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TECHNICAL NOTE

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THE LOW FREQUENCY POWER SPECTRUM OF COSMIC-RAY VARIATIONS DURING IGY

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SUMMARY

The power spectra of cosmic-ray variations, as recorded by neutron monitors at various locations, is computed and analyzed herein. The frequency range of the variations covered (from 1/200 to 1/2 per day) is found to contain only one significant peak, contributed by the well known 27-day variation. By use of the power spectrum, a quantitative estimate of the latitude dependence of the 27-day variation may be made. From this, the average rigidity dependence of the modulation amplitude is deduced, and is found to be essentially the same as that of Forbush decreases. The average was taken over the period July 1957 to December 1958, during which the peak decreased significantly.

Irregular variations were also investigated, as was the correlation with magnetic activity. It is found that the irregular variations have approximately the same latitude dependence as the 27-day peak, decreasing roughly exponentially with increasing frequency. There was some correlation between magnetic activity and cosmic-ray variations, but it showed no clear recurrence tendencies.

A general review of power spectral analysis, with emphasis on points relevant to this work, is included here, and the conclusions along with the connection between Forbush decreases and the 27-day variation are discussed.

CONTENTS

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INTRODUCTION

Examination of the record of a high counting rate cosmic-ray neutron monitor immediately reveals its large variability. At least four definite sources of variation can be traced: the 11-year cycle; the Forbush decreases accompanying sudden commencement (SC) magnetic storms; the 27-day variation; and the daily variation. Of these, three are periodic in character: (1) the 11-year variation, which is connected with the cycle of solar activity and will not be discussed further; (2) the 27-day variation, which is believed to be associated with the period of solar rotation; and (3) the daily variation, which reflects anisotropy in the cosmic radiation reaching the earth. Both the 27-day and the daily variation are often observed to undergo large changes in amplitude within a few cycles. In addition, the 27-day variation seems to vary in both phase and frequency (References 1, 2 and 3). If the sources of the 27-day periodicity are located upon the sun, such changes are indeed expected since all features of the solar surface have a transient nature. Thus, a source of 27-day variation may in time be superseded by another, at a different latitude (which affects the rotation period), and at a different longitude (which affects the phase). Changes of this kind, unfortunately, complicate the study of the phenomenon.

Apart from these well-defined variations, the counting rate undergoes various irregular fluctuations. Comparison between stations confirms the fact that these are genuine fluctuations, and not of instrumental origin. Some of the questions associated with these fluctuations are:

(1) What is their typical time scale?

(2) What is their energy dependence?

(3) Do there exist in the primary flux any periodic variations other than those listed?

To answer these and similar questions, and to analyze phase-unstable periodic variations quantitatively, the method of power spectrum analysis is very useful. This method will now be briefly reviewed.

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REVIEW OF POWER-SPECTRAL ANALYSIS

Conventional Fourier analysis is not suitable for the analysis of a time-dependent counting rate $x(t)$, which tends to be periodic with frequency f_0, but randomly changes its phase, now and again. Indeed, it can be shown that as the length of the given record tends to infinity, the corresponding estimated Fourier transform of $x(t)$ for any frequency approaches zero. This is true even for f_0; over the long run, $x(t)$ will have equal probability for being in or out of phase. Two main approaches exist for frequency analysis in this case.

One approach is based on the fact that even though the Fourier transform of $x(t)$ approaches zero in the limit, its mean square, under very general assumptions, tends to a finite limit $P(f)$:

$$P(f) \quad = \quad \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t) e^{2\pi f i t} \, dt \right|^2 \quad .$$

The function $P(f)$ is called the power spectral density, and will be strongly peaked at $f = f_0$. It differs only in minor details from the "Periodogram" introduced by A. Schuster (Reference 4; also see, for instance, Reference 5, Section 16.30). It can be intuitively understood as follows: let $x(t)$ be regarded as a voltage signal, and let it be passed through a filter network sharply tuned to pass only a narrow band Δf around the frequency f. Let the output signal be fed into a resistance of one ohm. The mean power of the output signal, which is proportional to the mean square of the output voltage, will then be $P(f)\Delta f$ (hence the name *power spectrum*). Power spectral analysis of a time-varying quantity is, therefore, similar to analysis of an unknown voltage signal for its frequency content by means of frequency filters (Reference 6).

An alternative approach is the investigation of recurrence tendencies. An early method attributed to Chree (Reference 7) consists of selecting, according to a predetermined criterion, times at which $x(t)$ was highest (or lowest), and then superimposing the record upon itself so that all selected points overlap. If there is a tendency for a maximum (or minimum) of $x(t)$ to recur after a period τ, this will generally show up in the sum of the superimposed records. This method has clearly demonstrated, among other things, the 27-day periodicity in cosmic-ray variations (see, for example, References 2, 3, 8, 9, 10, and 11). Unfortunately, it is not suitable for quantitative evaluations.

A more satisfactory measure for recurrence is the autocorrelation or autocovariance function, defined as:

$$C(\tau) \quad = \quad \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [x(t) - \bar{x}] \; [x(t-\tau) - \bar{x}] \, dt \quad .$$

If we first normalize $x(t)$ so that $\bar{x} = 0$, then $C(\tau)$ simplifies to

$$C(\tau) \quad = \quad \overline{x(t) \; x(t-\tau)} \quad .$$

Assuming that the process is symmetric about its mean, $x(t)$ has equal probability to be of either sign. The same holds for $x(t-\tau)$, provided it is totally unrelated to $x(t)$; in this case $C(\tau)$ will, over a long run, approach zero. On the other hand, if τ represents a recurrence period, whatever the sign of $x(t)$, then $x(t-\tau)$ will have more than even probability of being of the same sign, so that $C(\tau)$ tends to a positive limit.

Wiener and Khintchine (Reference 12, footnote 16) showed that the two measures described here contain equivalent information, and each can be derived from the other by means of a Fourier transformation:

$$C(\tau) = \int_{-\infty}^{\infty} P(f) e^{2\pi i f \tau} df \quad ;$$

$$P(f) = \int_{-\infty}^{\infty} C(\tau) e^{-2\pi i f \tau} d\tau .$$

The ordinary method of estimating the power spectrum, though not the only one (see, for instance, Reference 13), therefore involves prior estimation of the autocorrelation function by means of the finite record on hand, from which an estimate of $P(f)$ is obtained by transformation. Though both functions contain equivalent information, it is useful to consider both for complete understanding of the behavior of $x(t)$. If there is a recurrence tendency with period τ_0 lasting more than one cycle, $C(\tau)$ will have peaks not only at τ_0 but also at $2\tau_0$, $3\tau_0$, etc.; it will, therefore, indicate the average number of oscillations between phase jumps. On the other hand, $P(f)$ has the advantage of concentrating all the information about the component with frequency f at one point. This is especially important when there is more than one frequency involved, in which case, $C(\tau)$ is often rather irregular.

More details can be found in a number of books and reviews dealing with the subject (References 12, 14, 15, 16, 17, 18, 19 and 20). Of these, the one by Blackman and Tukey (Reference 18) is most useful in dealing with the practical problems connected with actual estimation of the power spectrum; the present computation, using SHARE program 574 adapted for the 7090 computer, essentially follows their method. Some relevant points, quoted here without details, are the following:

Frequency Resolution: If the data are sampled at intervals Δt, $C(\tau)$ can only be estimated for integral multiples of Δt. Furthermore, if $\tau_{max} = m\Delta t$ so that $C(\tau)$ is estimated for $m+1$ values of τ, the resulting estimates of $P(f)$ will cover $m+1$ points, equally spaced in frequency, from zero to $f_{max} = ½\Delta t$. Each estimate of $P(f)$ will represent an average of the power spectrum over a band of the order $f_{min} = ½ m\Delta t$ around the frequency it represents; the exact width and shape of the band depend on the relative weights given to the estimates of $C(\tau)$. "Hanning windows" were used in this case. In the present computation, daily averages of the cosmic-ray intensity were used and usually 100 estimates were taken, covering the spectrum for periods between 2 and 200 days.

Aliasing: If the power spectrum does not vanish above f_{max}, higher frequencies will contribute to the estimated spectrum in a way which cannot be resolved (aliasing). The spectrum of cosmic-ray variations has been found to fall off with increasing frequency rapidly enough as to make this source of error negligible. Even if this were not so, the fact that data points represent daily *averages*, and not momentary samples taken at daily intervals, strongly suppresses contributions from higher frequencies.

The Accuracy of Estimation: The method here described assumes that the process is stationary in time — i.e., insensitive to a shift of the time axis, and this assumption may be only an approximation. Indeed, the power spectral density of cosmic-ray variations changes over the period investigated, and therefore, any estimate of it gives only a time-averaged result (Reference 21). Secondly, the question arises as to how closely the spectral estimates drawn from a limited sample approximate the actual values. Blackman and Tukey show that the ratio between a sample obtained by m estimates from n data points, and the actual value, approximately follows a χ^2 distribution with $\nu = 2n/m$ degrees of freedom. For analyses covering the IGY period (July 1957 to December 1958), $n = 549$, giving $\nu = 11$. Finally, it should be remembered that because of the statistical fluctuations in the counting rate, our record is not only limited in length, but also has not been sampled with ideal accuracy. It can be shown, however, (Appendix A) that at least in the present case, this source of error is totally negligible.

Power spectrum analysis has been used in the investigation of diverse processes, such as the free oscillations of the earth, recurrence of magnetic storms, frequency analysis of sea waves, turbulence, and many others. Only in a few cases, however, has it been used to analyze cosmic-ray

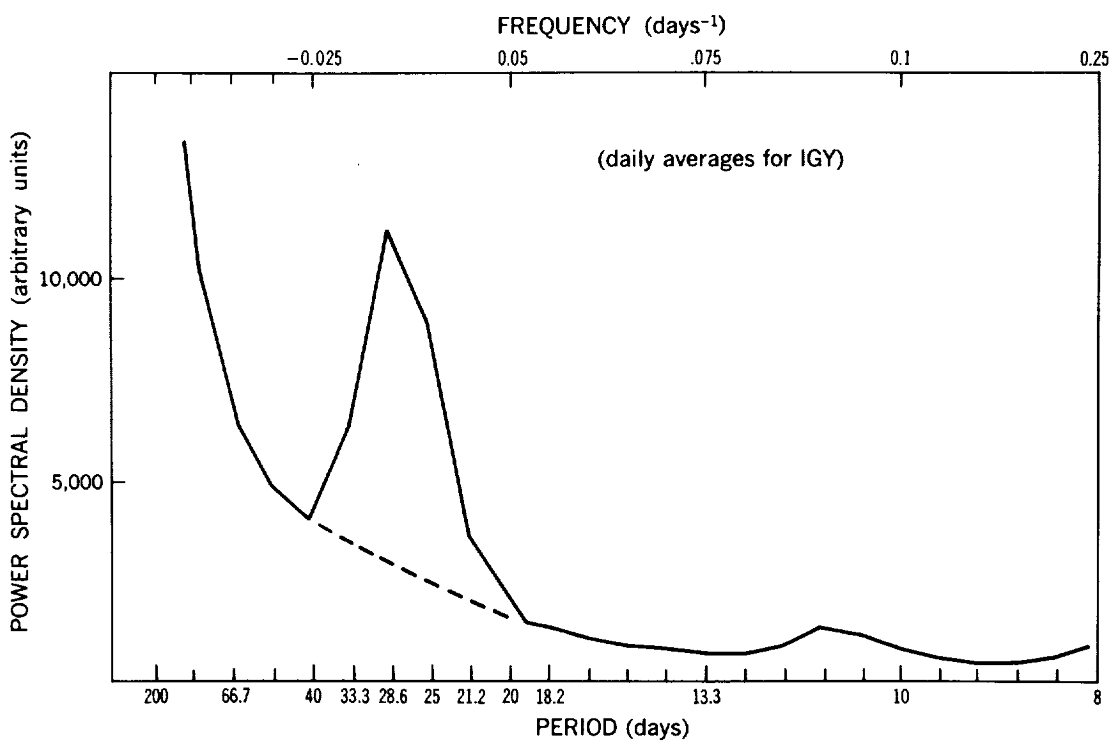

Figure 1—The low frequency power spectrum of cosmic-ray variations during IGY as recorded by the Climax neutron monitor.

variations. Fonger (Reference 22) used the autocorrelation to demonstrate a 27-day recurrence, but since his data covered only a three month period, they are not very significant statistically. Panofsky, Lethbridge and Neuberger (Reference 23) obtained power spectra of neutron monitor rates, and cross correlated them with various meteorological data. In the present work, an attempt has been made to deduce detailed properties of low frequency variations of the cosmic-ray intensity (as measured by neutron monitors during IGY), and especially of the 27-day variation.

THE POWER SPECTRUM AT LOW FREQUENCIES

As Figure 1 shows, the power-spectral density exhibits a marked peak between 27 and 28 days. The location of the peak is not fixed (see Figure 2), and over the period July 1957 – December 1959 its variation does not exhibit any marked trend (cf. Reference 2, Table III). None of the other peaks in the spectrum is believed to be significant; in particular, the absence of a conspicuous second harmonic is noted. Throughout the period analyzed, the amplitude of the 27-day peak rapidly decreases with time, as can be seen in Figure 2. The results described here are averages computed over the IGY period; during the first half year of the IGY, the amplitudes were about twice this average magnitude.

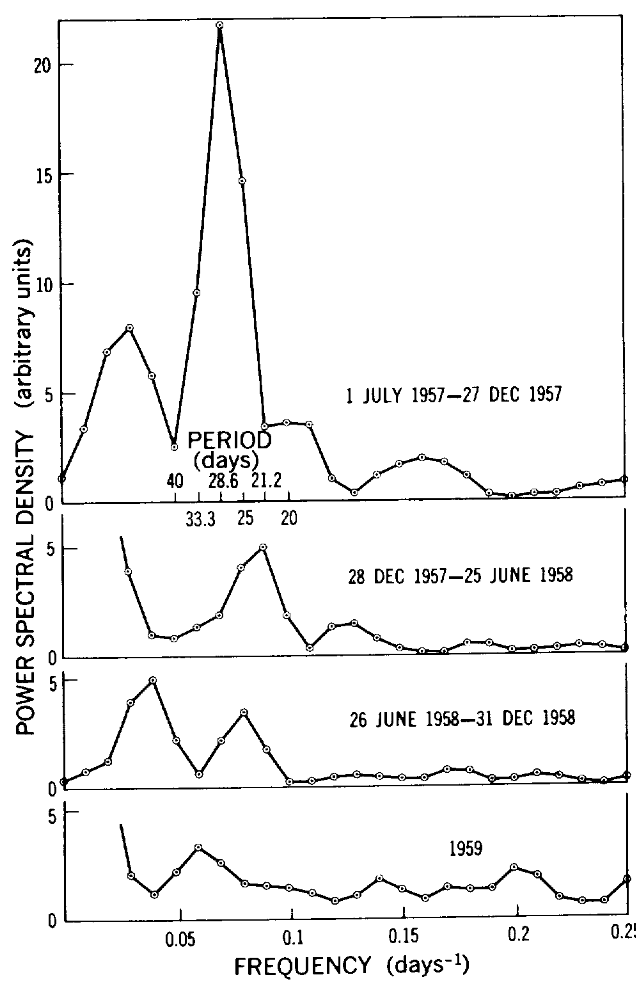

Figure 2–The power spectrum of cosmic-ray variations as recorded by the Zugspitze neutron monitor for different parts of IGY and for 1959

The autocorrelation has been plotted at daily intervals for 200 days (see Figure 3), and it is evident that the recurrence tendency is relatively stable.

The counting rate was also cross-correlated with the magnetic activity C indices, tabulated in the reports of Solar-Geophysical data by the National Bureau of Standards. A sharp negative peak is observed in the cross-correlation function:

$$C_c(\tau) = \overline{X(t)\ Y(t-\tau)} ,$$

where $X(t)$ is the counting rate and $Y(t)$ is the magnetic activity index. It is located near $t = 1$ day (see Figure 4), indicating that high geomagnetic activity is likely to be followed within one day (on the average) by low cosmic-ray rates. This agrees with known properties of SC magnetic storms; a 27-day recurrence tendency was

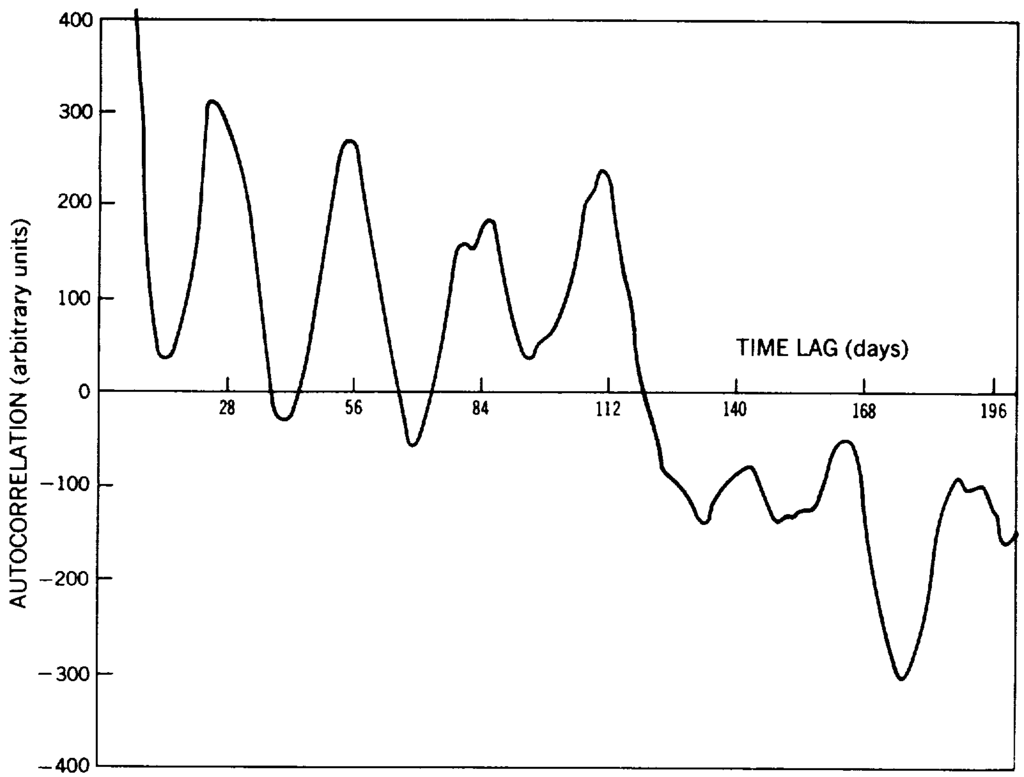

Figure 3—The autocorrelation function of cosmic-ray variations during IGY as recorded by the Climax neutron monitor.

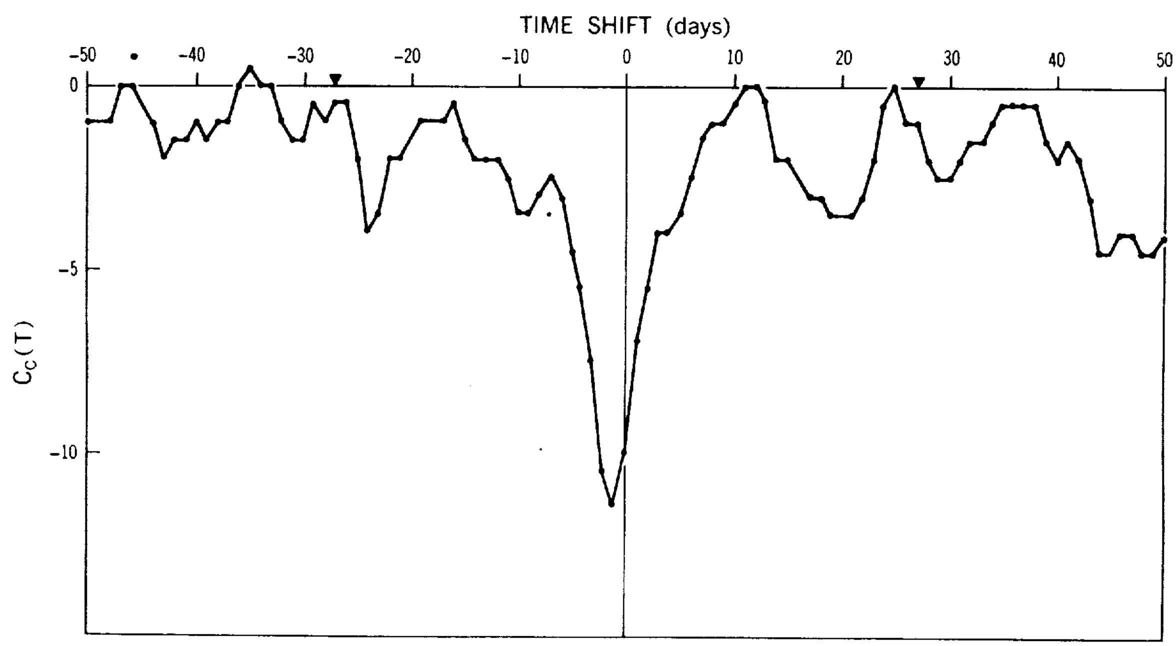

Figure 4—The cross-correlation function between magnetic activity and cosmic-ray intensity. On the right-hand side, cosmic-ray intensity precedes magnetic activity; on the left, the order is reversed.

not conspicuous in this case. There seems to be, on the whole, quite a difference between the 27-day recurrence tendencies of cosmic radiation and that of magnetic activity (References 23 and 24). While the spectrum of cosmic-ray variations shows a single well defined peak, that of magnetic activity not only has a second harmonic often exceeding the fundamental, but also shows higher harmonics, up to the sixth (Figure 5).

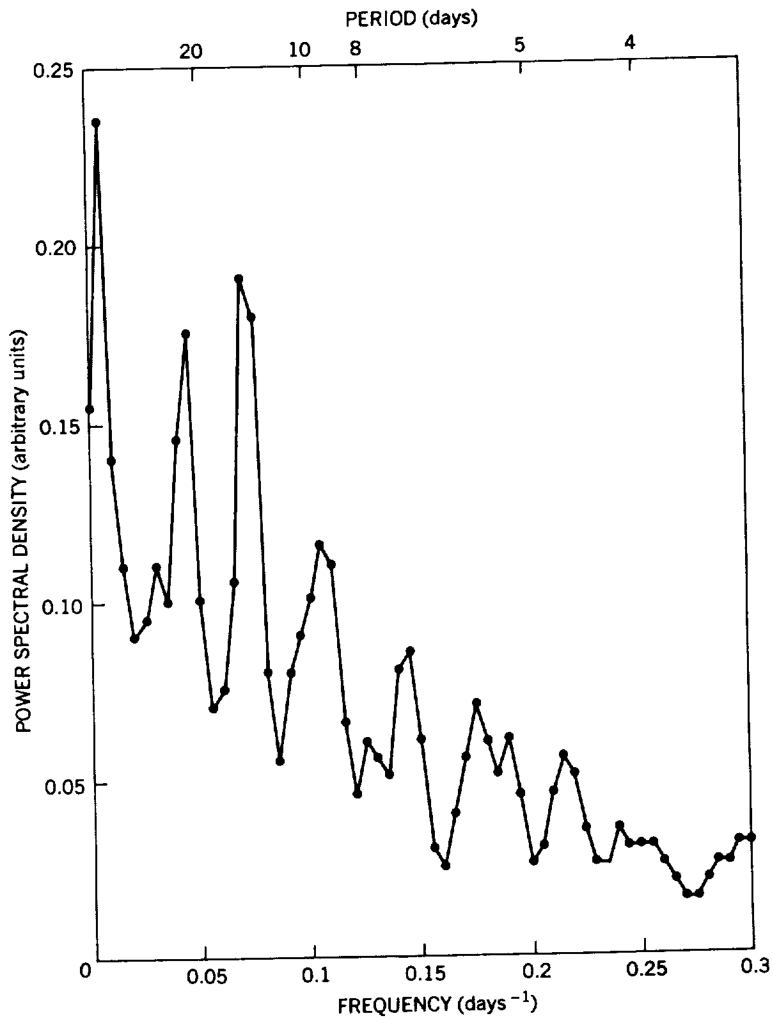

Figure 5—The power spectrum of magnetic activity indices during IGY.

THE RIDIGITY DEPENDENCE OF THE 27-DAY VARIATION

Regarding the counting rate as a voltage signal, we can define the "power" contained in it, between the frequencies f_1 and f_2 as the area contained by the power-spectral density graph between the two frequencies:

$$\text{Power} = \int_{f_1}^{f_2} P(f)\, df .$$

The power contained in the 27-day variation will be proportional to the area enclosed by the 27-day peak. It should be noted however, that this quantity will not be obtained by passing the data through a filter which selects a limited band around 27 days. In that case, the power would also contain a major contribution from the "pedestal" upon which the peak is superimposed, which presumably is due to irregular variations. We now define as the equivalent amplitude A of the variation the amplitude of a pure sinusoidal variation; containing the same power as is contained in the peak: the method by which A is calculated is described in Appendix B. The values of A thus obtained are between 0.5 and 2 percent.

The equivalent amplitudes have still to be corrected for altitude. Since the latitude dependence of the variation strongly resembles that of Forbush decreases, we adopt the correction proposed for

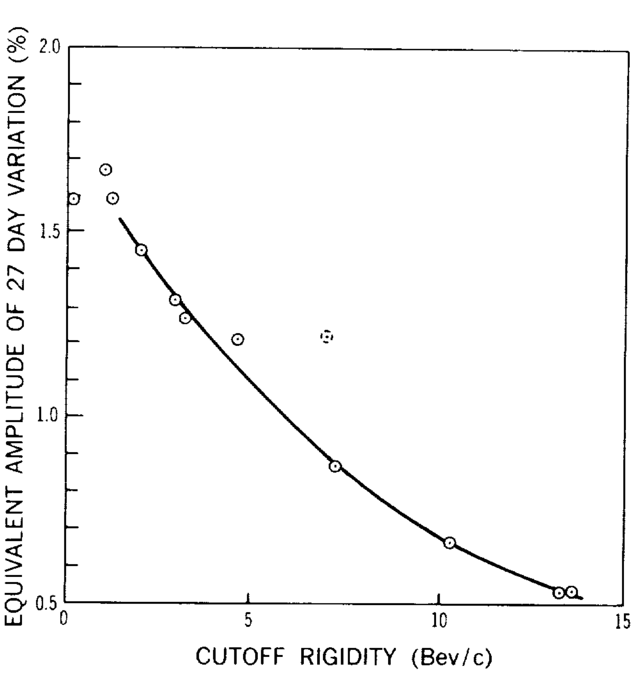

Figure 6—Latitude dependence of 27-day amplitude, averaged over IGY.

Table 1

Average Amplitudes of the 27-Day Variation for a Number of IGY Neutron Monitors.

Station	Amplitude (%)	Corrected Amplitude (%)	Cutoff Rigidity (Bev/c)
Mawson	1.59	1.59	0.2
Sulphur Mtn.	2.05	1.66	1.1
Uppsala	1.58	1.58	1.3
Mt. Wellington	1.62	1.50	2.0
Climax	1.84	1.305	3.0
Zugspitze	1.71	1.26	3.3
Rome	1.20	1.20	4.7
Hermanus	1.21	1.21	7.0
Alma Ata	0.94	0.85	7.3
Mt. Norikura	0.88	0.66	10.3
Huancayo	0.73	0.52	13.2
Lae	0.53	0.53	13.5

the latter case by McCracken and Johns (Reference 25), amounting to 12 percent per 1000 meters. This is further justified by the fact that corrected amplitudes of some high altitude stations (e.g., Sulphur Mountain; Huancayo) fall close to those of sea level stations (e.g., Uppsala; Lae), having approximately the same cutoff. In all, 12 amplitudes were found and plotted against cutoff rigidities obtained from the eccentric dipole model by Kodama, Kondo, and Wada (Reference 26); they are given in Table 1 and in Figure 6.

It will be seen that most of the points fall on a smooth curve, with the exception of Hermanus. This station is likely to have its cutoff lowered by local anomalies in the geomagnetic field, and was therefore not considered. It will also be noted that there seems to be no flattening of the curve down to a cutoff rigidity of about 1 Bev/c — this implies that the primary radiation in the low energy region undergoes very large variations.

From the latitude dependence of the counting rate, it is possible to deduce the modulation experienced by various portions of the primary spectrum. Let

$N(p)$ = The sea level counting rate at cutoff rigidity p,

$\delta N(p)$ = The 27-day equivalent amplitude,

$S(p)$ = The primary differential proton spectrum,

$\delta S(p)$ = The amplitude of the 27-day variation undergone by the primary spectrum at rigidity p, and

$Y(p)$ = The "gross" yield function at rigidity p (Reference 27, Equation 11).

The experimental data consist of the equivalent amplitudes $A(p) = \delta N/N$, and the latitude dependence $N(p)$. Neglecting penumbral effects, we can write

$$N(p) = \int_p^\infty S(p') Y(p') \, dp' , \tag{1}$$

$$\delta N(p) = \int_p^\infty \delta S(p') Y(p') \, dp' = N(p) A(p) . \tag{2}$$

Differentiating these expressions, and dividing Equation 2 by Equation 1, we have

$$\frac{\delta S}{S} = \frac{d}{dN} [N(p) \; A(p)] .$$

We now need two experimentally determined functions of the cutoff rigidity — the equivalent amplitude $A(p)$, and the total counting rate $N(p)$. Unfortunately, not many sea level measurements of the latitude dependence of $N(p)$ exist for IGY. We shall, therefore, base our calculations on the quiet-time rates $N_0(p)$ used by Quenby and Webber (Reference 27); they are arbitrarily normalized to $N_0(15 \text{ Bev}) = 100$.

Let us denote the ratio between the differential primary spectra during IGY and during the solar minimum (1954-5) by

$$\alpha(p) = \frac{S(p)}{S_0(p)} ;$$

values of $\alpha(p)$ are taken from the work of F. McDonald (Reference 28). The total counting rate $N(p)$ changes relatively little over the solar cycle; therefore, no correction is applied to it. On the other hand, dN/dp is proportional to $S(p)$, and receives a correction factor α. Substituting in Equation 3, we obtain

$$\frac{\delta S}{S} = A(p) + N_0(p) \frac{\frac{dA}{dp}}{\alpha(p) \frac{dN_0}{dp}} .$$

The results are presented in Table 2 and in Figure 7.

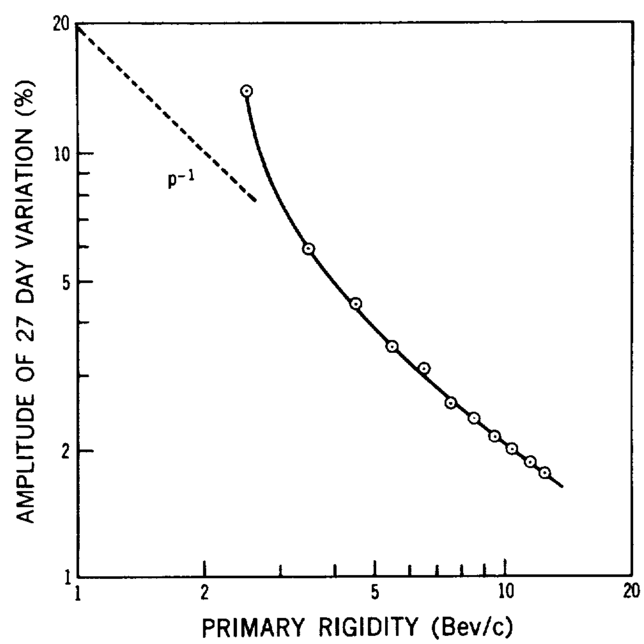

Figure 7—Rigidity dependence of the 27-day modulation undergone by the primary spectrum, averaged over IGY.

The rigidity dependence obtained here may be compared to other experimental results. It can be seen from Figure 7 that between 12.5 and 3.5 Bev/c, it roughly follows a p^{-1} relation; this agrees with results found for Forbush events (Reference 29). The dependence

Table 2

The Rigidity Dependence of the 27-Day Variations.

p (Bev/c)	N_0	$\frac{dN_0}{dp}$	α	$10^2 A(p)$	$\frac{dA}{dp}$	$\delta S/S$ uncorrected (%)	$\delta S/S$ corrected (%)
2.5	172.5	3.2	0.58	1.38	0.135	8.79	14.0
3.5	168.0	5.8	0.68	1.26	0.110	4.44	5.93
4.5	161.3	7.6	0.76	1.15	0.115	3.57	4.38
5.5	153.4	8.2	0.81	1.045	0.105	3.00	3.48
6.5	145.2	8.1	0.84	0.935	0.100	2.73	3.08
7.5	137.5	7.5	0.855	0.843	0.080	2.32	2.56
8.5	130.3	6.7	0.87	0.770	0.070	2.13	2.34
9.5	124.0	6.0	0.875	0.705	0.060	1.93	2.12
10.5	118.4	5.2	0.88	0.648	0.055	1.90	1.98
11.5	113.5	4.6	0.89	0.603	0.045	1.72	1.85
12.5	109.1	4.1	0.895	0.555	0.040	1.61	1.74

does not seem to be a power-law one, however, and it steepens at low rigidities. The rigidity dependence should also be compared with the large 27-day amplitudes found by other workers, especially by the Russians (References 30-33) at moderate balloon altitudes and high latitudes. Since balloon-borne instruments are relatively more sensitive to low rigidities, their results are in general agreement with the results obtained here. We cannot very well extrapolate the rigidity dependence to low energies; it seems, however, quite possible that the low energy cutoff of primary cosmic radiation undergoes a 27-day modulation at solar maximum, and that this may be a cause for experimental discrepancies.

IRREGULAR VARIATIONS

The 27-day peak is superimposed upon a continuum, rėpresenting the contribution of irregular variations to the power spectrum. This continuum (plotted against frequency) is roughly exponential in shape, reaching a relatively constant noise level (see Appendix B) at a frequency of about 5/day.

The latitude dependence of the continuum seems to be approximately the same, at first glance, as that of the 27-day peak. This in itself is not surprising, since it is bound to contain a major contribution from Forbush events, which have practically the same latitude dependence. The comparison is complicated, however, by the fact that the continuum also contains contributions from instrumental drifts, which are not latitude dependent; this will tend to make the latitude dependence less steep than that of the 27-day variation. Because of this effect, one should discount stations in which the continuum is abnormally high.

Table 3 gives the ratio (in arbitrary units) of the power-spectral density to the area of the 27-day peak for various frequencies. If the continuum has the same latitude dependence as the 27-day

Table 3

Latitude Dependence of Irregular Variations in Cosmic Ray Intensity During IGY.

Station	Power Spectral Density / Area of 27-day Peak (arbitrary units)						
	Freq. = .005/day	.010/day	.015/day	.020/day	.055/day	.060/day	.065/day
Mawson	1095	358	272	277	106	71	48
Sulfur Mtn.	930	477	305	219	81	47	37
Uppsala	734	409	341	292	91	70	55
Mt. Wellington	958	440	327	282	107	65	48
Climax	919	467	358	316	93	66	53
Zugspitze	753	351	314	247	107	73	49
Rome	989	425	281	232	109	75	52
Hermanus	5635	1357	422	333	215	133	51
Alma Ata	9042	1886	1108	810	316	181	89
Norikura	1495	513	363	334	177	98	70
Huancayo	1808	422	340	362	158	66	53
Lae	2777	1309	679	696	198	214	175

variation, this ratio at any given frequency should be constant for all stations. It can readily be seen that for three stations (Lae, Hermanus and Alma Ata) the ratios are unusually large; it is possible that these stations experience considerable drifts. For the other stations, there may be some increase towards the equator, but generally, the ratio seems to be fairly constant. It is, therefore, reasonable to assume that the latitude dependence of the continuum at low frequencies is close to that of the 27-day peak.

CONCLUSION

Several theories exist about the cause of the 27-day variation. One approach has been investigated in detail by Alfvèn (References 34 and 35). Alfvèn assumes that the interplanetary magnetic field near the solar equatorial plane contains "beams" of high plasma flux and magnetic field density, which co-rotate with the sun. These beams last for several rotations, and every time they intercept the earth, magnetic and cosmic-ray disturbances occur.

Alternative approaches are discussed by Dorman, who examined and rejected various explanations based on (1) a solar magnetic dipole noncoincident with the solar rotation axis, (2) atmospheric effects, and (3) high energy particles produced by solar flares. In Reference 37, Section 32, Paragraph 9(d), he suggests that "the effect of the decrease in cosmic-ray intensity during the time of geomagnetic disturbances, is the basis of the phenomenon of 27-day variations of the cosmic rays." The Forbush decreases, to which Dorman refers, are obviously nonperiodic phenomena, as they can generally be traced back to solar flares occurring a day or two previously. However, flares tend to be associated with centers of solar activity, and these are not evenly distributed in solar longitude. As is shown by the 27-day variation of sunspot numbers, there will generally be one center much more active than the rest. Every time this center faces the earth, there is a marked tendency for sudden commencement-type magnetic storms and their associated Forbush decreases to occur, leading to an apparent 27-day periodicity.

The results obtained here support this hypothesis. The energy dependence of the variation approximates that obtained for the Forbush effect (References 29 and 37). The variation in period length (see Figure 2) can be explained by noting that the "favorable interval" for an active area to cause a Forbush decrease is quite wide (Reference 38); the cosmic-ray record for the beginning of IGY (Figure 8), when the 27-day variation was very high, shows that the separation of the main Forbush events was indeed of the order of 27 days.

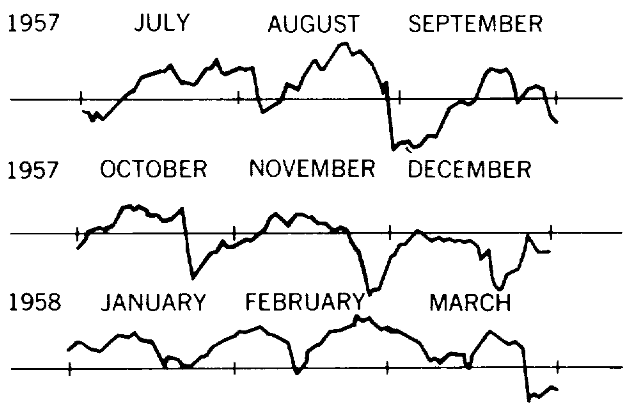

Figure 8—Cosmic-ray intensities during the first half of IGY as recorded by the Climax neutron monitor.

Examining this point more closely, we find that the main Forbush events alone cannot account for the 27-day variation. For one thing, the second harmonic of the variation would have been higher in that case. We also note that the cosmic-ray intensity (Figure 8) often starts decreasing before the main Forbush event. These "predecreases" (Reference 39) may be tentatively identified as Forbush decreases, the main impact of which misses the earth; they cause considerable smoothing of the 27-day variation, especially at high altitudes (Reference 30, Figure 1).

When we take into account the rigidity dependence of the 27-day variation obtained here, its correlation with magnetic activity, and its time dependence, it appears very likely that this variation is associated with nonrecurrent magnetic storms, showing spurious periodicity because of their origin in solar activity centers. Perhaps recurrent magnetic storms may be responsible for a 27-day variation different from the one discussed here, with presumably a smaller amplitude (possibly conforming with Alfvèn's model). This can only be established by analyzing periods of low solar activity when Forbush events are rare and magnetic activity shows strong recurrence tendencies (Reference 40).

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Appendix A

Estimation of the Fluctuation Noise

In practice, the cosmic-ray neutron rate $x(t)$ is estimated at fixed intervals Δt. Since the number of counts in any interval is subject to statistical fluctuation, this estimation is subject to error. We now wish to find the extent to which this will affect the estimation of power-spectral density $P(f)$.

Instead of dealing with an imperfectly sampled signal $x(t)$, it is found convenient to analyze a signal $x'(t) = x(t) + n(t)$ which is sampled without error—where $n(t)$ is the noise, duplicating the effects of statistical fluctuations. Since the measurements consist of averaging the counting rate over time intervals Δt, we choose $n(t)$ to be a "histogram" function (see Figure B1), each column of which has the width Δt. We assume that the heights of the columns are normally distributed around the mean; actually, they obey a Poissonian distribution, but it is only at very high sampling rates, when the average number of counts per interval is small, that the distinction is significant. Henceforth, we shall assume that both $x(t)$ and $n(t)$ have been normalized to average zero.

The noise is correlated with itself only for points within the same column; hence, its autocorrelation function is:

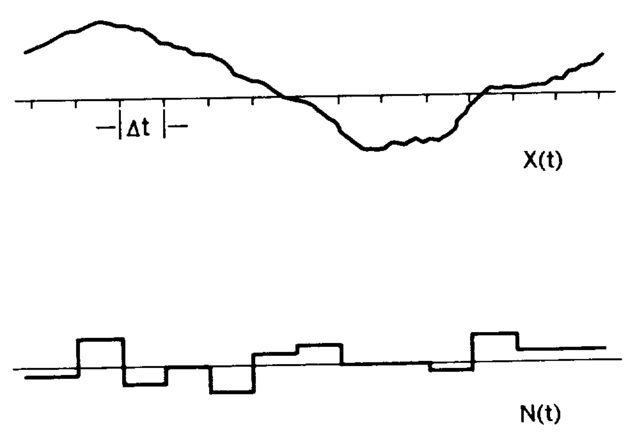
$$C_n(t) = \begin{cases} \overline{n_0^2}\,(1 - t/\Delta t) & \text{for } t < \Delta t. \\ 0 & \text{for } t > \Delta t. \end{cases}$$

The noise-power spectrum is then

$$P_n(f) = \int_{-\infty}^{\infty} e^{-2\pi i f t}\, C_n(t)\, dt,$$

$$= \overline{2n^2} \int_0^{\Delta t} \cos 2\pi f t \left(1 - \frac{t}{\Delta t}\right) dt,$$

$$= \overline{n^2}\, \Delta t\, [\mathrm{dif}\, (f\Delta t)]^2,$$



where

$$\mathrm{dif}\, x = \frac{\sin \pi x}{\pi x}.$$

is the well-known diffraction function.

Since the signal and noise are uncorrelated, the autocorrelation of $x'(t)$ will be the sum of those of the signal and of the noise, and the power spectrum will retain this additive property. The noise spectrum $P_n(f)$ is thus superimposed upon the desired spectrum. This in itself would cause no trouble if we had an infinite run of data at hand, since the true spectrum can then be obtained by subtracting the calculated value of $P_n(f)$. In a finite run, however, $P_n(f)$ undergoes unpredictable fluctuations, as derived from a χ^2 distribution (Reference 18), and it is this variability which constitutes the noisiness introduced by statistical fluctuations.

For the low frequency end of the spectrum, we may approximate $\mathrm{dif}\,(f\Delta t) \approx 1$. Let us assume that the cosmic-ray rate shows no variation apart from statistical fluctuations. Then, defining W, N_0, and N, as in Appendix A, we have

$$\overline{n^2} = \overline{(W - \overline{W})^2}$$

and

$$\overline{N_1^{\,2}} = \left(\frac{N_0}{1000}\right)^2 (\overline{n^2}) .$$

For purely statistical fluctuations, however,

$$\overline{N_1^{\,2}} = N_0 ,$$

hence,

$$\overline{n^2} = \frac{10^6}{N_0} .$$

The noise level will then be

$$P_n(f) \approx \frac{10^6 \Delta t}{N_0} .$$

The result is evidently inversely proportional to the counting rate. In actual practice, it seems to be somewhat larger than the above estimate indicates, probably because of nonstatistical fluctuations.

For daily averages of a typical neutron monitor station, such as Rome, $N_0 = 7.5 \times 10^5$. If time is measured in days, the noise level turns out to be of the order of unity. This is completely negligible in comparison to the amplitude of the 27-day peak, which on the same scale is close to 4500.

Appendix B

Power Spectra of Logarithmically Reduced Data

The IGY data used in most of the present computation (Reference 41) do not give the counting rate N, but instead give

$$W(t) = 1000 \log\left(\frac{N}{N*}\right) = 1000 \log N + \text{constant} ,$$

where $N*$ is a conveniently chosen constant. If we regard the counting rate as the sum of a constant rate N_0 and a small fluctuation N_1, averaging zero, we get (by expanding)

$$W \approx 1000 \frac{N_1}{N_0} + \text{constant} .$$

$$\overline{N_1^2} = \left(\frac{N_0}{1000}\right)^2 \overline{(W - \overline{W})^2} ,$$

$$= \left(\frac{N_0}{1000}\right)^2 C(0) ,$$

$$= \left(\frac{N_0}{1000}\right)^2 \int_{-\infty}^{\infty} P(f)\, df ,$$

$$= 2\left(\frac{N_0}{1000}\right)^2 \int_{0}^{\infty} P(f)\, df ,$$

where $C(t)$ and $P(f)$ are the autocorrelation and the spectral density obtained by analyzing $W(t)$. The power contained in any finite frequency band of $N(t)$ is proportional to the integral of P over the band, with the same proportionality factor as above.

Assume a frequency peak of area Q; if it were entirely due to a harmonic variation of frequency f and amplitude a, its power would equal $a^2/2$. Even if the peak is not infinitely sharp, we can define an equivalent amplitude a containing the same power; then

$$\left(\frac{a}{N_0}\right)^2 = 4 \times 10^{-6}\, Q.$$

In the text, relative equivalent amplitudes $A = a/N_0$ are usually given.