19

N63-15302

NASA TN D-1625

code 1

NASA TN D-1625

TECHNICAL NOTE

D-1625

THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES

By Robert W. Fralich and Edwin T. Kruszewski

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON April 1963

Cable-1

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-1625

THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES

By Robert W. Fralich and Edwin T. Kruszewski

SUMMARY

15302

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

INTRODUCTION

A critical problem area in the design of a space vehicle is the landing system. As conventional landing gears would be quite heavy, new methods for absorbing landing impacts have to be devised. One technique that contains little weight chargeable to the landing system is the one referred to as the skid-rocker landing system. (See, for example, refs. 1 and 2.) In this system the vehicle is permitted to slide on its curved lower surface, and the friction between this surface and the landing surface is relied on to stop the vehicle eventually.

An application of this landing concept is discussed in reference 1. The skid-rocker landing characteristics of two proposed space vehicles with different undersurfaces were evaluated experimentally through the use of dynamic-model testing techniques. The results of this investigation showed that upon initial contact with the landing surface, the friction force at the contact point could couple with the inertia forces in such a way as to cause the vehicle to tumble. As such an instability could preclude the use of a specific configuration, the stability characteristics of a landing vehicle become an important design consideration.

It would appear to be unrealistic to consider determining experimentally the stability characteristics of all competing vehicle shapes since the number of such vehicles is great and each vehicle should be optimized with respect to variations in the shape of the sliding surface and its weight distribution. Hence some analytical method of determining the stability of sliding bodies is needed. The presentation of such an analysis is the purpose of this paper.

This paper is divided into two parts. In the first part the governing equations for an arbitrary rigid body sliding on a surface are presented, along with the initial conditions both before and after contact. These nonlinear differential equations are solved exactly to obtain closed-form expressions for the angular position in terms of the initial conditions. These expressions in turn are used to derive the stability criterion. The second portion of the paper consists of numerical results obtained by the application of the derived stability criterion to one of the vehicles used in the experimental investigation discussed in reference 1. Numerical results showing the influence of coefficient of friction, sinking speed, and center-of-gravity location on the stability of the vehicle are discussed. In addition, comparisons between theory and experiment are presented.

SYMBOLS

b — perpendicular distance from body reference line to center of gravity (fig. 1)

$\bar{b} = b/D$

c — distance from base of body reference line to center of gravity, measured parallel to body reference line (fig. 1)

$\bar{c} = c/D$

D — base diameter of vehicle (fig. 1)

d — horizontal distance from contact point to center of gravity (fig. 1)

F — friction force

g — acceleration due to gravity

h — vertical distance from contact point to center of gravity (fig. 1)

h_o — value of h at initial contact

I — polar mass moment of inertia

N — normal force

P,Q — functions of θ defined in equations (10) and (11)

t — time

V_h — horizontal velocity

V_v — vertical velocity (positive downward)

V_θ rotational velocity

W weight of vehicle

x coordinate of center of gravity, measured parallel to landing surface

x_o value of x at initial contact

z,ξ coordinates of undersurface

α,η dummy variables

θ attitude of vehicle (angle between body reference line and the perpendicular at the point of contact, fig. 1)

θ_L angular limit of stability

θ_o initial-contact angle

μ friction coefficient

$\Omega = \omega^2$

ω angular velocity, $\dot{\theta}$

ω_o value of ω at initial contact

$\omega_{o,L}$ critical value of ω at initial contact

Dots on symbols denote differentiation with respect to t; primes denote differentiation with respect to θ.

ANALYSIS

Description of Problem

The problem to be considered is that of the motion of a rigid body sliding on a landing surface in such a way that continuous contact is maintained between the body and the landing surface. Only bodies having sliding surfaces with no reverse curvature are considered; thus only one point is in contact with the landing surface at any time. The body is assumed to have no roll or yaw upon contact, so that all forces including the inertia forces are coplanar.

A typical cross section through the center of gravity of such a body is shown in figure 1. The position of the body at any time t can be given by the coordinates x and θ, where x is the coordinate of the center of gravity parallel to the landing surface and θ is the angle between some reference line in the body and the perpendicular at the point of contact. The position of the center of gravity with reference to the point of contact is given by h, the vertical distance above the surface (positive up), and d, the horizontal distance from the contact point (positive aft). Note that for a given shape of undersurface both h and d are known functions of θ only. The location of the center of gravity with reference to the base of the body reference line is given by the distances b and c, measured perpendicular and parallel to the body reference line respectively. The quantities b, c, d, h, and θ are shown in the positive sense in figure 1. Note that the contact angle θ is the negative of the conventional angle of attack in aerodynamics.

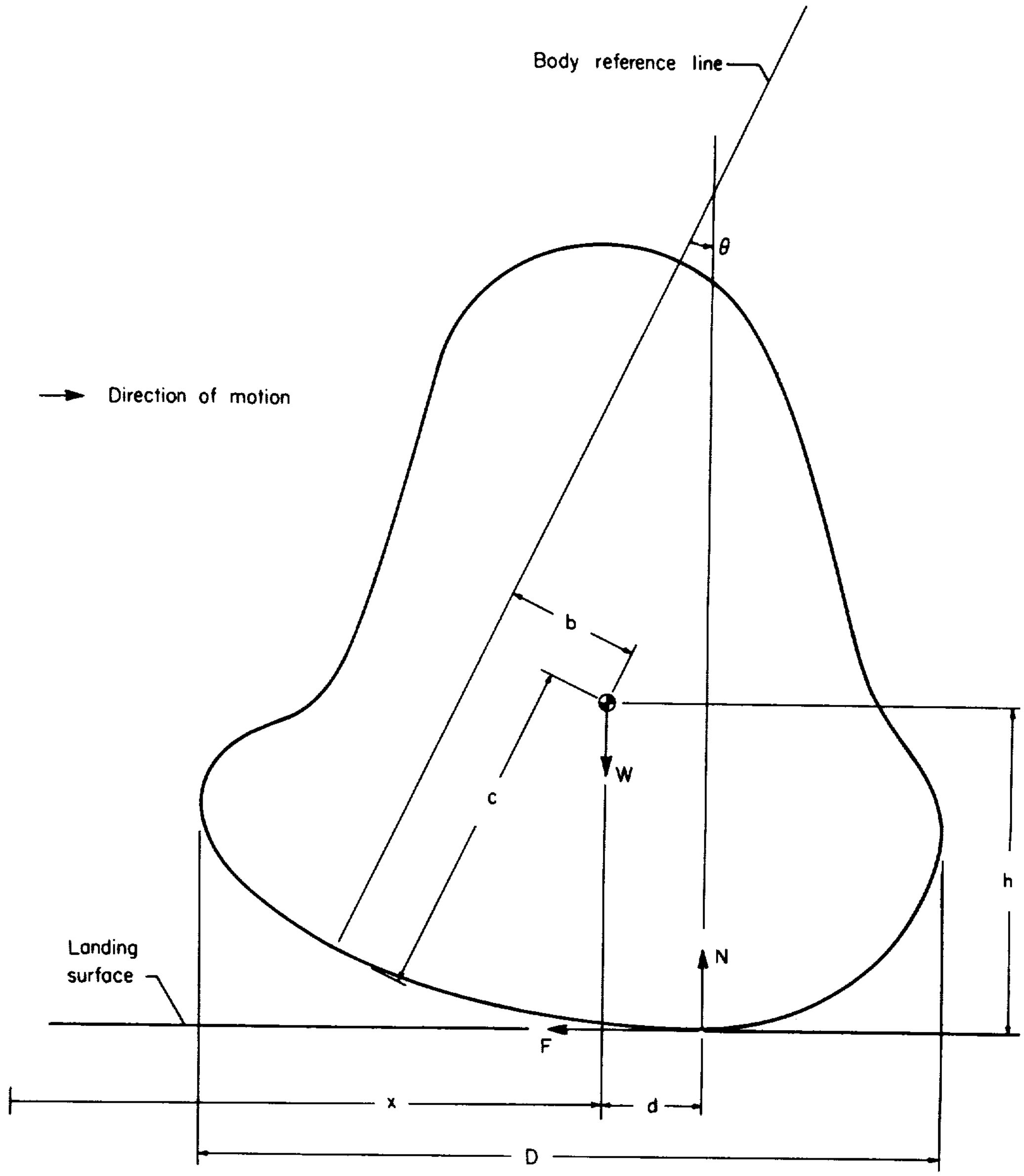

Figure 1.- Body with force system.

The forces acting on the body consist of the normal force N, the friction or drag force F, and the weight of the body W. All forces are positive in the directions shown in figure 1.

Governing Equations

Application of Newton's laws of motion results in the following equations:

$$F = -\frac{W}{g}\ddot{x} \tag{1}$$

$$Fh - Nd = I\ddot{\theta} \tag{2}$$

$$N - W = \frac{W}{g}\ddot{h} \tag{3}$$

where g is the acceleration due to gravity and I is the mass moment of inertia about the axis through the center of gravity and perpendicular to the plane of motion. The dots over the symbols refer to differentiation with respect to time t.

The drag force F is related to the normal force by the equation

$$F = \mu N \tag{4}$$

where μ is the coefficient of friction, which is assumed to be constant. Note that since h is a function of θ alone, and θ is a function of t, the vertical velocity and acceleration of the center of gravity are given by

$$\dot{h} = h'\dot{\theta} \tag{5}$$

and

$$\ddot{h} = h''\dot{\theta}^2 + h'\ddot{\theta} \tag{6}$$

where the primes denote differentiation with respect to θ.

Angular Motion of Vehicle

Solution of equation (2) after substitution of equation (4) results in an expression for the normal force in terms of θ:

$$N = -\frac{I}{d - \mu h}\ddot{\theta} \tag{7}$$

With this expression for N and equation (6), equation (3) can be written as follows:

$$\left(\frac{I}{d - \mu h} + \frac{W}{g}h'\right)\ddot{\theta} + \frac{W}{g}h''\dot{\theta}^2 = -W \tag{8}$$

Since the quantities d and h are functions of θ and the quantities I, W, μ, and g are constants, equation (8) constitutes the sole governing equation for θ. The solution of this equation gives the orientation of the body at any time. Note that equation (1), which involves the acceleration in the horizontal direction, does not enter into the derivation of equation (8). Hence the angular motions of the vehicle are entirely uncoupled from the horizontal motion. The horizontal velocity, however, must at all times be high enough to insure that the resultant velocity of the contact point is positive so that the friction force does not change direction.

Before proceeding with the solution of equation (8), it is convenient to rewrite the equation as

$$2\ddot{\theta} + P\dot{\theta}^2 = Q \tag{9}$$

where both P and Q are functions of θ defined by

$$P = \frac{2\frac{W}{g}h''}{\frac{I}{d - \mu h} + \frac{W}{g}h'} \tag{10}$$

$$Q = -\frac{2W}{\frac{I}{d - \mu h} + \frac{W}{g}h'} \tag{11}$$

Although θ, the angular coordinate, is necessary in obtaining the orientation of the body, a more important variable for the stability problem is the angular velocity ω where

$$\dot{\theta} = \omega \tag{12}$$

and

$$\ddot{\theta} = \dot{\omega} = \omega\omega' \tag{13}$$

With equations (12) and (13), equation (9) can be written in terms of ω:

$$2\omega\omega' + P\omega^2 = Q \tag{14}$$

This nonlinear equation (eq. (14)) can be reduced to a linear differential equation by the transformation

$$\Omega = \omega^2 \tag{15}$$

so that

$$\Omega' = 2\omega\omega' \tag{16}$$

Substitution of equations (15) and (16) into equation (14) results in the following linear differential equation in Ω:

$$\Omega' + P\Omega = Q \tag{17}$$

With the use of the integrating factor $e^{\int_0^\theta P(\alpha)d\alpha}$ the solution to equation (17) can be written as

$$\Omega e^{\int_0^\theta P(\alpha)d\alpha} - \Omega(0) = \int_0^\theta Q(\alpha)e^{\int_0^\alpha P(\eta)d\eta} d\alpha$$

or, in terms of ω,

$$\omega^2 e^{\int_0^\theta P(\alpha)d\alpha} - \omega^2(0) = \int_0^\theta Q(\alpha)e^{\int_0^\alpha P(\eta)d\eta} d\alpha \tag{18}$$

The value of $\omega^2(0)$ is obtained by evaluating equation (18) for the initial conditions at $t = 0$; that is, for $\theta = \theta_o$ and $\omega(\theta_o) = \omega_o$:

$$\omega^2(0) = \omega_o^2 e^{\int_0^{\theta_o} P(\alpha)d\alpha} - \int_0^{\theta_o} Q(\alpha)e^{\int_0^\alpha P(\eta)d\eta} d\alpha \tag{19}$$

With the use of equation (19), equation (18) can be solved for ω^2:

$$\omega^2 = e^{-\int_0^\theta P(\alpha)d\alpha}\left[\omega_o^2 e^{\int_0^{\theta_o} P(\eta)d\eta} + \int_{\theta_o}^\theta Q(\alpha)e^{\int_0^\alpha P(\eta)d\eta} d\alpha\right] \tag{20}$$

Equation (20) can be used to obtain the value of the angular velocity ω for any value of θ in terms of known initial values of θ_o and ω_o. The angular acceleration can be obtained by differentiating equation (20) with respect to θ and making use of equation (13). Thus

$$\ddot{\theta} = -\frac{1}{2}P(\theta)e^{-\int_0^{\theta}P(\eta)d\eta}\left[\omega_o^2 e^{\int_0^{\theta_o}P(\eta)d\eta} + \int_{\theta_o}^{\theta} Q(\alpha)e^{\int_0^{\alpha}P(\eta)d\eta}d\alpha\right] + \frac{1}{2}Q(\theta) \qquad (21)$$

or

$$\ddot{\theta} = -\frac{1}{2}P\omega^2 + \frac{1}{2}Q \qquad (22)$$

Equation (22) could also have been obtained directly from equation (9) by noting the relationship in equation (12).

Stability Criterion

Upon initial contact of the vehicle with the landing surface, friction force and normal force are generated at the contact point. These forces couple with the vertical inertia forces to create angular accelerations of the vehicle. If the initial conditions and geometry of the vehicle are such that these accelerations are large enough, the vehicle will topple over. If these forces are not large enough, the vehicle will reach a maximum angle of tip and then oscillate about some mean position until the friction forces bring it to a halt.

From this description of the vehicle motion it can be seen that for a vehicle to be stable two conditions must be met. First, the vehicle must oscillate between two extreme values of θ; consequently, there must be two values of θ at which the angular velocity vanishes:

$$\dot{\theta}\left(\theta_{E+}\right) = \omega\left(\theta_{E+}\right) = 0 \qquad (23a)$$

$$\dot{\theta}\left(\theta_{E-}\right) = \omega\left(\theta_{E-}\right) = 0 \qquad (23b)$$

Second, the angular acceleration at these points must be such as to cause the vehicle to return to the mean position; that is,

$$\ddot{\theta}\left(\theta_{E+}\right) < 0 \qquad (24a)$$

$$\ddot{\theta}\left(\theta_{E-}\right) > 0 \qquad (24b)$$

In these expressions θ_{E+} is the value of the extreme on the positive side of the mean position and θ_{E-} the value of the extreme on the negative side. The angular limit of stability θ_L is obtained when at one of these extremes the angular acceleration attains a value of zero. Thus θ_L is given by

$$\dot{\theta}\left(\theta_L\right) = \omega\left(\theta_L\right) = 0 \tag{25a}$$

$$\ddot{\theta}\left(\theta_L\right) = 0 \tag{25b}$$

For each vehicle considered, an investigation must be made of the accelerations at the extreme angular positions in order to determine which extreme yields the limit of stability.

Equation (25a), upon substitution from equation (20), yields

$$e^{-\int_{\theta_o}^{\theta_L} P(\eta)d\eta}\left[\omega_{o,L}^2 + e^{-\int_0^{\theta_o} P(\eta)d\eta}\int_{\theta_o}^{\theta_L} Q(\alpha)e^{\int_0^{\alpha} P(\eta)d\eta}d\alpha\right] = 0$$

from which the critical initial angular velocity $\omega_{o,L}$ can be determined. Since

$$e^{-\int_{\theta_o}^{\theta_L} P(\eta)d\eta} \neq 0$$

the equation for $\omega_{o,L}$ becomes

$$\omega_{o,L}^2 = -e^{-\int_0^{\theta_o} P(\eta)d\eta}\int_{\theta_o}^{\theta_L} Q(\alpha)e^{\int_0^{\alpha} P(\eta)d\eta}d\alpha \tag{26}$$

In addition, as θ_L must satisfy equations (25a) and (25b) simultaneously (see eq. (22)),

$$Q\left(\theta_L\right) = 0 \tag{27}$$

From the definition of Q in equation (11) it can be seen that equation (27) is satisfied only if either

$$h' = \infty$$

or

$$d\left(\theta_L\right) - \mu\, h\left(\theta_L\right) = 0 \tag{28}$$

The former condition is impossible when it is noted that

$$h' = d \tag{29}$$

Consequently the latter (eq. (28)) must be one of the governing stability requirements.

Note that when d and h are related as in equation (28), the resulting force at the point of contact intersects the center of gravity of the body. For most practical shapes of undersurfaces used as skid rockers there are three solutions to equation (28) (that is, three positions at which eq. (28) is satisfied). The intermediate solution is associated with a position that is always stable while the extrème values are the possible limits of stability.

The attitude of a given vehicle at the limit of stability is obtained from equation (28) and is a function of the coefficient of friction and the shape of the body. Once this limiting angle is known, the critical initial angular velocity $\omega_{o,L}$ is found as a function of initial contact angle θ_o from equation (26). The integrands appearing in this equation, however, are relatively complex functions of the shape in terms of h, h', and h'' and consequently must be integrated numerically.

Initial Conditions

The initial conditions to be used in equation (26) are, of course, the conditions just after contact is made with the landing surface. In many instances, however, the values of the velocities which are known are those just prior to contact. These values are related to the initial conditions (just after contact) through impulse momentum equations:

$$-\mu\int_0^{\Delta t} N\,dt = \frac{W}{g}\left(\dot{x}_o - V_h\right) \tag{30}$$

$$\int_0^{\Delta t} N(\mu h - d)dt = I\left(\omega_o - V_\theta\right) \tag{31}$$

$$\int_0^{\Delta t} (N - W)dt = \frac{W}{g}\left(\dot{h}_o + V_v\right) \tag{32}$$

where V_h, V_v, and V_θ are, respectively, the horizontal, vertical, and rotational velocities just prior to contact, and $\dot{x}_o$, $\dot{h}_o$, and ω_o are the corresponding initial velocities (just after contact). The integrals on the left of equations (30) to (32) represent the impulse existing between the landing surface and the vehicle. Their actual value depends on the resilient properties of both the vehicle and the landing surfaces. However, the time interval Δt, representing the duration of the impulse, is assumed to be small enough so that

$$\int_0^{\Delta t} N(\mu h - d)dt = (\mu h - d)\int_0^{\Delta t} N\ dt \tag{33a}$$

$$\int_0^{\Delta t} W\ dt = W\int_0^{\Delta t} dt = 0 \tag{33b}$$

The initial vertical and rotational velocities are related through equation (5):

$$\dot{h}_o = h'\left(\theta_o\right)\omega_o \tag{34}$$

With the use of equations (33b) and (34), equation (32) reduces to

$$\int_0^{\Delta t} N\ dt = \frac{W}{g}\left[h'\left(\theta_o\right)\omega_o + V_v\right] \tag{35}$$

The initial angular velocity ω_o, in terms of the velocities just prior to contact, is obtained from equation (31) after making use of equations (33a) and (35):

$$\omega_o = \frac{-\left[d\left(\theta_o\right) - \mu\ h\left(\theta_o\right)\right]\frac{W}{g}V_v + IV_\theta}{I + \left[d\left(\theta_o\right) - \mu\ h\left(\theta_o\right)\right]\frac{W}{g}h'\left(\theta_o\right)} \tag{36}$$

A similar expression for $\dot{x}_o$ is obtained with the use of equations (30), (35), and (36):

$$\dot{x}_o = V_h - \mu I\frac{V_v + h'\left(\theta_o\right)V_\theta}{I + \left[d\left(\theta_o\right) - \mu\ h\left(\theta_o\right)\right]\frac{W}{g}h'\left(\theta_o\right)} \tag{37}$$

Although the values of the impulse quantities depend on the resilient properties of the contacting surfaces, the initial velocities just after contact can be obtained in terms of the velocities just prior to contact without knowing these resilient properties.

Solution Neglecting Friction

As was previously stated, the integrals (see eq. (26)) that must be evaluated to obtain the critical initial angular velocity are integrable only for vehicles with specific simplified sliding surfaces. If the effects of friction are neglected, however, the quantities inside the integral signs become exact differentials; consequently, the integrals can be evaluated exactly for the general case. As the stability criterion is obtained in a simple closed-form expression, the solution will be presented herein.

As seen from equation (28), the attitude at which instability occurs when friction is neglected, θ_L, is the position at which the center of gravity is directly above the point of contact - that is, the attitude for which d, and consequently h', is zero. With the attitude of the vehicle at the point of instability known, the critical initial angular velocity at which the instability occurs, $\omega_{o,L}$, is obtained from equation (26).

With the assumption that the friction coefficient μ is zero, the expressions for P and Q (eqs. (10) and (11)) reduce to

$$P = \frac{2Wh'h''}{Ig + W(h')^2} \tag{38}$$

and

$$Q = \frac{-2Wgh'}{Ig + W(h')^2} \tag{39}$$

Note that P is now an exact derivative; therefore

$$\int_0^{\alpha} P \, d\eta = \log_e \frac{1}{R_o}\left[Ig + W(h')^2\right] \tag{40}$$

where

$$R_o = Ig + W\left[h'(o)\right]^2 \tag{41}$$

Also

$$e^{\int_0^{\alpha} P \, d\eta} = \frac{1}{R_o}\left[Ig + W(h')^2\right] \tag{42}$$

By the use of equations (39) and (42), equation (26) can be integrated to yield

$$\omega_{o,L}^2 = \frac{2Wg\left[h(\theta_L) - h(\theta_o)\right]}{Ig + W\left[h'(\theta_o)\right]^2} \tag{43}$$

Consequently, all values of initial velocities before contact that result in initial angular velocities ω_o (as calculated by eq. (36) with $\mu = 0$) such that $\omega_o^2 > \omega_{o,L}^2$ result in instability.

NUMERICAL RESULTS

The purpose of this section is twofold: first, to illustrate the application of the stability criterion in obtaining a stability boundary for a typical sliding body, and second, to show the quantitative effects of various parameters on the stability limit of a typical vehicle.

Description of Vehicle

The cross section of the vehicle whose stability is to be investigated in this section is shown in figure 2. The shape was taken to be the same as that of

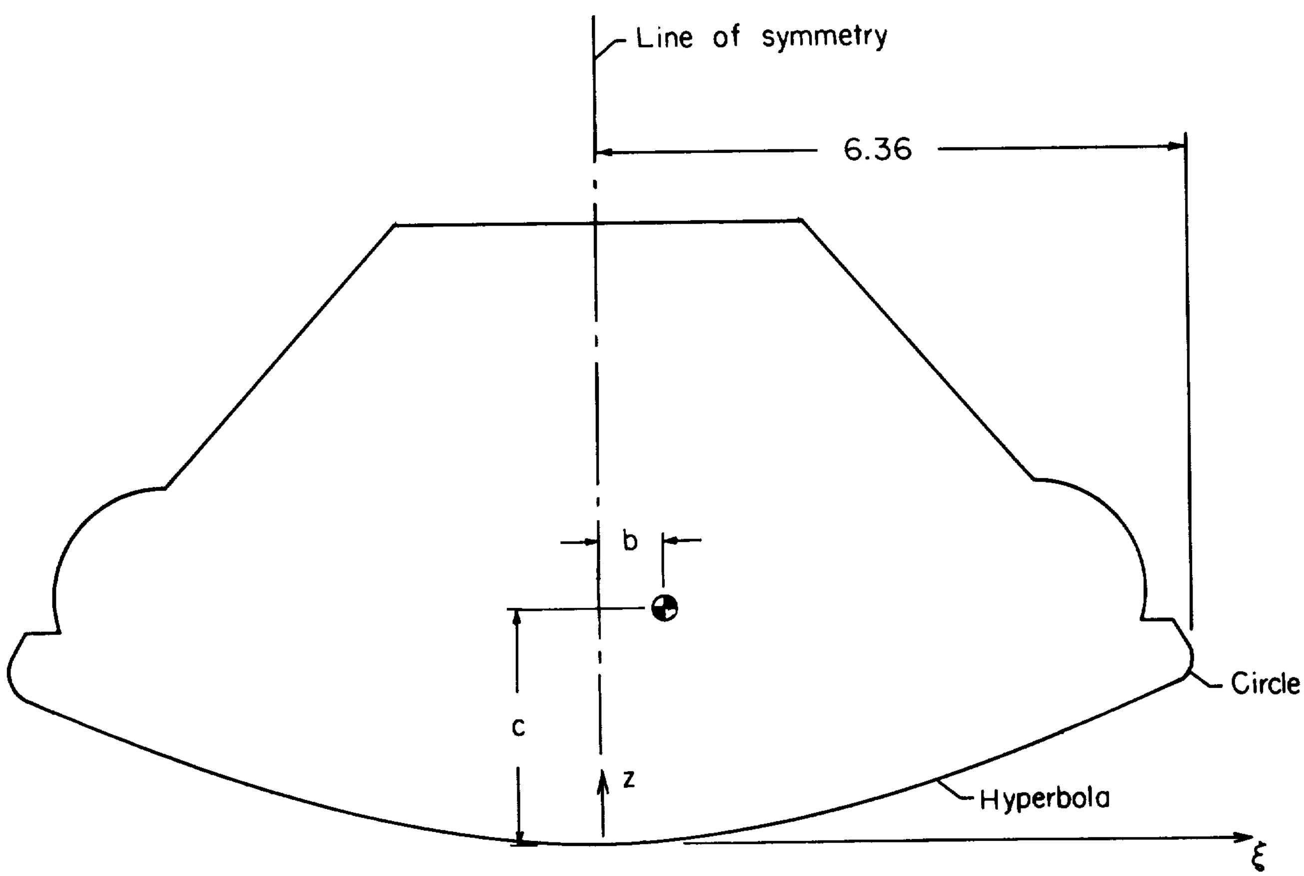

Figure 2.- Cross section of configuration C of reference 1.

configuration C used in the experimental investigation discussed in reference 1. The undersurface is symmetrical and consists of hyperbolic and circular segments having a common tangent at their junctions. The equations for the curves and the expressions for h in the three segments of the undersurface shown in figure 2 are as follows:

For the hyperbola $\left(-22\frac{1}{3}^{\circ} < \theta < 22\frac{1}{3}^{\circ}\right)$:

$$z = -1.784 + \sqrt{0.2305\xi^2 + 1.784^2}$$

$$h = (c + 1.784)\cos\theta - b\sin\theta - 1.784\sqrt{\cos^2\theta - 4.3383\sin^2\theta}$$

For the circle at $\theta > 22\frac{1}{3}^{\circ}$:

$$z = 1.977 - \sqrt{(0.3333)^2 - (\xi - 6.032)^2}$$

$$h = (c - 1.977)\cos\theta - (b - 6.032)\sin\theta + 0.3333$$

For the circle at $\theta < -22\frac{1}{3}^{\circ}$:

$$z = 1.977 - \sqrt{(0.3333)^2 - (\theta + 6.032)^2}$$

$$h = (c - 1.977)\cos\theta - (b + 6.032)\sin\theta + 0.3333$$

The value of the ratio of the mass to the mass moment of inertia is

$$\frac{W}{Ig} = 0.0944 \text{ per sq ft}$$

Application of Stability Criterion

The first step in obtaining a stability boundary is to calculate the angular limit of stability θ_L from equation (28) for the assumed value of the coefficient of friction. As the vehicle in figure 2 is symmetrical, it is convenient to use the line of symmetry as the angular reference line. For this symmetrical body, with the center-of-gravity positions considered, the highest of the three values of θ_L from equation (28) yields the limit of stability. Values of θ_L for various values of the coefficient of friction are shown in figure 3. With the value of θ_L known, $\omega_{o,L}$ (the critical initial angular velocity after contact for which instability or tumbling occurs) is calculated for a given contact angle θ_o from equation (26). The integrals appearing in equation (26) are evaluated numerically by use of trapezoidal integration with $2/3^\circ$ intervals of θ.

Any combination of sinking velocity V_v and of pitching velocity V_θ that results in a value of ω_o^2 (as calculated from eq. (36)) greater than $\omega_{o,L}^2$ leads to instability. As the experimental technique used in reference 1 does not permit the vehicle to have a pitching velocity, V_θ is assumed to be zero in the numerical analysis used herein. Consequently, the value of ω_o in the following discussion will be a function only of the sinking velocity V_v and the attitude at impact as given by θ_o.

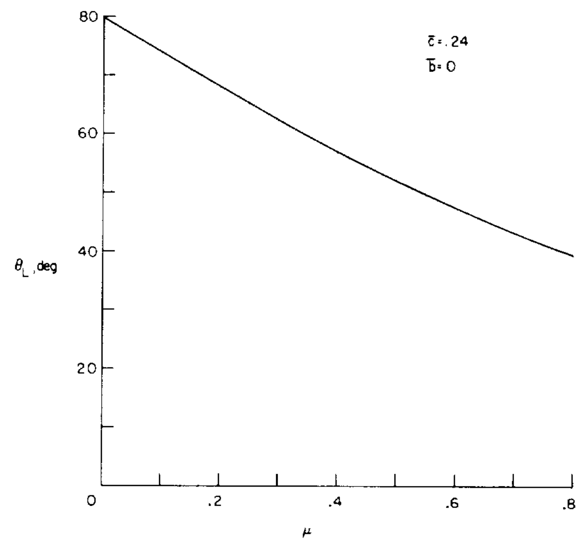

Figure 3.- Variation of angular limit of stability θ_L with friction coefficient μ.

Discussion of Results

Stability boundaries were calculated for the vehicle of figure 2 for a range of values of sinking velocity, coefficient of friction, center-of-gravity location, and contact attitude. The results of these calculations are shown in figures 4 to 7.

In figure 4 the critical stability curve is shown as a function of sinking velocity and contact angle for a vehicle with given center-of-gravity location and coefficient of friction. For all combinations of contact angle and velocity that fall below this curve the vehicle is stable. For values that fall above this curve the vehicle tumbles. The distance between the two curves for a given velocity represents the range of contact angles for which a stable landing is possible, and is referred to as the region of safe touchdown.

The amount of attitude control necessary for the landing vehicle decreases with an increase in the size of this region of safe touchdown. For example, at a sinking velocity of 40 feet per second, the controls must be sufficiently accurate to land the vehicle within a range of 10° of contact angle. At a velocity of 20 feet per second this range increases to 20° and thus less control is necessary. For velocities below 20 feet per second there is a sudden increase in the size of the safe touchdown region.

Note that for sinking velocities greater than 20 feet per second some kind of attitude control is always necessary. As can be seen in figure 4, for velocities above 20 feet per second the only stable contact angles are positive; consequently the landing of the vehicle must be controlled to assure such an angle.

Even at lower sinking velocities the desirability of some kind of attitude control is indicated by the fact that the range of positive safe contact angles is considerably larger than the range of negative angles.

The influence of the coefficient of friction and the height of the center-of-gravity location (c in fig. 2) on the region of safe touchdown is shown in figures 5 and 6, respectively. In figure 5 the critical stability curve of the vehicle is shown as a function of the contact angle and coefficient of friction for a sinking velocity of 10 feet per second and the same center-of-gravity location used to obtain the results of figure 4. In figure 6 the stability curve is shown as a function of the contact angle and a dimensionless center-of-gravity height $\bar{c} = c/D$, for the sinking velocity of 10 feet per second (same as used in fig. 5) and coefficient of friction of 0.40 (same as in fig. 4).

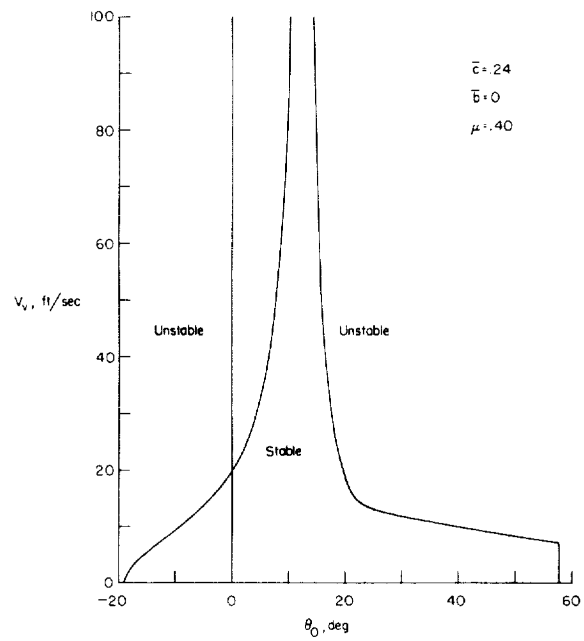

Figure 4.- Influence of sinking velocity V_v on stability.

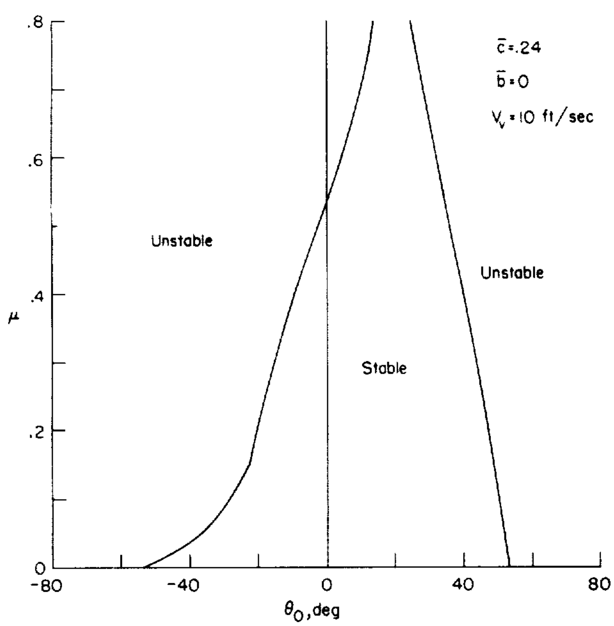

Figure 5.- Influence of friction coefficient μ on stability.

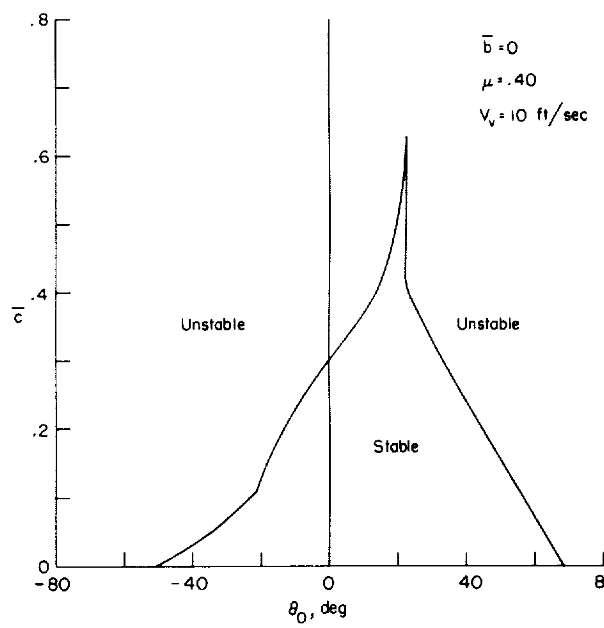

Figure 6.- Influence of center-of-gravity height $\bar{c}$ on stability.

From figures 5 and 6 it can be seen that, the region of safe touchdown becomes larger with a decrease in both the coefficient of friction and the height of the center of gravity. Furthermore, in both figures, for reasonable values of μ and $\bar{c}$ the range of positive safe contact angles is considerably larger than the range of negative angles. This fact supports the desirability of attitude control.

The influence of offsetting the center of gravity from the line of symmetry is shown in figure 7. In this figure the critical stability curve is given as a function of the contact angle and a dimensionless offset distance $\bar{b} = b/D$. The values of $\bar{c}$ and μ are the same as in figure 4 and V_V is equal to 10 feet per second. This figure indicates that, for a sufficiently high negative offset, the range of negative safe contact angles might be made large enough to eliminate the necessity for attitude control. This situation, however, would require that the location of the center of gravity with respect to the motion of the vehicle be made a design criterion.

For completeness the comparison between theory and experiment presented in reference 1 is shown in figure 8. In this figure the curves show the calculated stability boundaries as functions of center-of-gravity height and landing attitude for three values of friction coefficient and a sinking velocity of 10 feet per second. The symbols show the results of the experimental investigation; the open symbols represent stable landing attitudes while the closed symbols represent unstable ones. In general the agreement between theory and experiment is good.

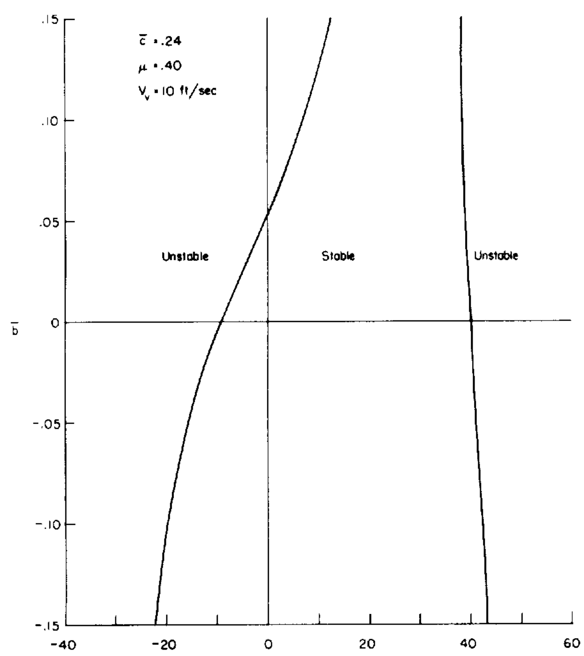

Figure 7.- Influence of center-of-gravity offset $\bar{b}$ on stability.

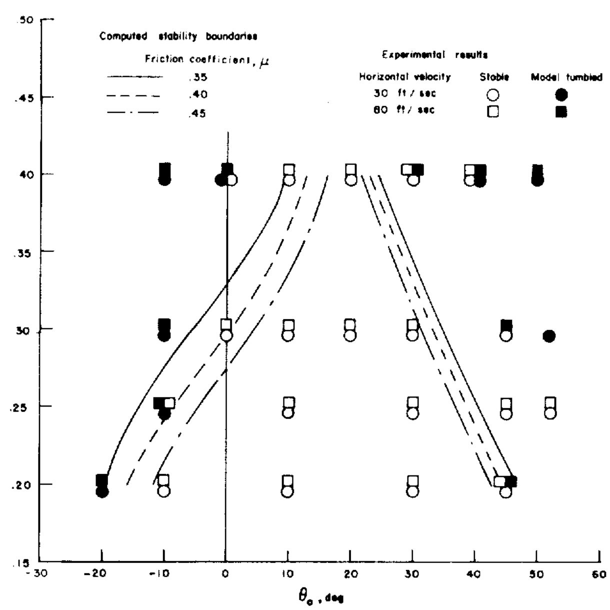

Figure 8.- Comparison of theoretical results with experimental results for configuration C of reference 1 showing the influence of center-of-gravity height $\bar{c}$ on stability. Vertical velocity V_V = 10 ft/sec; center-of-gravity offset $\bar{b}$ = 0.

CONCLUDING REMARKS

A method of calculating a stability criterion for sliding bodies has been developed. The method resulted from an exact solution of the nonlinear equations of an arbitrary rigid body sliding on a surface in such a way that continuous point contact is maintained. The range of values of initial contact angles which result in stable motion is obtained from the solution of two equations, one of which involves integrals that, for most cases, must be integrated numerically.

The method was used to obtain the stability boundaries of a vehicle used in an experimental investigation. From the numerical results it was concluded that although the region of safe touchdown could be increased by proper choice of center-of-gravity location and sinking velocity, some attitude control for the landing vehicle is desirable.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 15, 1963.

REFERENCES

1. Stubbs, Sandy M.: Investigation of the Skid-Rocker Landing Characteristics of Spacecraft Models. NASA TN D-1624, 1963.

2. Mayo, Wilbur L.: Skid Landings of Airplanes on Rocker-Type Fuselages. NASA TN D-760, 1961.

NASA-Langley, 1963 L-2066

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski. Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA

NASA TN D-1625
National Aeronautics and Space Administration.
THEORETICAL STABILITY ANALYSIS OF SKID-ROCKER LANDINGS OF SPACE VEHICLES.
Robert W. Fralich and Edwin T. Kruszewski. April 1963. 18p. OTS price, $0.50.
(NASA TECHNICAL NOTE D-1625)

The governing equations for an arbitrary rigid body sliding on a landing surface are used to derive a stability criterion which relates the critical values of initial velocities to the coefficient of friction, center-of-gravity location, and initial angle of contact. A numerical application of the stability criterion is made for a vehicle used in an experimental investigation.

I. Fralich, Robert W.
II. Kruszewski, Edwin T.
III. NASA TN D-1625

NASA