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# ANALYSIS OF THE EFFECT OF ALTIMETER-SYSTEM ACCURACY ON COLLISION PROBABILITY

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# ANALYSIS OF THE EFFECT OF ALTIMETER-SYSTEM ACCURACY ON COLLISION PROBABILITY

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## SUMMARY

The probability of collision between two airplanes has been determined for their normal assignment to adjacent flight levels and for their very rare misassignment to the same flight level. Collision probabilities were computed for a flight-level separation minimum of 500 feet, airplane vertical dimensions of 20 and 40 feet, altitude-misassignment probabilities of 1/100,000 and 1/1,000,000, and altimeter-system errors ($3\sigma$ values) ranging from 0 to 2,000 feet.

The results of the analysis showed that minimum collision probabilities occurred at altimeter-system errors of about 200 feet, and that for altimeter-system errors from 500 to 2,000 feet, the collision probability was several orders of magnitude greater than that for altimeter-system errors of 250 feet or less.

## INTRODUCTION

The amount by which an airplane deviates from an assigned flight level, that is, its vertical-displacement error, depends on the altimeter-system error (combined static-pressure and altimeter errors) and on the flight technical error (random deviations from stabilized cruise altitude (ref. 1)). The probability of collision between two airplanes flying adjacent flight levels depends, therefore, on their vertical displacement errors and, in addition, on (1) the size of the airplanes, (2) the flight-level separation minimum, (3) the lateral displacement of the airplanes, and (4) the collision exposure time. The collision exposure time can vary from very small to very large values depending on the direction of flight of the two airplanes; if they are flying in the same direction, the exposure time becomes a function of the longitudinal separation and the relative speeds of the airplanes.

In a study performed by the Federal Aviation Agency in 1961, the relationship between collision probability and vertical-displacement error was determined for the case of two airplanes assigned to adjacent flight levels. More recently, questions have been raised regarding the relation between altimeter-system errors and collision probability for the case where airplanes, through an error on the part of the ground controller, are assigned to the same flight level. The possibility of altitude misassignments poses a conflicting problem with regard to altimeter-system accuracy in that the small errors which would prevent collision between airplanes assigned to adjacent flight levels would increase the probability of collision if the airplanes were misassigned to the same flight level. To provide an answer to this problem, an analysis has been made to determine the effect of altimeter-system accuracy on the probability of collision between two airplanes for the case of assignment to adjacent flight levels or misassignment to the same flight level. In this analysis the probabilities of collision were computed on the assumption that the airplanes were located along a vertical line.

## SYMBOLS

$\sigma$ standard deviation of altimeter-system error

$h$ vertical dimension of airplane

$S$ flight-level separation minimum

$P_m$ altitude misassignment probability

$P_{1,2,3,\ldots n}$ probability of the altimeter-system error being within error increments numbered 1, 2, 3, . . . n, equal to the corresponding area of the increment under a normal curve

$P_{c_s}$ collision probability for two airplanes assigned to same flight level

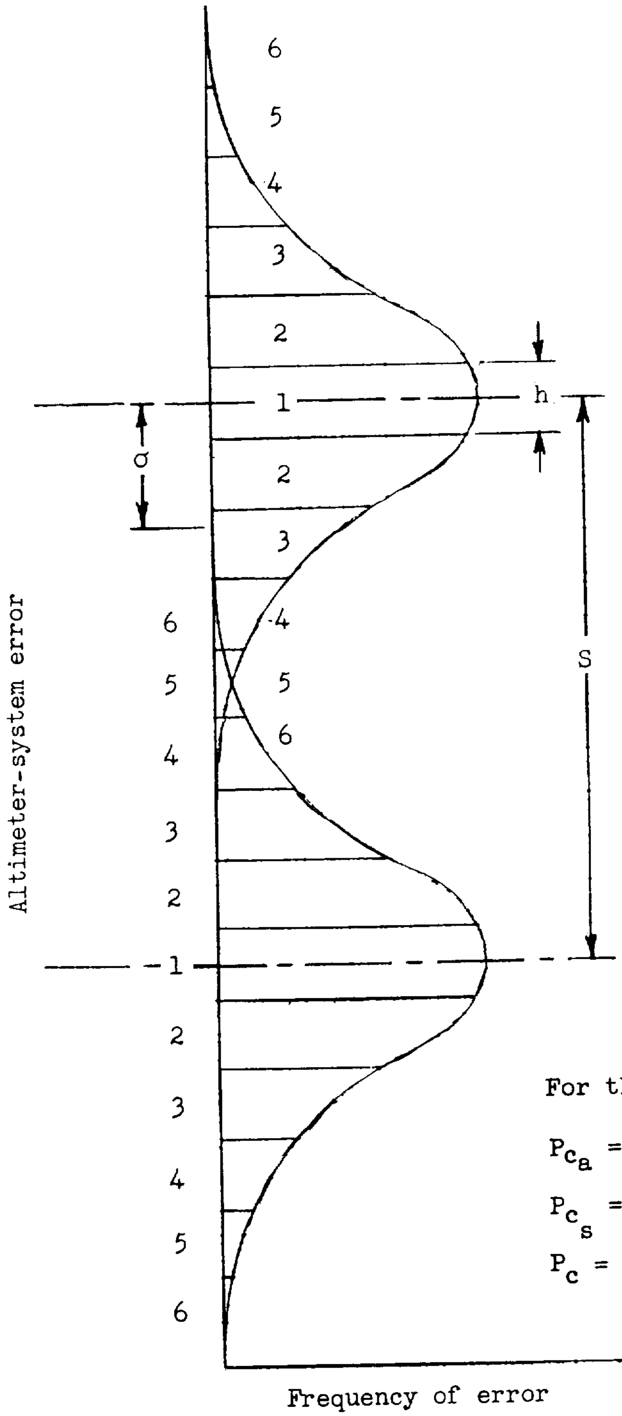
$P_{c_a}$ collision probability for two airplanes assigned to adjacent flight levels

$P_c$ total collision probability for two airplanes assigned to same flight level or to adjacent flight levels, $P_{c_s} + P_{c_a}$

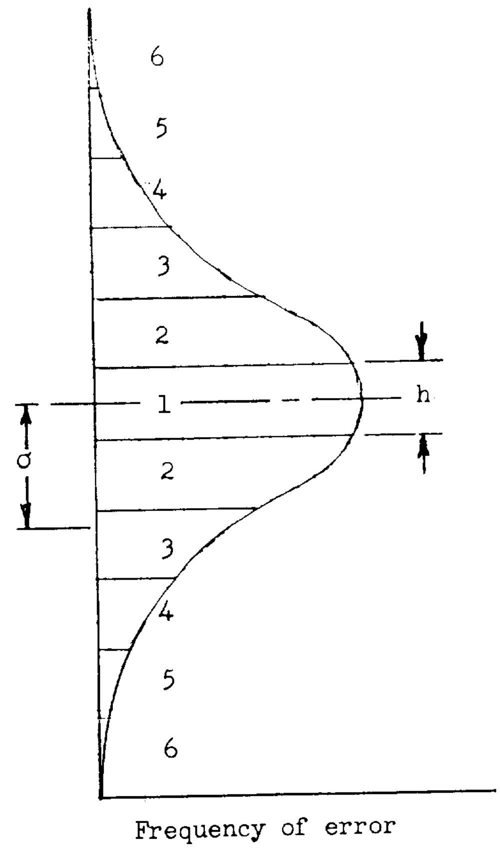
## ANALYSIS

The method of analysis used in the present study is based on the assumption that the altimeter-system errors of all airplanes have a normal distribution. For any one airplane, therefore, the probability that the error of its altimeter system will be within a given error increment can be determined from the corresponding area under a normal curve; these incremental areas are readily determined from tables of probability developed for this type of distribution.

The application of normal curves to the determination of collision probabilities is illustrated in figure 1. Shown in this figure are normal curves representing the altimeter-system error distribution for two airplanes assigned to flight levels separated by a distance $S$ (fig. 1(a)) and for two airplanes assigned to the same flight level (fig. 1(b)). The altimeter-system error scales




(a) Two airplanes assigned to adjacent flight levels.




(b) Two airplanes assigned to same flight level.

For the illustrative cases shown in this figure:

$$P_{c_a} = P_5^{\,2} + 2P_4P_6$$

$$P_{c_s} = (P_1^{\,2} + 2P_2^{\,2} + 2P_3^{\,2} + 2P_4^{\,2} + 2P_5^{\,2} + 2P_6^{\,2})\,P_m$$

$$P_c = P_{c_a} + P_{c_s}$$

Figure 1.- Diagram of method for calculating collision probabilities for two airplanes assigned to adjacent flight levels or to the same flight level.

of each of the curves are divided into increments equal to the vertical dimension $h$ of the airplanes. This dimension $h$ represents the largest error increment within which collision between the two airplanes is certain to occur since the values of $h$ are assumed to be equal for the two airplanes. For the illustrative example shown in figure 1, $h$ is an even increment of $S$; however, it is not essential that this be the case for the application of the method.

As can be seen from figure 1, collision will occur when the altimeter-system errors of the two airplanes fall within intersecting or corresponding error increments at the same time. For example, for the case of the two airplanes assigned to adjacent flight levels, the probability that the error of one airplane will fall within increment 4 is represented by the area of that increment and may be designated $P_4$; similarly, the probability that the error of the other airplane will fall within intersecting increment 6 is represented by the area of that segment and may be designated $P_6$. The probability that the errors of the airplanes will fall within these increments simultaneously is equal to the product of the two error-increment probabilities or $P_4P_6$. For the case of the two airplanes assigned to the same flight level, the collision probability for a given error increment is equal to the product of the probabilities (i.e., areas) of corresponding error increments. For example, for error increment 1, the collision probability would be the product of the two error increment probabilities $P_1$, that is, $P_1^2$.

In the illustration in figure 1(a) the collision probability, $P_{c_a}$ for the two airplanes assigned to adjacent flight levels is the sum of the collision probabilities of error segments 4, 5, and 6 and is equal to $P_5^2 + 2P_4P_6$. For the case of two airplanes assigned to the same flight level (fig. 1(b)) the collision probability $P_{c_s}$ is determined not only by the sum of the collision probabilities of the corresponding error increments but also by the probability of the two airplanes being assigned to the same flight level, that is, the altitude-misassignment probability $P_m$. For the case illustrated in figure 1, therefore, the collision probability, $P_{c_s}$, is equal to $\left(P_1^2 + 2P_2^2 + 2P_3^2 + 2P_4^2 + 2P_5^2 + 2P_6^2\right)P_m$. With the introduction of $P_m$, the value of $P_{c_a}$ should, to be exact, be multiplied by the factor $(1 - P_m)$; however, since the value of $P_m$ is extremely small, the factor $(1 - P_m)$ can be considered unity with negligible error.

The total collision probability $P_c$ for a given set of values of $\sigma$, $h$, $S$, and $P_m$ is the sum of the collision probabilities for the two cases of airplanes assigned to adjacent flight levels or misassigned to the same flight level, that is, $P_c = P_{c_a} + P_{c_s}$.

In the present study, $S$ is assigned a value of 500 feet (the flight-level separation minimum specified for combined IFR (Instrument Flight Rules) and VFR (Visual Flight Rules) operations in the altitude range below

29,000 feet). Values of 20 and 40 feet were assigned to the airplane vertical dimension $h$, since 20 feet can be considered representative of the vertical dimension of medium-size airplane and 40 feet represents the approximate vertical dimension of the largest of present-day turbojet transports. Values of 1/100,000 and 1/1,000,000 were assigned to the altitude-misassignment probability $P_m$ because the rate at which altitude misassignments occur in present-day operations is estimated to fall within the range of these values; this estimate is based on an approximation of the number of altitude assignments per year in the United States and on information relating to near-miss incidents and ground controller error reported in Project Scan (ref. 2). For the values of $S$, $h$, and $P_m$ noted above, collision probabilities $P_c$ were computed for altimeter-system errors which, expressed as $3\sigma$ values (that is, values within which the probability is 99.7 percent), ranged from 0 to 2,000 feet.

In the present analysis, the assumption is made that the airplanes fly their indicated altitudes exactly, that is, flight technical errors are zero. In addition, it is assumed that the two airplanes are located along the same vertical line, that is, the effects of lateral separation and collision exposure time are not included.

## RESULTS AND DISCUSSION

The results of the analysis are presented in figures 2 and 3. Figure 2 shows the variation of collision probability $P_c$ with altimeter-system error ($3\sigma$ values) up to 300 feet; variations are presented for four combinations of airplane vertical dimension $h$ and altitude-misassignment probability $P_m$. In figure 3, one of the curves in figure 2 has been extended through a range of altimeter-system errors up to 2,000 feet.

It should be noted that the $P_c$ scales in figures 2 and 3 are logarithmic and that the scale of figure 3 is compressed with respect to that of figure 2. Since, as noted earlier, the values for $P_c$ were computed on the assumption that the airplanes were located along a vertical line, these values would be reduced if the probabilities for lateral separation and collision exposure time were included. Thus, although the values of $P_c$ do not represent collision probabilities for the general case of collision occurrence, the values are valid for the purpose of showing the relative effects of variations of $\sigma$, $h$, and $P_m$.

The curves in figure 2 show that the collision probability decreases as the altimeter-system error approaches values somewhat less than 250 feet or one-half the flight-level separation minimum; the collision probabilities in this range of altimeter-system errors are determined solely by the probability of collision of the two airplanes when they are assigned to the same flight level. At higher values of altimeter-system errors, the collision probability increases rapidly (because of the probabilities of collision when the two airplanes are assigned to adjacent flight levels) and, as shown in figure 3, reaches a peak value at an altimeter-system error of about 1,000 feet or twice the flight-level separation

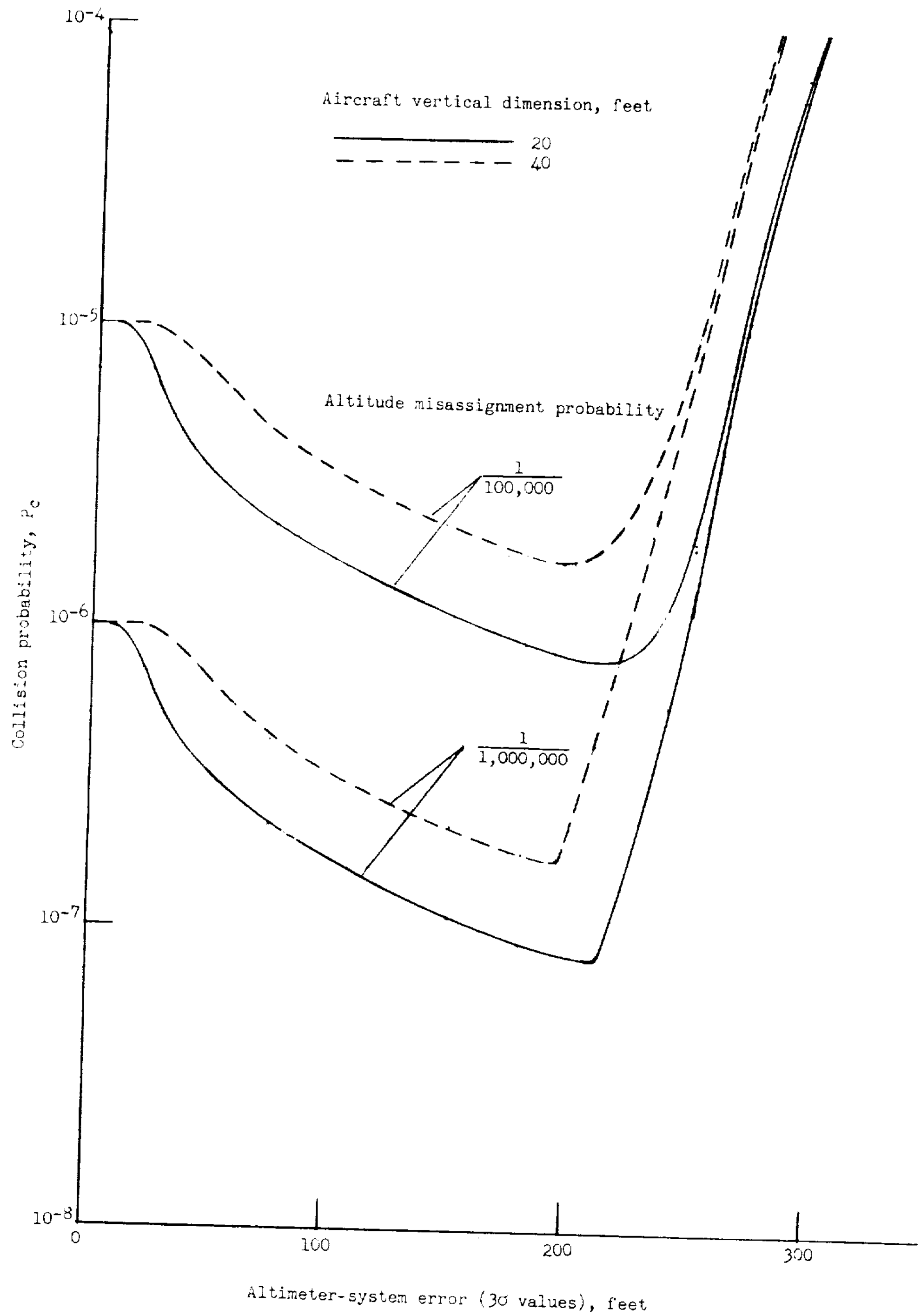



Figure 2.- Variation of collision probability with altimeter-system error (up to 300 feet) for two airplanes assigned to the same flight level or to flight levels 500 feet apart; values of $P_c$ apply to collision along a vertical line only, i.e., the probabilities for lateral separation and collision exposure time are not included.

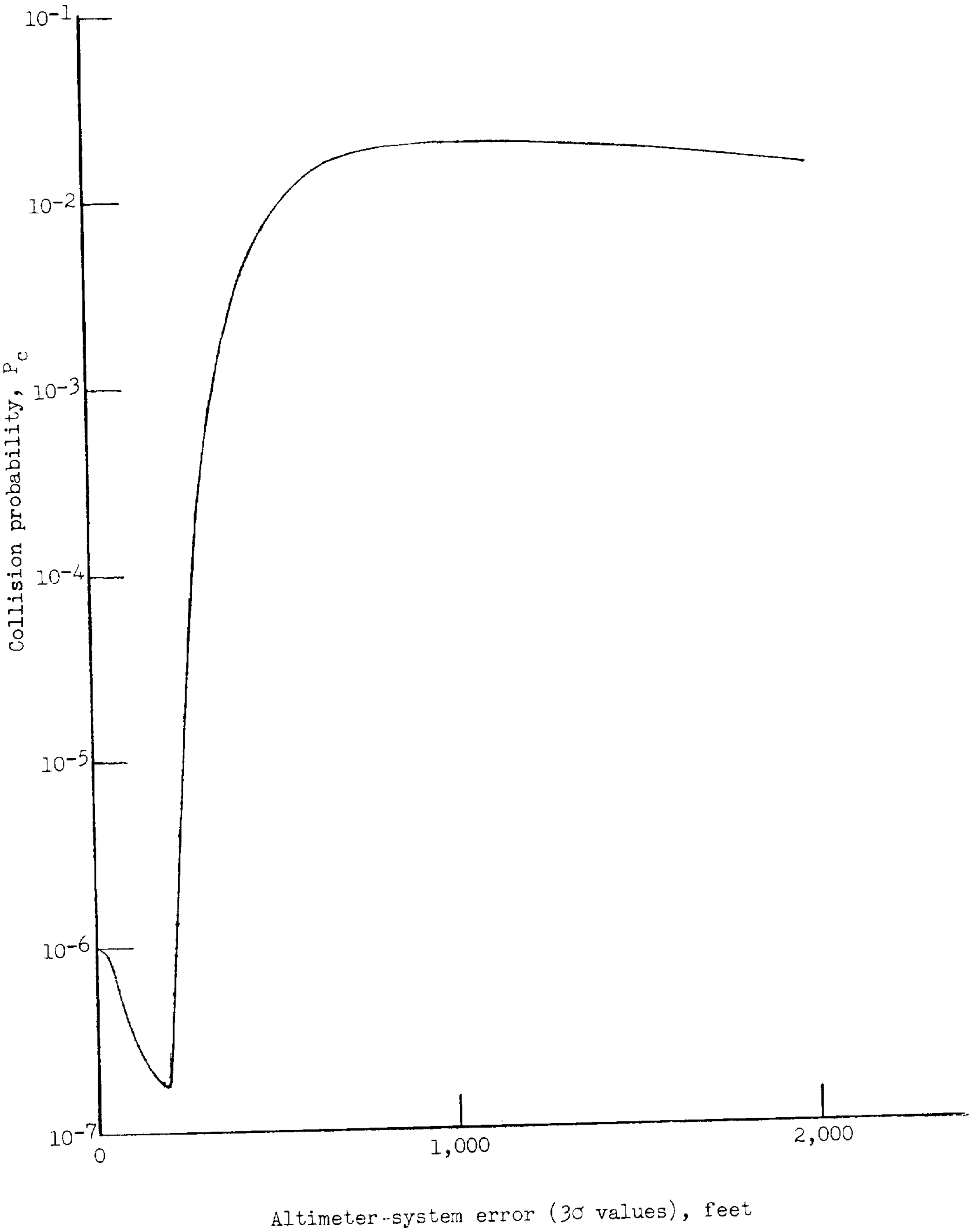



Figure 3.- Variation of collision probability with altimeter-system error (up to 2,000 feet) for two airplanes assigned to the same flight level or to flight levels 500 feet apart; values of $P_c$ apply to collision along a vertical line only, i.e., the probabilities for lateral separation and collision exposure time are not included. $h = 40$ feet; $P_m = 1/1,000,000$.

minimum. Despite the decrease in collision probability beyond this point, the collision probabilities for a range of altimeter-system errors of from one to as high as four times the separation minimum are several orders of magnitude greater than the collision probabilities for system errors less than one-half the separation minimum.

As shown in figure 2, minimum collision probabilities occur at altimeter-system errors ranging from about 195 feet (for the 40-foot airplane vertical dimension) to about 210 feet (for the 20-foot airplane vertical dimension). For the values of $P_m$ and h considered in this analysis, therefore, the value of the altimeter-system error for minimum collision probability is unaffected by changes in $P_m$ and affected to only a small extent by changes in h. From a consideration of these results, it might appear that a minimum collision probability could be realized if altimeter-system errors ($3\sigma$ values) were on the order of 200 feet. As noted earlier, however, the altimeter-system error is only a part of the vertical-displacement error of an airplane and for actual operations the flight technical error must also be taken into account.

Some preliminary information on the flight technical error has recently been acquired from operations of 13 turbine-powered transports in the altitude range from sea level to 40,000 feet. These data have shown that for flight under autopilot (altitude-hold) control, the random deviations from cruise altitude were less than 100 to 250 feet for 99.7 percent of the cruise time. For operations of two of the airplanes under manual control, the altitude deviations were found to be about 350 feet. On the basis of this information, the $3\sigma$ value of flight technical error would be expected to be generally on the order of 200 feet. As this value is about the same as that computed in the present study for minimum collision probability, the altimeter-system error would have to approach zero if the vertical-displacement error is not to exceed a value of 200 feet. Since zero or near-zero altimeter-system errors are not within the present state of the art of altimetry, however, the flight technical error would have to be reduced to a value less than 200 feet if minimum collision probability is to be realized with present-day altimeter systems. If, for example, the flight technical error could be reduced to 150 feet (by maximum use of autopilot control and minimum operation under manual control), the altimeter-system error could have a value of 90 feet and still produce a combined value for the vertical displacement error of 200 feet (since the combined value is the square root of the sum of the squares of the two individual errors). Although the errors of the uncorrected altimeter systems of many present-day airplanes are greater than 90 feet, altimeter-system errors of this order of magnitude can be achieved by applying corrections for the static-pressure and instrument errors by means of correction cards or by the use of static-pressure compensators and altimeter scale-error correctors.

## CONCLUDING REMARKS

The variation of collision probability with altimeter-system error has been computed for two airplanes assigned to adjacent flight levels or misassigned to the same flight level. For a flight-level separation minimum of 500 feet, airplane vertical dimensions of 20 and 40 feet, and altitude-misassignment probabilities of 1/100,000 and 1/1,000,000, minimum collision probabilities were found to occur at altimeter-system errors ($3\sigma$ values) of about 200 feet. For altimeter-system errors from 500 to 2,000 feet, the collision probability was several orders of magnitude greater than that for altimeter-system errors less than 250 feet.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 18, 1963.

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