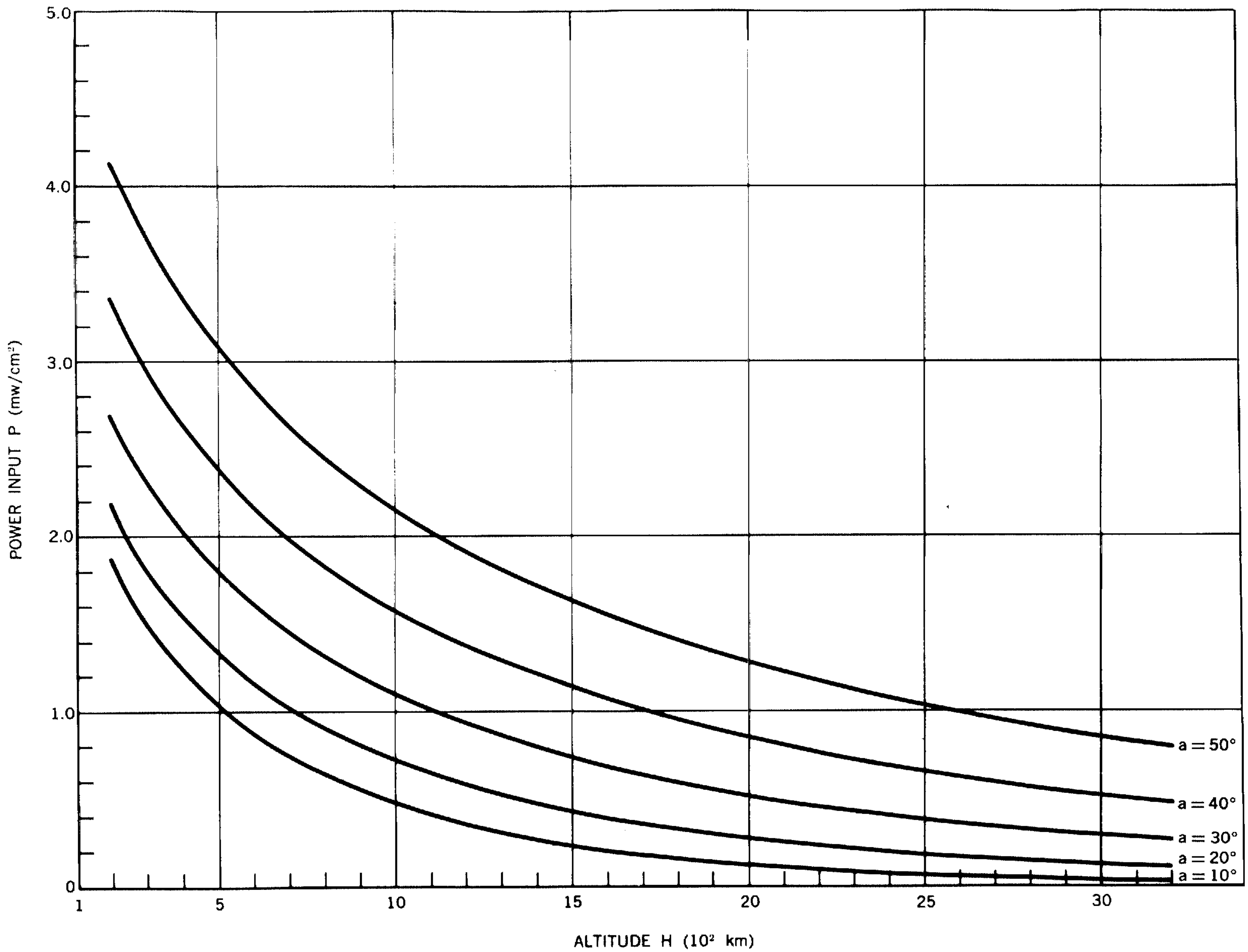

Figure A13—b = 130°

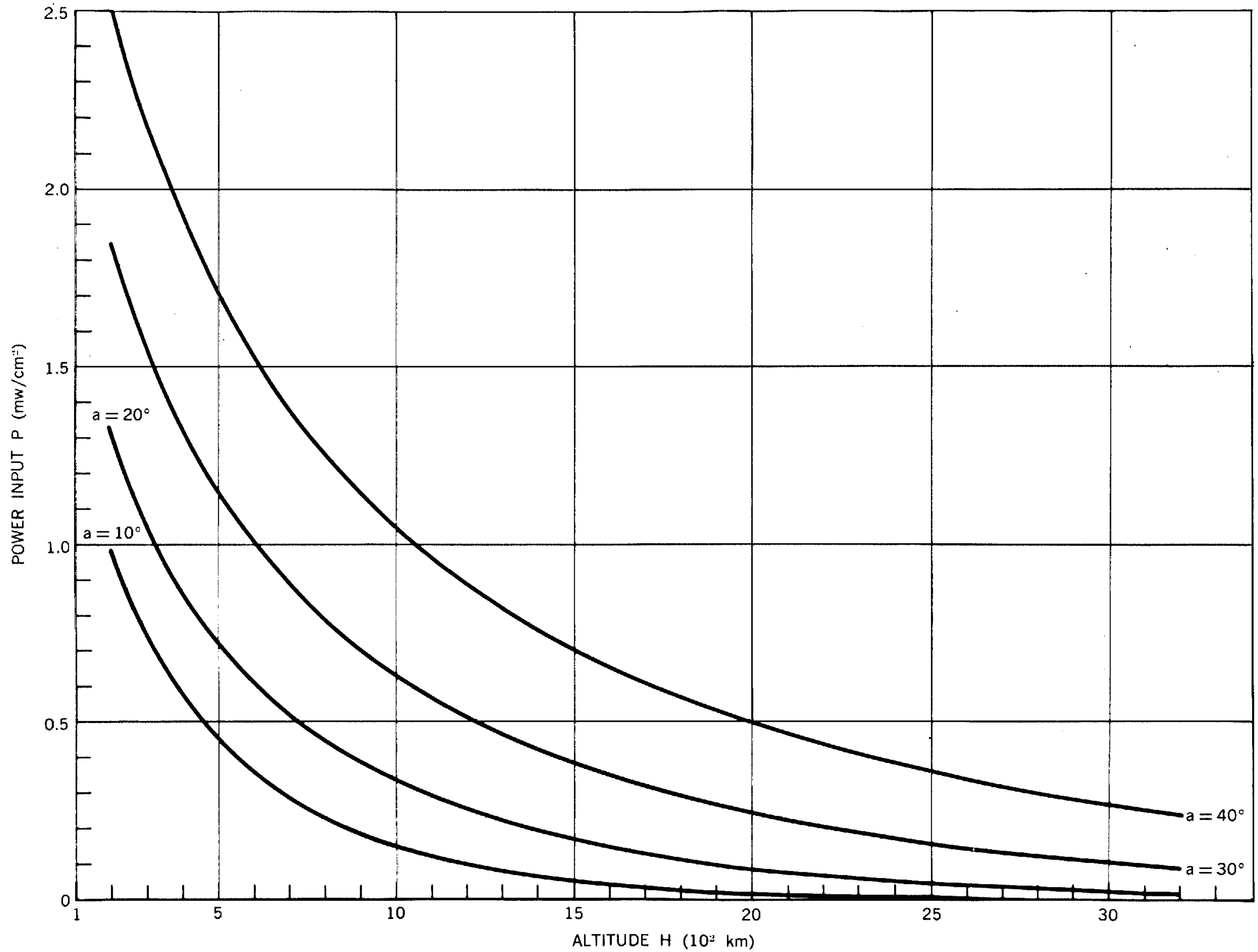

Figure A14—b = 140°

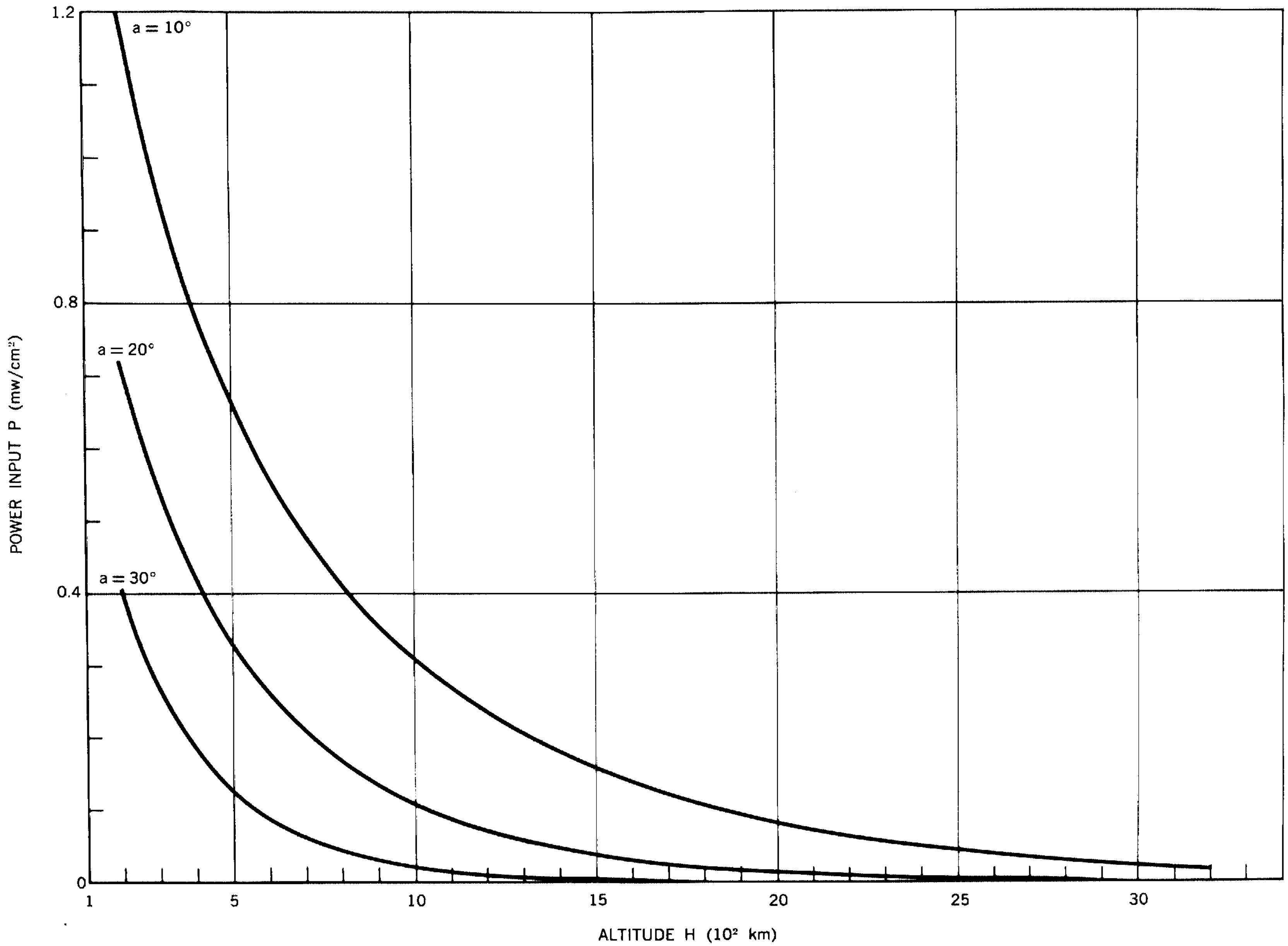

Figure A15—b = 150°

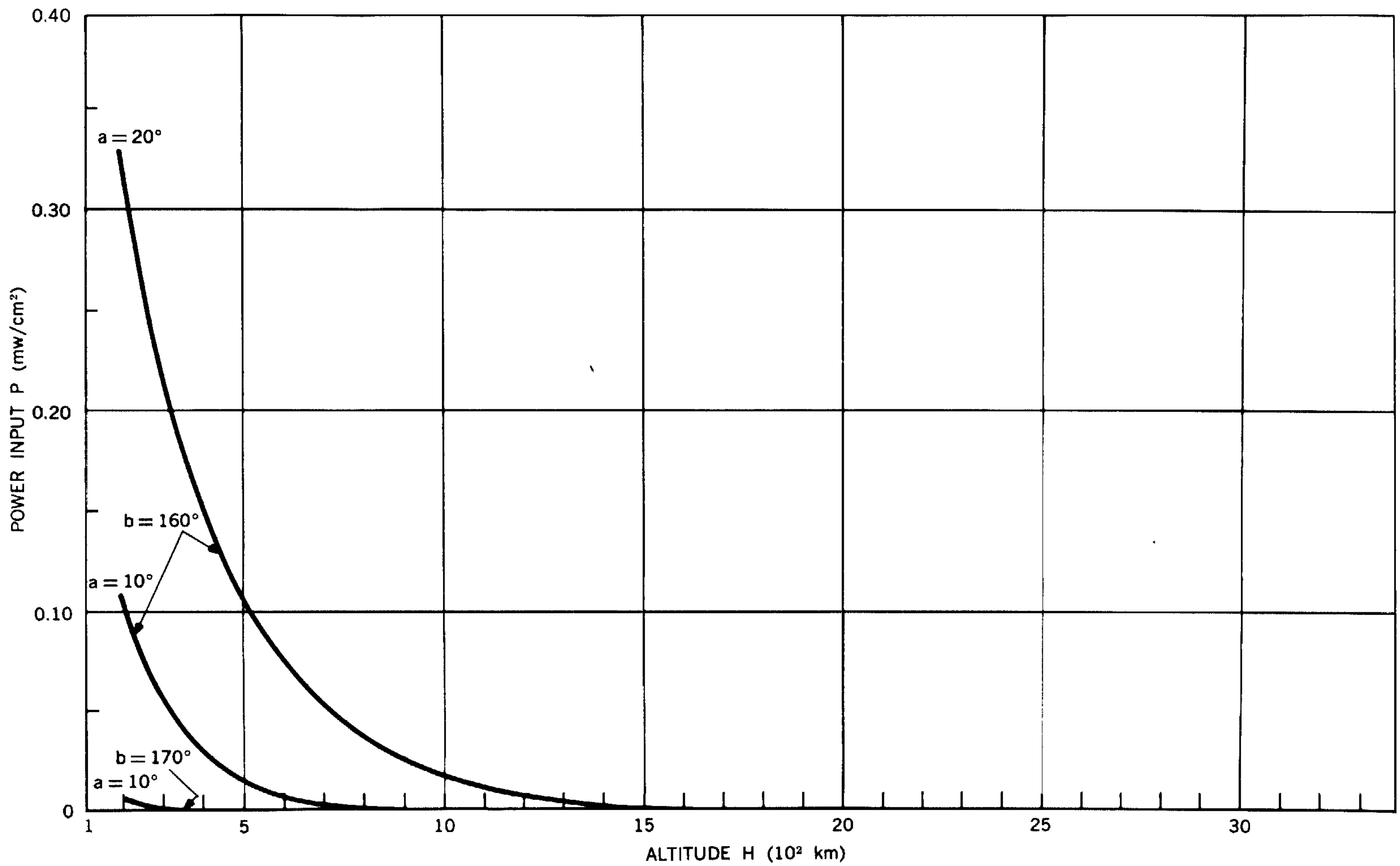

Figure A16—b = 160°, 170°

Appendix B

The Average Incident Power to a Flat Plate per Spin Period for Various Angles a and b as a Function of Altitude for the Range $200 \leq H \leq 32000$ km

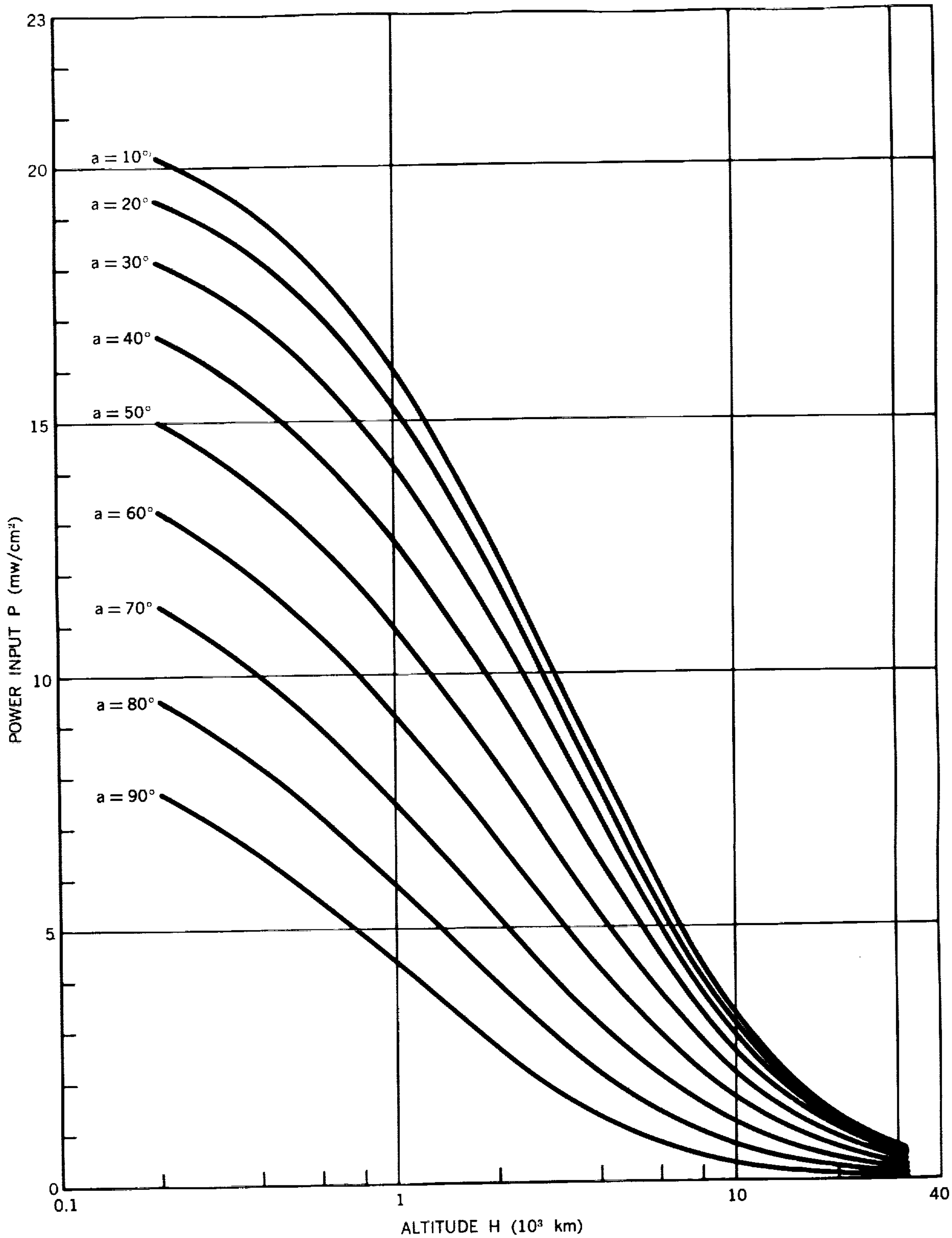

Figure B1—b = 10°