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SIMILITUDE IN THERMAL MODELS OF SPACECRAFT

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SUMMARY

Scaling criteria for the design and testing of thermal models of spacecraft are discussed. Four dimensionless similitude parameters are derived concerning radiation, internal heat generation, thermal conductivities of materials, and heat capacities of materials. Difficulties in achieving accurate simulation are pointed out and methods of effecting compromises without seriously affecting the validity of the data are suggested. The most difficult problems appear to be the accurate scaling of thermal conductivity and heat capacity. For manned spacecraft, some additional discussion is given of similitude criteria for the convective heat transfer.

INTRODUCTION

Because the temperatures of many components of spacecraft must be maintained within fairly narrow limits, the problem of predicting these temperatures and, particularly, of predicting their variations with time as the spacecraft passes alternately through sunlight and shadow has acquired considerable importance. The approach that has been most used in the past is the analytical one in which a set of simultaneous differential equations for selected areas, involving the geometry of the spacecraft, the geometry and the intensity of the irradiation and its variation with time, and the thermal and optical characteristics of the spacecraft, is set up and solved. Another approach, toward which considerable effort is now being directed, is the experimental one, in which the spacecraft is placed within a cold-wall vacuum chamber and irradiated with simulated sunlight, simulated reflected sunlight from the Earth, and simulated Earth thermal radiation. The two approaches are not mutually exclusive, however; as in other problems of engineering design, analytical and experimental studies are complementary, and, in addition, possibilities of useful compromise techniques exist in both approaches.

A particular problem associated with the experimental approach is that, with increasing booster capacity, spacecraft will become so large that the correspondingly sized space-simulation chambers complete with high-vacuum pumps, refrigerated walls, beams of simulated sunlight, and associated equipment will become unreasonably large, complex, and expensive. Accordingly, increasing attention will be given to the possibility of getting the desired information by testing scaled models of the spacecraft in space chambers of more convenient size.

The present paper derives and discusses the fundamental similarity criteria that have to be taken into account in constructing and testing such thermal models of spacecraft. In addition, because the similarity criteria may sometimes be difficult to satisfy, some discussion is given of possible compromises that might be effected without seriously invalidating the results. Special emphasis is laid on the problem of adjusting or compromising the thermal conductivities of the materials used in the model, inasmuch as this property may turn out to be one of the most difficult to scale adequately.

SYMBOLS

C	heat capacity per unit volume, $\frac{Q}{L^3T}$
C_p	heat capacity at constant pressure, $\frac{Q}{Tm}$
g	acceleration due to gravity, $\frac{L}{t^2}$
Q	heat
h	convective-heat-transfer rate, $\frac{Q}{L^2t}$
H	radiation intensity, $\frac{Q}{L^2t}$
k	thermal conductivity, $\frac{Q}{LtT}$
L	length
m	mass
N_{Gr}	Grashof number
N_{Nu}	Nusselt number
N_{Pr}	Prandtl number
N_{Re}	Reynolds number
q	rate of internal heat production per unit volume, $\frac{Q}{L^3t}$
T	absolute temperature
t	time

V velocity, $\frac{L}{t}$

α absorptance

β coefficient of thermal expansion, $\frac{1}{T}$

ϵ emittance

μ viscosity, $\frac{m}{Lt}$

ρ density, $\frac{m}{L^3}$

σ Stefan-Boltzmann constant, $\frac{Q}{L^2tT^4}$

DERIVATION OF DIMENSIONLESS SCALING PARAMETERS

Assumptions

It is assumed in this analysis that heat transfer occurs only by radiation from external sources (Sun or planet) to the spacecraft, by radiation from the spacecraft to cold space, by radiation between different areas of the spacecraft, and by solid conduction within the spacecraft. Since most spacecraft do not contain fluids, convective heat transfer will not be considered in this section; it will be discussed separately in a later section.

It is also assumed that a correspondingly ideal experimental technique for the model tests has been achieved. In particular, it is assumed that: (1) The pressure within the chamber is so low that heat conduction from the model to the walls through the residual air is negligible in comparison with the radiation from the model to the wall. (2) The mechanism that supports the model within the chamber similarly does not conduct a significant amount of heat from the model to the walls; nor are significant amounts of heat transferred back and forth between the mechanism and the model. In addition, the mechanism is not so bulky that it interferes appreciably with the transfer of heat by radiation. (3) The chamber walls are so cold that radiation from the walls to the model is negligible. In addition, the walls are so black and the chamber is so large that radiation reaching the walls from the model or from the simulated Sun or planet is not reflected to the model to any significant degree.

Fundamental Quantities

Let the basic dimensions be heat Q, length L, time t, and temperature T. The dimensions of the material properties and thermal inputs that are involved in the thermal behavior of the spacecraft are heat capacity per unit volume,

$$C = \frac{Q}{L^3T}$$

thermal conductivity,

$$k = \frac{Q}{L^2t\ \frac{T}{L}} = \frac{Q}{LtT}$$

rate of internal heat production per unit volume,

$$q = \frac{Q}{L^3t}$$

and radiation intensity,

$$H = \frac{Q}{L^2t}$$

With regard to this last quantity, radiation intensity, the radiant energy absorbed by a surface is determined not only by H but also by the surface absorptance for the given radiation. The absorptance α, however, is already a nondimensional ratio (the ratio of the radiation absorbed to the incident radiation). Like other nondimensional ratios that characterize the spacecraft - for example, the length-width ratio - absorptance is here assumed to be the same for the model as for the full-scale spacecraft; and it does not enter explicitly in the present analysis. The relative distributions of energy in the irradiation spectra fall in the same category; they are also assumed to be correctly duplicated for the model tests. Some further discussion of these matters will be given in a later section.

A final quantity, involved with the radiation emitted from a surface, is the Stefan-Boltzmann constant,

$$\sigma = \frac{Q}{L^2tT^4}$$

The emitted radiation is also determined by the dimensionless surface emittance ϵ. As with absorptance, emittance is assumed to be the same for the model as for the full-scale spacecraft, and it does not enter explicitly in the present analysis.

Solution of the Equations

In the differential equations governing the thermal behavior of the spacecraft, the following eight of the preceding quantities would occur: L, t, T, q, H, C, k, σ. Powers a, b, c, d, e, f, g, and h, respectively, must be found such that $L^at^bT^cq^dH^eC^fk^g\sigma^h$ is dimensionless. This condition gives the following four equations (for the basic dimensions) for the eight powers a to h:

$$
\begin{array}{llllllllllllll}
Q: & & & & d & + & e & + & f & + & g & + & h & = 0 \\
L: & a & & & -3d & - & 2e & - & 3f & - & g & - & 2h & = 0 \\
t: & & b & & -d & - & e & & & - & g & - & h & = 0 \\
T: & & & c & & & & - & f & - & g & - & 4h & = 0
\end{array}
$$

Four linearly independent solutions can be obtained as follows:

(1) Let $d = f = g = 0$, $h = 1$; then $a = 0$, $b = 0$, $c = 4$, $e = -1$

(2) Let $f = g = h = 0$, $d = 1$; then $a = 1$, $b = 0$, $c = 0$, $e = -1$

(3) Let $d = f = h = 0$, $g = 1$; then $a = -1$, $b = 0$, $c = 1$, $e = -1$

(4) Let $d = g = h = 0$, $f = 1$; then $a = 1$, $b = -1$, $c = 1$, $e = -1$

These four solutions give the following four independent dimensionless groups: $\frac{\sigma T^4}{H}$; $\frac{qL}{H}$; $\frac{k}{L}\frac{T}{H}$; $\frac{CL}{t}\frac{T}{H}$. With various combinations of these four independent dimensionless groups, other sets of four independent dimensionless groups may be obtained; however, the four shown here probably constitute the most convenient basis for discussion of the simulation problem. The reason for writing the third and fourth groups as products of two fractions will appear presently.

In the following, the problems associated with duplicating each of these four dimensionless groups will be discussed in separate sections. An additional section discusses possible compromise approaches for dealing with the third group. A final section discusses some special problems introduced by spacecraft containing fluids.

THE GROUP $\frac{\sigma T^4}{H}$

Radiation Intensity

The first dimensionless group $\frac{\sigma T^4}{H}$ makes no demands on the model itself. It indicates merely that if the intensity of the simulated sunlight differs from true sunlight intensity (the intensities of any simulated Earth-reflected sunlight and Earth radiation must then also differ from the corresponding true values by the same ratio), all model temperatures will be affected in proportion to the fourth root of the intensity. For example, if all radiation intensities in the model tests were only half of the true values in space, then when a point on the spacecraft is at 300° K, the corresponding point on the model is $\left(\frac{1}{2}\right)^{1/4} \times 300^\circ \text{ K} = 252^\circ \text{ K}$.

If full-intensity solar simulation is difficult to obtain, the experimenter might be tempted to proceed along the lines just indicated. Several practical considerations, however, limit the extent to which one may safely reduce the radiation intensities and temperatures. These considerations may be listed as follows:

1. Space-chamber walls are normally cooled with liquid nitrogen, and thus do not quite represent the cold of interstellar space. The error due to radiation from the walls is negligible if the model surface temperatures are of the order of room temperature, but it would begin to be appreciable if the model surface temperatures dropped toward 200° K.

2. Thermal conduction from the model to the chamber walls through the residual gas in the chamber is always a potential source of error. In order that this conduction be negligible in comparison with the radiation heat transfer from the model to the walls, the pressure should not exceed 10^{-4} torr if the model surface has a high emittance or 10^{-5} torr if the model surface has a low emittance. Because of minute leaks and outgassing, the pumping equipment may barely suffice to maintain the pressure below these permissible limits. Reducing the model temperature increases the ratio of the heat transferred by gas conduction (which varies linearly with temperature) to the heat transferred by radiation (which varies with the fourth power of the temperature), thus increasing the relative error due to conduction. The same applies to the heat conduction through the model supports.

3. As previously mentioned, the optical properties (in particular α and ϵ) of the model surfaces should be the same as for the spacecraft itself. Generally, it would be most convenient merely to use the same coatings and finishing techniques for both. Theoretically, however, both α and ϵ can change with temperature, not only because spectral absorptances and emittances of materials can change with temperature but also because the black-body spectrum shifts with temperature. Although the corresponding inaccuracy associated with reduced model temperatures will probably be too small, in general, to be of more than academic interest, two specific exceptions might be worth mentioning. It may be noted, first, that some research is now being directed toward developing surface coatings with optical properties that are sensitive to temperature variations, in the hope that such coatings would simplify the thermal-balance problem. If such a coating were being used, the model temperatures would have to be the same as the temperatures of the actual spacecraft. Second, the emittance of a shiny metal surface varies roughly as the temperature, so that if appreciable areas of the spacecraft are to be provided with such surfaces, the model would have to be tested at very nearly the correct temperatures (unless some appropriately different metal could be used).

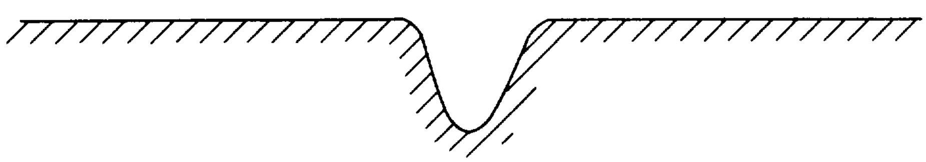
4. As will be shown in the later discussion of the third dimensionless group, decreasing the radiation intensity increases the difficulty in scaling the thermal conductivity k.

On the whole, then, one may conclude that although quantitative duplication of the radiation intensity in space is not essential, large deviations should not be accepted without careful consideration of the implications, especially with regard to the four items just mentioned.

Radiation Spectrum

It has already been indicated that for an accurate experiment the solar spectrum, the Earth-reflection spectrum, and the Earth-radiation spectrum should all be duplicated in the model tests. The problem of duplicating the first and second of these spectra has been the cause of considerable concern. Because an intense beam of simulated sunlight is difficult to produce in a vacuum chamber, it has been proposed that some inaccurate but more convenient spectrum (as that from a tungsten-in-quartz lamp) be used and that the intensity be adjusted so that the radiant energy absorbed per unit area is the same as that for sunlight irradiation. Specifically, the product of beam intensity and absorptance for the particular radiation is made equal to the product of sunlight intensity and absorptance for sunlight.

Some experimental difficulties may appear, as when two adjacent areas with different surface coatings have to be irradiated with the same beam and the product $H\alpha$ cannot be simultaneously duplicated for both. This problem might be solved by using some different coating on one of the two areas. A further objection is that the variation of α with angle of incidence might not be the same for both kinds of light; however, the difference would scarcely be significant. A more important source of concern exists when the spacecraft is not everywhere convex, or, in general, when light reflected from one area falls on another area. Suppose, for example, that the surface contains a depression, as in the sketch, and that the absorptance α of the coating for sunlight is 0.25; and



suppose also, as an extreme case, that the coating is black ($\alpha = 1.0$) to the substitute radiation, which is accordingly adjusted to one-fourth the intensity of sunlight. Then the heating rate per unit area over the main external surface is correctly duplicated with the substitute radiation; but the depression absorbs 2 or 3 times as much radiation from the sunlight as from the substitute light, inasmuch as a depression tends to be highly absorbing even though the surface coating itself is fairly reflective. The objections would be met if the model α for the substitute radiation were equal to the spacecraft α for sunlight; also, when α varies throughout each spectrum, the equality should hold for corresponding portions of the two spectra.

Electrical Heating in Lieu of Radiation Heating

Certain spacecraft are required to remain oriented toward the Sun, or at a fixed orientation relative to the Sun, for long periods of time. If the absorptances of the sunlit surfaces are known, then the thermal inputs to these surfaces due to solar radiation can be easily calculated. In such cases it might be well to consider whether the solar heat input to these surfaces might better be supplied electrically rather than by a beam of simulated sunlight. However, if the spacecraft configuration is such that appreciable amounts of sunlight are reflected from one surface to another - that is, if the sunlit part is not everywhere convex - this method must be used with caution.

The precise nature of the heating element used would depend on the type of surface and on the characteristics of the available power supply. If the surface is a thin metal sheet, for example, it might be cut into a continuous ribbon, and the electric current then passed through the sheet itself from one end of the ribbon to the other. Another approach is to cement Nichrome ribbon to the surface to be heated, or even to make the surface itself of Nichrome ribbon covered with the desired coating. Conducting rubbers or other polymers and metal-coated materials might also be applied for this purpose.

THE GROUP $\frac{qL}{H}$

Simulating the second group $\frac{qL}{H}$ should present no basic difficulties. If the radiation intensity H is about the same for the model as for the spacecraft, this group merely prescribes that q vary inversely as L; that is, the volume rate of heat production within the model instruments (or other heat sources) must be inversely proportional to the scale factor. Because volume is proportional to the cube of the scale factor, the rate of heat production within the entire model instrument should then be proportional to the square of the scale factor; and the same square proportionality obviously applies to the rate of heat production within the entire model spacecraft.

In the spacecraft, the internal heat is generated by the crew, instruments, mechanical equipment, heaters, and primary power sources, such as radioactive isotopes; and internal cooling may be provided by Peltier coolers or by systems using external radiators. In the model, internal heating will generally be provided simply from a laboratory electrical power supply; and internal cooling, if it must be simulated, can be provided similarly with a Peltier cooler. The detail with which the various items are simulated in the model must remain within the discretion of the experimenter. There are, no doubt, cases in which simply providing the correct total heat, as with a long Nichrome coil strung within the model, would be adequate. In many other cases, however, the details of the internal heating may have to be simulated fairly closely. Thus, a large, slightly warm instrument might transfer its heat to the walls mainly by conduction along the supports, whereas a small glowing heater absorbing the same power would transfer its heat to the walls mainly by radiation, and the data obtained with it would not necessarily provide useful information on the temperatures of either the instrument or the wall.

At present, power for operating spacecraft is generally obtained from externally carried silicon solar cells. For a model, as just indicated, power can be obtained from the laboratory power supply, and the solar-cell panels can be simulated with some appropriate sheet materials. An interesting question arises with regard to such simulation. Suppose, for example, that a solar-cell panel reflects 20 percent of the incident solar radiation and absorbs 80 percent, of which 10 percent is converted to electricity and dissipated within the spacecraft. Then the remaining 70 percent is reradiated from the panel as thermal radiation. Now, if the simulated panels are made with the same absorptance and emittance as the solar-cell panels, their temperatures will be too high, since they must reradiate the entire 80 percent. If the spacecraft is essentially convex in form and covered with solar cells, a change to a surface absorptance of only 70 percent for the model would provide more nearly correct data. However, if the solar-cell panels extend outward from the main spacecraft, any surfaces getting reflected light from the solar-cell panels will be slightly overheated, since the reflectance of the panels is now 30 percent (100 - 70) instead of 20 percent (100 - 80). There seems to be no theoretically correct simple way around this impasse. Fortunately, however, the importance of this reflected radiation in the thermal balance of the spacecraft is normally so small that the matter is largely academic, especially since maximum power is not usually drawn from the solar cells.

THE GROUP $\frac{k}{L}\frac{T}{H}$

Problems in Scaling the Conductivity

The third dimensionless group $\frac{k}{L}\frac{T}{H}$ presents the main problem in the design of the model. If, following the previous recommendation, H and hence T are about the same for the model as for the spacecraft itself, this group dictates that k must be proportional to L. For example, in a 1/10-scale model, the thermal conductivity of every material in the model must be one-tenth the thermal conductivity of the corresponding material in the spacecraft. Thus, if a certain part of the spacecraft is made of aluminum, it might be modeled in stainless steel. Similarly, stainless steel or titanium might be replaced with a ceramic, a ceramic might be replaced with a rubber or plastic, and a plastic might be replaced with a foamed plastic, felt, or so-called "composition" material.

Areas of concern are immediately apparent. Several are listed as follows:

1. Some of these substitutions, such as plastic for metal or foam for solid, obviously result in weaker structures. If, as is typical, many parts of the spacecraft shell and structure are only about 1 or 2 mm thick, the model may then be too fragile to be either constructed or tested.

2. Obtaining the same optical properties for the model surface as exist on the spacecraft might involve considerable effort and ingenuity. For example, if the spacecraft has a bright metal surface, it would be necessary to provide a similar surface on the plastic substitute, perhaps by vapor deposition of a thin film of the metal, by application of metal foil to the surface, or by application of a thin plastic sheet having a vapor-deposited metal film on one side. (Actually, these methods do not necessarily assure exact similarity, since the

optical properties of vapor-deposited metal films depend on the technique of deposition and on both the film thickness and the nature of the substrate.)

As a further example, if a smooth plastic is replaced with foam or some other porous material, the smooth finish attainable on the spacecraft surface may be difficult to reproduce on the model. Possibly a very thin plastic sheet might be applied over the surface before applying the coating.

Incidentally, coating thicknesses cannot normally be scaled without changing the optical properties, and some corresponding allowance for the coating mass, conductance, and heat capacity may have to be made when a fairly heavy coating is applied to relatively thin sheet material.

3. The thermal conductivity of porous materials and foams drops with decreasing air pressure, in some cases until the pressure is down to 10^{-3} torr or less. Accordingly, the conductivities in the vacuum chamber may be far less than the handbook values obtained by ordinary measuring procedures. This difficulty, in itself, is not a basic one, since thermal conductivity can be measured in vacuum without undue additional experimental difficulty. However, for such materials in vacuum, part of the heat transfer through them occurs by radiation across the cell spaces, so that the apparent conductivity falls off rapidly with decrease in temperature, in effect changing the form of the partial differential equation that governs the heat flow through the material. The effect may be significant if a large temperature difference exists across the material or if temperatures vary widely during orbit.

4. If the spacecraft already comprises some highly insulating material, the still better insulator required for the model may not even exist. The problem raised, however, is not necessarily as frustrating as it may seem. Consider, for example, the superinsulators, consisting of many layers of highly reflecting foil (or of metallized plastic sheet) with or without separators. In theory, if the heat flow through such materials occurs only by radiation, k is not involved, so that the full-scale superinsulator may be used for the model. The multilayer superinsulator, however, may be too voluminous to use for the model, and the investigator might well consider whether a thinner substitute consisting of only a few layers of foil might not be adequate for the purpose. It may be that, in most cases in which these materials are used on spacecraft, model studies of the thermal balance would be satisfactory in most respects if the superinsulator is only approximately simulated, and detailed information on the heat flow through it and on the consequences of such heat flow (for example, the rate of evaporation of liquid hydrogen within insulated tanks) would be obtained with the aid of separate studies.

Theoretical Possibility of Increasing H

It should be mentioned that the model can, in theory, be constructed of the same materials as the actual spacecraft if the radiation intensity can be made high enough. Thus, for a $\frac{1}{10}$- scale model, the value of $\frac{k}{L}\frac{T}{H}$ can be retained without changing the value of k if $\frac{T}{H}$ can be reduced by a factor of 10. Constancy of the first dimensionless group $\frac{\sigma T^4}{H}$ then requires that the radiation intensity H be increased by a factor of $10^{4/3}$, or approximately 21, with T correspondingly increased by a factor of $10^{1/3}$, or about 2.1.

The practical difficulties associated with such an approach are apparent. First, providing a large collimated beam 21 times as intense as sunlight inside a cryogenic vacuum chamber is certainly a formidable undertaking. Second, the materials and coatings involved may not be able to withstand the higher temperatures. Third, the optical properties of the surfaces, in particular α and ϵ, might be appreciably different at the higher temperature even if the materials are otherwise essentially unaffected.

Contact Resistance

In built-up spacecraft structures, heat transfer from one element to an adjacent one occurs at the contact surface. If one defines a contact-heat-transfer coefficient as the heat transfer per unit area per unit time per unit temperature difference, it can easily be shown that a corresponding dimensionless group is

$$\text{Contact-heat-transfer coefficient} \times \frac{L}{k}$$

Then, if L/k is approximately the same for the model as for the full-scale spacecraft, this coefficient should also be approximately the same.

The coefficient is actually a function of the mechanical precision with which the two mating surfaces are formed and the pressure with which they are held together. Accordingly it may be somewhat vaguely defined for the spacecraft itself, and even more uncertainly reproducible in a model made of different materials. Possibly a tight construction and a thin layer of vacuum grease or of some convenient cementing material between the mating surfaces in both the spacecraft and the model can provide such low contact resistance as to eliminate it from consideration.

Possible Compromises With Sheet or Built-Up Structures

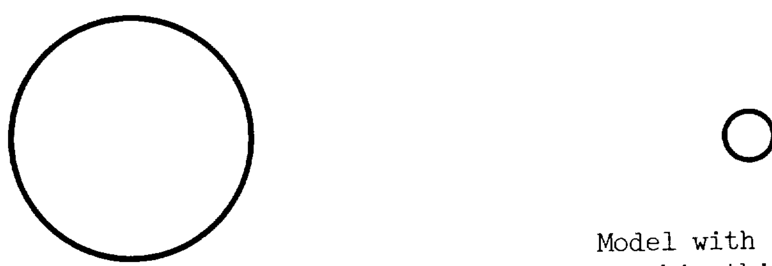
Many spacecraft consist, in their basic structure, of an outer skin with internal bulkheads, partitions, and sheet stiffeners. For a model of such a spacecraft, certain compromises may be permissible that would greatly simplify its construction without introducing unacceptable inaccuracies in the test results.

Thin sheets.- Consider first, as an extreme example, the 100-foot-diameter Echo I balloon, made of 1/2-mil Mylar with an outer film of vapor-deposited aluminum 2,200 angstroms thick. Heat conduction through this skin is so rapid that inner and outer surface temperatures at any part of the balloon are essentially identical. On the other hand, heat conduction along the skin is negligible, so that, in effect, the only transfer of heat from one area to another is by radiation through the inside of the balloon.

Thus, in this extreme example, thermal conductivity would, in effect, not be involved in a thermal study of the balloon. Within limits, a thermal model could be made of almost any sheet material; and furthermore, the sheet thickness need not be correctly scaled, provided it is thin enough so that heat flow across it is very rapid and heat flow along it is very slow. For cases that are not quite as extreme as the Echo I balloon, one might still consider whether extreme effort in duplicating the model is worth the additional accuracy of 2° to 3° where less rigor in duplicating the model might greatly simplify its construction and testing.

Thick sheets.- There also exist spacecraft designs in which a considerable temperature difference exists across the skin. If, as in the preceding case, the lateral heat flow along the skin is negligible, then the only values of k and L that are involved in the heat conduction are the conductivity k in the direction through the skin and the thickness L of the skin. Then, within limits, as long as the ratio k/L is correctly duplicated in the model, the thickness need not be correctly scaled. For example, if precisely the same material and skin thickness are used for the model as for the full-scale spacecraft, k/L will be correctly duplicated as far as heat flow through the skin is concerned, and the model will be satisfactory in this respect. If the full-scale skin cannot be used, a useful compromise might still be effected by using, say, a 1/4-scale skin thickness where the model scale is otherwise 1/10, provided the conductivity of this skin material is 1/4 as much as that of the spacecraft skin material.

Caution must be exercised in at least two respects, however. First, the skin thickness in the model may now become so large relative to the other dimensions (as indicated in the sketch) that heat flow through the skin is no longer one dimensional. Second, lateral heat flow along the skin may now, in the



Full scale

Model with full-scale skin thickness

model, become appreciable, whereas in the full-scale spacecraft it was negligible. Some further discussion of these two questions is given in the following paragraphs.

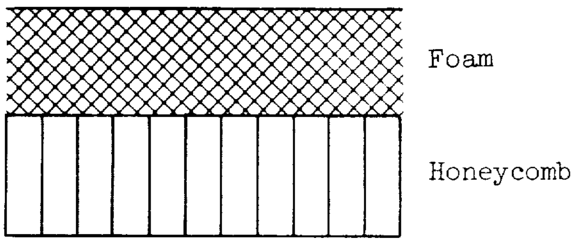
Heat flow through a thick highly curved wall.- As just noted, the radial heat flow through a thick highly curved wall, such as is shown on the right of the preceding sketch, is hardly one dimensional. For example, in the case of a cylinder having an inner diameter/outer diameter ratio of 0.9, the resistance to heat flow through the wall is 1.054 times that of a similarly constructed flat plate of area equal to that of the outer surface of the cylinder; and it is 0.949 times that of a similarly constructed flat plate of area equal to that of the inner surface of the cylinder. Following is a small table of these resistance ratios for several values of diameter ratio, for both cylinders and spheres.

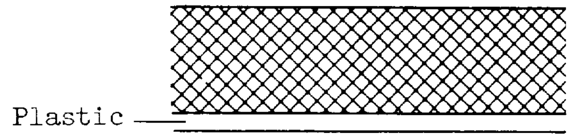
Inner diameter / Outer diameter	Resistance ratios			
	Cylinders		Spheres	
	Based on outer surface	Based on inner surface	Based on outer surface	Based on inner surface
0.6	1.277	0.766	1.667	0.6
.7	1.189	.832	1.428	.7
.8	1.116	.893	1.250	.8
.9	1.054	.949	1.111	.9

It is clear from this table that if the wall thickness becomes an appreciable fraction of the model dimension, the results will not be very accurate. The resistance ratio, of course, can be made equal to unity by basing it on neither the outer nor the inner surface but on some imaginary surface between them. In any case, however, the thermal resistance and the radiating areas (that is, the inner and outer surface areas) cannot all be correctly simulated when the geometry itself is not accurate.

The reason for emphasizing this matter here is that if the spacecraft has a complicated composite wall through which heat flows by both conduction and radiation (and perhaps also along various parallel but dissimilar paths), developing an equivalent thin wall for the model may be a considerable project. In fact, if the composite wall is already a very good thermal insulator, it may be practically impossible to develop a practical model wall structure with the greatly reduced thermal conductivity required by the exact theory. For such cases, one might seriously consider using the actual intended spacecraft wall structure for the model, in accordance with the preceding discussion. Some simple steady-state heat-flow analysis based on the thermal and surface-optical properties of the composite wall could indicate whether a model made of the full-scale wall structure could give useful information.

A useful compromise may be possible in the case of a multilayer wall construction in which the problem just discussed characterizes only some of the layers. Consider, for example, the two-layer construction shown in the left-hand sketch (next page), consisting of an outer layer of highly insulating foam between thin metal sheets and an inner honeycomb plate that provides the basic structure. The outer highly insulating layer cannot be readily scaled down in thickness, at least not without some research. The honeycomb plate, however, has a well-defined thermal conductance across it; and hence it may be quite feasible to replace the honeycomb core of the model with a thin plastic sheet having the same thermal conductance and the same heat capacity. This construction is indicated in the right-hand portion of the following sketch. For a curved wall, this construction would be more practical and accurate than that of the left-hand sketch, in the sense of the earlier discussion.





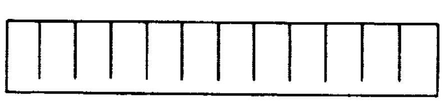
<u>Heat flow along the skin</u>.- The second caution mentioned with regard to the use of a skin that is thicker than true scale thickness is that the heat flow along the skin may become significantly greater than it would be in a true scale model. For example, if the full-scale material and thickness are used in an otherwise 1/10-scale model, then the heat conduction along the skin will be too high by a factor of 100, because both the conductivity of the material and the cross-sectional area of the heat-flow path along the skin will each be too high by a factor of 10.

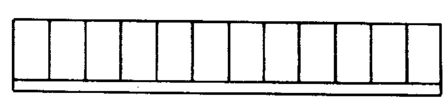
If, as in the case of the Echo I balloon, the lateral flow of heat is minute, increasing it by a factor of 100 may still leave it negligible. In many other cases, even a much smaller increase could seriously affect the validity of the data. In general, if the flow of heat along the skin is not negligible, it will be correctly duplicated only if the product of conductivity and skin thickness correctly duplicates that for an exact model. In other words, instead of the group $\frac{k}{L}\frac{T}{H}$, the group

$$\frac{k(\text{thickness})}{L}\frac{T}{LH}$$

has to be duplicated. If the skin is disproportionately thick, the conductivity k will then have to be especially low. But such overreduction of k will then cause the heat flow through the skin to be much slower than that corresponding to exact simulation; and the model will accordingly be unacceptable if the heat flow through the skin has to be accurately simulated. It is important, then, to consider what might be done to reduce the flow of heat along the skin without affecting the flow of heat through it.

The general approach to reducing the flow of heat along a sheet is to cut slits in the material normal to the surface, almost but not quite deep enough to cut through the sheet completely, as shown in the left-hand sketch. A modification of this method is to cut through the material completely, obtaining many small strips, and then to cement the strips to a thin sheet of plastic, as shown



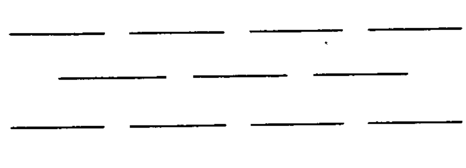


in the right-hand sketch. When the model is assembled and the skin is drawn tight, the strips will be separated enough to prevent heat transfer across the cuts (except for a small amount transferred by radiation), and the heat flow along the sheet will have to occur by way of the small connecting areas at the base.

The increase in the resistance to heat flow along the sheet that is caused by the slits is shown in figure 1. These results were obtained analytically by conformal transformations, as outlined in the appendix. Tables I and II show results for some other configurations of cuts, namely, V-shaped grooves and rectangular slots, intended to represent types that might be machined in metal or plastic sheets. These results were obtained by means of the two-dimensional electrical analogy method; conducting paper (Teledeltos recording paper) was used. (See ref. 1.) The effect of these slots and grooves on the resistance to heat flow through the sheet has not been investigated. If the effect appears to be significant for any desired configuration, it can probably be estimated with adequate accuracy; or it can be measured by standard techniques.

The results just presented correspond to cuts, grooves, and slots arranged so as to obstruct the heat flow in only one direction along the sheet. Such arrangements will be adequate for cases in which the main heat-flow direction along the sheet is fairly obvious. For example, if the spacecraft spins about an axis and has approximately circular symmetry about this axis, there will be practically no circumferential temperature gradients; and any slits needed for the model of such a spacecraft would be cut circumferentially, in order to restrict the longitudinal heat flow. If the main heat-flow direction along the sheet is uncertain, sets of slits must be cut in two perpendicular directions. Then the increase in thermal resistance shown in figure 1 will apply for heat flow in any direction along the sheet. The same is not true for the results given in tables I and II, however, because machining slots or grooves (of finite width compared with the slits just mentioned) in the second direction will increase the resistance to heat flow in the first direction. The increase is not readily amenable to analysis; however, it is probably fairly small, especially for the V-shaped grooves.

If the skin is made of metal or a stiff plastic sheet, one might also consider cutting series of short slits completely through the material, leaving enough distance between adjacent slits to provide adequate structural strength and rigidity, as indicated in the sketch. The increase in thermal resistance



(in the direction normal to the slits) of such a sheet is shown in figure 2. These results were obtained by the two-dimensional electrical analogy method with conducting paper.

THE GROUP $\frac{CL}{t}\frac{T}{H}$

The group $\frac{CL}{t}\frac{T}{H}$ may appear to present no problem to the model designer since it merely indicates that times t will be proportional to CL $\frac{T}{H}$; or, with T/H about the same for the model as for the spacecraft, times t will be about proportional to CL. For example, if the true orbital period is 100 minutes, the simulated orbit for an exact 1/10-scale model made of material having the same heat capacity per unit volume as that of the spacecraft itself would have to be run through in only 10 minutes. If the spacecraft is mainly a sheet structure and, as discussed previously, the model is made of the same sheet material as the spacecraft, with full-scale thickness, the simulated orbit will take the full 100 minutes. For this case, L is taken as the sheet thickness.

The heat capacity per unit volume may present some very serious problems, however, in the design of the model. Suppose, for example, that the spacecraft is made of a variety of materials having, in general, different values of k and C, and suppose that a group of materials has been selected for an exact model such that their conductivities k are all less than the k values of the corresponding materials of the spacecraft by the scale factor. Then the values of C for the various model materials must bear the same ratios to each other as do the corresponding spacecraft materials; else the ratio of CL for the model materials to CL for the corresponding spacecraft materials will not be uniform. Accordingly, if the spacecraft is constructed of more than one material, the model designer must consider both k and C in selecting materials for the model.

Here again the model designer may be afforded some relief if the model is basically of sheet construction and if some deviation from accurate scaling of sheet thicknesses would not seriously affect the model geometry. Where such deviation is permissible, the designer has the additional freedom to adjust the local sheet thickness L in seeking materials such that (1) k/L is the same for the model as for the spacecraft and (2) the ratio of CL for the model to CL for the spacecraft is everywhere the same. It is not necessary that the thicknesses of the various sheet materials all be scaled similarly. Thus, in a nominal 1/10-scale model, one area might be made of an accurately scaled sheet one-tenth as thick as the corresponding sheet on the spacecraft and with k correspondingly one-tenth as much as that of the spacecraft material, whereas another area might be made of a sheet half as thick as the corresponding sheet on the spacecraft, with k correspondingly half as much. Of course, if an area on the spacecraft is so thin or has such high conductivity that inner and outer surface temperatures are practically identical, the value of k for the model need not be accurately scaled at all as long as it is large enough, and only consistency with regard to CL has to be maintained. It is assumed in this paragraph, as in the earlier discussion of k, that the lateral flow of heat in the sheets either is negligible or can be kept from introducing serious error by the methods previously discussed.

The product CL has the dimensions of heat capacity per unit area, and as used in the preceding paragraph it actually is the heat capacity per unit area of the skin or other sheet material being considered. If, as just discussed, the sheets used for the model construction are disproportionately thick, their values of CL may be rather large. This fact in itself would not be objectionable, but it may create a problem with regard to achieving proportionately high values for the internal components of the model. Accurate simulation of the heat capacities of the internal components may not always be essential; but if it appears to be necessary, the model components might be formed as solid mock-ups made of some high-specific-heat material, or perhaps as scaled metal shells filled with water. One might also consider making a model instrument somewhat oversize in order to get more mass and heat capacity, especially if the heat transfer between it and the skin (or other elements) is mainly by conduction rather than by radiation, so that its external area (radiating area) is not an important factor.

SPACECRAFT CONTAINING FLUIDS

Similarity Criteria for Convective Heat Transfer

Manned spacecraft will contain breathable atmospheres and will also be provided with forced ventilation for the comfort of the crew and for cooling the instruments and equipment. Some of this heat will be delivered, by convective heat transfer, directly to the wall, thence to be radiated to space. The present section will discuss the problem of simulating this heat transfer.

Convective heat transfer is defined by the dimensionless Nusselt number

$$N_{Nu} = \frac{hL}{k\ \Delta T}$$

where h is the heat transfer per unit area per unit time, k is the thermal conductivity of the gas, L is some characteristic length in the flow such as tube diameter or boundary-layer thickness, and ΔT is generally the difference between the temperature of the wall and the temperature of the main body of fluid outside the boundary layer. The Nusselt number is a function of the Reynolds number and the Prandtl number, which are defined as

$$N_{Re} = \frac{\rho V L}{\mu}$$

and

$$N_{Pr} = \frac{C_p \mu}{k}$$

where ρ is fluid density, V is the velocity of the main body of fluid outside the boundary layer, μ is the viscosity of the fluid, and C_p is the heat capacity of the fluid at constant pressure. Since all gases have about the same

Prandtl number, the Nusselt number is essentially determined by the Reynolds number. Incidentally, the similarity of the Nusselt number to the previously discussed group $\frac{k}{L} \frac{T}{H}$ will be noted. As will be seen, it presents similar problems.

In order that the flow within the model have the same aerodynamic characteristics as the flow within the spacecraft, it should have the same Reynolds number. The Nusselt numbers will then be the same; that is, for a given ΔT, the heat flow per unit area will be proportional to k/L. If this convective heat flow per unit area must (like the radiation input per unit area) be about the same for the model as for the spacecraft, the thermal conductivity of the gas k must be reduced by the scale factor. Unfortunately, there exists no gas for which k is even as low as one-half that of air, so that rigorous convective-heat-transfer similitude in this respect is not possible. Possible approaches to the problem of obtaining the necessary information are discussed in the following two sections.

Changing the Reynolds Number

One possible approach to solving the problem just presented is simply to lower the Reynolds number. The inaccuracy in flow similitude thereby introduced may be insignificant, because, as long as the flow is turbulent, the nature of the flow is, in general, only slightly affected by even large changes in Reynolds number. Since convective heat transfer to the walls of a tube (presumed roughly comparable to the form of the main living quarters of the spacecraft) is about proportional to $\left(N_{Re}\right)^{0.8}$ (see p. 211 of ref. 2), the Reynolds number in a 1/10-scale model would thus have to be reduced by a factor of about 18 in order to get the same convective heat transfer per unit area in the model as in the spacecraft.

Use of Full-Scale Mock-Up

The wall of a manned spacecraft will be a fairly good thermal insulator in order that the inner wall temperatures might remain fairly uniform and constant. This wall temperature might average, say, 70° F. The internal air temperature will be, say, 75° F, so that there will be some net heat loss to space by way of the walls (with an external radiator to remove the remainder of the internally generated heat). Now, if one considers just what useful information, relative to the present discussion, is to be obtained from the model tests, two particular items might be presented.

First is the heat flow through the wall to its outer surface when its inner surface temperature averages 70° F. But this information can be found by adding heat internally in any reasonable way, such as by a heater coil and fan or by a radiating wire or coil along the center of the chamber, and by determining the total power input (including the fan power) required to maintain an average inner wall temperature of 70° F. Second is the quantitative description of the internal heat transfer from the air to the wall. However, since air flow and heat transfer are greatly affected by such items as furniture, equipment, shelves, and so on, a separate study, with perhaps a full-scale mock-up of the chamber, complete with air-conditioning system and ducting, would probably be made in any case. Accordingly, precise simulation of the internal flow and convective heat transfer within the model may not be necessary.

Artificial Gravity

It was pointed out previously that flow similitude requires, at least in theory, that the internal-flow Reynolds number be the same for the model as for the spacecraft. If the manned spacecraft spins in order to provide artificial gravity, the rotational Reynolds number (in which V is taken as the peripheral speed of the model) should also be the same for both. Since it may not be obvious that the characteristics of free thermal convection in this artificial gravity field will then also be the same for both, the following brief analysis is given.

In general, when free thermal convection is significant, the Nusselt number is determined not only by the Reynolds number and the Prandtl number, but also by the Grashof number

$$N_{Gr} = \frac{gL^3\beta\rho^2\Delta T}{\mu^2}$$

where g is the acceleration due to gravity and β is the volume coefficient of thermal expansion of the gas. In the spacecraft, the centrifugal acceleration, or artificial gravity, is proportional to V^2/L. If this expression is substituted for g, the Grashof number takes the form

$$\frac{V^2L^2\rho^2}{\mu^2}\beta\,\Delta T = N_{Re}{}^2\beta\,\Delta T$$

Thus, for the given rotational Reynolds number and given temperatures, the Grashof number would be duplicated automatically and, accordingly, so also would the Nusselt number.

It should perhaps be emphasized that the preceding discussion is largely academic, intended only to show that if both the rotational and the internal-flow Reynolds numbers are duplicated, the flow characteristics and the Nusselt number will be duplicated. The fact remains, however, that since there exists no gas with a sufficiently low value of k, duplicating the Nusselt number must still provide a heat-transfer rate h that is much too high. One might suggest that, if the model is to spin, its rotational Reynolds number be reduced along with, and in proportion to, the internal-flow Reynolds number (as discussed earlier) in order that the main flow characteristics might be roughly duplicated. The correspondingly reduced artificial gravity in the model, however, will then be small relative to the natural gravity on the Earth, where the model is to be tested; thus, the problem cannot be resolved in this way.

Actually, the experimental difficulties associated with getting data from a spinning model are such that spinning would generally not even be considered. Furthermore, the low values of artificial gravity proposed for such spacecraft are probably not enough to produce significant free convection where any appreciable forced convection is present. Some rotation of the model will have to be provided in order to assure circumferentially uniform irradiation; but this uniformity can be achieved merely by oscillating the model through several hundred degrees of arc, which requires only that some flexibility be provided in the wiring and tubing that are connected to the model.

CONCLUDING REMARKS

The scaling criteria for the design and testing of thermal models of spacecraft have been derived and discussed. There are no basic theoretical inconsistencies associated with satisfying the various similarity criteria simultaneously; nevertheless, the practical problems in constructing and testing an accurate thermal model can be formidable. It may be possible, however, to simplify the problems by effecting certain compromises with the similarity criteria without seriously affecting the validity of the results. In any case, from the present discussion of the similarity criteria and of these possible compromises, it appears that the design and test of an adequate thermal model will involve careful study, generally along with some auxiliary experimental and analytical investigations.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 25, 1963.

APPENDIX

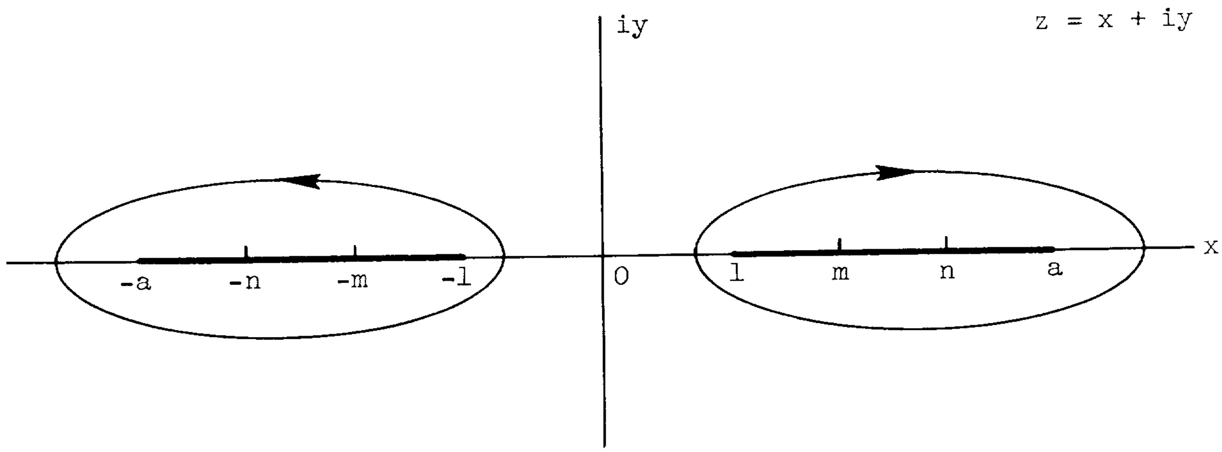
DERIVATION OF THE RESISTANCE INCREASES PLOTTED IN FIGURE 1

Figure 1 gives the relative increase in the resistance to heat flow along a sheet caused by a series of equally spaced cuts extending partially through the sheet. This appendix outlines the derivation of the method of calculating this increase. All the indicated integrals can be found on page 45 of reference 3. The standard elliptic-integral nomenclature, as used in reference 3, for example, will be followed.

The complex velocity for the circulating flow about the two lines indicated in sketch (a) is

$$\frac{dw}{dz} = \frac{1}{\sqrt{\left(1 - z^2\right)\left(z^2 - a^2\right)}}$$

where w is the complex potential, $\phi + i\psi$; ϕ is the velocity potential; ψ is the stream function; and $z = x + iy$.



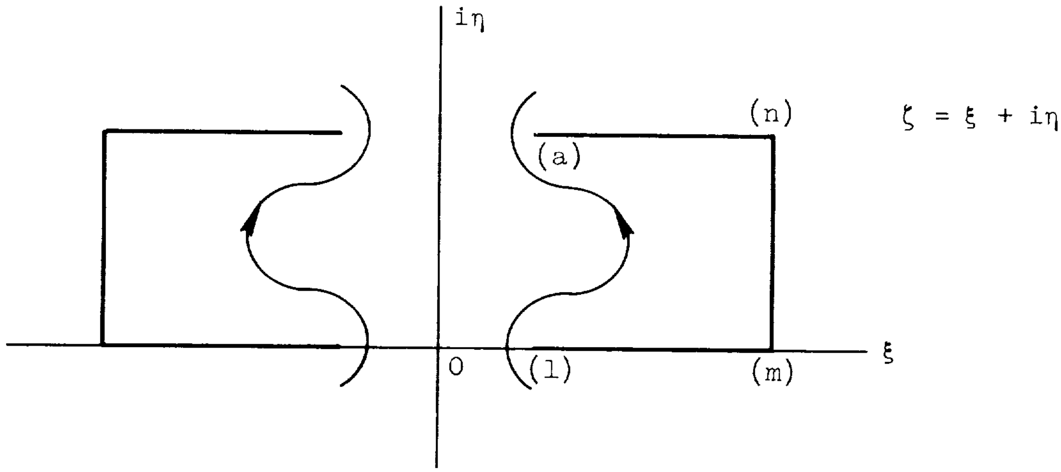
Sketch a

The potential difference between the origin and infinity is found by integrating dw/dz along the y-axis from the origin to infinity. Its value is $\frac{1}{a} K'\left(\frac{1}{a}\right)$. ($K$ and K' are the complete elliptic integrals of the first kind.) The flow through the slot is given by the integral of dw/dz along the x-axis between $x = -1$ and $x = 1$. The integral is $\frac{2}{a} K\left(\frac{1}{a}\right)$.

The upper half of the z-plane is transformed to the inside of a rectangle in the ζ-plane (sketch (b)) by the transformation

$$\frac{d\zeta}{dz} = \frac{1}{\sqrt{(m^2 - z^2)(n^2 - z^2)}}$$

Corresponding points in the z-plane and ζ-plane are indicated in sketches (a) and (b).



Sketch b

The half-width of the rectangle is $\frac{1}{n} K\left(\frac{m}{n}\right)$, which is found by integrating $d\zeta/dz$ along the x-axis from the origin to $x = m$. The height of the rectangle is $\frac{1}{n} K'\left(\frac{m}{n}\right)$, found by integrating $d\zeta/dz$ along the x-axis from $x = m$ to $x = n$. The half-width of the lower slot is $\frac{1}{n} F\left(\frac{m}{n}, \sin^{-1} \frac{1}{m}\right)$, found by integrating $d\zeta/dz$ along the x-axis from the origin to $x = 1$. (F is the incomplete elliptic integral of the first kind.) The half-width of the upper slot is $\frac{1}{n} F\left(\frac{m}{n}, \sin^{-1} \frac{n}{a}\right)$. The upper and lower slots will have the same width if $\frac{1}{m} = \frac{n}{a}$; accordingly, a must be equal to mn.

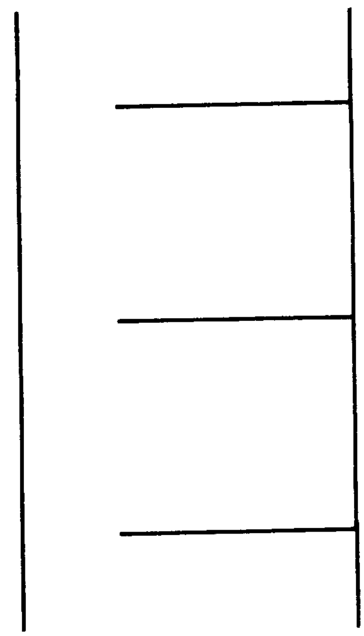
If the total flow through the slot $\left(\frac{2}{a} K\left(\frac{1}{a}\right)\right)$ were to flow along a simple uncut strip of width equal to that of the rectangle $\left(\frac{2}{n} K\left(\frac{m}{n}\right)\right)$, its velocity would be $\frac{n}{a} \frac{K\left(\frac{1}{a}\right)}{K\left(\frac{m}{n}\right)}$. Then the potential difference along a path equal to the height of the rectangle would be

$$\left[\frac{n}{a} \frac{K\left(\frac{1}{a}\right)}{K\left(\frac{m}{n}\right)}\right]\left[\frac{1}{n} K'\left(\frac{m}{n}\right)\right] = \frac{K\left(\frac{1}{a}\right)K'\left(\frac{m}{n}\right)}{aK\left(\frac{m}{n}\right)}$$

When this same flow passes through the rectangle from the lower slot to the upper slot, the potential difference between the two slots is $\frac{1}{a} K'\left(\frac{1}{a}\right)$, as derived previously. The ratio of these two potential differences is

$$\frac{K'\left(\frac{1}{a}\right)K\left(\frac{m}{n}\right)}{K\left(\frac{1}{a}\right)K'\left(\frac{m}{n}\right)}$$

It will be recognized that half of the rectangle represents one segment of the cut sheet, here sketched to the same scale. Accordingly, the preceding expression is the desired relative increase in resistance to heat flow along the sheet caused by the cuts.



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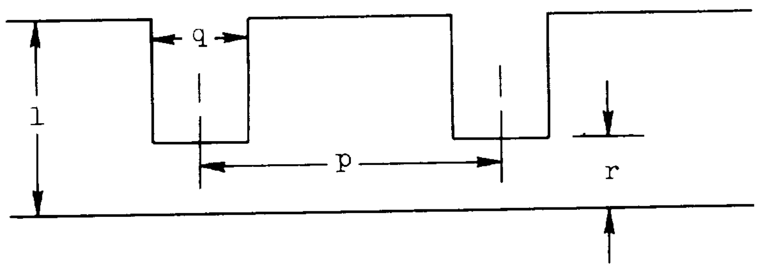

TABLE I.- THERMAL RESISTANCE INCREASE DUE TO NOTCHES

p	q	r	Resistance ratio[a]
1.5	0.0625	0.0625	3.913
	.1875		5.277
	.3125		6.624
	.7500		11.265
1.5	.0625	.1250	2.923
	.1875		3.517
	.3125		4.125
	.7500		6.115
1.5	.0625	.2500	2.057
	.1875		2.319
	.3125		2.564
	.7500		3.344
1.5	.0625	.5000	1.347
	.1875		1.435
	.3125		1.513
	.7500		1.757
3.0	.1250	.1250	2.095
	.3750		2.676
	.6250		3.212
	1.5000		5.262
3.0	.1250	.2500	1.578
	.3750		1.833

p	q	r	Resistance ratio[a]
3.0	0.6250	0.2500	2.080
	1.5000		2.954
3.0	.1250	.5000	1.216
	.3750		1.304
	.6250		1.385
	1.5000		1.668
.5	0	.0625	6.286
	.0625		8.496
	.1875		11.831
.5	0	.1250	3.704
	.0625		5.377
	.1875		6.594
.5	0	.2500	2.488
	.0625		3.205
	.1875		3.611
.5	0	.5000	1.597
	.0625		1.780
	.1875		1.900

[a]Ratio of resistance to that of uncut sheet.

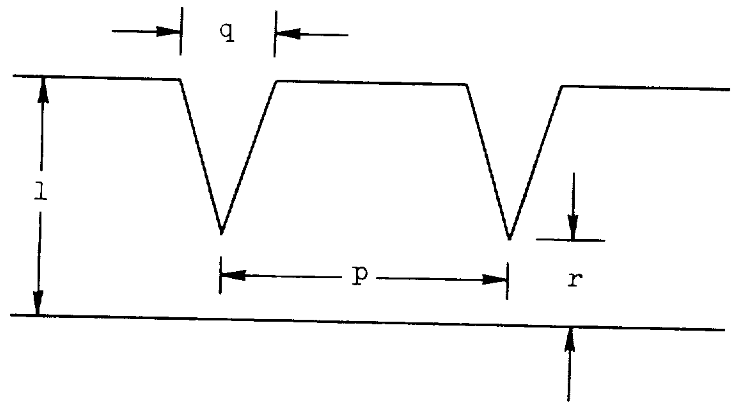

TABLE II.- THERMAL RESISTANCE INCREASE DUE TO V-SHAPED GROOVES[a]

p	q	r	Resistance ratio
1.5	0.502	0.0625	3.556
	.469	.1250	2.758
	.402	.2500	1.976
1.5	1.083	.0625	3.940
	1.010	.1250	2.957
	.866	.2500	2.088
.5	.5	.0625	7.109
		.1250	4.701
		.2500	3.044

[a]Ratio of resistance to that of uncut sheet.

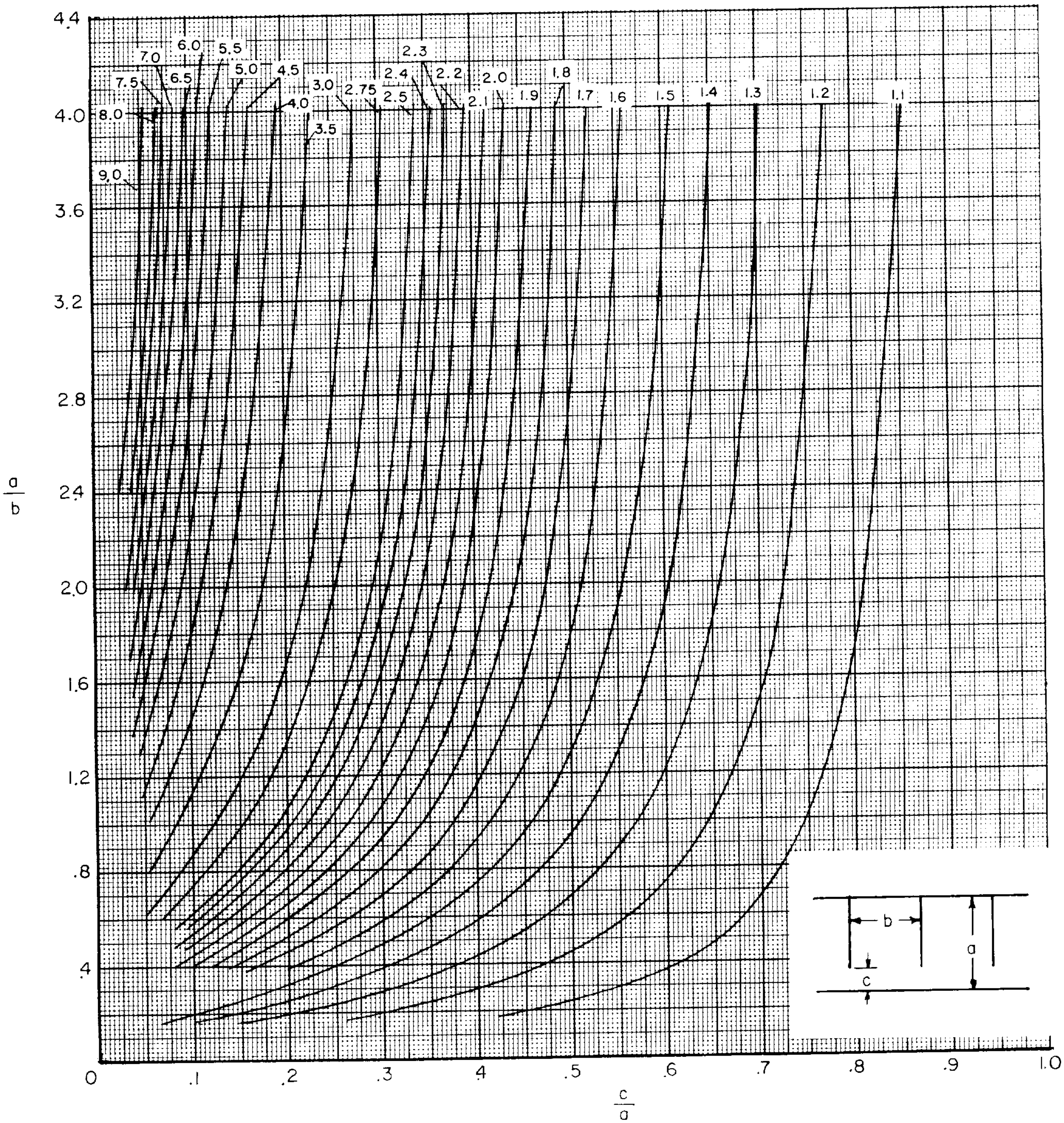

Figure 1.- Relative increase in resistance to heat flow along a sheet caused by cuts made partially through sheet. Symbols are defined on the sketch, which shows a section through the sheet, normal to the cuts. Numbers on curves give the following ratio: $\frac{\text{Resistance to heat flow along cut sheet}}{\text{Resistance to heat flow along uncut sheet}}$.

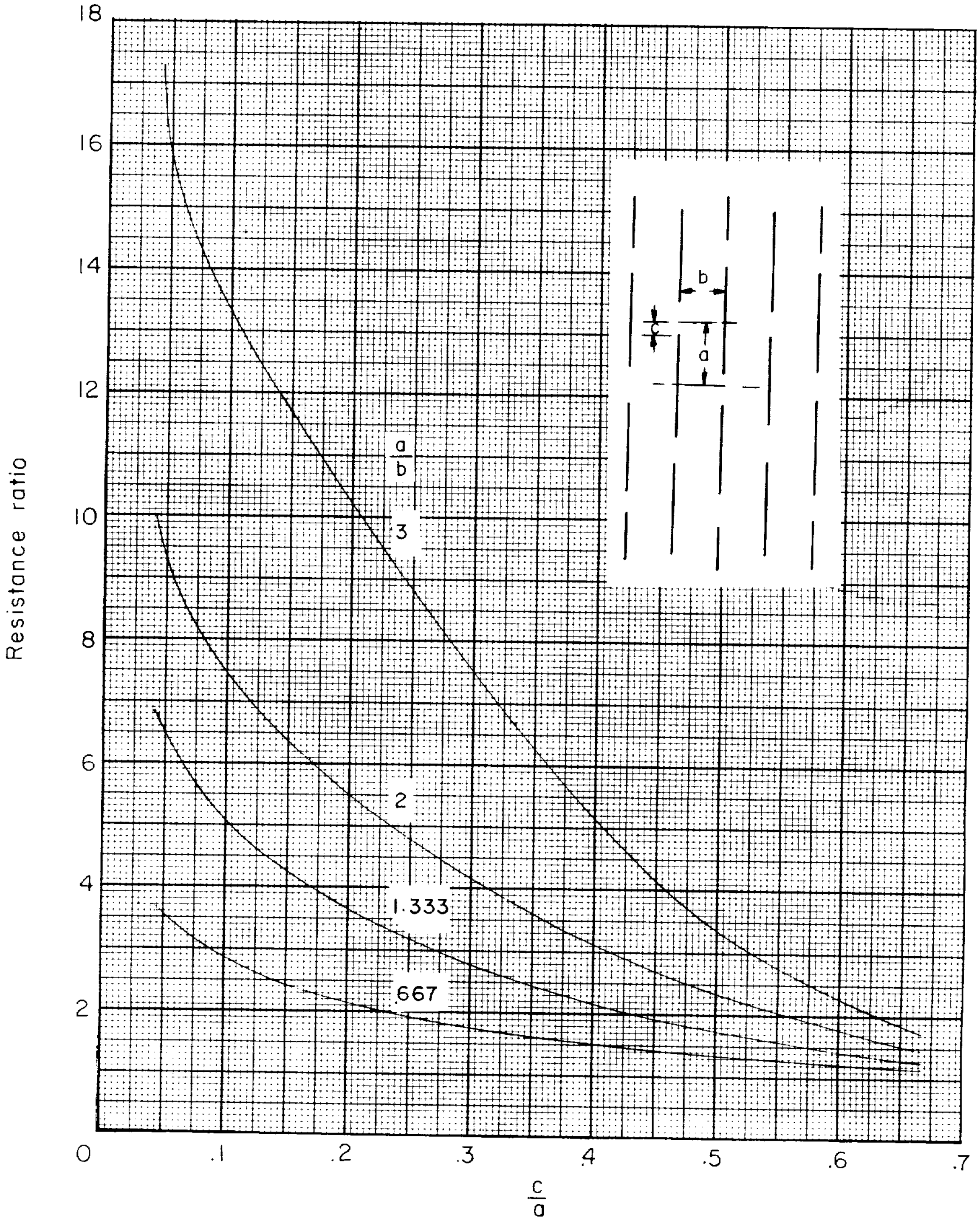

Figure 2.- Relative increase in resistance to heat flow along a sheet caused by cuts made completely through sheet. Symbols are defined on the sketch, which shows the surface of the sheet and the pattern of cuts.

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