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ORBITS RETURNING FROM THE MOON TO A SPECIFIED GEOGRAPHIC LANDING AREA

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SUMMARY

This paper develops a method of computing approximate trajectories returning from the Moon to a fixed landing site. The gravitational field of a spherical Earth is assumed to govern orbital motion and the entry phase of the trajectories is described by a linear relation between entry range and flight time in the atmosphere.

As an example, data were computed for trajectories returning to Edwards Air Force Base during the month of February 1966 and an analysis of these data is presented.

INTRODUCTION

Return from the Moon to Earth is the last major phase of a successful manned lunar mission. Many of the phases in the mission impose trajectory constraints; for example, constraints that result from launch site and launch azimuth restrictions, boost vehicle operations, tracking considerations, operations in the vicinity of the Moon, lighting conditions at various points in the mission, etc.

The return phase also imposes an important trajectory constraint that arises from the need for control over the terrestrial landing area. One possible interpretation of this control problem is to restrict operations to trajectories that return to a fixed landing site. The literature has included substantial contributions to the midcourse guidance (e.g., ref. 1) and entry phases (e.g., ref. 2) of the return but has tended to ignore the analysis of satisfactory return trajectories. The present work investigates trajectories which return from the Moon to a fixed landing site on the Earth but are otherwise unrestricted.

A large amount of data is required to determine the effects of the many variables in the problem so that a rapid and reasonably accurate method of finding return trajectories is necessary. The method of solution described in the text follows the general approach of references 3 and 4, in which orbital motion is described by the two-body approximation; that is, multibody effects are neglected and only the gravitational field of a spherical Earth is considered.

The accuracy of the solutions obtained by this approximation is such that when the perigee position and velocity are entered in an n-body numerical integration computer program and integrated backwards in time to the Moon, the trajectory originates on the surface of the Moon.

Data were obtained with a view to developing an understanding of the general nature of return trajectories and to establishing the effects of the variables in the problem. The data presented largely concern trajectories landing at Edwards Air Force Base during February 1966, but are typical of other landing sites in the Southwestern United States and other time periods, and illustrate the general nature of return trajectories.

SYMBOLS

A_Z	azimuth, measured from local North
D	declination
e	orbital eccentricity
E, F	eccentric anomaly for ellipses and hyperbolas
I	orbital plane inclination angle
INT	truncation function
R	distance from Earth's center
RA	right ascension, measured from the first point of Aries
T	time (Time is given in mean solar units. Calendar date is given in Greenwich mean solar time unless otherwise specified.)
TF	flight time
T_L	time of landing
T_M	time of launch from the Moon
V	speed
XYZ	inertial coordinate frame
θ	orbital true anomaly from vacuum perigee
$\Delta\theta$	true anomaly of entry point

φ	entry range angle
μ_E	gravitational constant, 398613.50 km^3/sec^2
ξ	in-plane angle from the nearest ascending node
$\Delta\xi$	geocentric angle, Moon to landing
Ψ	equatorial angle from the nearest ascending node
$\Delta\Psi$	equatorial angle, Moon to landing
$(\ )_L$	conditions at landing
$(\ )_M$	conditions at the Moon
$(\ )_P$	conditions at vacuum perigee
$(\bar{\ })$	vector
$(\ )^D$	desired value
$(\dot{\ })$	time derivative

ANALYSIS

Method of Solution

The problem is to compute those trajectories which return from a point in the Earth-Moon system and allow a vehicle to land at a specified site on the Earth. The point in space is specified by its position, $\bar{R}$, and the time of departure, T, and the landing site, by its right ascension at some reference time and its geographic latitude.

The method of solution can be divided into two steps: First, by choosing the azimuth at landing it is possible to compute from geometrical considerations the required total geocentric angle in the plane of motion from the point of departure to the landing site, and the required time of landing and corresponding total flight time. The dynamics of the return trajectory fall into two regimes, an orbital phase and an entry phase. The second step requires an iterative procedure to find the combined entry trajectory and Keplerian orbit which match the required flight time and geocentric angle computed from the first step. This results in the solution orbit and the required atmosphere entry range.

Geometrical Considerations

Total geocentric angle.- The return trajectory will be in a single plane as a result of the assumption of Keplerian orbital motion. It will therefore be useful to consider first the geometry associated with the intersection of an orbital plane with the celestial sphere, as shown in figure 1 where X, Y, Z form the usual inertial coordinate frame centered at the Earth. For any point on this intersection the angles A_Z (azimuth), D (declination), Ψ (equatorial angle from the ascending node), RA (right ascension), and ξ (in-plane angle from the ascending node) are defined.

The orbital plane inclination angle, I, is the angle between the North Pole and the normal to the orbital plane, where the normal is taken in the positive direction of orbital angular velocity. Only easterly orbits (i.e., orbital motion from west to east with respect to the Earth) will be considered, for which the inclination angle and azimuth are restricted to the ranges $0 \leq I \leq \pi/2$ and $0 \leq A_Z \leq \pi$.

Some convenient relations among the angles are

$$\cos I = \cos D \sin A_Z \tag{1a}$$

$$\sin \Psi = \tan D/\tan I \tag{1b}$$

$$\cos \Psi = \cos A_Z/\sin I \tag{1c}$$

$$\cos \xi = \cos \Psi \cos D \tag{1d}$$

$$\sin \xi = \sin D/\sin I \tag{1e}$$

The total geocentric angle can be determined after obtaining the angles associated with two points on the orbital track. These points are to be specified by their declinations, D_L (the landing site latitude) and D_M (the declination of the Moon at the time of departure). The necessary relations result from application of equations (1) first at the landing site and then at the Moon.

The free choice of one of the geometrical parameters other than D_L and D_M is available; for example, entry range angle, orbital true anomaly (or orbital energy), inclination angle, etc. The choice is a matter of convenience to the purpose of the computations and in the present case the method of solution will be formulated with the azimuth at landing, A_{ZL}, as the independent parameter. This fixes the inclination angle through use of equation (1a). Vehicles on trajectories having the same landing azimuth will approach the landing site from the same direction and will have nearly identical tracks over the rotating Earth in the final phase of the return.

The orbital plane inclination and equatorial angle of the landing site are computed from equations (1) applied at the landing site.

$$
\left.\begin{aligned}
\cos I &= \cos D_L \sin A_{ZL} \\
0 &\leq I \leq \pi/2 \\
\sin \Psi_L &= \tan D_L/\tan I \\
\cos \Psi_L &= \cos A_{ZL}/\sin I
\end{aligned}\right\} \qquad (2)
$$

The first of these equations, which relates inclination at landing to the azimuth angle, is plotted in figure 2(a) for landings at Edwards AFB ($D_L = 34.9^\circ$). In general, the inclination angle cannot be less than the maximum required declination on the orbit, which in this case is the latitude of Edwards. The declination of the Moon, plotted in figure 2(b) for the month of February 1966, can vary between 28.5° North and 28.5° South over a month. A minimum inclination orbit will have a due East heading at landing and polar orbits will refer to those with zero landing azimuth.

The same equations are now applied at the Moon, giving:

$$
\left.\begin{aligned}
\sin A_{ZM} &= \cos I/\cos D_M \\
\sin \Psi_M &= \tan D_M/\tan I \\
\cos \Psi_M &= \cos A_{ZM}/\sin I
\end{aligned}\right\} \qquad (3)
$$

The first equation gives two values of the heading angle at the Moon for the given values of I and D_M, and the two corresponding values of Ψ_M are obtained from the remaining two equations. Geometrically, these two sets of angles correspond to the two points on the orbital track of figure 1 which have the specified lunar declination, D_M. The procedure that follows is the same for either set of angles.

Finally, the total geocentric angle from the Moon to the landing site, $\xi_L - \xi_M$, can be computed from the following equation, obtained by substitutions of equations (2) and (3) in equations (1d) and (1e).

$$
\begin{aligned}
\cos \Delta\xi &= \cos D_M \cos D_L \cos \Delta\Psi + \sin D_M \sin D_L \qquad (4) \\
\Delta\xi &= \xi_L - \xi_M \\
\Delta\Psi &= \Psi_L - \Psi_M
\end{aligned}
$$

To obtain the correct quadrant for the geocentric angle, it should be noted that $\sin \Delta\xi$ and $\sin \Delta\Psi$ have the same sign.

The relations among the various angles are summarized in figures 2(c) and 2(d), which show $\Delta\xi$ vs. D_M for various values of $\Delta\Psi$ and landing azimuth. The geocentric angle, $\Delta\xi$, is given only in the range from π to 2π. This is the only range of interest in the present case because of restrictions on the orbital true anomaly and atmosphere entry range.

<u>Times of landing and flight times</u>.- Once the landing azimuth, D_L, and D_M are specified, the angles, I, A_{ZM}, $\Delta\Psi$, and $\Delta\xi$ can be computed as in the preceding section. The orbital plane can next be located inertially, since its inclination is known and it must pass through the Moon's position at the specified time of departure with the correct heading (fig. 3). The required inertial direction of the landing site at the time of landing is then located. Since the landing site

occupies a given inertial direction on its track once each sidereal day, the landing times and corresponding flight times can be found.

The equatorial angle east from the Moon at the time of departure to the landing site at the time of landing is the difference in right ascension of these two inertial directions; that is,

$$RA_L - RA_M = \Psi_L - \Psi_M$$

and the right ascension of the landing site is

$$RA_L = RA_M + \Delta\Psi \tag{5}$$

The times at which the landing site has this value of right ascension are given by:

$$T_L(i) = \frac{RA_L - RA_{LO}}{2\pi\omega} + \frac{i}{\omega}, \qquad i = \ldots -2, -1, 0, 1, 2, \ldots \tag{6}$$

Here, time is measured in mean solar days and is related to sidereal time by the factor $\omega = 1.0027379$ sidereal days per mean solar day. Time may be taken as zero at any convenient Greenwich calendar date and RA_{LO} is the right ascension of the landing site at that reference time. A convenient relation for the reference right ascension is

$$RA_{LO} = RA_{GO} + l_L$$

where RA_{GO} is the right ascension of Greenwich at the reference time and l_L is the east longitude of the landing site. Finally, the required total flight time for any value of "i" is

$$TF(i) = T_L(i) - T_M \tag{7}$$

Equation (6) only specifies the times at which the landing site has the desired inertial position, which occurs once every sidereal day, that is, once for each value of "i." Only some of these times are of interest owing to obvious restrictions on the flight time from the Moon to the Earth. Except for orbits that leave the Moon heading away from the Earth, the maximum flight time is given by the trajectory for which the Moon's position is at apogee. In this case the orbital flight time is

$$TF = \pi\sqrt{\left(\frac{R_P + R_M}{2}\right)^3 \frac{1}{\mu_E}} \cong 5 \text{ days}$$

where

$$R_P = 6{,}430 \text{ km} = \text{safe entry vacuum perigee}$$

$$R_M = 384{,}000 \text{ km} = \text{mean lunar distance}$$

This estimate of the maximum flight time of interest neglects the entry flight time, which is comparatively small. Although it is theoretically possible to compute orbits of zero flight time, energy considerations place a practical lower limit on flight times (cf. fig. 4(a)). The limits used in this study were taken as 1.5 and 5 days. In general, if the flight times of interest are in the range

$$\tau_{min} \leq TF \leq \tau_{max}$$

then the values of i of interest in equation (6) are given by

$$\left.\begin{aligned} & i_{min} \leq i \leq i_{max} \quad (i \text{ an integer}) \\ & i_{min} = 1.0 + \mathrm{INT}\left[\omega(T_M + \tau_{min}) - \frac{RA_L - RA_{LO}}{2\pi}\right] \\ & i_{max} = \mathrm{INT}\left[\omega(T_M + \tau_{max}) - \frac{RA_L - RA_{LO}}{2\pi}\right] \end{aligned}\right\} \tag{8}$$

The derivation of this result is given in appendix A. The function $\mathrm{INT}(y)$ simply truncates y to the next lower integer; for example, $\mathrm{INT}(3.6) = 3$.

Dynamics

The return trajectory must be separated into two phases: orbital and entry. The required total flight time and geocentric angle of the return trajectory are given by equations (4) and (7) and the complete solution is given by that combination of orbital motion and entry maneuver which matches both constraints; that is, the combination which satisfies the equations

$$TF_O - \Delta TF + TF_E = TF^D \tag{9a}$$

$$\theta - \Delta\theta + \varphi = \Delta\xi^D \tag{9b}$$

where the various quantities are

TF_O orbital flight time, Moon to vacuum perigee

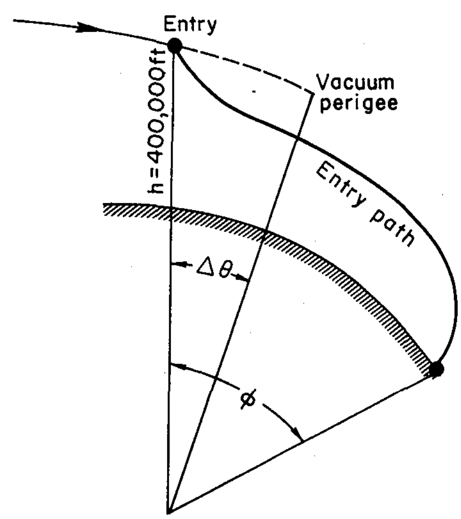
TF_E entry flight time, atmosphere entry to landing

ΔTF orbital flight time, atmosphere entry to vacuum perigee

θ orbital true anomaly, Moon to vacuum perigee

φ entry range angle

$\Delta\theta$ true anomaly of entry location



Sketch (a).- Entry phase parameters.

The quantities, TF^D and $\Delta\xi^D$, are the desired values of flight time and geocentric angle obtained from equations (7) and (4).

Entry phase.- The entry phase begins when the vehicle is at an altitude of 400,000 feet (6,500 km) and terminates at landing. Vacuum perigee of the return orbits was fixed at 6,430 km, the middle of the entry corridor, in order to obtain suitable entry conditions.[1] With this value of vacuum perigee the flight-path angle at entry is very nearly fixed at -6.0° for all trajectories returning from the Moon with flight times in the range of interest. Further, the true anomaly, $\Delta\theta$, from entry to vacuum perigee is nearly fixed at 12.0° with a corresponding orbital flight time, TF, of 122 seconds.

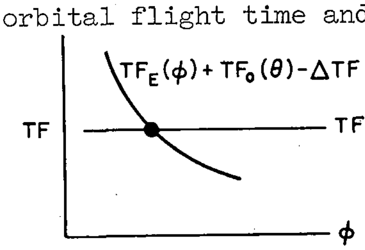
A relation between entry flight time and entry range is necessary. Work on entry from circular orbits (refs. 5 and 6) has indicated a linear relation between these two parameters, and an unpublished study which extends the work of reference 6 to entry from parabolic orbits indicates a satisfactory linear approximation of the relation between entry range and entry flight time. The data of this unpublished study were obtained in both variable and fixed L/D skipping entry flight paths for Apollo type vehicles, and provide, to within about one minute, the following linear relation:

$$TF_E = 0.00933\ \varphi + 0.00254 \tag{10}$$

valid approximately for all entry trajectories of interest. In equation (10) TF_E and φ are taken in days and radians, respectively. The solution to the problem is relatively insensitive to any errors in the approximation to entry characteristics given by equation (10) because entry flight times are small compared to the total flight time from the Moon; for example, an entry range of 10,000 nautical miles requires about 43 minutes flight time. In the present work, upper and lower limits were placed on entry range, namely, 1,000 and 10,000 nautical miles or range angle limits of 16.6° to 166.2°. The possibility of establishing a parking orbit after skip-out and the effects of lateral range control were not considered.

Orbital flight time.- Flight time from the Moon to vacuum perigee as a function of true anomaly is readily computed from the equations describing Keplerian orbits (cf. fig. 4(b)). The form of these equations found convenient for the computer program used in obtaining the numerical results of this paper is given in appendix B.

Remarks on the solution of equations (9).- Equations (9) must be solved simultaneously and, because Kepler's equation for orbital flight time is transcendental, an iterative procedure is required. One way to do this is to vary the entry range angle, φ, computing the true anomaly from (9b) in each case. The orbital flight time and entry maneuver times are then found from equations (10) and (B6) to (B9). This is done until some value of φ is found that satisfies equation (9a) also; that is, it is necessary to find the intersection of the two curves in sketch (b).



Sketch (b).

In programming the above process of solution some caution is necessary. There are limits on the true anomaly for orbits having a given value of

[1]The solution is insensitive to choice of altitude within the entry corridor.

perigee radius and passing through some other required range, in this case, the distance to the Moon. Values of true anomaly outside these limits cannot be used in the equations of appendix B. The limits are:

$$\cos^{-1}\left(\frac{R_P}{R_M}\right) \leq \theta \leq \pi$$

The lower value is the minimum possible true anomaly, corresponding to an orbit of infinite energy, and the upper limit eliminates consideration of orbits which depart the Moon heading away from the Earth and corresponds to a flight time of about 5 days. These limits can be combined with equation (9a) to obtain corresponding limits on the value of φ that may be used in the search for the solution once $\Delta\xi^D$ has been calculated:

$$\Delta\xi^D - \pi + 12^\circ \leq \varphi \leq \Delta\xi^D - \cos^{-1}\left(\frac{R_P}{R_M}\right) + 12^\circ$$

In addition, limits on the entry range capability of the vehicle have been assumed, which may be combined with the above limits to give

$$\left.\begin{aligned} \varphi_{min} &= \begin{cases} 16.6^\circ \\ \Delta\xi^D - \pi + 12^\circ \end{cases} \quad \text{whichever is larger} \\ \varphi_{max} &= \begin{cases} 166.2^\circ \\ \Delta\xi^D - \cos^{-1}\left(\frac{R_P}{R_M}\right) + 12^\circ \end{cases} \quad \text{whichever is smaller} \end{aligned}\right\} \tag{11}$$

In the process of searching for a solution, the angle φ may be varied between the limits given by equation (11). Outside this region, either the entry range angle would exceed the assumed vehicle capability, or the true anomaly would be outside the region for which orbits returning from the Moon are possible or have flight times less than 5 days.

A second consideration is that the existence of a solution (that is, the occurrence of an intersection as in sketch (b)) must first be checked before searching for a solution. As is evident in the sketch, a solution exists only if the required flight time, TF^D, is bracketed by the flight times corresponding to the limits in entry range angle given by equations (11) above; that is, only if

$$TF_O(\Delta\xi^D - \varphi_{min} + 12^\circ) + TF_E(\varphi_{min}) > TF^D + \Delta TF > TF_O(\Delta\xi^D - \varphi_{max} + 12^\circ) + TF_E(\varphi_{max})$$

Once the values of φ, θ, and TF are determined, the solution is defined and any other parameter of interest can be generated, for example, entry position, entry speed, eccentricity, etc.

Except for details, the method of solution described above is common to other problem areas in the lunar mission. To compute approximate trajectories launched from a specified site on the Earth to arrive at the Moon, the equations governing atmosphere entry are replaced by those describing a boost and parking orbit phase. The problem of aborts to a specified landing site from midcourse points on a lunar mission (ref. 4) may also be investigated by similar methods.

RESULTS AND DISCUSSION

The method of solution described above was programmed for use on a digital computer and results were obtained for trajectories returning from the Moon during February 1966 to Edwards AFB. The Moon's time history of position was taken from the Naval Observatory ephemerides tapes. The trends obtained are determined largely by the latitude of the landing site, rather than its longitude, and are therefore typical of landing sites near 35° latitude.

For purposes of the following discussion, the time of leaving the Moon, T_M, is associated with the time of injection onto a return orbit. However, the event associated with T_M is, strictly speaking, undefined since the computations neglected the presence of the Moon.

Times of Landing and Flight Time

The times of landing for lunar launches during the first half of February 1966 are given in figure 5. The landing times fall into rather narrow bands; the lower line of each band is given by orbits with a landing azimuth of 5° from North (nearly polar orbits), and the upper line by orbits having an easterly heading at landing (minimum inclination orbits). The width of the bands varies from 3 to 8 hours during the month, but could be extended to 12 hours in every case by considering the complete range of landing azimuth from 0° to 180°. However, landing at Edwards at azimuths above 90° will require entry ranges in excess of 10,000 nautical miles for launches from the Moon over some portion of the month.

For a fixed time of departing from the Moon and a particular value of landing azimuth there may be three or four discrete landing times. For example, the following landing times occur for a lunar launch on February 8 on minimum inclination orbits:

T_M = 0 hr 8 Feb.; $A_{ZL} = 90°$

Landing time		Flight time
5.31 hr	10 Feb.	2.2211 days
5.24 hr	11 Feb.	3.2184 days
5.18 hr	12 Feb.	4.2157 days

There are three minimum inclination orbits into which the vehicle may launch at this time. The landing times correspond to the several times at which the landing site occupies the correct inertial position, as in equation (6), and for which the dynamics (eqs. (9)) can be satisfied. These times are therefore one sidereal day apart and the corresponding flight times differ by a sidereal day.

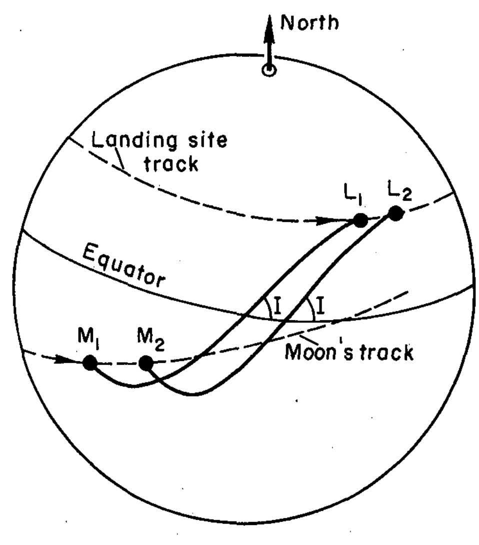
If the time of launching from the Moon is varied, there is very little change in the time of landing for return orbits of the same inclination, for example:

$A_{ZL} = 90^{\circ}$

Lunar launch time		Landing time		Flight time
0 hr	2 Feb.	16.90 hr	6 Feb.	4.7042 days
0 hr	3 Feb.	17.96 hr	6 Feb.	3.7485 days
0 hr	4 Feb.	19.06 hr	6 Feb.	2.7940 days
0 hr	5 Feb.	20.13 hr	6 Feb.	1.8389 days

A change in landing time of 3.23 hours occurs for a delay in launch time of 3 days. Thus, any launch delay may be taken up almost entirely by a corresponding reduction in flight time.

The source of this behavior is the slow angular motion of the Moon compared to the angular motion of the landing site. This can be recognized from the sketch of the celestial sphere, sketch (c). The orbital tracks of two orbits having the same inclination are shown leaving the Moon at two different times one day apart. The angular motion of the Moon in its orbit is about 13° per day so that the Moon takes a full day to move from the position M_1 to M_2. The Earth, however, rotates 15° per hour so that less than one hour is needed for the landing site to move from L_1, its required inertial position when the Moon is at M_1, to L_2. The net result is that the landing time changes about 1/30th as rapidly as the time of departing from the Moon. This result can also be derived mathematically and a general formula is reported in appendix C.



Sketch (c).- Celestial sphere.

There is, therefore, no launch-time problem for departing from the Moon to return to a specified landing site and launch can take place at any time provided the required variation in flight time is acceptable. Figure 6 shows the flight time for launches from the Moon during the first half of February into minimum inclination orbits. Launch at any time requires that flight time variations up to one day be acceptable, but it is possible to choose the one day period of variation arbitrarily (e.g., 2.5 to 3.5 day orbits). In this case, if the planned launch time required a 3.5-day orbit, then any launch delay would be taken up by a corresponding reduction in flight time, until, after a delay of one day, the flight time was reduced to 2.5 days. During this period, the corresponding landing time would remain almost fixed and the vehicle would still arrive at the landing site at the same time. Any further delay in launch, however, would return the required flight time to 3.5 days and the corresponding landing time would change by one day by passing to the next curve in figure 6. In this way, it is possible to operate over the entire month with a flight time variation of no more than one day.

If a single value of flight time is required, it is possible to launch only once each day. For example, if the required flight time is three days, then launch can take place only at about 4 a.m., Feb. 3, 4, 5, etc., for minimum inclination return orbits. If it is necessary that the launch occur when the Moon is in view of some particular station on Earth, then launch would be restricted to a short period of the day and the return flight time would be specified from figure 6.

Over the period of a month the possible landing times occur during a short period once each day, so that a requirement that the vehicle land at a particular time of day will restrict the time of the month during which return from the Moon may take place. The time of day at landing for the month of February is given in figure 7. Curves for minimum and near-polar inclination are given; intermediate inclinations give intermediate curves in the shaded area. For example, if it is required to land at 6 a.m. on a minimum inclination orbit, then the time of lunar launch is restricted to noon on Feb. 21.

The variation of flight time with landing azimuth is given in figure 8 for several launch times during the month. Alternatively, the difference in flight time between minimum inclination and near-polar orbits may be taken directly from figure 7 as the difference in times of landing, which varies from 3 to 8 hours during the month. In general, the required flight time increases with landing azimuth; that is, it is less for polar orbits of zero landing azimuth than for minimum inclination orbits.

The entry speed varies with the flight time and the distance to the Moon. Figure 4(a) gives the vacuum perigee speed versus flight time for the minimum and maximum values of the lunar distance. For any given distance to the Moon, lower flight times correspond to higher energy trajectories. Polar return orbits have slightly greater energies than minimum inclination orbits and will therefore have slightly higher speeds both at entry and at departure from the Moon.

The higher speeds required on polar orbits at departure from the Moon are of some interest since they will occasion an increase in fuel requirements for injection into the return orbit. To illustrate the size of the penalty involved, some precision trajectories were obtained which depart from an altitude of 266 km above the Moon at a point behind the Moon on the Earth-Moon line. The speed with respect to the Moon at departure for various inclination angles is given in the right-hand column of the following table:

T_M = 12 hr; 1 February 1966

I, deg	TF, days	V_P, km/sec	V_M (Lunar injection speed), km/sec
34.9	2.506	11.0834	2.5296
39.5	2.402	11.0913	2.5678
51.2	2.310	11.0958	2.6146
65.0	2.242	11.1063	2.6629
85.0	2.174	11.1141	2.7229

The nearly polar orbit requires a departure speed 193.3 m/sec greater than that of the minimum inclination orbit. The differences in flight time and vacuum perigee speed are 8 hours and 30.7 m/sec, respectively. However, only a part of the difference in departure speed is due to the higher energy of the polar orbit. The major part is due to kinematic considerations. Trajectories with the same flight time have the same energy with respect to the Earth whatever the inclination. Their energy with respect to the Moon varies, however, with inclination because of the motion of the Moon with respect to the Earth. Vehicles which return in the Moon's orbital plane may take advantage of the Moon's velocity in order to obtain the required energy with respect to the Earth, but vehicles returning in orbits that are polar to the Moon's orbital plane must remove this component of velocity. The result is that among orbits with respect to the Earth that have the same energy, the polar orbits require a higher velocity with respect to the Moon and, consequently, greater maneuver fuel costs in the vicinity of the Moon than orbits that lie closer to the Moon's orbital plane.

Entry Range Requirements

The entry range requirements for minimum and near-polar inclinations are given in figures 9(a) and 9(b) for the entire month of February. Each curve in these figures corresponds to one of the fixed azimuth lines of figure 5 and therefore to a very nearly fixed time of landing. Flight times of 4 days and 2 days are associated with the left and right end points, respectively, of each curve in the figures.

Both figures show a general monthly variation in the required entry range which parallels the variation in lunar declination; that is, the entry range decreases during the first half of the month when the Moon's declination decreases (cf. fig. 2(b)), and increases during the second half of the month with the lunar declination. This behavior stems from the geometry of the problem; the total geocentric angle, $\Delta\xi$, from Moon to landing site increases with the declination of the Moon. In figure 2(d) in the region of possible lunar declinations, from -28.5° to $+28.5^\circ$, it is seen that the angle, $\Delta\xi$, increases with the declination for every value of landing azimuth. The total geocentric angle is made up of the orbital true anomaly and the entry range angle, as in equation (9b);

$$\Delta\xi^D = \theta - 12^\circ + \varphi$$

Consider next trajectories of fixed flight time and, hence, of fixed true anomaly. In this case, as D_M increases the angle, $\Delta\xi$, increases, and, by the above equation, the entry range angle, φ, must also increase.

This argument is not entirely correct since, for a fixed landing azimuth, the return flight time and corresponding true anomaly have a daily variation as in figure 6. But, since it is possible to have a fixed value of flight time once each day, the argument is correct from day to day and therefore a general trend must occur in which the required entry range varies with the lunar declination as observed in figures 9(a) and 9(b).

The variation of entry range with lunar declination also depends on the latitude of the landing site. The trend noted above is correct for landing sites in the northern hemisphere above the maximum declination of the Moon and therefore applies to typical sites in the Southwest United States. For landing sites in the southern hemisphere below the minimum declination of the Moon the trend is opposite; that is, the angle, $\Delta\xi$, decreases with increasing lunar declination and the entry range is less at northerly declinations of the Moon than at southerly declinations. Such a case is illustrated in figure 10(a) which gives the entry range requirements for minimum inclination returns to Woomera, Australia (-31.4° lat., 136.9° long.).

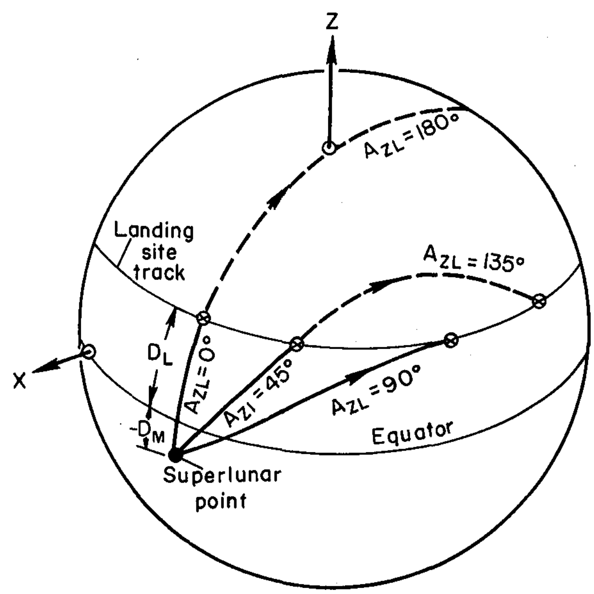
Figure 9(a) indicates that a return to Edwards AFB at any time of the month with minimum inclination orbits requires entry ranges from 3,000 to 9,500 nautical miles. If the entry range capability of the vehicle is much less than 9,500 nautical miles there are several alternatives. The return from the Moon could be restricted to that portion of the month for which the entry range requirements are within the capability of the vehicle; in the case of landing sites in the Southwest United States, this would correspond to times when the Moon is at negative declinations. However, operation over the entire month can be restored by having two landing sites; a second site at southern latitudes would have low entry range requirements for positive lunar declinations (fig. 10(a)). A second landing site could also be placed part way along the entry track for minimum inclination returns to Edwards AFB. Such a case is illustrated in figure 10(b) for landings at 20.2° latitude, 189.8° east longitude, in the area west of Hawaii about 3,200 nautical miles from Edwards. The entry range requirements are about 3,000 miles less than for Edwards but there is a portion of the month when the Moon is near or below -20.2° latitude during which no return solutions occur.

It is evident that the use of a fixed landing site requires a varying entry range for operations at any time of the month. The use of two or more fixed landing sites allows the amount of variation to be reduced. If this procedure is extended to the limit, that is, if a mobile landing area is used, then the required entry range can be held fixed. However, an investigation of this alternative method of landing area restriction is beyond the scope of the present work.

Another alternative is to use polar return trajectories for which the entry range requirements are substantially less than those for minimum inclination return orbits. The requirements for nearly polar return to Edwards are given in figure 9(b), which indicates an entry range capability of 1,500 to 5,000 nautical miles is necessary, or, roughly, half of the entry range capability required for minimum inclination orbits; for example,

Time of launch	Declination of the Moon	Inclination	Entry range, nautical miles
0 hr 3 Feb.	26.1°	34.9° 85.9°	9500 5500
0 hr 9 Feb.	1.0°	34.9° 85.9°	6550 3275
0 hr 16 Feb.	-26.2°	34.9° 85.9°	3500 1680

A general trend is illustrated by the above table; entry range requirements increase with increasing landing azimuth. The reasons for this behavior can be determined from sketch (d) which shows the superlunar point on the celestial sphere together with the tracks of return trajectories having various landing azimuths from 0° to 180°. The superlunar point is 180° from the direction of the Moon at the time of departure and has a declination of $-D_M$. Once the time of departure from the Moon is specified, the superlunar point is fixed inertially and all the return trajectories must pass through it. The landing site track is shown in the sketch and its intersections with the tracks of the various return orbits locate the landing site inertially at the time of landing. It is evident from this sketch that the total geocentric angle from the Moon to landing increases with landing azimuth, from a value of $\pi + D_M + D_L$ to $2\pi + D_M - D_L$ as landing azimuth varies from 0° to 180°. The total change is $\pi - 2D_L$. If the return flight time and orbital true anomaly were fixed, the increase in total geocentric angle with landing azimuth would require an equal increase in entry range angle by equation (9a).



Sketch (d).- Effects of landing azimuth.

Although the return flight time is not fixed, it can be shown that the change in true anomaly is comparatively small, with the result that the increase in total geocentric angle does, in fact, require an almost equal increase in entry range angle. It is evident from the sketch that the landing site rotates halfway around the celestial sphere as the landing azimuth is increased from 0° to 180° and the time of landing must therefore increase by half a sidereal day with an equal increase in return flight time. This increase in flight time is effected almost entirely by an increase in orbital true anomaly, but an increase in flight time of half a day only requires an increase in true anomaly of about 5° for flight times in the range of interest (cf. fig. 4(b)).

Two points are noted from this discussion; first, the required entry range increases with landing azimuth; it is a minimum for polar orbits with a due-north heading at landing and a maximum for polar orbits with a due-south heading at landing. In the case of landings at Edwards AFB, the necessary entry range capability for operations over the entire month is under 10,000 nautical miles for landings at azimuths up to 90°, that is, for approaches from the southwest. For approaches from the northwest the entry range requirements will exceed 10,000 nautical miles over some portion of the month.

Second, the return flight time increases locally with landing azimuth, being half a day longer for polar orbits with a due-south heading at landing than for orbits with a due-north heading at landing.

The variation of entry range and flight time with landing azimuth also depends on the landing site latitude. The trends noted in the preceding discussion apply to landing sites at latitudes greater than the declination of the Moon. In the case of Edwards AFB and other sites in the Southwest United States above the maximum declination of the Moon, these trends occur at all times of the month. The opposite trends would obtain for landing sites in the southern hemisphere at latitudes below the minimum declination of the Moon.

The longitude and latitude of the atmosphere entry points for all the solutions departing the Moon at 0 hr, 3 Feb. are given in figure 11(a). There are three lines of solutions corresponding to landing times one day apart. Entry locations for departures at several times of the month are given in figure 11(b) where only the solutions having 2.5 to 3.5 day flight times are given. The two constant inclination lines in this figure give the locus of entry points for minimum and polar inclinations. These lines are very nearly the track of the vehicle over the Earth during the final portion of the return trajectory for all orbits having the corresponding inclinations.

CONCLUSIONS

An analysis has been given for computing trajectories returning to a specified landing site, assuming a two-body description of orbital motion and an approximate flight time - entry range relation for the entry phase. The method of solution was programmed for an IBM 7090 digital computer and general results covering returns from the Moon to Edwards AFB for the month of February 1966 were obtained. Generally, the trends reported here will be the same for other time periods and landing sites in the Southwest United States.

The trends indicated are as follows:

1. For a specified orbital inclination angle the times of landing are restricted to a short period once each day.

2. The return flight time is a function of the time of leaving the Moon. For a specified orbital inclination angle, launch into an orbit with a specified flight time can take place once a day. Launch at any time requires a one-day variation in flight time.

3. Return polar orbits are of substantially higher energy with respect to the Moon than minimum inclination orbits.

4. It is possible to return from the Moon to a specified landing site at any time with any desired inclination angle greater than the minimum possible value, provided the necessary entry range capability is available.

5. Entry range requirements vary over the month, showing a close relation to the lunar declination. For return to Edwards AFB, the entry range requirements are 3,000 to 9,500 nautical miles for minimum inclination orbits and 1,500 to 5,000 nautical miles for polar orbits.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Dec. 10, 1962

APPENDIX A

LANDING TIMES OF INTEREST

If a trajectory is to leave the Moon at T_M and the flight times of interest are limited to the range of values

$$\tau_{min} < TF < \tau_{max} \tag{A1}$$

and if the times at which the landing site occupies the correct inertial position are given by equation (6) of the text as

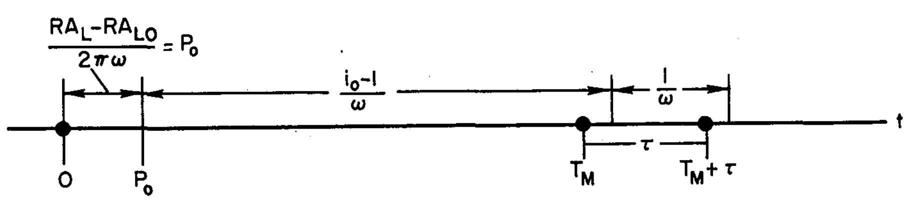
$$T_L(i) = \frac{RA_L - RA_{LO}}{2\pi\omega} + \frac{i}{\omega}, \quad i = \ldots -2, -1, 0, 1, 2, \ldots \tag{6}$$

then it is necessary to find the values of i which satisfy the flight time limits of (A1).

The flight times corresponding to the required landing times are:

$$TF(i) = T_L(i) - T_M \tag{7}$$

Consider the following sketch of the time scale (mean solar time), showing the reference origin and the points T_M and $T_M + \tau$. Between P_O and $T_M + \tau$ there are $(i_O - 1)$ complete sidereal days (or $(i_O - 1)/\omega$ mean solar days) plus a fraction of another sidereal day. The number of sidereal days from P_O to $T_M + \tau$ can be computed from



Sketch (e).- Time scale.

$$t_{sidereal} = \omega(T_M + \tau - P_O) \tag{A2}$$

and hence the integral number of sidereal days from P_O to $T_M + \tau$ is

$$i_O - 1 = INT\left[\omega(T_M + \tau - P_O)\right] \tag{A3}$$

where $INT(y)$ truncates y to the next lower integer value as explained in the text. Finally, the minimum value of i which gives a flight time in the range of interest corresponds to the lowest value of $T_L(i)$ which is greater than $T_M + \tau_{min}$; whence

$$i_{min} = INT\left[\omega(T_M + \tau_{min}) - \frac{RA_L - RA_{LO}}{2\pi\omega}\right] + 1 \tag{A4}$$

Similarly, the maximum value of i which gives a flight time in the range of interest corresponds to the highest value of $T_L(i)$ which is less than $T_M + \tau_{max}$; or

$$i_{max} = \text{INT}\left[\omega(T_M + \tau_{min}) - \frac{RA_L - RA_{LO}}{2\pi\omega}\right] \tag{A5}$$

The landing times of interest are those given by equation (6) with values of i in the range.

$$i_{min} \leq i \leq i_{max} \tag{A6}$$

$$i \text{ an integer}$$

APPENDIX B

ORBITAL FLIGHT TIME FORMULAS

A subroutine which computes orbital flight time was necessary in the computer program for point-return trajectories. The most convenient subroutine is one that accepts values of perigee radius, departure point radius, and true anomaly, and computes the orbital flight time from the departure point to vacuum perigee. The perigee radius for the return trajectories is fixed at 6,430 km and the departure point radius is, for trajectories returning from the Moon, the distance to the Moon. This distance varies from about 356,000 km to 407,000 km as shown in figure 4(c) for February 1966. Although the usual flight time formulas can always be used, a more convenient set, as derived below, was used with the computer program.

The solution orbit will be one of the possible orbits which have the required value of R_P and pass through a point of radius, R_M. For Keplerian orbits

$$\left.\begin{aligned} R_M &= \frac{P}{1 + e\cos\theta_M} \\ R_P &= \frac{P}{1 + e} \end{aligned}\right\} \tag{B1}$$

Define the parameter

$$\rho = \frac{R_P}{R_M} \tag{B2}$$

This parameter has a value from 0.0158 to 0.0180 for orbits leaving the Moon. It follows, after dividing the equations of (B1), that all the possible orbits are described in the formula

$$\rho = \frac{1 + e\cos\theta_M}{1 + e}$$

or

$$e = \frac{1 - \rho}{\rho - \cos\theta_M} \tag{B3}$$

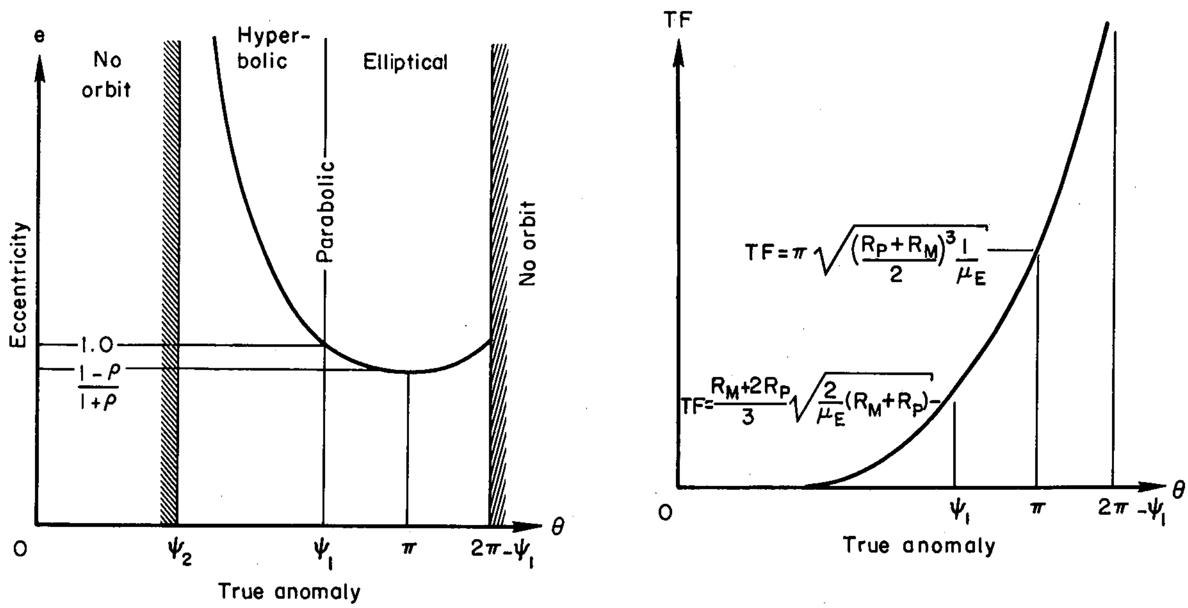
Some points of interest given by equation (B3) are:

$$\left.\begin{aligned} &\text{i) minimum eccentricity} \\ &\qquad e_{min} = \frac{1 - \rho}{1 + \rho} \qquad \text{at} \quad \theta_M = \pi \\ &\text{ii) parabolic orbits} \\ &\qquad e = 1.0 \qquad \text{at} \quad \theta_M = \cos^{-1}(2\rho - 1) \equiv \Psi_1 \\ &\text{iii) minimum true anomaly} \\ &\qquad e \to \infty \qquad \text{at} \quad \theta_M = \cos^{-1}(\rho) \equiv \Psi_2 \end{aligned}\right\} \tag{B4}$$

The angles Ψ_1 and Ψ_2 are taken between zero and π. The points given in (B4) give the following restrictions on some of the orbital parameters for orbits passing through R_M and having the required value of R_P:

$$\left.\begin{array}{ll} \frac{1-\rho}{1+\rho} \leq e < \infty & \\ \Psi_2 \leq \theta_M < \Psi_1 & \text{hyperbolic orbits } (e > 1) \\ \theta_M = \Psi_1 & \text{parabolic orbits } (e = 1) \\ \Psi_1 < \theta_M < 2\pi - \Psi_1 & \text{elliptic orbits } (e < 1) \\ \left.\begin{array}{l} \theta_M < \Psi_2 \\ \theta_M \geq 2\pi - \Psi_1 \end{array}\right\} & \text{no orbits possible} \end{array}\right\} \tag{B5}$$

Restrictions on other orbital parameters can be derived from (B5) and other orbital relations. The restrictions of (B5) are illustrated in the sketch below and a plot of these same parameters is given in figure 4(b).



Sketch (f).

The eccentric or hyperbolic anomaly at R_M can be computed as a function of the true anomaly.

$$\frac{1}{\rho}(\cos\theta + 1 - \rho) = \begin{cases} \cosh F & \text{for } \Psi_2 \leq \theta \leq \Psi_1 \quad \text{hyperbolic orbits} \\ \cos E & \text{for } \Psi_1 \leq \theta \leq \pi \quad \text{elliptic orbits} \end{cases} \tag{B6}$$

$$0 \leq F,\ E \leq \pi$$

Although equation (B6) is correct for the entire range of elliptic orbits ($\Psi_1 < \theta < 2\pi - \Psi_1$), the range of true anomaly discussed here is limited to values below π since higher values correspond to orbits on which the vehicle would leave the Moon headed away from the Earth.

A formula for flight time can be given in terms of any three independent orbital parameters by substitutions of the appropriate orbital relations into Kepler's equation for flight time. In the present case, the desired result is:

$$TF = \left(\sqrt{\frac{R_M{}^3}{\mu_E}(\rho - \cos\theta)}\right)\beta \tag{B7}$$

where β is computed from the following equations:

$$\beta = \begin{cases} \dfrac{E(1 - \rho\cos E) - (1 - \rho)\sin E}{(1 - \cos E)^{3/2}} & \text{elliptic region} \\[2ex] \dfrac{F(1 - \rho\cosh F) - (1 - \rho)\sinh F}{(\cosh F - 1)^{3/2}} & \text{hyperbolic region} \end{cases} \tag{B8}$$

For parabolic orbits, where $e = 1$ and $E, F = 0$, both expressions for β have a singularity. A nonsingular expression for β in the region near the parabolic condition is readily obtained by expansion of the trigonometric functions to give a series expression for β,

$$\beta = \sqrt{2}\,\frac{\frac{1}{3}(1 + 2\rho) + 2\sum_{n=1}^{\infty}\frac{1 + 2(n + 1)\rho}{(2n + 3)!}x^n}{\left[1 + 2\sum_{n=1}^{\infty}\frac{x^n}{(2n + 1)!}\right]^{3/2}} \tag{B9}$$

$$x = \begin{cases} -E^2 & \theta \geq \Psi_1 \\ F^2 & \theta \leq \Psi_1 \end{cases}$$

For computation, (B9) may be used for all orbits provided a sufficient number of terms is taken in the series to give the required accuracy.

In particular, for parabolic orbits, equations (B4), (B7), and (B9) give

$$TF = \frac{R_M + 2R_P}{3}\sqrt{\frac{2}{\mu_E}(R_M - R_P)}$$

The flight time subroutine accepts values of R_P, R_M, and θ and uses equations (B2) and (B6) with (B7), (B8), or (B9) to compute the flight time.

APPENDIX C

VARIATION OF LANDING TIME WITH TIME OF DEPARTING THE MOON FOR FIXED INCLINATION

From equations (5) and (6) of the text it follows that

$$\frac{dT_L}{dT_M} = \frac{1}{2\pi\omega}\left(\frac{dRA_M}{dT_M} + \frac{d\Psi_L}{dT_M} - \frac{d\Psi_M}{dT_M}\right) \tag{C1}$$

where the derivatives are to be computed for fixed landing azimuth and, hence, fixed inclination. Equations (2) and (3) of the text may be used to derive:

$$\frac{d\Psi_L}{dT_M} = 0$$

$$\frac{d\Psi_M}{dT_M} = m \frac{\cos I}{\cos^2 D_M} \frac{\frac{dD_M}{dT_M}}{\sqrt{1 - \frac{\cos^2 I}{\cos^2 D_M}}} \tag{C2}$$

where $m = \pm 1$, depending on which of two possible heading angles at the Moon is to be used.

$$m = +1 \quad \text{for} \quad 0 < A_{ZM} < \pi/2$$

$$m = -1 \quad \text{for} \quad \pi/2 < A_{ZM} < \pi$$

The Moon in its orbit is considered next; the Moon's orbit is assumed to lie in a single plane and the general formulas of equations (1) of the text are applied. Noting that the right ascension of the Moon, RA_M, is related to the equatorial angle of the Moon from its nearest ascending line of nodes, Ψ_{MM}, by the addition of a constant, then by analogy to the second equation of (C2) above,

$$\frac{dRA_M}{dT_M} = \frac{d\Psi_{MM}}{dT_M} = n \frac{\cos I_M}{\cos^2 D_M} \frac{1}{\sqrt{1 - \frac{\cos^2 I_M}{\cos^2 D_M}}} \frac{dD_M}{dT_M} \tag{C3}$$

$$n = +1 \quad \text{for } D_M \text{ increasing}$$

$$n = -1 \quad \text{for } D_M \text{ decreasing}$$

$$I_M = \text{inclination of Moon's orbital plane}$$

It is convenient to introduce the orbital angular velocity of the Moon, $\dot{\theta}_M$, in the above expressions in order to simplify the results, and because $\dot{\theta}_M$ is nearly constant. It varies about 2°/day around a mean value of approximately 13°/day over the lunar month. The following relations among the angles describing the Moon's orbit occur from spherical trigonometry (eqs. (1d) and (1e) of the text):

$$\sin D_M = \sin I_M \sin \theta_M$$

$$\cos \theta_M = \cos \Psi_{MM} \cos D_M \tag{C4}$$

where θ_M is measured from an ascending node. Together with equations (1) these provide the result

$$\frac{dD_M}{dT_M} = n \sqrt{1 - \frac{\cos^2 I_M}{\cos^2 D_M}}\, \dot{\theta}_M \tag{C5}$$

The rate of change of the Moon's declination is a maximum ($n\dot{\theta}_M \sin I_M$) when the Moon is at a node, and is zero at the extreme declinations, $\pm I_M$. Equation (C3) becomes

$$\frac{dRA_M}{dT_M} = \frac{\cos I_M}{\cos^2 D_M} \dot{\theta}_M \tag{C6}$$

This has a minimum value at a node where ($\dot{RA}_M = \dot{\theta}_M \cos I_M$) and a maximum value at the extreme declinations ($\dot{RA}_M = \dot{\theta}_M \sec I_M$).

Finally, the combination of (C2), (C5), and (C6) in equation (C1) gives the result

$$\frac{dT_L}{dT_M} = \frac{\dot{\theta}_M}{2\pi\omega} \frac{\cos I_M}{\cos^2 D_M} \left[1 - mn \sqrt{\frac{(\cos^2 D_M/\cos^2 I_M) - 1}{(\cos^2 D_M/\cos^2 I) - 1}} \right] \tag{C7}$$

where m and n are ±1 according to the circumstances described above.

Equation (C7) may be written in the following alternative form, noting (C6):

$$\frac{dT_L}{dT_M} = \frac{\dot{RA}_M}{2\pi\omega} \left[1 - mn \sqrt{\frac{(\cos^2 D_M/\cos^2 I_M) - 1}{(\cos^2 D_M/\cos^2 I) - 1}} \right] \tag{C8}$$

Since $2\pi\omega$ is the angular velocity of the Earth then (C8) gives the rate of change of landing time with departure time as the ratio of the equatorial angular velocity of the Moon to the angular velocity of the Earth times the modifying factor in the brackets.

In particular, for polar orbits $(I \to \pi/2)$

$$\frac{dT_L}{dT_M} = \frac{R\dot{A}_M}{2\pi\omega}$$

or, stated differently,

$$\frac{dRA_L}{dT_M} = \frac{dRA_M}{dT_M}$$

This last is an expected result from simple physical considerations.

For orbits having the inclination of the Moon's orbital plane $(I = I_M)$, equation (C8) becomes

$$\text{for} \quad D_M \neq I_M \qquad \frac{dT_L}{dT_M} = \frac{R\dot{A}_M}{2\pi\omega}(1 - mn)$$

$$\text{for} \quad |D_M| = I_M \qquad \frac{dT_L}{dT_M} = \frac{1}{\cos I_M}\,\frac{\dot{\theta}_M}{2\pi\omega}$$

more generally, for $I_M < I < \pi/2$ then

$$0 < \frac{dT_L}{dT_M} < 2\left(\frac{R\dot{A}_M}{2\pi\omega}\right)$$

which has a maximum value of about 0.06. This is the result already expected from the physical considerations given in the text; if the inclination of the return orbit is fixed, the landing time changes very slowly with time of departure from the Moon.

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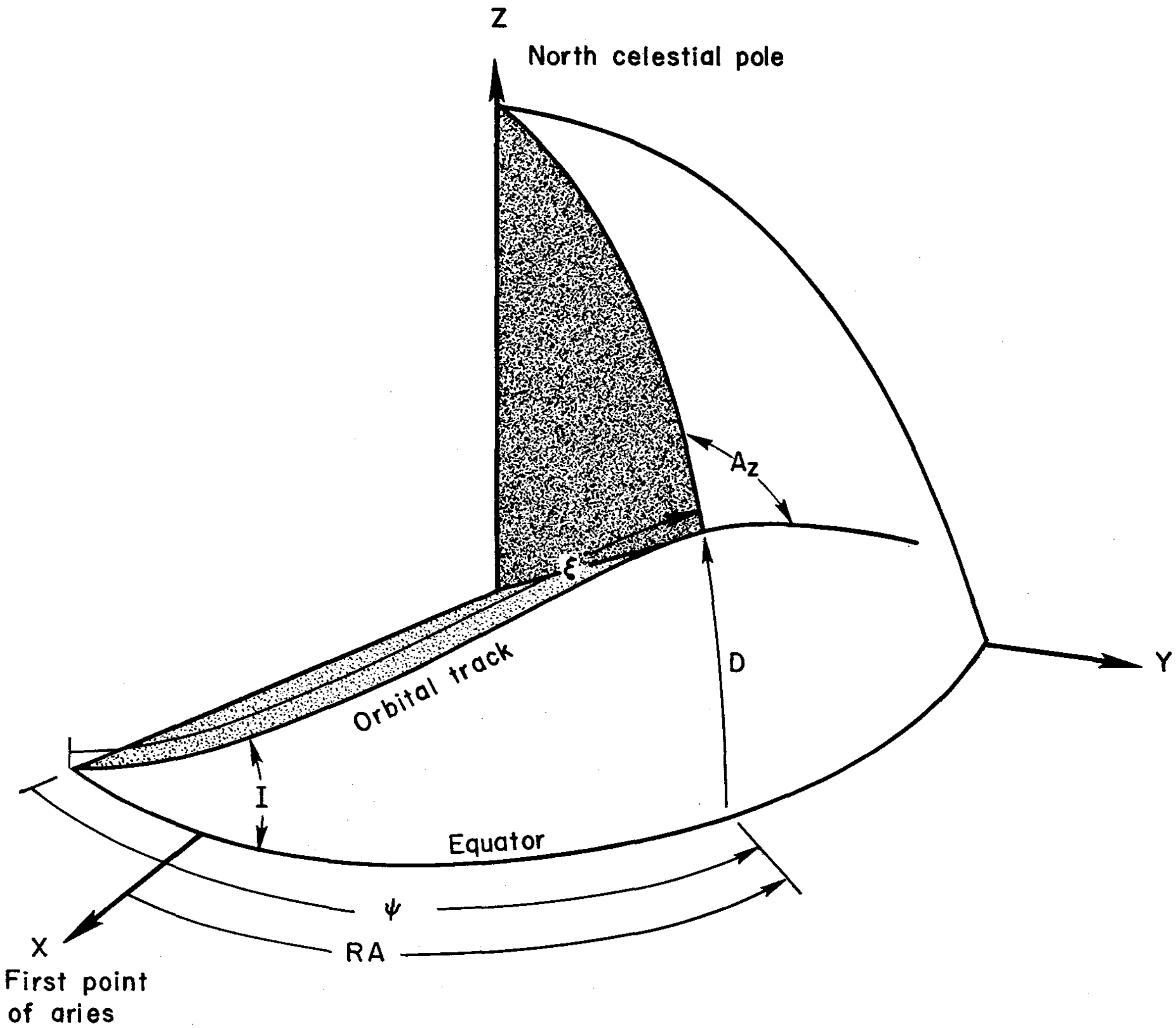
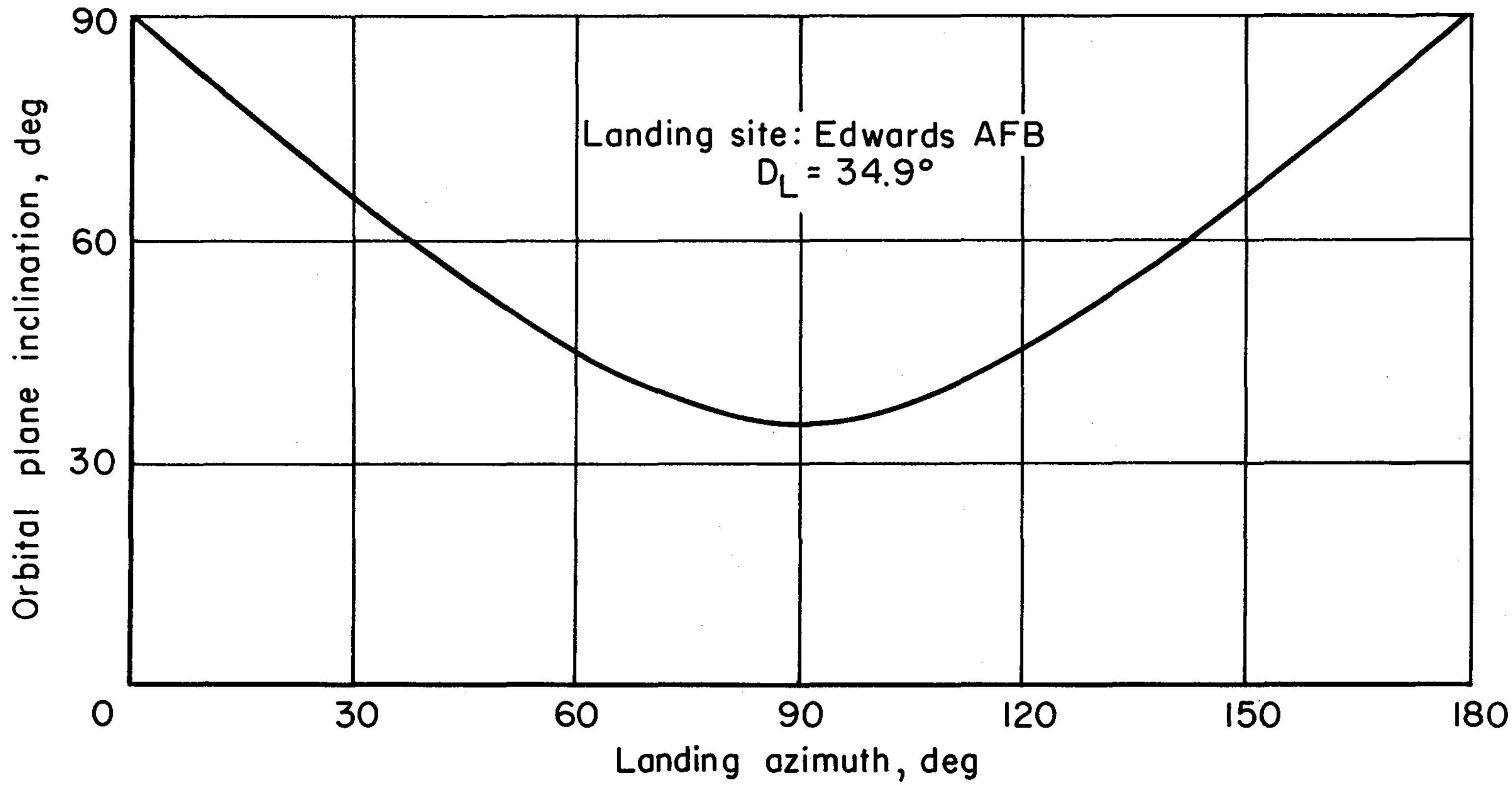
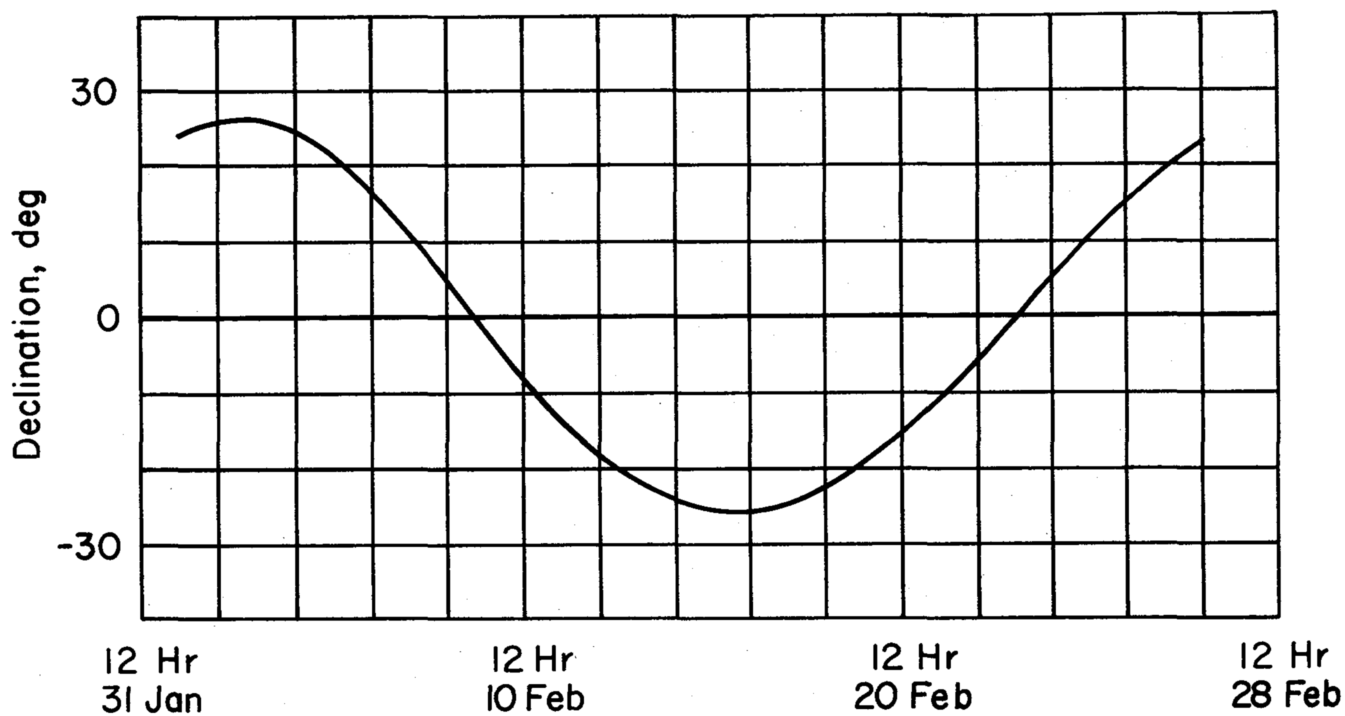


Figure 1.- Orbital track parameters.



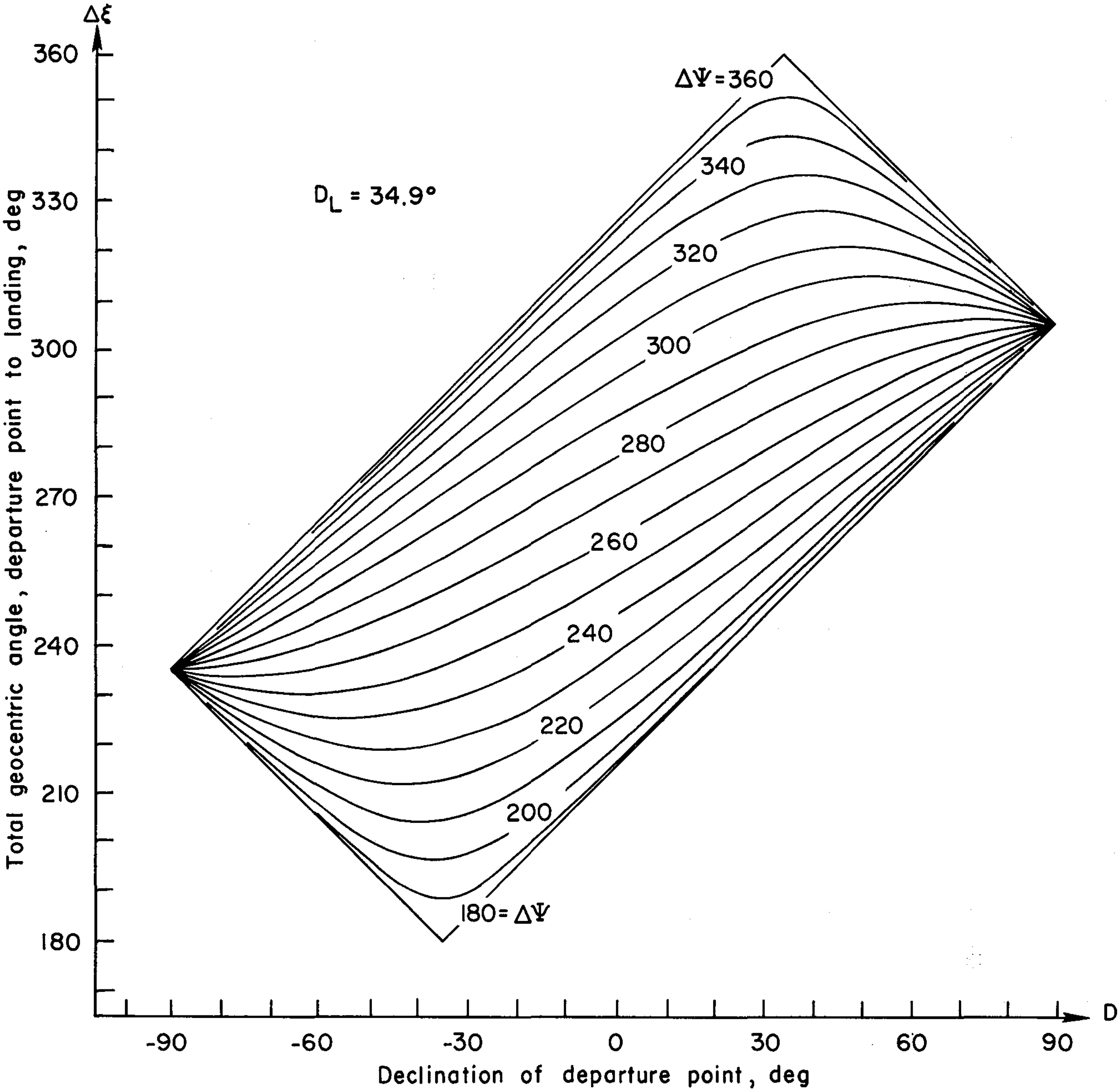
(a) Landing azimuth versus orbital plane inclination.

Figure 2.- Geometrical parameters of the orbital plane.



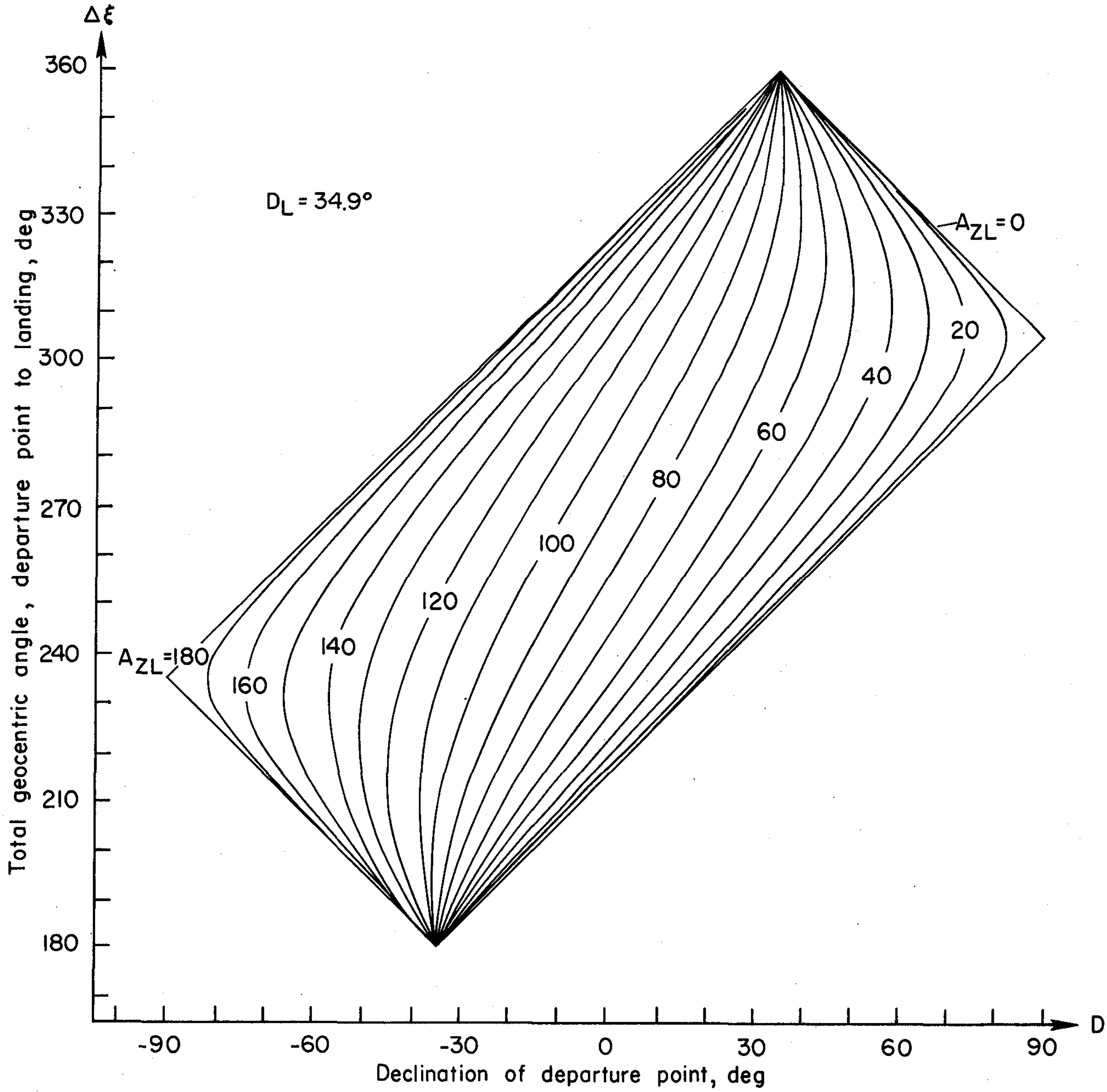
(b) Lunar declination; Feb. 1966.

Figure 2.- Continued.



(c) Lines of constant equatorial angle.

Figure 2.- Continued.



(d) Lines of constant landing azimuth.

Figure 2.- Concluded.

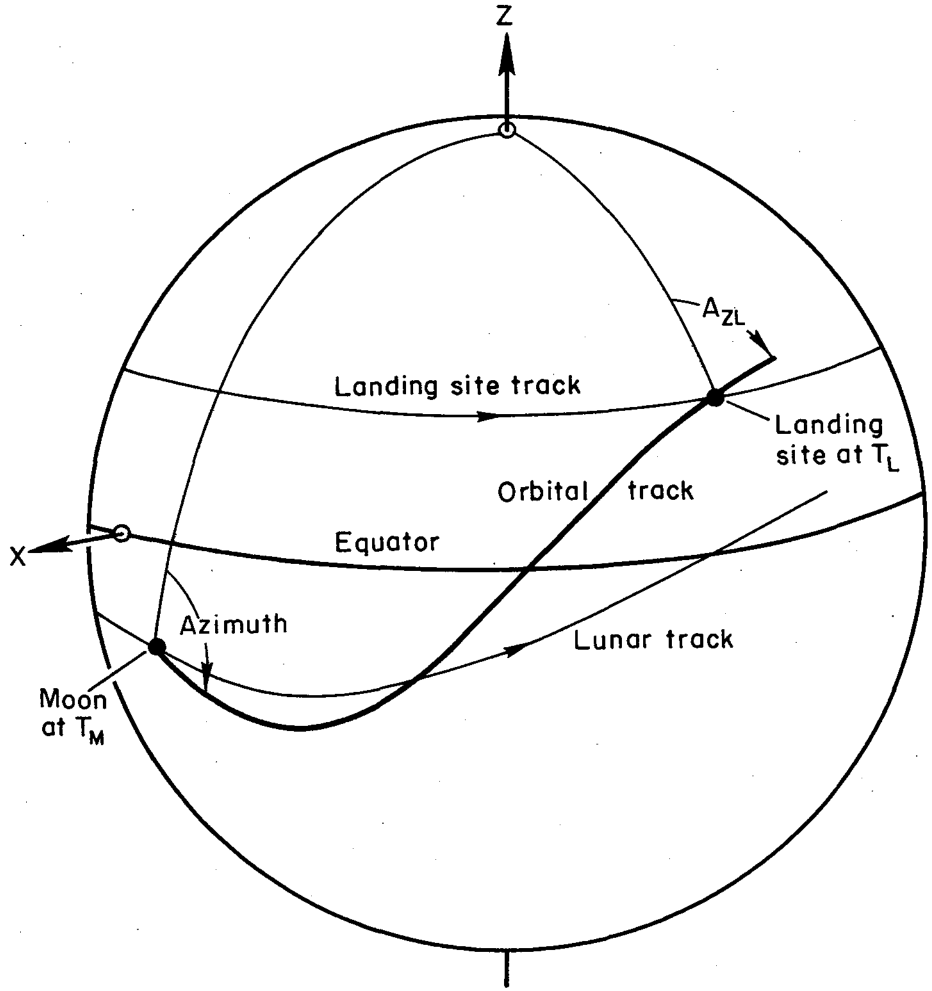
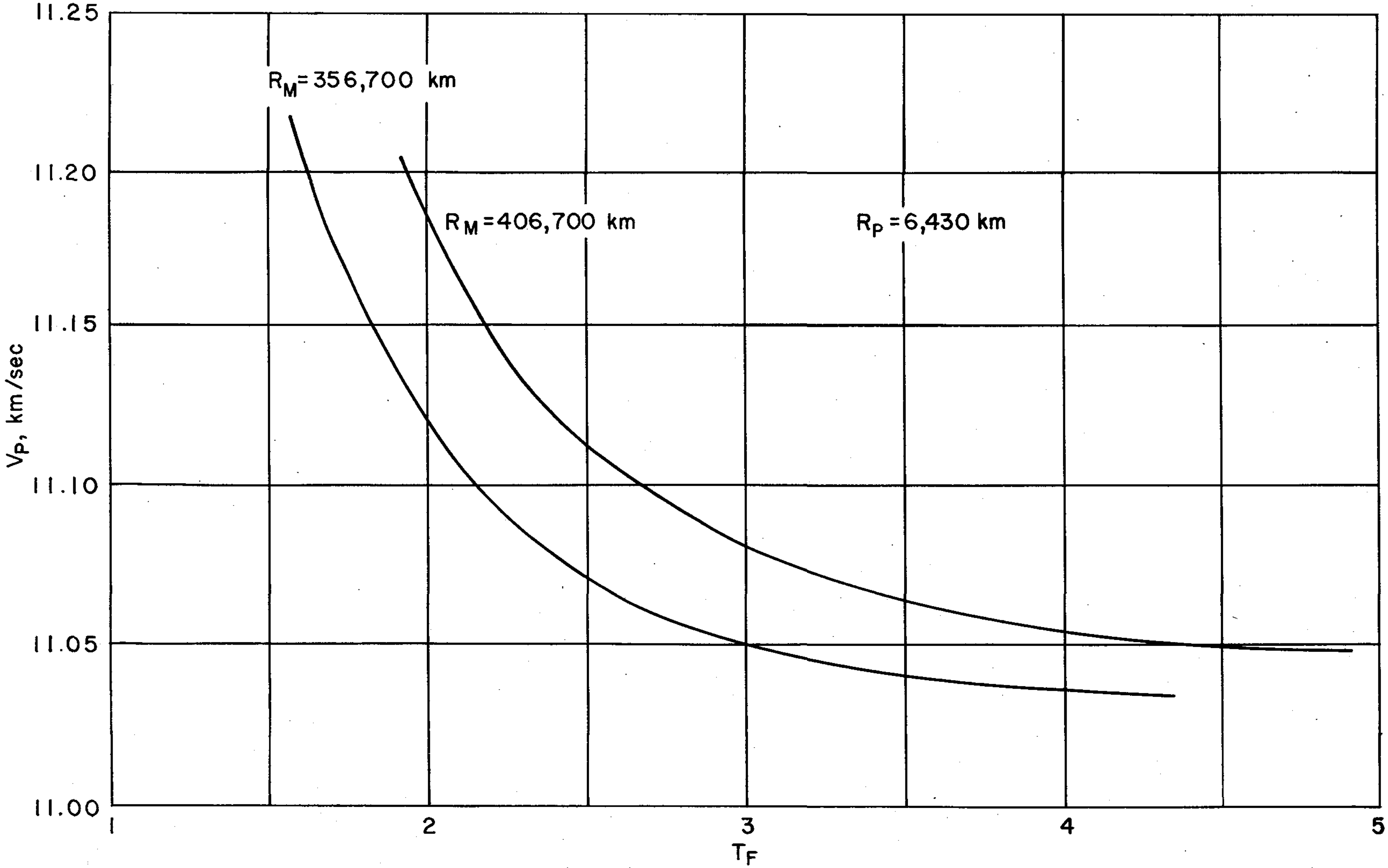
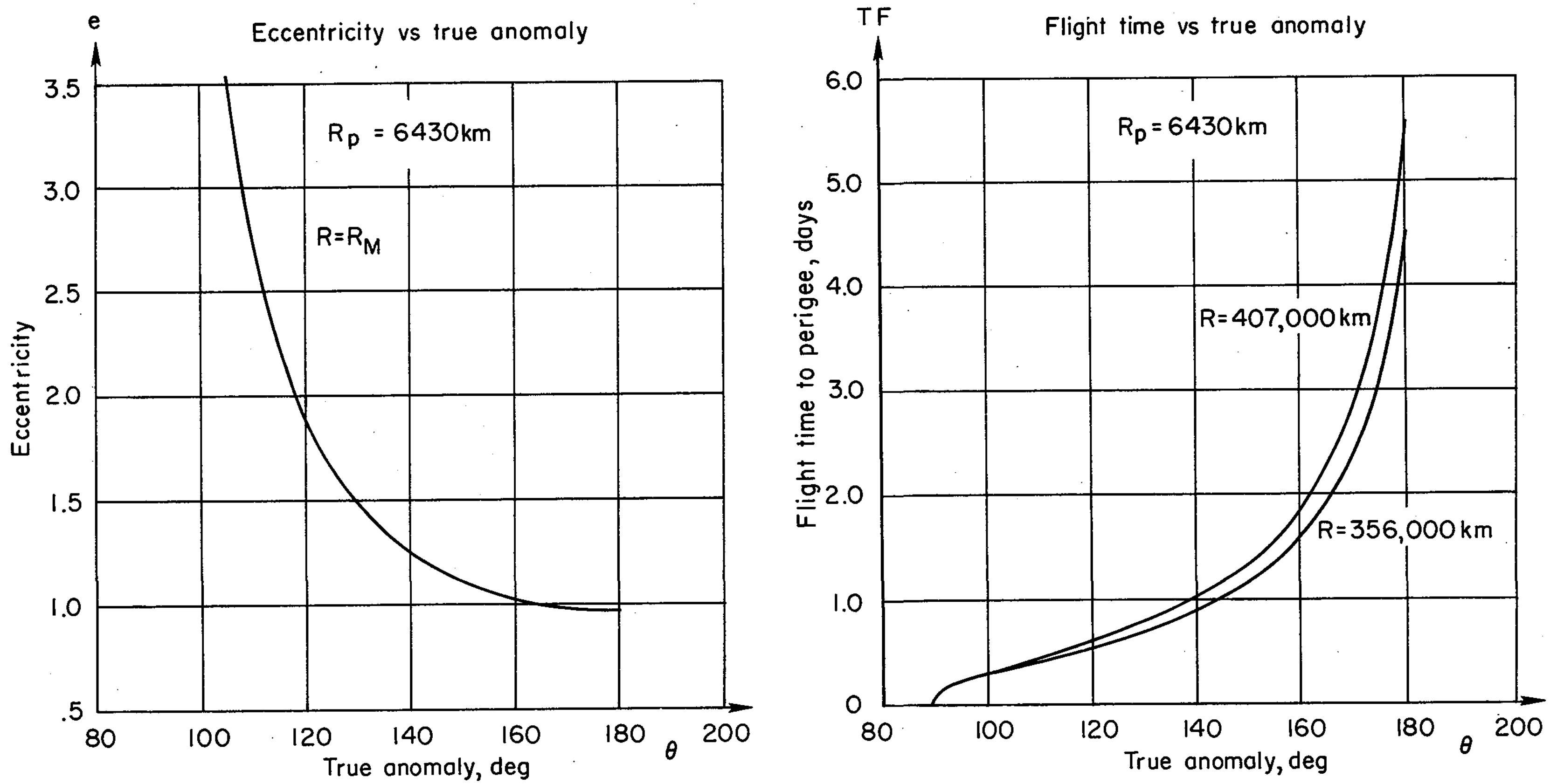


Figure 3.- Return orbit track on the celestial sphere.



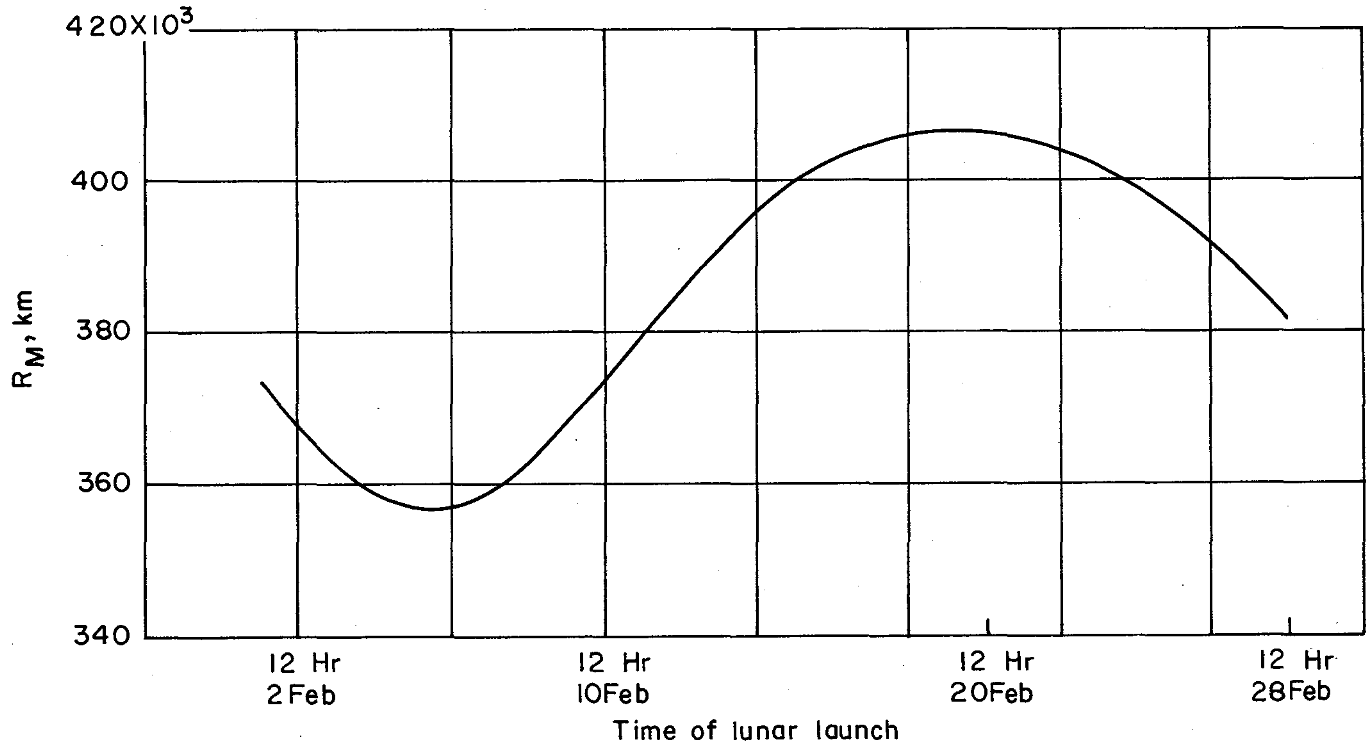
(a) Perigee speed versus flight time from the Moon to perigee.

Figure 4.- Some orbital parameters for orbits returning from the Moon.



(b) Eccentricity and flight time versus true anomaly.

Figure 4.- Continued.



(c) Lunar distance; Feb. 1966.

Figure 4.- Concluded.

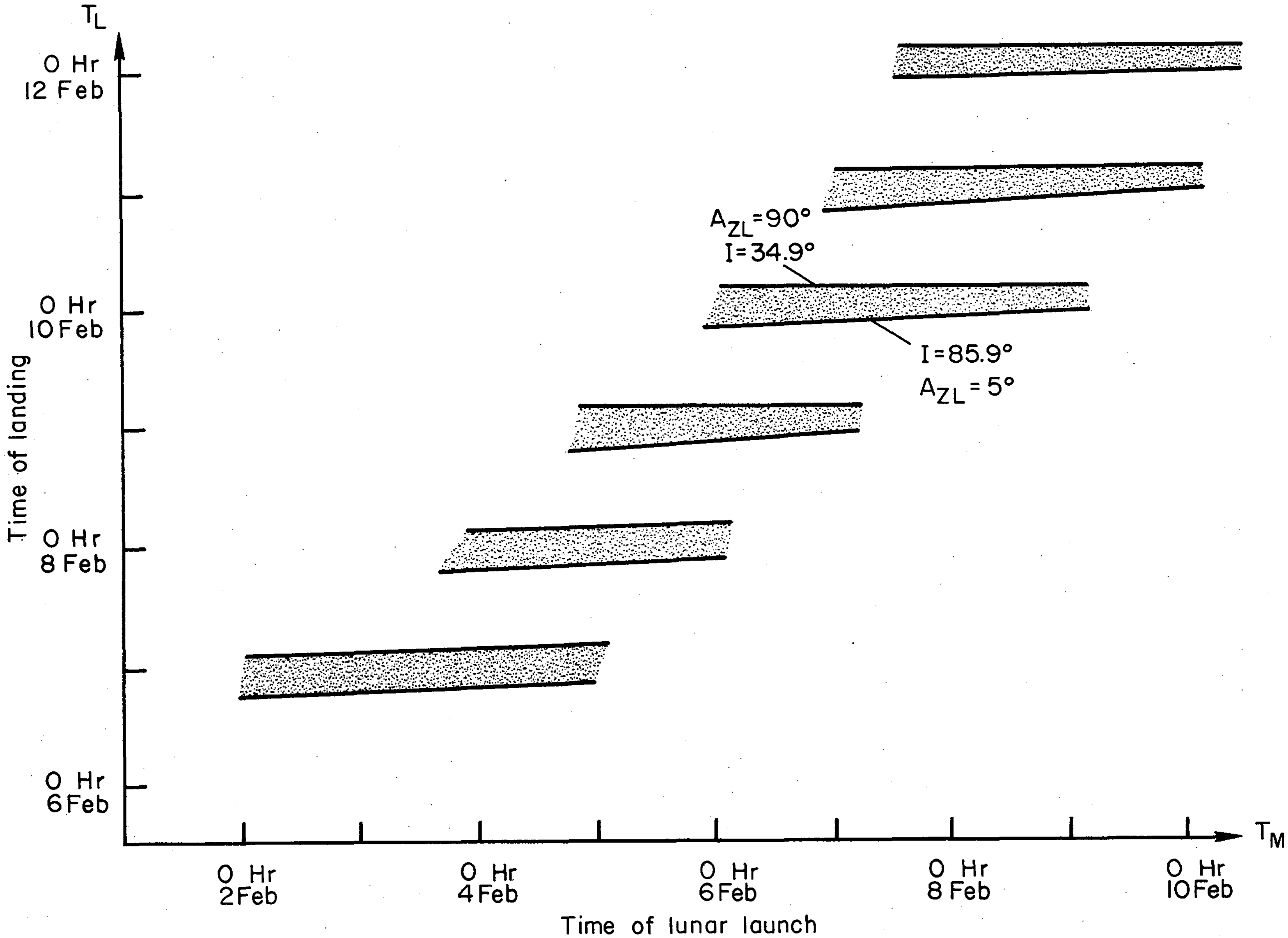


Figure 5.- Landing time at Edwards Air Force Base versus time of lunar launch, Feb. 1966.

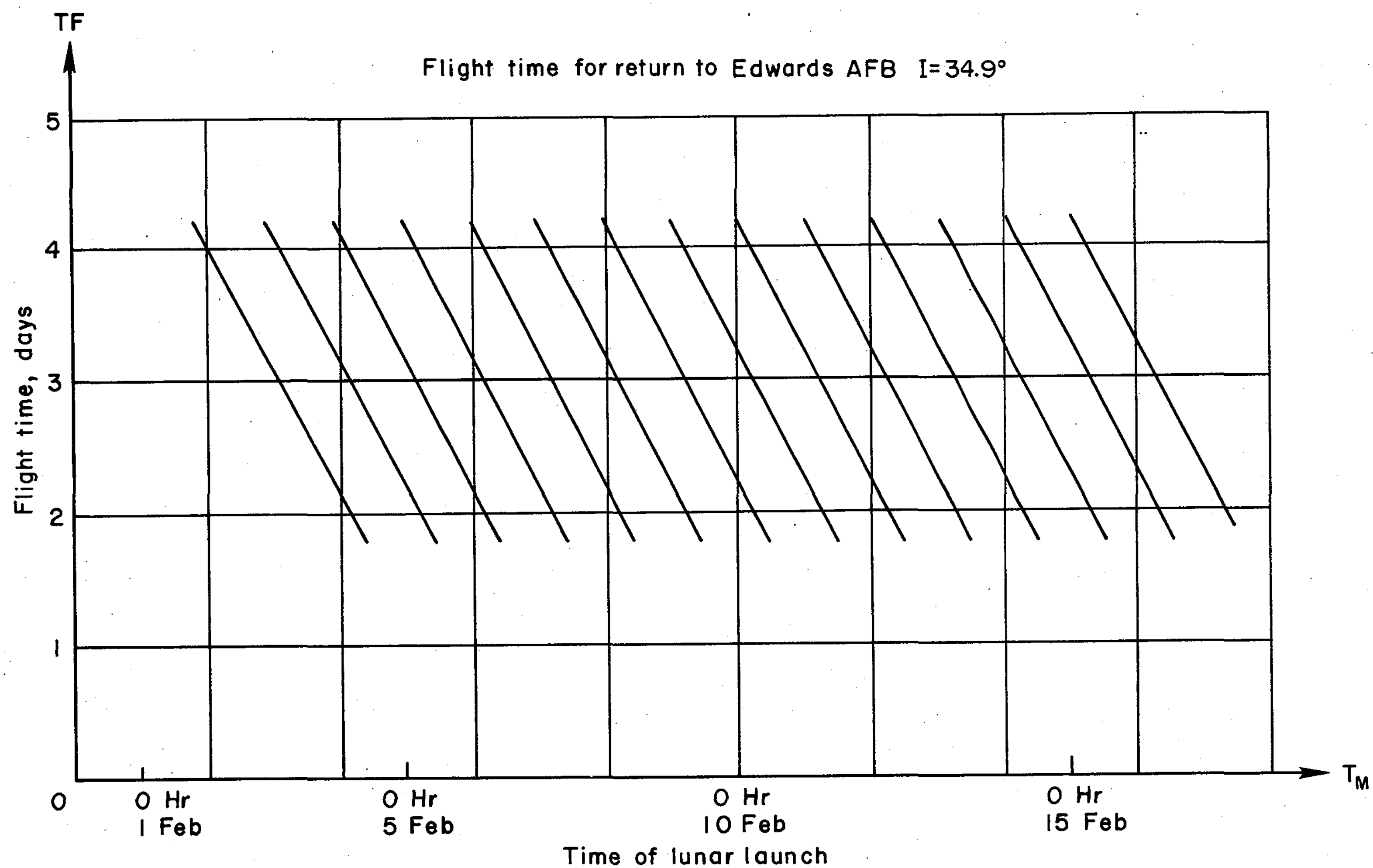


Figure 6.- Flight time versus time of lunar launch, Feb. 1966; I = 34.9°.

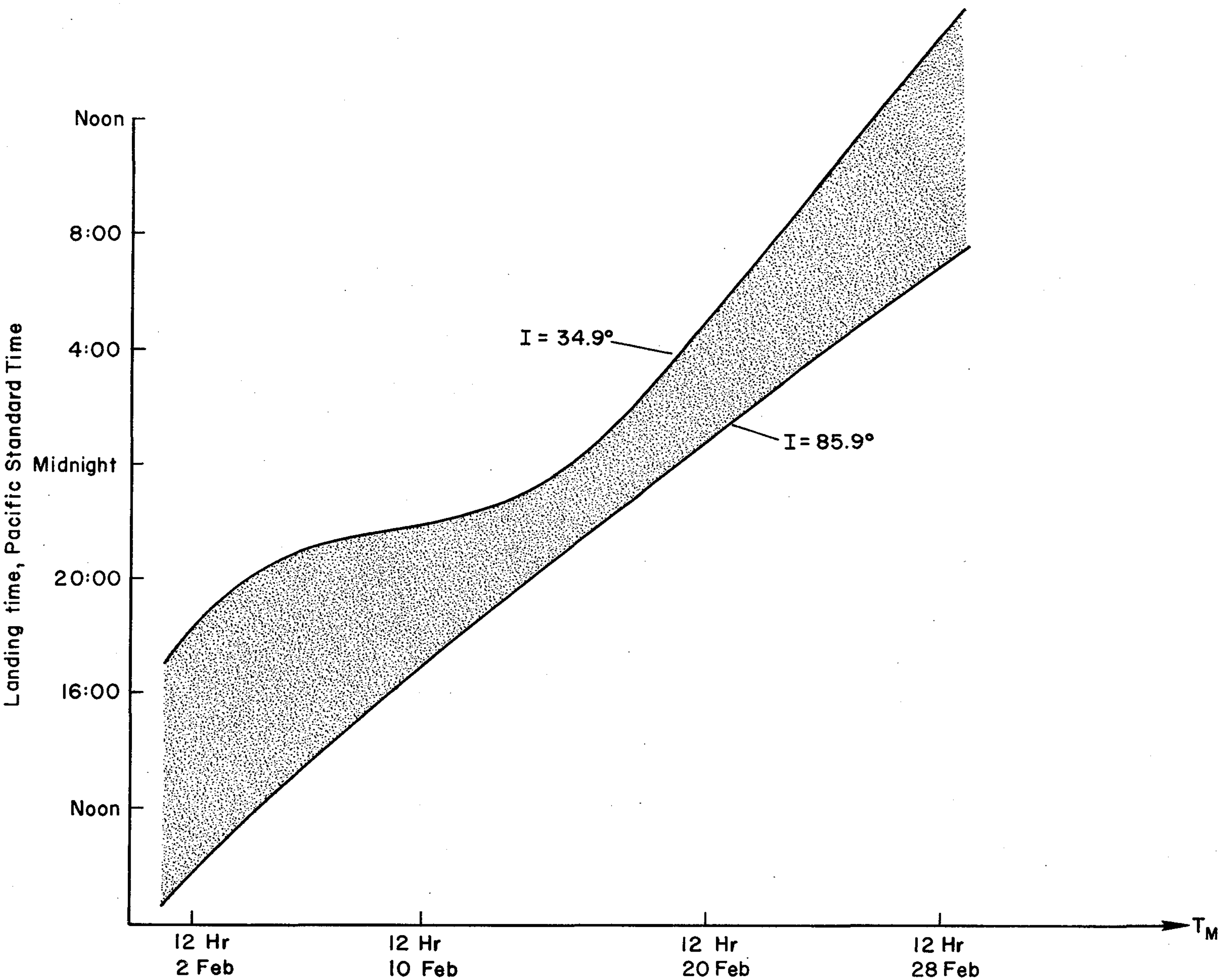


Figure 7.- Time of day at landing at Edwards Air Force Base versus time of lunar launch, Feb. 1966.

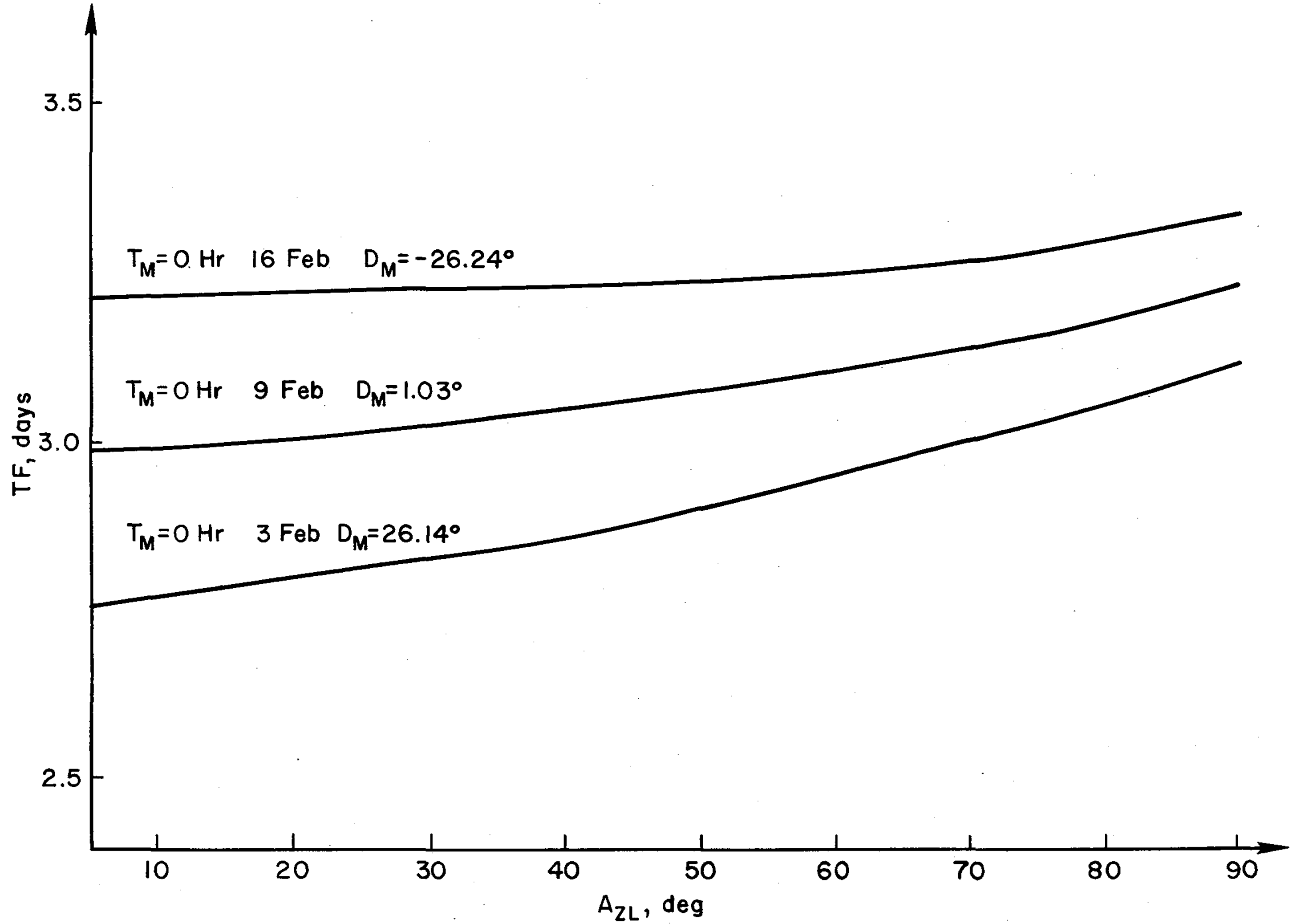
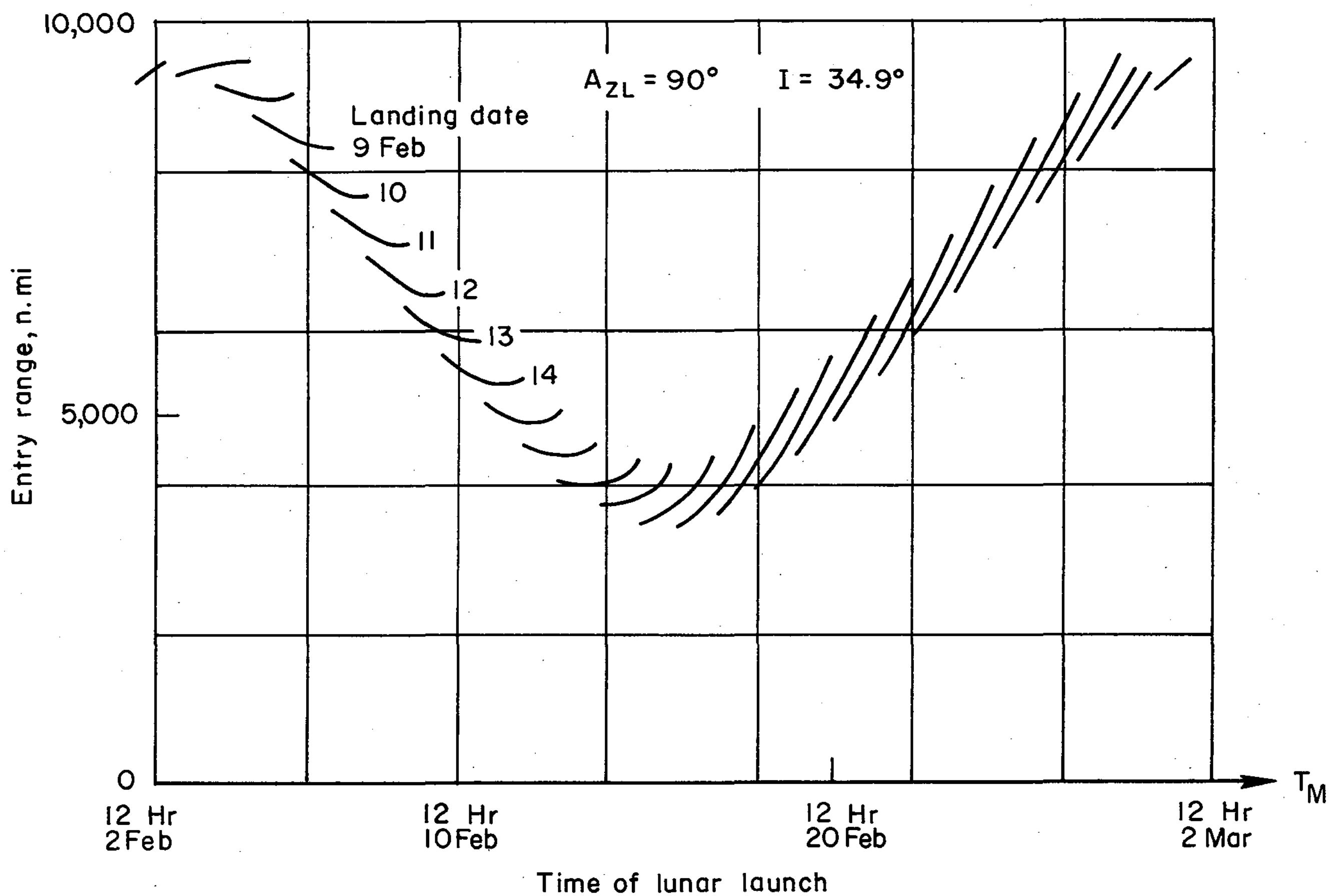
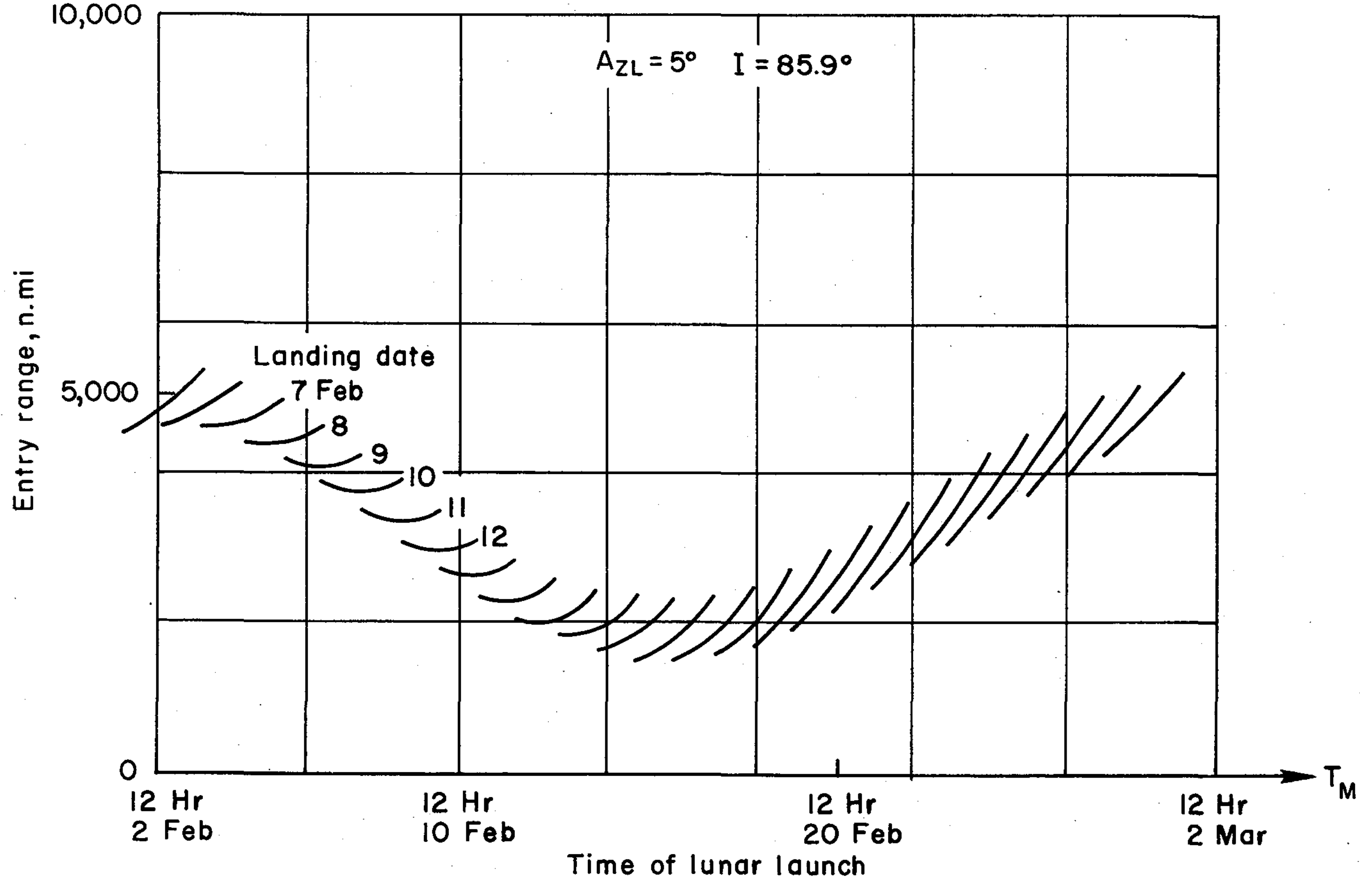


Figure 8.- Returns to Edwards Air Force Base; flight time versus azimuth at landing.



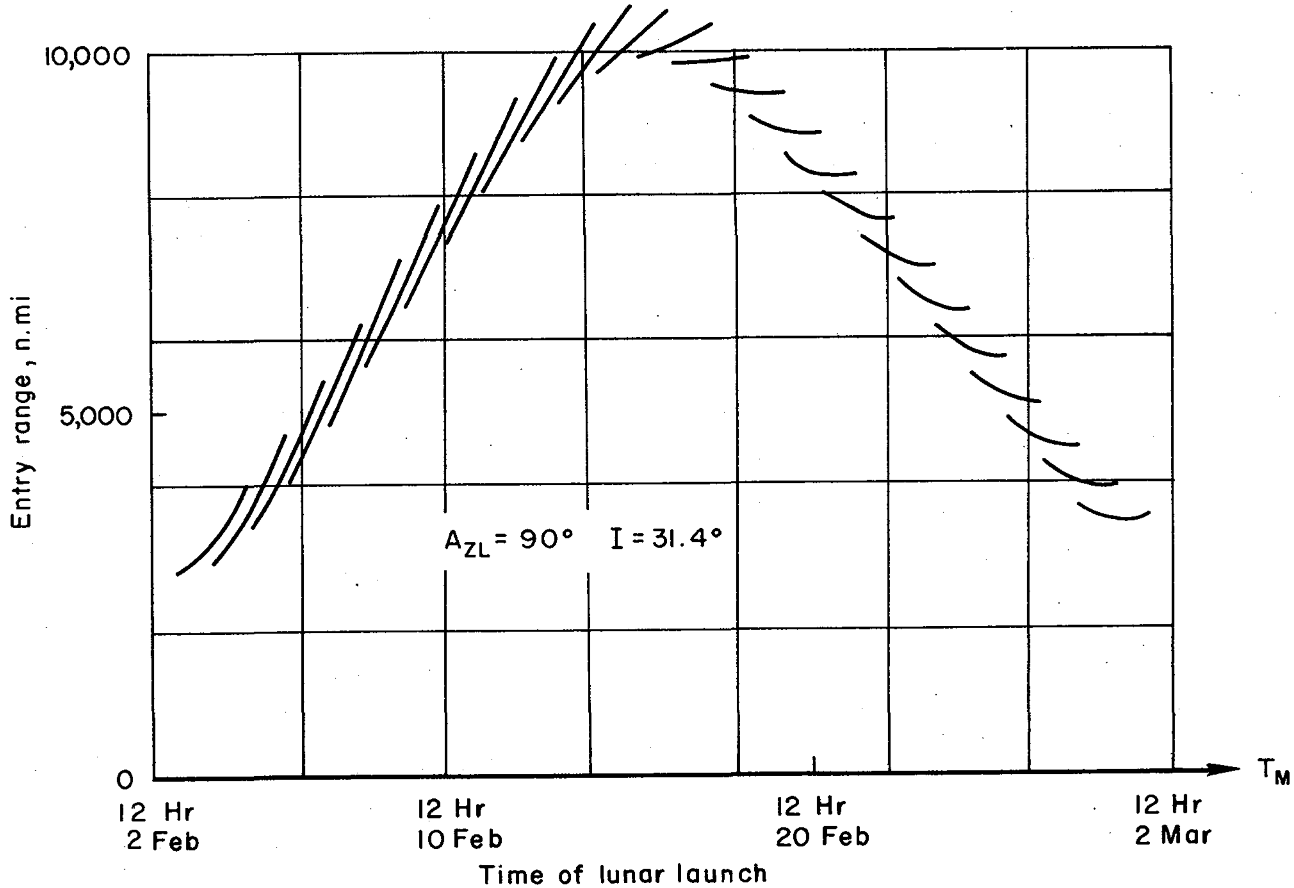
(a) $A_{ZL} = 90^{\circ}$, $I = 34.9^{\circ}$

Figure 9.- Entry range requirements for landing at Edwards Air Force Base, Feb. 1966.



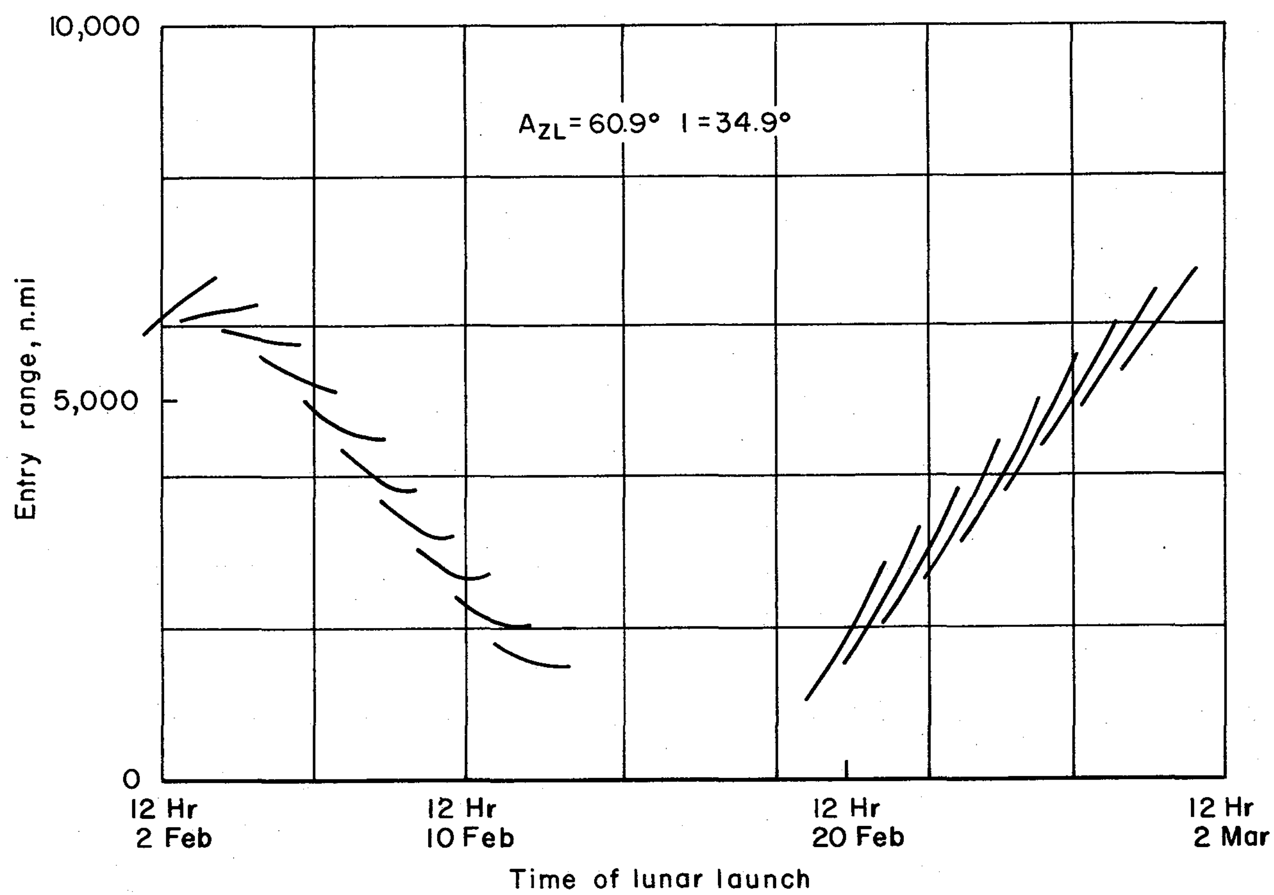
(b) $A_{ZL} = 5^{\circ}$, $I = 85.9^{\circ}$

Figure 9.- Concluded.



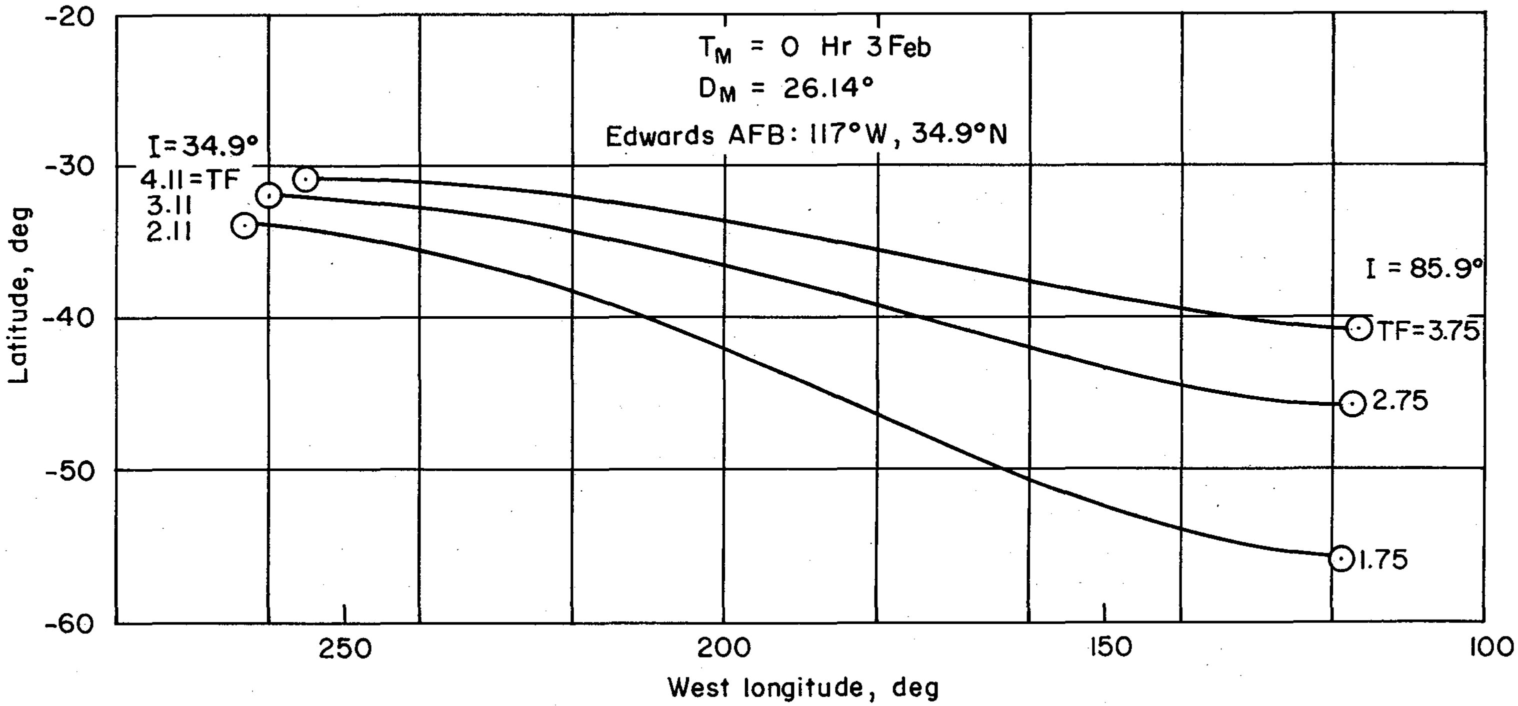
(a) Landing site: Woomera, Australia, Feb. 1966.

Figure 10.- Entry range requirements for alternative landing sites.



(b) Landing site: (170° W, 20.2° N), Feb. 1966.

Figure 10.- Concluded.



(a) Entry location for fixed time of departure.

Figure 11.- Returns to Edwards Air Force Base; longitude and latitude at entry.

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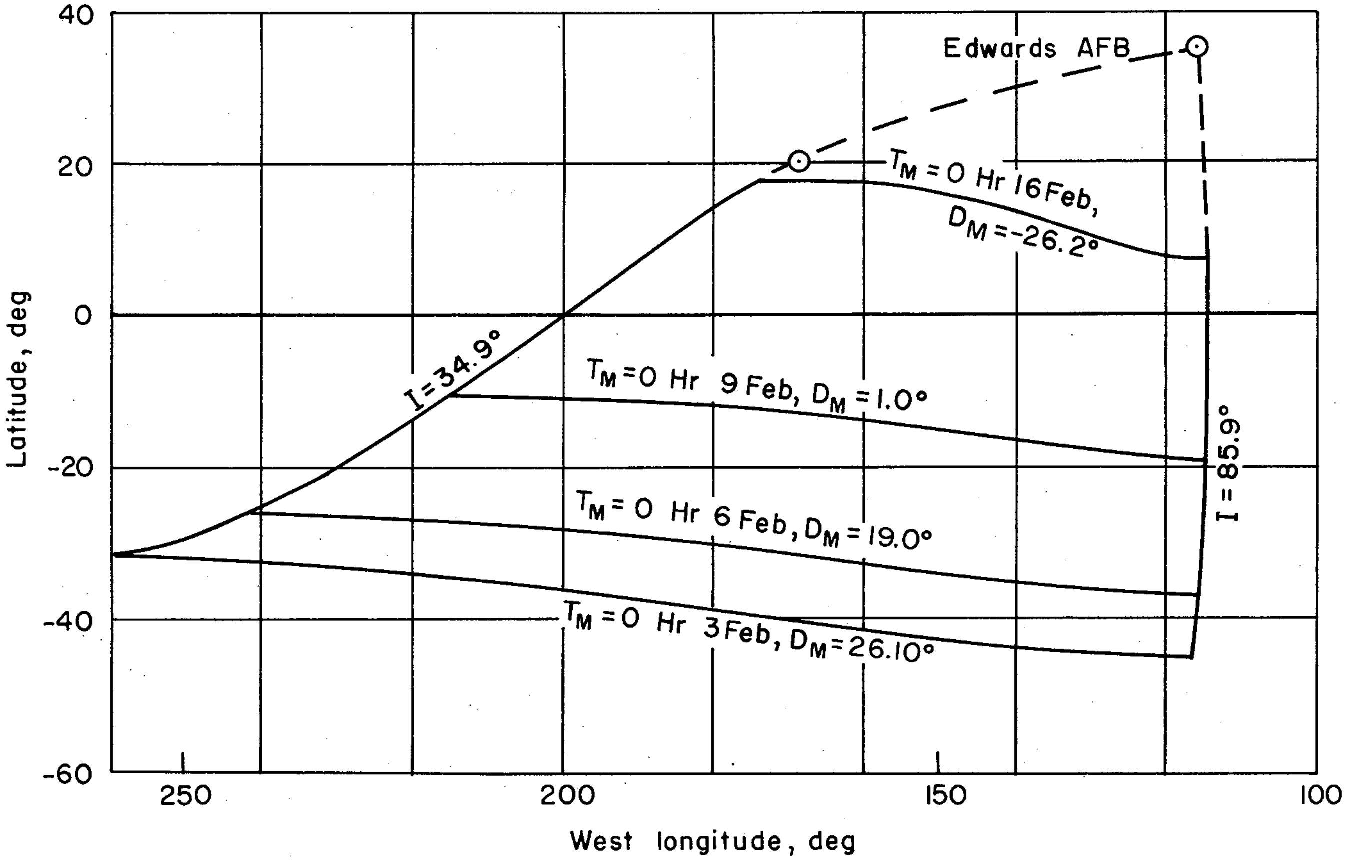

(b) Entry location for various times of departure.

Figure 11.- Concluded.