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ELECTRON DENSITY DISTRIBUTION
IN THE UPPER F-REGION

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SUMMARY

A modified Chapman function with a variable scale height gradient has been found to be in good agreement with the electron density distribution obtained experimentally within the height range of about 100 km below the F2 peak to an altitude of about 700 km. The scale height distribution derived from this model is also consistent with the neutral gas scale height of the 1961 COSPAR International Reference Atmosphere.

CONTENTS

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INTRODUCTION

Although ionospheric investigations have been carried on extensively for about three decades, observational data were confined to the peak of the F-region and below, while the region above the peak remained unexplored due to the limitations of ground-based radio sounding techniques. Only recently, with the advent of rocket and radar backscatter techniques, has it been possible to explore the region above the F peak and to obtain some reliable observational data.

On the basis of their ion trap experiment from Wallops Island, Virginia, Hanson and McKibbin (Reference 1) concluded that the ion-density profile between 350 and 750 km corresponds to a hydrostatic distribution with a constant temperature of 1240°K assuming an ion mass of 16. Their measurement corresponded to 8:40 PM, about two hours after sunset. Jackson and Bauer (Reference 2) obtained a daytime (3:00 PM) electron density profile between 220-620 km from the same station which supported the concept of a hydrostatic distribution above the F peak. Again assuming an ion mass of 16, they concluded the daytime temperature to be about 1640°K.

The purpose of this work is to examine electron density data in the light of the above findings and to estimate some of the atmospheric parameters. An analytical expression for the electron density distribution will be formulated in terms of the scale height which best fits the observational data and permits the determination of the scale height and other relevant parameters.

EMPIRICAL MODEL OF THE F-REGION

Wright (Reference 3) proposed a model of the ionosphere above the F2-maximum represented by a simple Chapman function with a constant scale height of 100 km. Yonezawa and Takahashi (Reference 4), who developed an extensive theory of F2 region, showed that such a simplified version of the

ionosphere was not consistent with experimental results in general. They attempted to fit an experimental profile obtained by Garriott (Reference 5) with a constant scale height gradient and found close agreement only up to 100 km above the F2 peak; the calculated and observed values were found to differ considerably in the higher region. Yonezawa and Takahashi suggested that a better fit with the experimental results might be obtained by assuming a variable scale height gradient.

With the availability of more reliable experimental data, it is now possible to examine these questions in more detail. In arriving at a model of the electron density profiles, from near the peak of the F2 layer to the region above, the following known features of the electron density distribution have been taken into account:

1. The region above 250 km is essentially isothermal and may be divided into three parts according to the relative concentrations of the ionic constituents. It is now generally believed that oxygen ions predominate up to an altitude of about 900 km, helium ions from 900 km to about 3000 km, and protons above 3000 km. The thicknesses of these regions vary with atmospheric temperature (Reference 6), but the various ionic constituents are in thermodynamic equilibrium with the neutral atmosphere.

2. The electron distribution near the peak of the F layer can be approximated by a parabolic layer and that above the peak by an exponential layer varying as $\exp -z/H'$, where H' is the scale height of electron ion gas. This region is usually called an isothermal diffusive equilibrium region. Since the predominant ionic constituent in this region is O^+ and the neutral atmosphere also consists predominantly of atomic oxygen, we may write, assuming thermodynamic equilibrium,

$$H' = 2H_O \tag{1}$$

where H_O is the scale height corresponding to atomic oxygen.

3. In the region near the F peak the scale height of the neutral atmosphere is height dependent because, even though the region is isothermal, the mean molecular mass changes with height. It approaches, H_O, the scale height of atomic oxygen at heights well above the peak.

We will begin with a Chapman function modified to include a variable scale height:

$$N = N_m \exp \frac{1}{2} \left[1 - \int_{h_m}^{h} \frac{dz}{H} - \exp\left(-\int_{h_m}^{h} \frac{dz}{H} \right) \right] , \tag{2}$$

where N corresponds to electron density at height h and h_m is the height of the maximum electron density, N_m. The physical significance of the scale height h, will be discussed later. The following analytical form for H has been found to be compatible with the aforementioned features of the F region:

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$$H = \frac{H_0 \left[1 - \alpha \, \exp\left(- \frac{\alpha z'}{2}\right)\right]^2}{\left[1 - \alpha \, \exp\left(- \frac{\alpha z'}{2}\right) - \frac{\alpha^2 z'}{2} \, \exp\left(- \frac{\alpha z'}{2}\right)\right]} \, , \tag{3}$$

where

$$z' = \frac{z}{H_0} = \frac{h - h_m}{H_0} \, ,$$

$$\alpha = \frac{H_0 - H_m}{H_0} \, ,$$

and H_m is the "scale height" at the peak $(z = 0)$. Equation 3 has been so formulated that with the increase of z', H approaches H_0 and reduces to a constant scale height for $\alpha = 0$. For $\alpha z' \ll 1$, Equation 3 assumes a linear form. Substituting Equation 3 in Equation 1 gives

$$N = N_m \exp \frac{1}{2} \left\{ 1 - \frac{z'}{1 - \alpha \, \exp\left(- \frac{\alpha z'}{2}\right)} - \exp\left[\frac{z'}{1 - \alpha \, \exp\left(- \frac{\alpha z'}{2}\right)}\right] \right\} . \tag{4}$$

It is easy to verify that Equation 4 reduces to the appropriate forms near the peak and well above the peak. In view of the different ion transition regions, Equations 3 and 4 are assumed to be valid a few hundred kilometers above the peak where O^+ ions are predominant.

COMPARISON OF EXPERIMENTAL AND EMPIRICAL DATA

From Equation 4, N/N_m may be plotted as a function of z', for arbitrary values of α. This is shown in Figure 1 for values of $\alpha = 0$, 0.3, 0.4, 0.5 and 0.6. The curve corresponding to $\alpha = 0$ is equivalent to the simple Chapman function. The parameter α may be interpreted as an index which measures the departure from the simple Chapman function.

In order to verify that Equation 4 is in agreement with the measurements, we shall consider the electron density profile obtained by Jackson and Bauer (Reference 2) and the ion-density profile by Hanson and McKibbin (Reference 1), shown in Figures 2 and 3 respectively. Equation 3 is completely determined for a given set of values $(N_m, h_m, \alpha$ and $H_0)$ which can be evaluated for any given experimental profile. The numerical values corresponding to the profiles shown are given in Table 1. The computation was performed on the Goddard Space Flight Center IBM 7090 computer by a differential correction least-square fit.

In Figures 2 and 3 Equation 4 is compared with the experimental results. It is seen that the empirical formulation of electron density distribution is in excellent agreement with the experimental data. Note that the value of α is higher in the day than in the evening. Since α is an index which measures the departure of Equation 4 from the Chapman function, it may be concluded that the electron
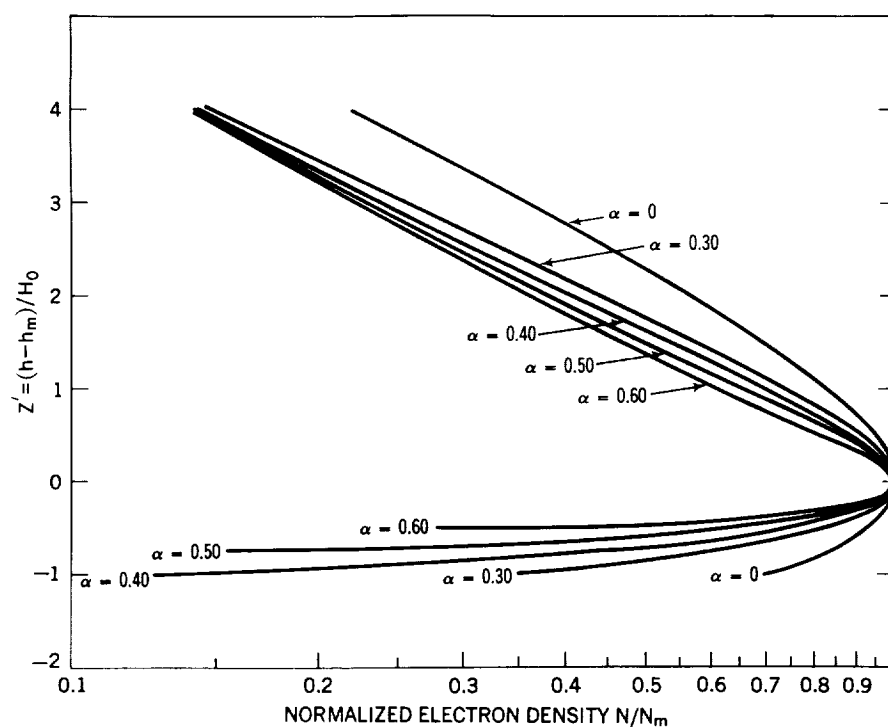
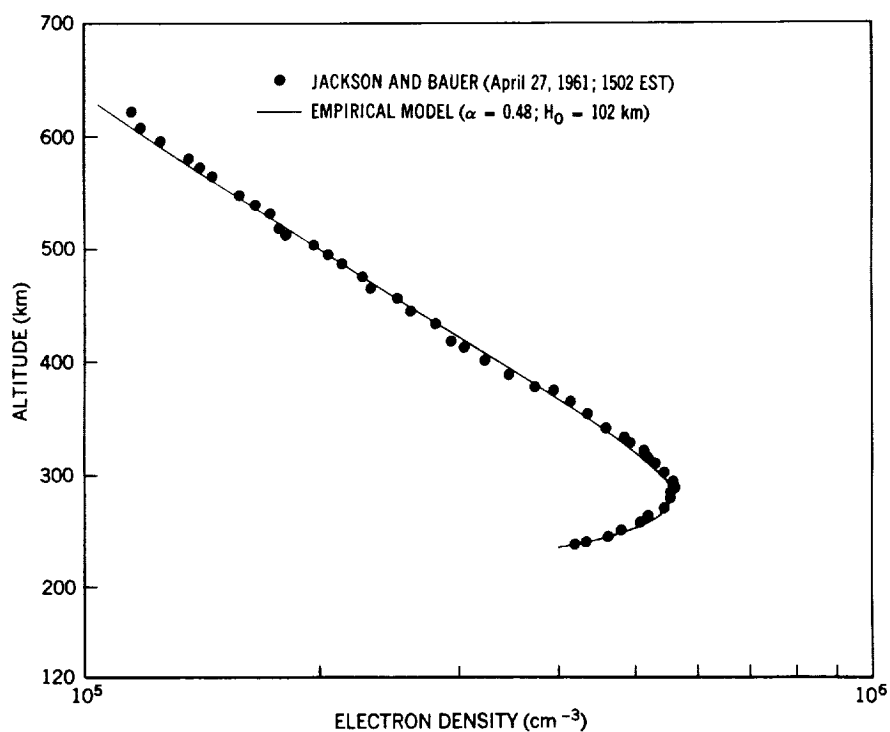Figure 1—Empirical Models of Electron Density Distribution.

Figure 2—Comparison of an Experimental Electron Density
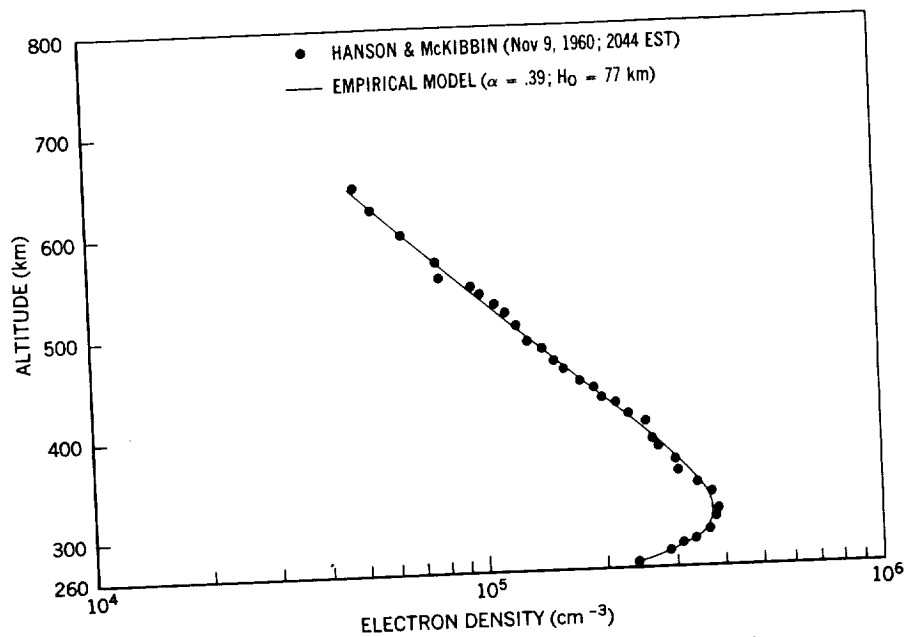Distribution (Reference 2) with the Empirical Model.

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Figure 3—Comparison of an Experimental Electron Density
Distribution (Reference 1) with the Empirical Model.

density distribution tends towards the Chapman function after sunset. This conclusion was also reached by Yonezawa, Martyn, Duncan and Dungey (References 7 through 10) from purely theoretical considerations.

In addition to the two experimental profiles shown in Figures 2 and 3, we have considered three sets of unpublished data (References 11 and 12) obtained during quiet sun conditions, to test the validity of Equation 4.

Table 1—Numerical Values
of the Parameters of Figures 2 and 3.

Profile	Time (EST)	N_m ($10^5 cm^{-3}$)	h_m (km)	h_m (km)	α
Electron Density* Ion Density †	1500 2040	5.55 3.73	280 313	102 77	0.48 0.39

*Reference 2
†Reference 1

In all three cases the Wallops Island Data (taken at 0947 and 2143 EST, July 13, 1960 and 1817 EST, June 24, 1961) were in excellent agreement with Equation 4 between the height range of about 100 km below the peak to an altitude of about 700 km.

SCALE HEIGHT DISTRIBUTION

After the values of h_m, α and H_o are obtained for a given profile, Equation 3 may be used to compute H as a function of height. The height variation of H is shown in Figure 4 for the Jackson and Bauer, Hanson and McKibbin profiles and the neutral scale heights of the COSPAR International Reference Atmosphere (Reference 13). Since H approaches the scale height of atomic oxygen in the region
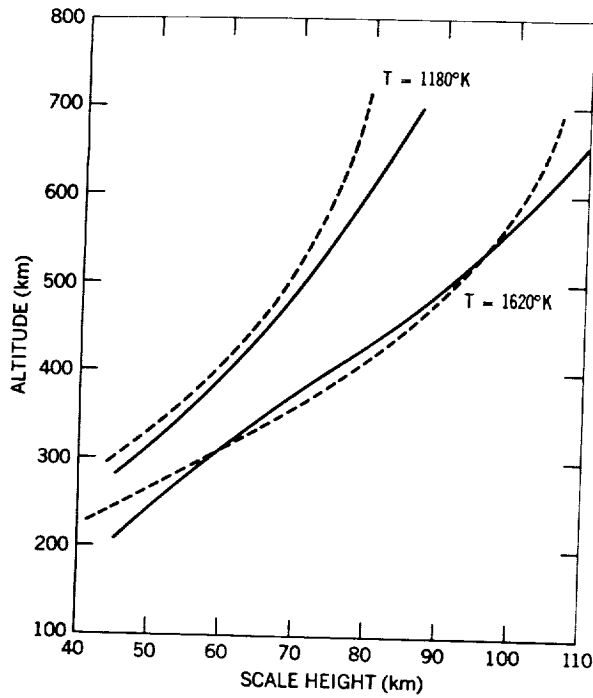
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Figure 4—Comparison of Neutral Scale Heights of CIRA (solid curves) with the Scale Heights Derived from Electron-Density Data (dashed curves).

well above the peak, the corresponding isothermal temperature can be computed from the following relation

$$H = \frac{kT}{mg},$$

(5)

where k is the Boltzmann constant, m the mean molecular mass, T the temperature and g the acceleration due to gravity. By taking $m = 16$ and the appropriate value of g at 600 km, the isothermal temperatures corresponding to the Jackson and Bauer and Hanson and McKibbin profiles are about 1620 °K and 1180 °K, respectively. These values are within the limits assigned by these authors to their temperature estimates. For the sake of proper comparison, the numerical values of the neutral scale heights of the reference atmosphere have been normalized to the temperatures in the isothermal region derived from the experimental data.

It is seen from Figure 4, that the agreement between the neutral scale heights of CIRA and the scale heights derived from the electron density data is very good considering the model atmosphere uncertainties. It may therefore be reasonable to assume that the scale height derived from the electron density profile is actually the scale height of the neutral atmosphere. In the subsequent discussion we shall use H for the neutral scale height.

EQUATION OF CONTINUITY

The electron density and neutral scale height distribution obtained in the preceeding sections may now be examined in the light of the equation of continuity. In a horizontally stratified ionosphere, under the influence of the gravitional field only, the continuity equation may be written:

$$\frac{dN}{dt} = q - L - \frac{d}{dz}\left(NW_D\right),$$

(6)

where q and L are the rates of electron production and loss, respectively, and W_D is the upward ambipolar diffusion velocity of the electron ion gas, given by the following expression (Reference 14):

$$W_D = -D\left[\frac{1}{N}\frac{dN}{dz} + \left(\frac{1}{T}\frac{dT}{dz} + \frac{1}{H'}\right)\right],$$

(7)

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where H' is the scale height of the electron-ion gas, T is the temperature and the diffusion coefficient

$$D = \frac{4.5 \times 10^{-17}}{n} T^{\frac{1}{2}} \sin^2 I , \qquad (8)$$

where I is the magnetic dip angle and n is the density of the neutral atmosphere. For an isothermal region it can be easily shown that:

$$n = n_m \exp\left(-\int \frac{dz}{H}\right) , \qquad (9)$$

where n_m is the number density at the height of the maximum. Equation 6, together with Equations 1, 2, 7, 8 and 9, may be written in the following form:

$$\frac{dN}{dt} = q - L + \frac{4.5 \times 10^{17} T^{\frac{1}{2}} \sin^2 T}{4H^2 n_m X} \left[X^2 - 2X \left(\frac{dH}{dz} - \frac{H}{2H_0} + 1 \right) + \left(\frac{H}{H_0} + 2 \frac{dH}{dz} - 1 \right) \right] , \qquad (10)$$

where

$$X = \exp\left(-\int \frac{dz}{H}\right) .$$

The last term in Equation 10 represents the effects of diffusion and may be computed for a given electron density profile and a given value of n_m.

Assuming that the loss term is an attachment type with a loss coefficient β given by Ratcliffe et al. (Reference 15):

$$\beta = 10^{-4} \exp\left(-\int \frac{h - 300}{50}\right) \sec^{-1}$$

between the height interval of 250-350 km, we can estimate the relative importance of the loss and diffusion terms. Further, for a quasi-equilibrium condition, it is also possible to estimate the rate of production. Thus, by using the values of N_m, h_m, α and T corresponding to Figure 2 and assuming

Table 2

Numerical Estimates of the Production, Loss, and Diffusion Terms for Wallops Island.

Altitude (km)	$L(=\beta N)$ (cm^{-3} sec^{-1})	div NW_D (cm^{-3} sec^{-1})	q = L + div NW_D (dm^{-3} sec^{-1})
255	124	18	142
280	84	37	121
306	47	36	83
331	25	29	54
357	14	22	36

$n_m = 2 \times 10^9 / cm^3$ at the peak ($h_m = 280$ km), the numerical estimates of the production, loss, and diffusion terms can be tabulated for Wallops Island, Virginia ($I = 70°$). In Table 2, the last column, which represents the production, is computed under the assumption of quasi-equilibrium.

The numerical values of q shown in the last column are in reasonable agreement with the estimates of the production rate of O^+ given by Watanabe and Hinteregger (Reference 16).

CONCLUSION

An analytical form of the electron density distribution has been found which can be made to fit observational data, with high accuracy, within the height range of about 100 km below the F2 peak to an altitude of about 700 km. This technique may provide a useful tool for matching — in the region around the F2 maximum — the bottomside observations with the data from the Topside Sounder Satellite. In principle it is possible to use Equation 4 to extrapolate the bottomside data of the quiet ionosphere up to an altitude of about 700 km.

The scale height distribution derived from the electron density data is consistent with the neutral scale heights of the COSPAR International Reference Atmosphere. By assuming that Equation 4 is a solution of the equation of continuity and using appropriate values for diffusion and loss coefficients, the estimate of electron production rate is found to be in reasonable agreement with the values obtained from solar flux data by Watanabe and Hinteregger (Reference 16). Consequently, it may be possible by further refinement of this method and definitive rocket flights containing simultaneous ionospheric, neutral atmosphere and solar radiation experiments, to use ionospheric data for studying the neutral atmosphere.

ACKNOWLEDGMENTS

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