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TRAJECTORIES IN THE EARTH-MOON SPACE WITH SYMMETRICAL FREE RETURN PROPERTIES

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SUMMARY

An extensive investigation has been made of the characteristics of so-called "free return" trajectories. For the purposes of the study, these trajectories are defined as having certain symmetric properties which afford flight to the vicinity of the moon and return to earth without need for propulsion after the initial boost phase. The restricted three-body model for the earth-moon-probe system is used throughout.

Two kinds of free return trajectories are shown to exist and are studied. Of particular interest is the fact that for one kind of free return path, the largest inclination which can be achieved between the flight plane at periselenum and the plane of the moon's orbit about earth is about 10.8 degrees while for the other kind of path the largest possible inclination is dependent on the radius at periselenum. In this case the inclination is limited to about 14 degrees or less for periselenum radius of 1938 km, but may be as great as 90 degrees with periselenum radius of 21150 km.

Trajectories are also demonstrated which pass in front of the moon. These exhibit inclination behavior very much like that given by trajectories which go behind the moon. The injection velocity for these trajectories also changes only slightly from the circumlunar trajectories (less than 2 m/s for periselenum radius of 1938 km). However, the position of injection is changed considerably and the flight time may be increased by as much as five times that for circumlunar flight.

SECTION I. INTRODUCTION

The utilization of trajectory shapes which bring an exploring vehicle from the earth to a point arbitrarily near the moon and which, if allowed to continue, return the vehicle to the earth with such conditions that reentry and recovery are feasible, might be attractive for both instrumented and manned flights. For unmanned flights such missions as photographing the moon's surface, determination of astronomical constants by observation of trajectory perturbation, and check-out of hardware to be used for manned flights are obvious examples. For manned exploration the first flights will very likely be such "fly-by" trips with no plan for landing on the moon. Indeed, when the manned mission is to land on the moon's surface, a "free return" trajectory may be used so that if unforeseen difficulties arise which would make landing undesirable or impossible (particularly a failure of the propulsion system to brake the vehicle speed so as to make landing possible) the astronauts will return safely to the earth.

These trajectories (which will be called "free return" trajectories here) impose by their nature some important restrictions on the flight geometry and travel times. This report attempts to establish in a general way what these restrictions are and what behavior various trajectories of this type have in common.

SECTION II. DISCUSSION

A. EARTH-MOON MODEL AND COORDINATE SYSTEM

In order to study the problem in a general way the restricted problem of three bodies was employed. The assumption is made that the moon and earth revolve about a barycenter in circular orbits, avoiding the complexity of including the exact ephemeris data. Additionally a coordinate system is established for measuring position and velocity components with respect to the line from the earth to the moon and the plane of the moon's orbit about the earth. This coordinate system is illustrated in Figure 1. An equator for earth and moon is defined as being in the moon-earth orbit plane. North direction is defined perpendicular to the plane in the direction of the rotational velocity vector of the system. East on either earth or moon is the direction of sunrise. Latitude is measured positive north and negative south of the equator. Longitude is measured positive east from the earth-moon line. Earth longitude is zero on the earth-moon line at the side of earth farthest from the moon. Moon longitude is zero on the line between earth and moon. Azimuth is measured positive east from north and path angle is measured from local vertical. This system has been used in previous studies of earth-moon trajectories and was called the MEP (moon-earth plane) system. The directions may be specified as MEP north or MEP east, for example, in places where there may be doubt as to whether the true direction is referred to. Also MEP equator is used to distinguish that defined above from the true equator of a body.

B. DEFINITION OF FREE RETURN TRAJECTORIES

For convenience we define here a rotating cartesian coordinate system which has its origin at the barycenter, the positive x axis in the direction of the moon, the positive z axis in the direction of the angular velocity vector of the moon-earth system (the MEP north direction), and the y axis in the right handed relation to x and z. This system is the same as the one used in Reference 1 and the Theorem of Image Trajectories developed in that work is used in the definition of free return trajectories.

Two kinds of trajectories which may be called free return are possible. The first kind results from the image trajectory which exists with respect to the x axis. If a trajectory is possible from earth to moon, the image reflected about the x axis is also possible, flown in the opposite sense, that is, from moon to earth. If a trajectory then, arrives near the moon on the earth-moon line and has velocity direction perpendicular to the earth-moon line, this trajectory is a "free return" trajectory which comes back to earth at a point opposite the earth-moon line from the injection point. Or, in other words, when altitude above earth on the return leg is equal to injection altitude, the latitude is the negative of injection latitude; velocity magnitude and direction (path angle and azimuth) are equal to the injection values, and longitude is 360 degrees minus the injection longitude. Figure 2 shows this kind of path.

The second kind of "free return" path also is derived from the symmetry conditions about the earth-moon line but, in addition, takes advantage of the symmetry about the moon-earth plane. The second kind of "free return" trajectory may be defined as one which passes near the moon, perpendicular to the xz plane. For example, an outbound trajectory may arrive at the xz plane near to the moon and in the southern hemisphere. This outbound leg will have an "inbound reflection" about the x axis. The near moon point on the inbound trajectory will therefore be in the northern hemisphere. It is clear, however, that either leg also has a reflection in the opposite hemisphere. This indicates that if the trajectory intersects the xz plane in a direction perpendicular to it, it will continue in a path which is symmetric with respect to the xz plane. When such a trajectory reaches a return altitude equal to the injection altitude, it lies at the same latitude as at injection; return path angle is equal to injection path angle, and longitude and azimuth are reflected about the xz plane. Figures 3 and 4 illustrate the geometry of this kind of trajectory.

Trajectories which lie always in the moon-earth plane are common to both kinds of "free return" paths. These trajectories will be referred to as "coplanar" free return trajectories.

Other trajectories may exist which afford close approach to the moon and free return to specified reentry conditions but are not symmetrical trajectories. These are not included in this study.

C. APPROACH TO THE PROBLEM

In order to reduce the number of parameters to be considered, injection (and, therefore, reentry) is assumed to occur at 100 nautical mile altitude or 6555 km radius from the earth's center with horizontal path angle.

Since the free return trajectories must by definition meet certain conditions at their approach to the moon, and, in addition, the path shape and transit time in the outbound leg are equal to those on the return leg, it is advantageous to calculate only the return leg of the trajectory. For example, trajectories of the first kind must pass through zero latitude with longitude 0 or 180 degrees and simultaneously with path angle horizontal with respect to the moon. Computation of the trajectory can be commenced then at a chosen radius from the moon's center with only velocity magnitude and azimuth as open parameters. If the azimuth is chosen, then velocity can be determined to isolate the specified perigee radius. One observes from this procedure that a limit exists for the azimuth beyond which the perigee radius cannot be obtained. As the azimuth at the moon is changed from 270 degrees (due west) to more northern directions, the velocity required increases. This further has the effect that at the earth the reentry latitude goes from 0 degrees to positive values while azimuth at earth goes from 90 degrees (due east) to more southern directions (the trajectory moves toward passing over the North Pole). As the approach to earth continues to sweep over the pole and then becomes retrograde at earth, the azimuth at the moon passes through a maximum value and then decreases back to 270 degrees, and the velocity continues to increase until it reaches a maximum where azimuth is 270 degrees again. In other words, for all azimuths between the limits which can be used at the moon, there are two velocities which will give return perigee at the desired altitude. The smaller velocity produces a return which has an eastward velocity component at earth; the larger one produces a westward velocity component at earth.

For the second kind of free return trajectory, the procedure is similar. This type of trajectory must pass through longitude 0 with azimuth 90 degrees or longitude 180 degrees with azimuth 270 degrees, and path angle horizontal. For a chosen radius of close approach to the moon then, the latitude and velocity magnitude are open parameters. Variation in latitude has the same effect here as the azimuth variation had for the first kind of trajectory. There is a limiting latitude which allows free return from a given close approach radius at the moon, and for each latitude between the extremes, there are two returns given by two velocities; one is co-rotational and the other counter-rotational at the reentry.

Although the study has been conducted by calculating trajectories starting at the close approach to the moon as described above, most data are presented in terms of injection conditions at earth as well as radius of close approach to the moon. The equations of motion are integrated by Cowell's Method exclusively throughout the study.

SECTION III. RESULTS

It was found that free return paths which pass arbitrarily close to the moon occur for periselenum on the near earth side of the moon, as well as on the far side. If the periselenum occurs at the far side, the trajectory passes all the way around the moon and is often called "circumlunar." For the cases where periselenum occurs on the near earth side of the moon, the trajectory path does go beyond the radius of the orbit of the moon; however, the trajectory comes back within the moon's orbit before passing near the moon and, thus, reaches periselenum between earth and moon. These trajectories which have periselenum "in front of the moon" will be called "cislunar" trajectories. Both circumlunar and cislunar free return paths of both the first and second kind can be produced for some range of radii of periselenum with any azimuth of injection at the earth. The exact effect of choice of periselenum radius is discussed later in this report. The coplanar cases are illustrated in Figures 5 and 6. Because all these trajectories approach the moon in a direction opposite the moon's orbital motion about earth, the circumlunar cases always have a counter-rotational (retrograde) component of motion at periselenum with respect to the moon's rotation on its axis, while the cislunar cases always have a co-rotational component.

A. INCLINATION OF FLIGHT PLANE WITH RESPECT TO LUNAR MEP EQUATOR

If a braking maneuver into orbit of the moon is to be attempted, it would be desirable to make this maneuver without any flight plane changes. Therefore, the inclination of the instantaneous flight plane at periselenum with respect to the MEP lunar equator as indicated by a two-body approach neglecting the earth's gravitation and considering the moon as the attracting body is of particular interest. In the data presented, then, inclination is defined as the angle between the MEP north-south axis and the angular momentum vector of the trajectory, taken with the moon as the central force. This definition allows inclination to vary between 0 and 180 degrees, and indicates the direction (co-rotational or counter-rotational) of the orbit transferred to.

In terms of the two-body consideration the line of nodes of the flight plane and MEP lunar equator may be specified. For free return paths of the first kind, the line of nodes of the moon's equator (the earth-moon orbit plane) and the trajectory plane is coincident with the earth-moon line while for paths of the second kind, the line of nodes is perpendicular to the earth-moon line.

Figure 7 shows inclination at the periselenum as a function of injection azimuth at earth for periselenum radius of 1938 km for both the cislunar and circumlunar trajectories of the first kind. At this radius the extreme inclination possible is about 169.4 degrees (10.6 degrees between planes) for circumlunar and about 10.8 degrees for the cislunar. The inclination decreases very slightly for the cislunar and increases very slightly for circumlunar flights as the radius of periselenum is increased, the change in extremes of inclination being less than 0.5 degrees for radii out to 20,000 km.

For trajectories of the second kind, a quite different behavior of inclination is demonstrated in Figure 8. Here, inclination at periselenum is shown as a function of injection azimuth at earth with periselenum radius as a parameter. At periselenum radius of 1938 km the extremes of inclination occur at 166 degrees for circumlunar flights and 14 degrees for cislunar flights. As the periselenum radius is increased (through varied injection velocities) the possible variation in inclination is also increased. Using a periselenum radius of about 21,150 km allows variation of inclination from 0 degrees through 180 degrees, while earth injection azimuth is swept from 90 degrees through 270 degrees. (A symmetrical situation is produced by azimuths at earth from 270 degrees to 360 degrees and from 0 degrees through 90 degrees as indicated in the figure.) When periselenum radius is larger than 21,150 km, the inclination at the periselenum is variable from 0 degrees through 180 degrees with less than 180 degrees variation in injection azimuth.

B. VELOCITY REQUIREMENTS AT INJECTION

Injection velocity at earth is given as a function of periselenum radius for the coplanar free return cases in Figure 9. For any azimuth at injection, the velocity required for trajectories of both first and second kinds is between the limits given by the coplanar cases. The injection velocity for trajectories of the second kind is shown as a function of injection azimuth with periselenum radius as a parameter in Figure 10.

C. TOTAL FLIGHT TIMES

Total flight time from injection to reentry is shown for coplanar cases as a function of periselenum radius in Figure 11. Non-coplanar trajectories of the first kind have flight times which, in general, lie between the limits shown by the coplanar cases; however, the crossing of the two curves, indicating equal flight time for co-rotational and counter-rotational injection to cislunar periselenum of about 3400 km, does not indicate equal flight time for all non-coplanar cases as well. Rather, in this region of periselenum radius, the non-coplanar cases require a few hours more time than either of the coplanar cases.

Trajectories of the second kind have flight times of 10 to 20 hours difference between the coplanar cases and the maximum inclination cases. Cislunar non-coplanar flights require smaller times than coplanar, while circumlunar noncoplanar flights require more time than the coplanar.

D. LOCUS OF INJECTION

The loci of injection at perigee for the various flights to periselenum radius of 1938 km are shown in Figures 12 and 13. The arrows indicate roughly the azimuth of injection for various points on the curves. For both kinds of trajectories, the loci for circumlunar flights appear to be nearly circles on the earth while for cislunar flights, the curve is egg shaped. As the periselenum radius is increased, these curves become more distorted and move along the equator. Figures 14 and 15 show the loci of injection perigee for several periselenum radii for the second kind of paths.

E. PERIODIC TRAJECTORIES

If an earth-moon transit trajectory crosses the xz plane perpendicular to it in two places, by the theory of image trajectories about the xz plane (Reference 1), it is clear that such a trajectory, when allowed to continue, closes on itself or may be said to be periodic. Such a trajectory is indicated in the data already presented.

We note first that injection from earth occurs for all data given at perigee. Further, only coplanar cases can have injection perpendicular to the xz plane because azimuth of 90 or 270 degrees is necessary for perpendicularity and this occurs only for the coplanar cases. Since all coplanar cases cross the xz plane perpendicular to it at periselenum, the only condition remaining to be satisfied is that injection longitude be 0 or 180 degrees (on the xz plane). Cross plotting of the coplanar injection longitude against periselenum radius (from Figure 15) indicates that one cislunar case is a periodic orbit which has counter-rotational injection at 180 degrees longitude and periselenum radius of about 2150 km. From the curve in Figure 11, the period is ascertained to be about 650 hours or about one month.

SECTION IV. SUMMARY AND CONCLUSIONS

Two kinds of free return trajectories can be defined. The first kind always has periselenum on the earth-moon line, and the line of nodes of the flight plane with the moon's MEP equator at periselenum lies on the earth-moon line. The second kind has periselenum at a point above or below the earth-moon line with direction such that the line of nodes of the flight plane at periselenum and the lunar equator is perpendicular to the earth-moon line.

For both kinds of trajectories the periselenum may occur at either the front or back side of the moon. With paths of the first kind the inclination of the flight plane at the moon is limited to about 10.8 degrees or less regardless of periselenum radius chosen. For paths of the second kind the inclination which can be achieved is strongly a function of the periselenum radius so that near the moon (1938 km radius) only about 14 degrees can be obtained between the planes, while at radius of 21,150 km or greater, any inclination is possible.

For periselenum radius within a few hundred kilometers of the moon's radius, the transit time for circumlunar flights is about 138 to 140 hours, while for cislunar flights the transit time is from 4-1/2 to 5 times as long (620 to 700 hours). The injection velocity for these flights with periselenum near the moon is nearly the same for cislunar or circumlunar cases but changes strongly as a function of injection azimuth and periselenum radius.

As periselenum radius is increased, the transit times for cislunar and circumlunar flights become more nearly equal, ranging from 220 to 260 hours at 22,000 km.

One of the free return cases feasible is actually a periodic trajectory which approaches the moon at about 2150 km radius approximately once each month.

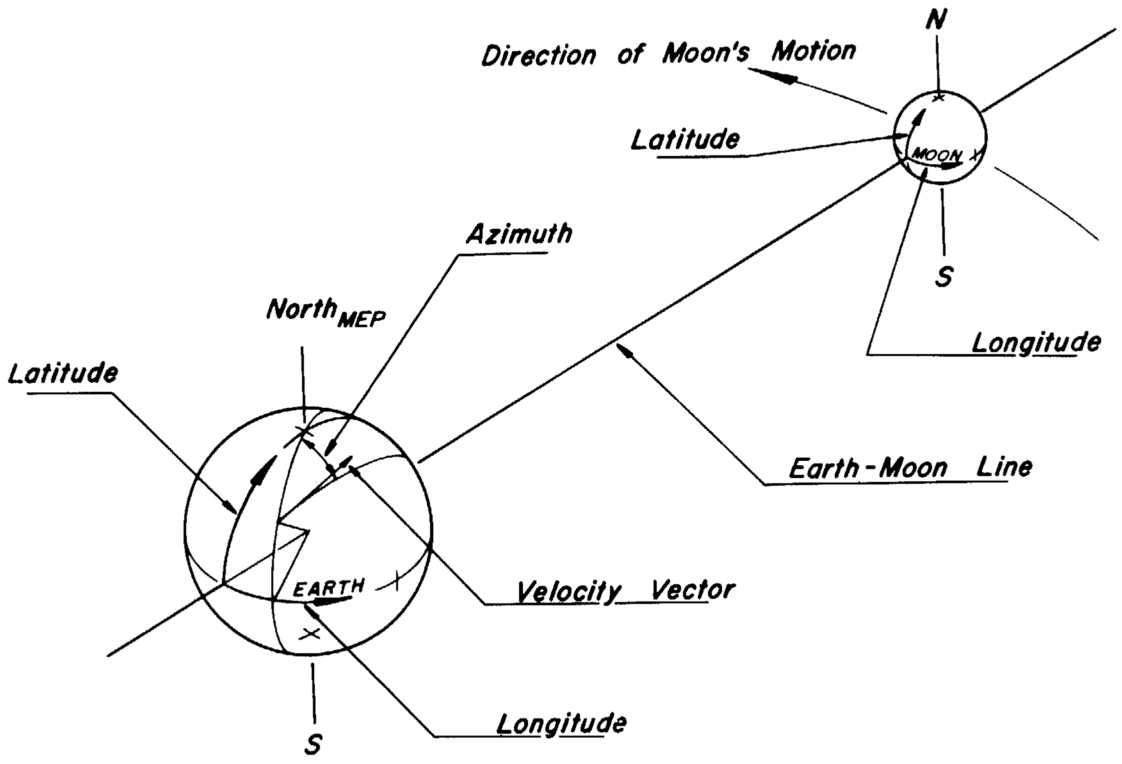


FIG. I. MEP COORDINATE SYSTEM

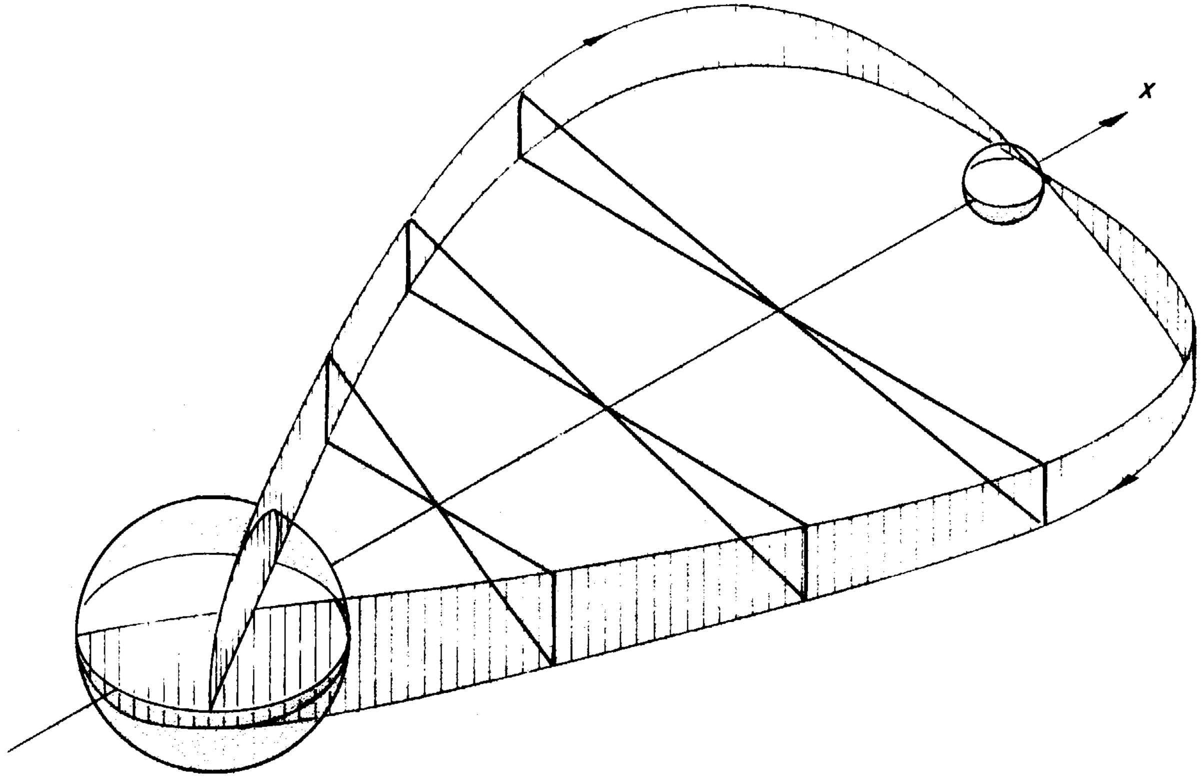


FIG. 2. FREE RETURN TRAJECTORY OF THE FIRST KIND

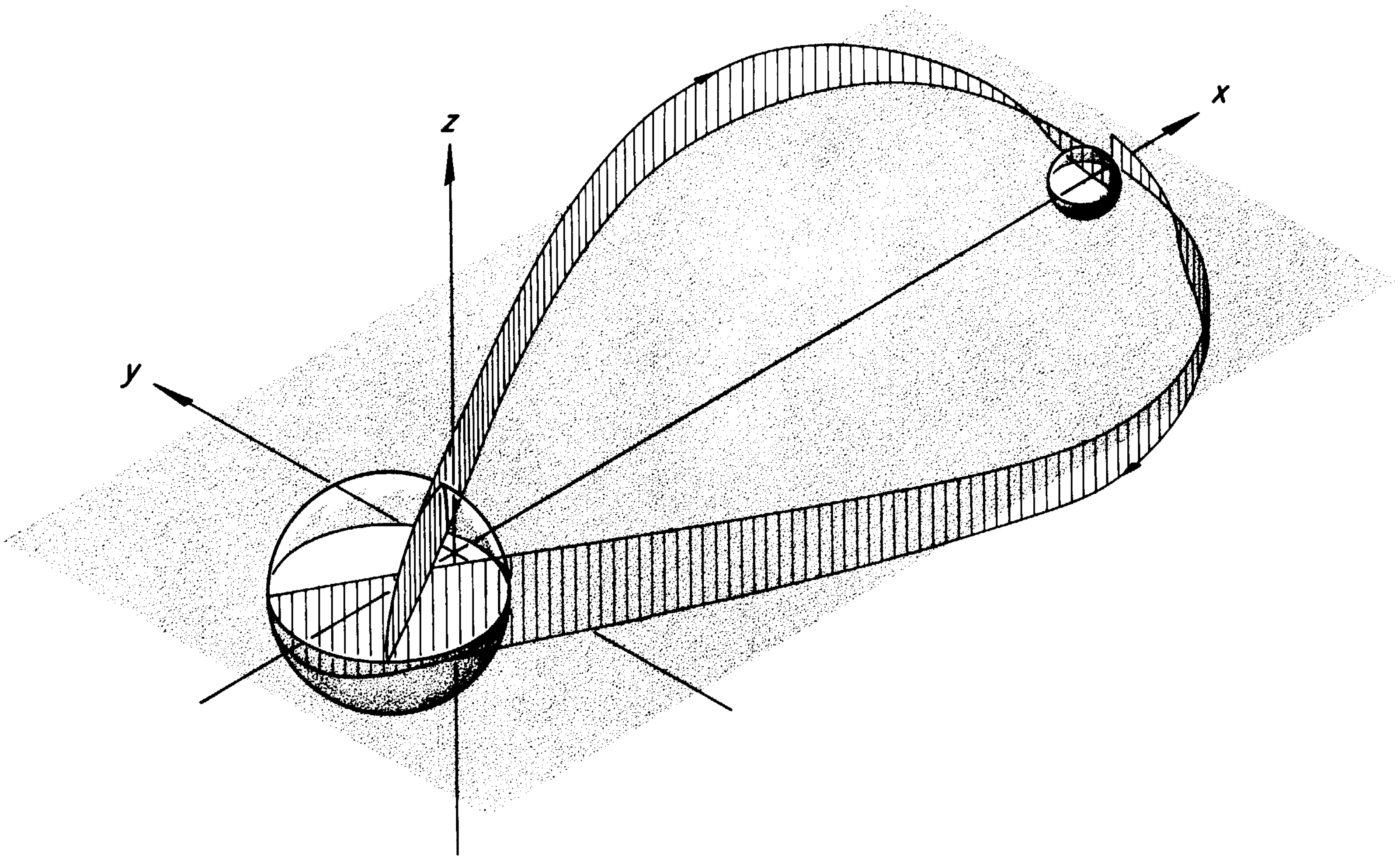


FIG. 3. REFLECTION ABOUT EARTH-MOON LINE OF OUTBOUND PATH

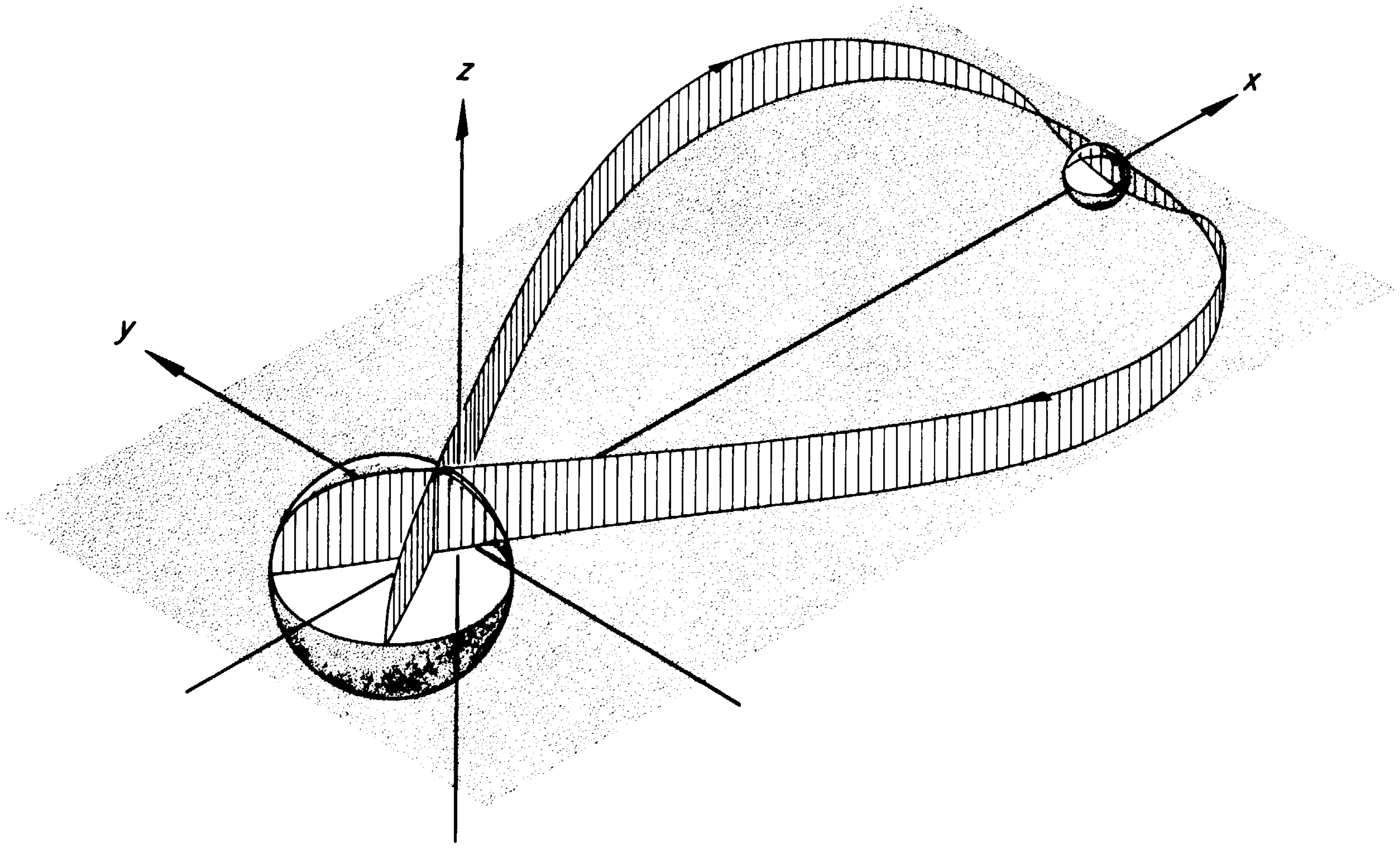


FIG. 4. FREE RETURN TRAJECTORY OF THE SECOND KIND

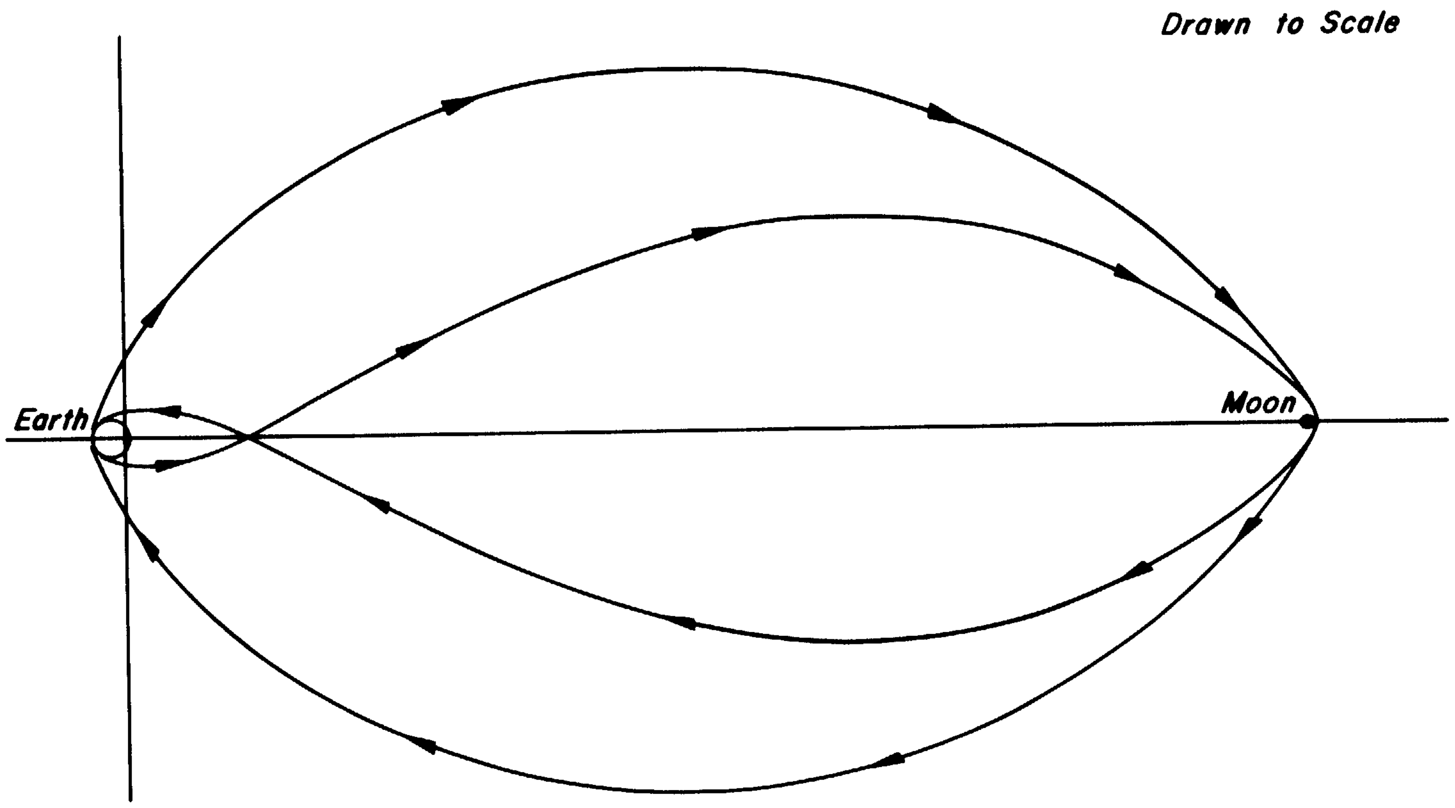


FIG. 5. COPLANAR CIRCUMLUNAR FREE RETURN TRAJECTORIES

(Reference Frame Rotating with Moon)

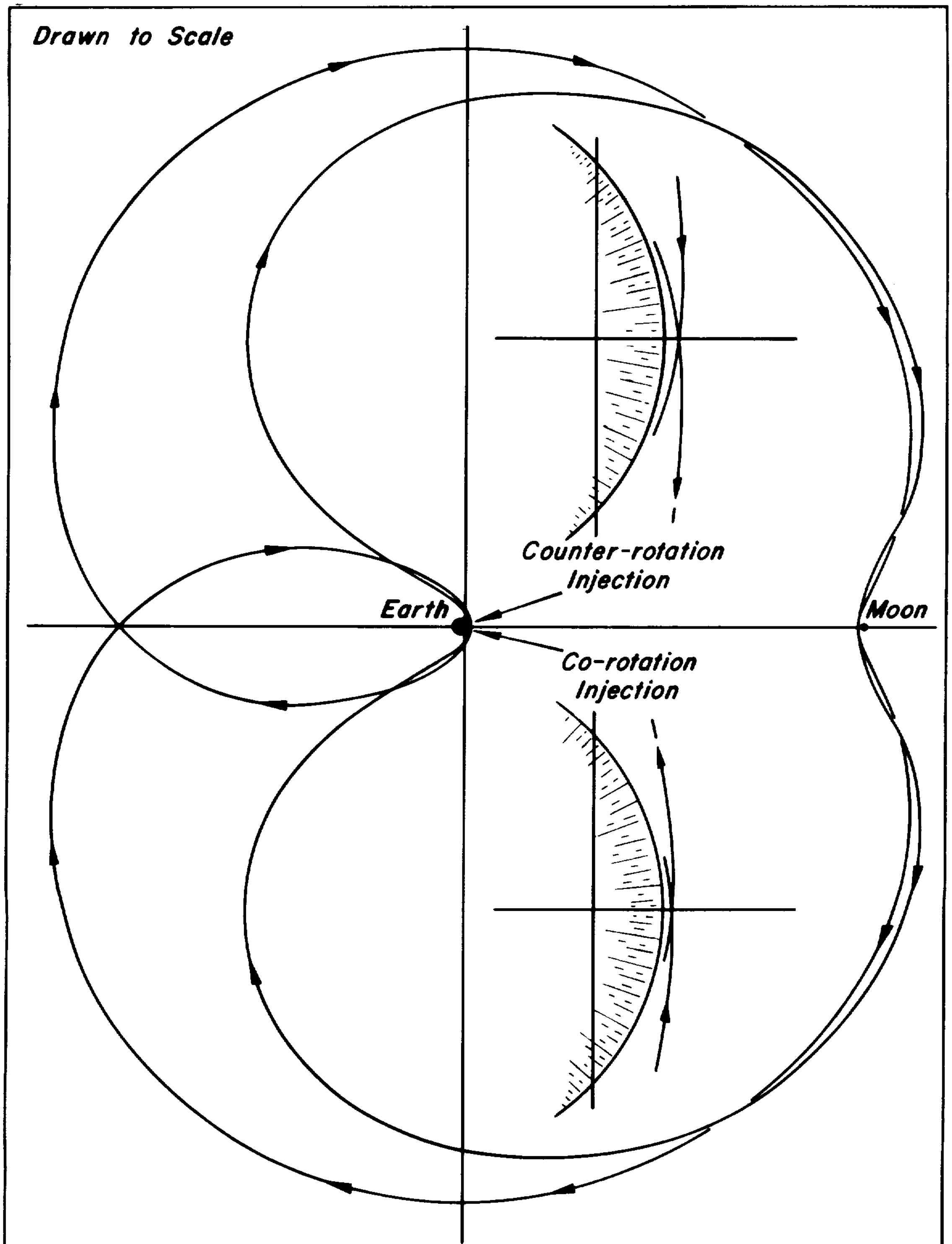


FIG. 6.
COPLANAR CISLUNAR FREE RETURN TRAJECTORIES
(Reference Frame Rotating with Moon)

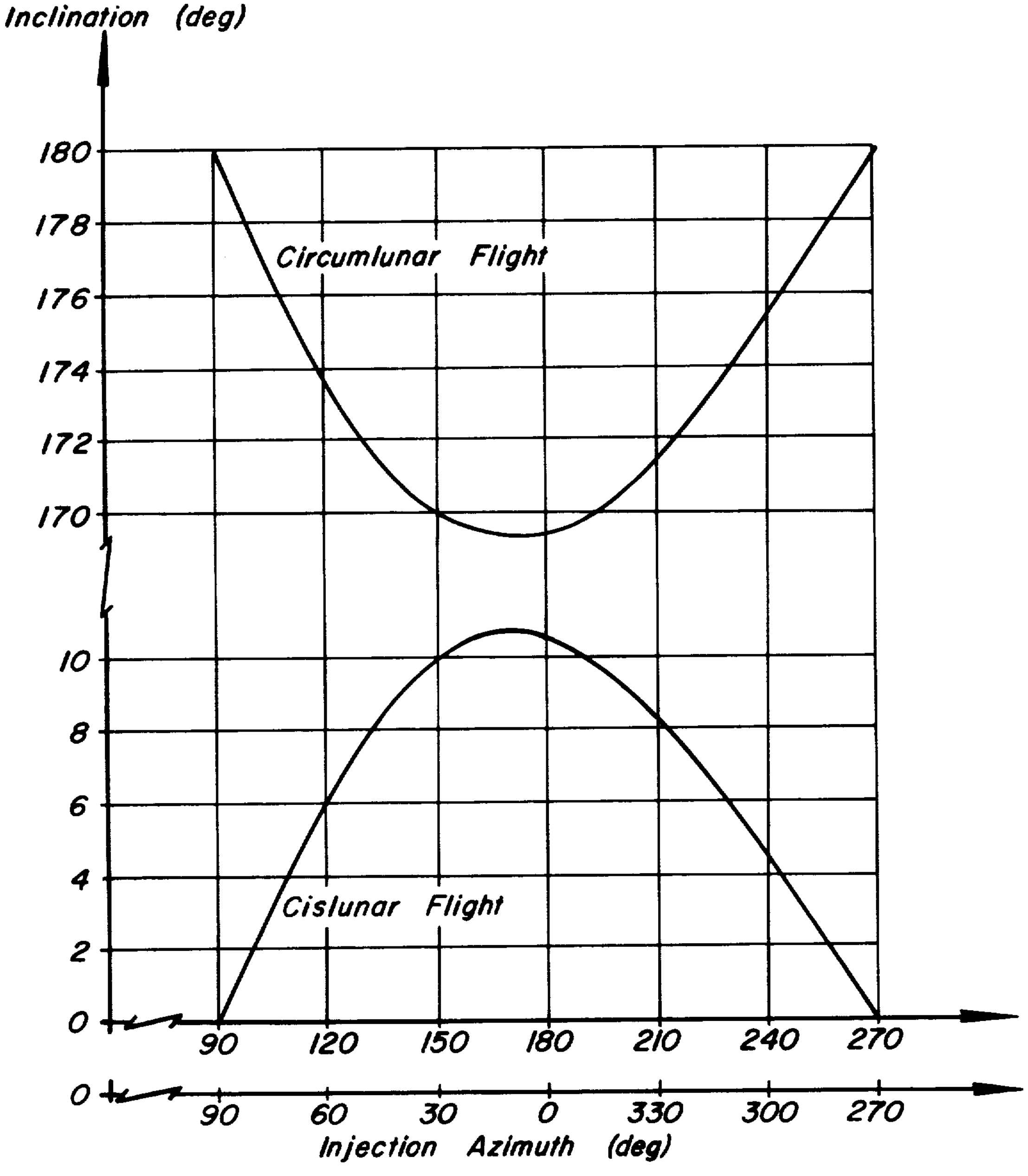


FIG. 7. INCLINATION OF FLIGHT PLANE AT PERISELENUM TO LUNAR MEP EQUATOR AS FUNCTION OF INJECTION AZIMUTH FOR FREE RETURN FLIGHTS OF THE FIRST KIND

Periselenum Radius = 1,938 km

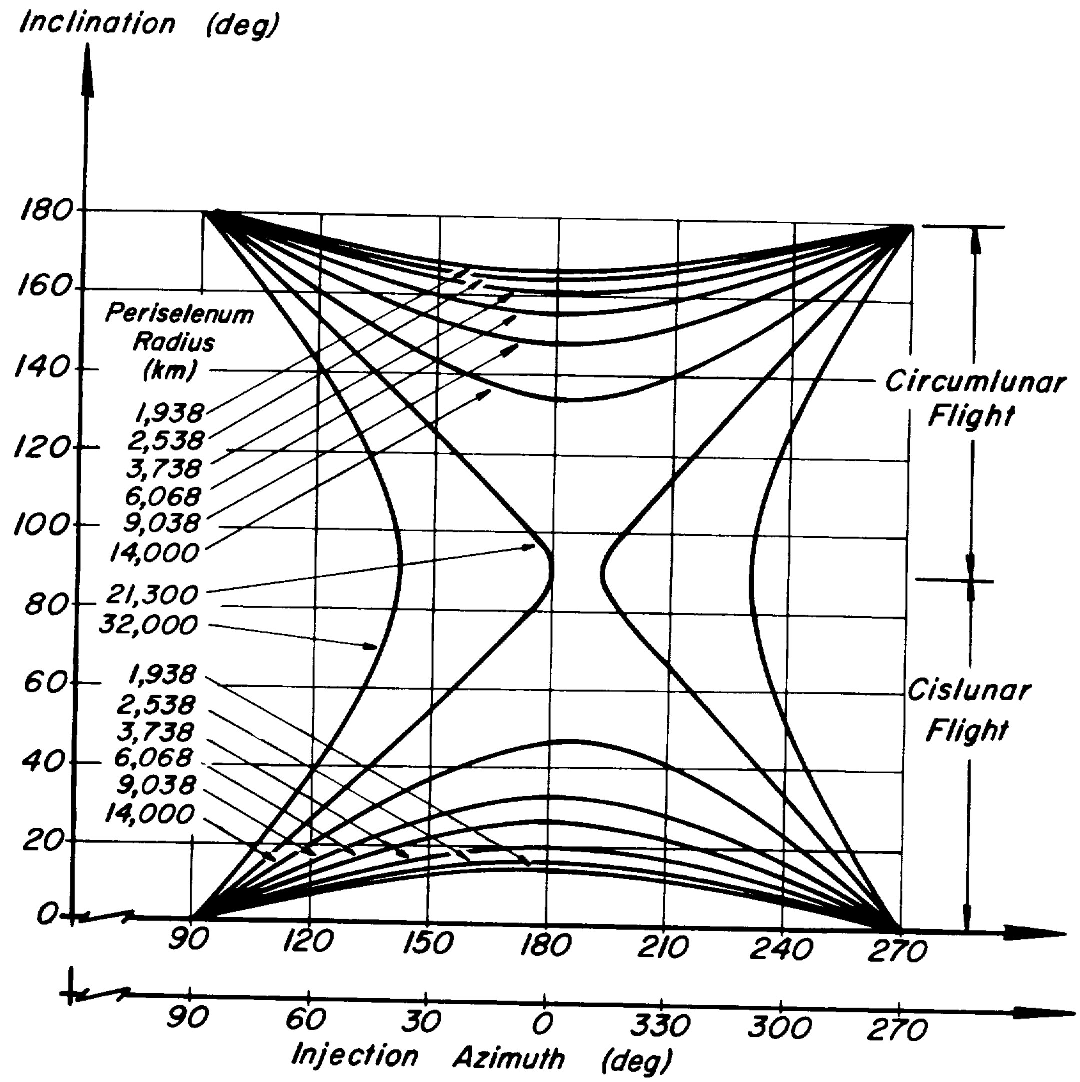


FIG. 8. INCLINATION OF FLIGHT PLANE AT PERISELENUM TO LUNAR MEP EQUATOR AS FUNCTION OF INJECTION AZIMUTH FOR FREE RETURN FLIGHTS OF THE SECOND KIND

Periselenum Radius As Parameter

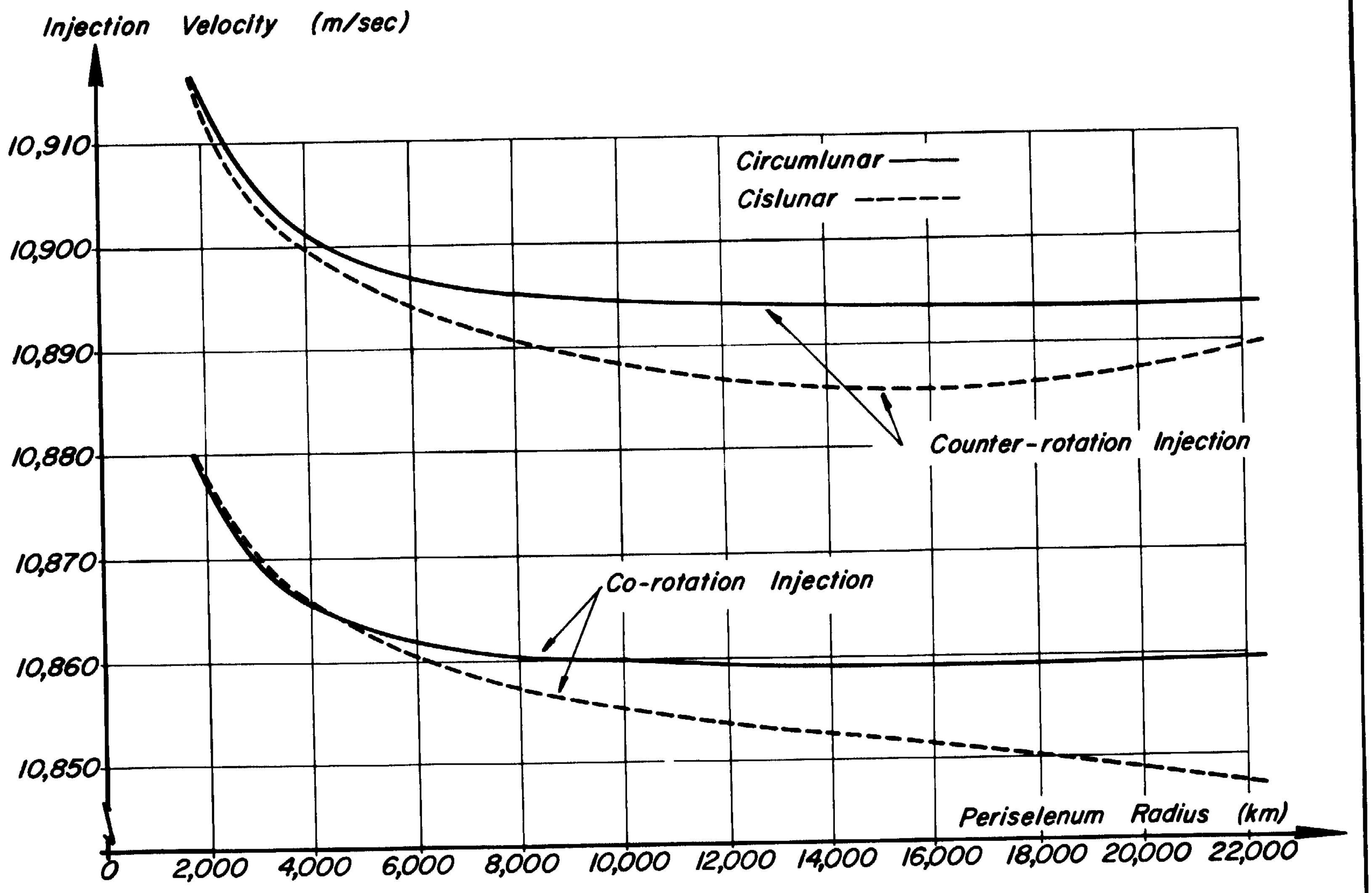


FIG. 9. INJECTION VELOCITY AT EARTH AS FUNCTION OF PERISELENUM RADIUS FOR COPLANAR FREE RETURN

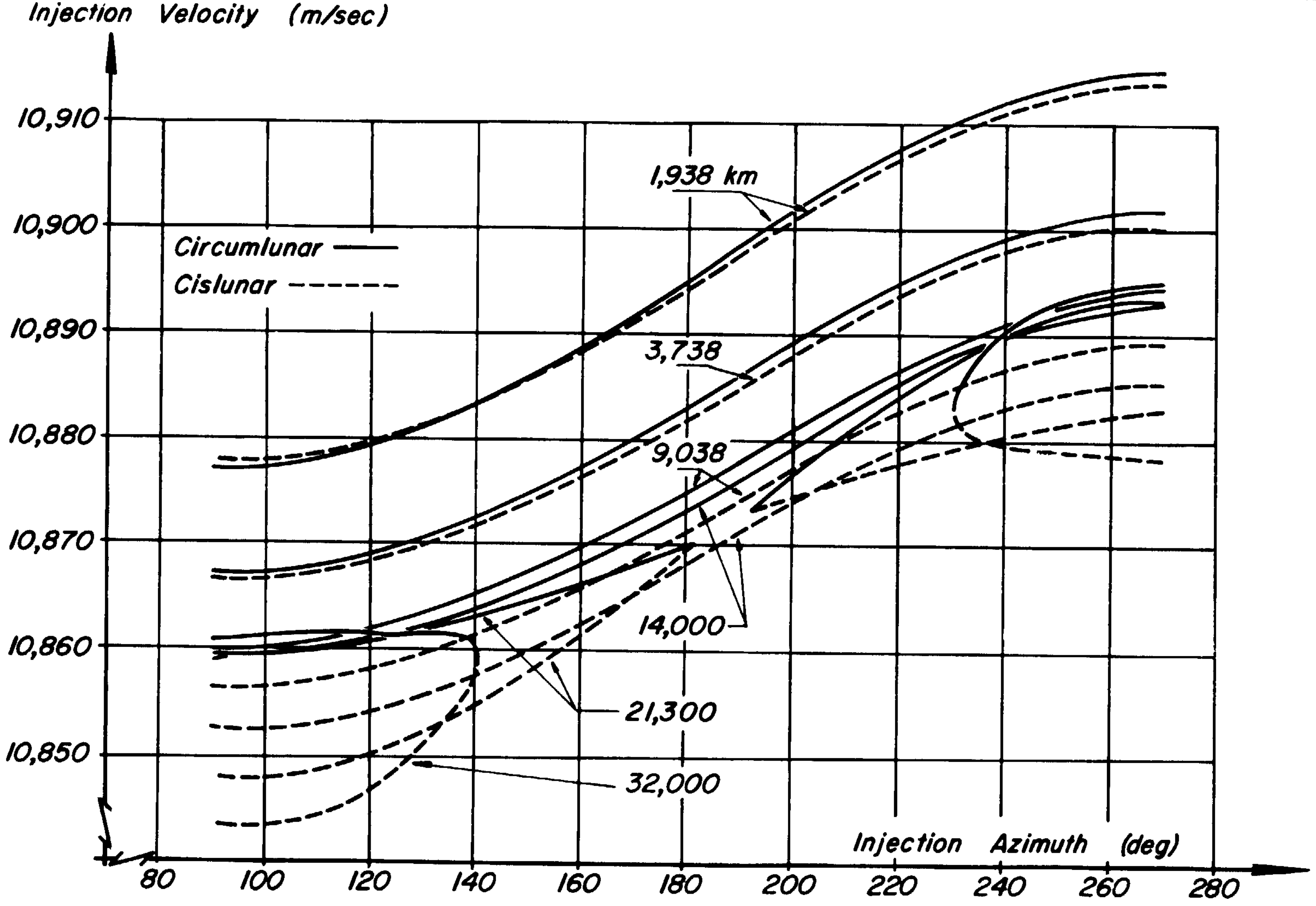


FIG. 10. INJECTION VELOCITY AS FUNCTION OF INJECTION AZIMUTH FOR FREE RETURN FLIGHTS OF THE SECOND KIND

Periselenum Radius As Parameter

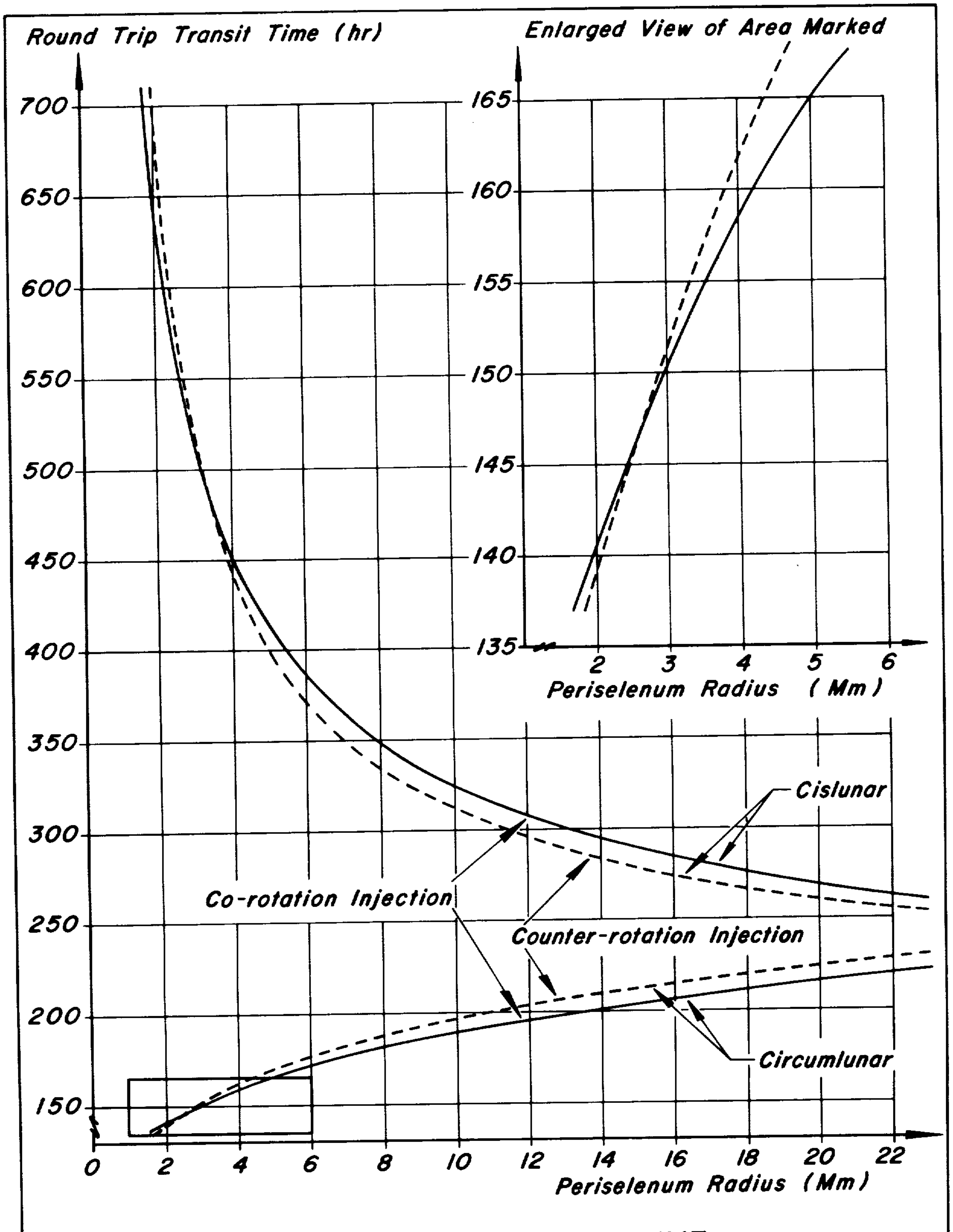
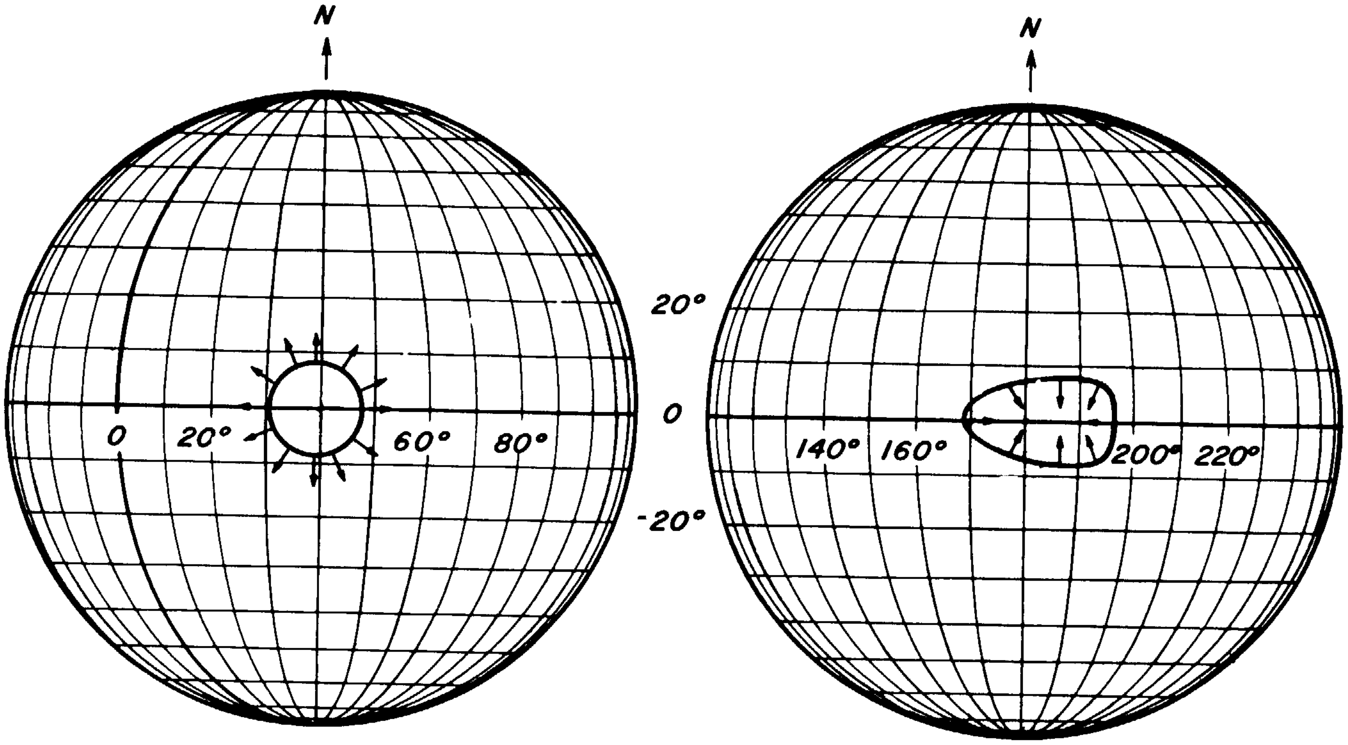


FIG. 11. TOTAL FLIGHT TIME FOR COPLANAR FREE RETURN TRAJECTORIES

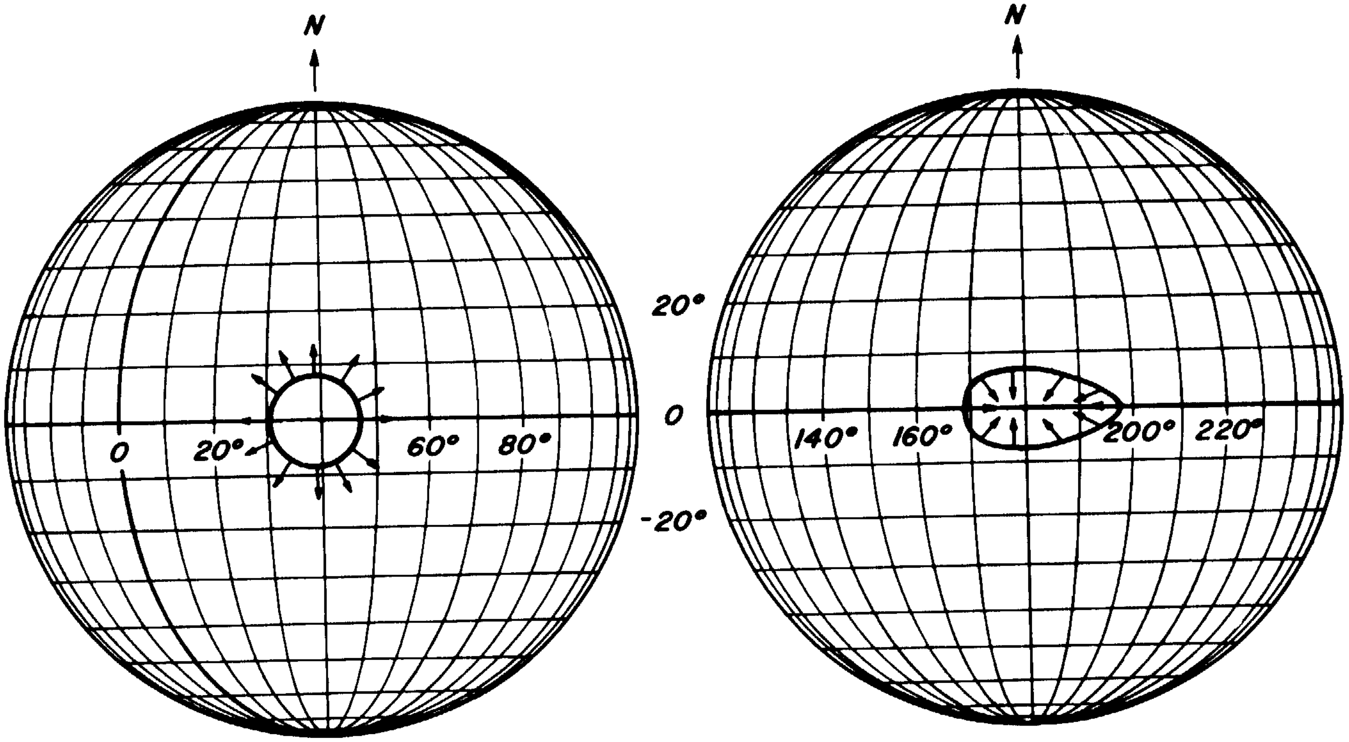


For Circumlunar Flights

For Cislunar Flights

FIG. 12. INJECTION LOCI FOR FREE RETURN TRAJECTORIES OF THE FIRST KIND

Periselenum Radius = 1,938 km ; Injection at Perigee Radius = 6,555 km



For Circumlunar Flights

For Cislunar Flights

FIG. 13. INJECTION LOCI FOR FREE RETURN TRAJECTORIES OF THE SECOND KIND

Periselenum Radius = 1,938 km ; Injection at Perigee Radius = 6,555 km

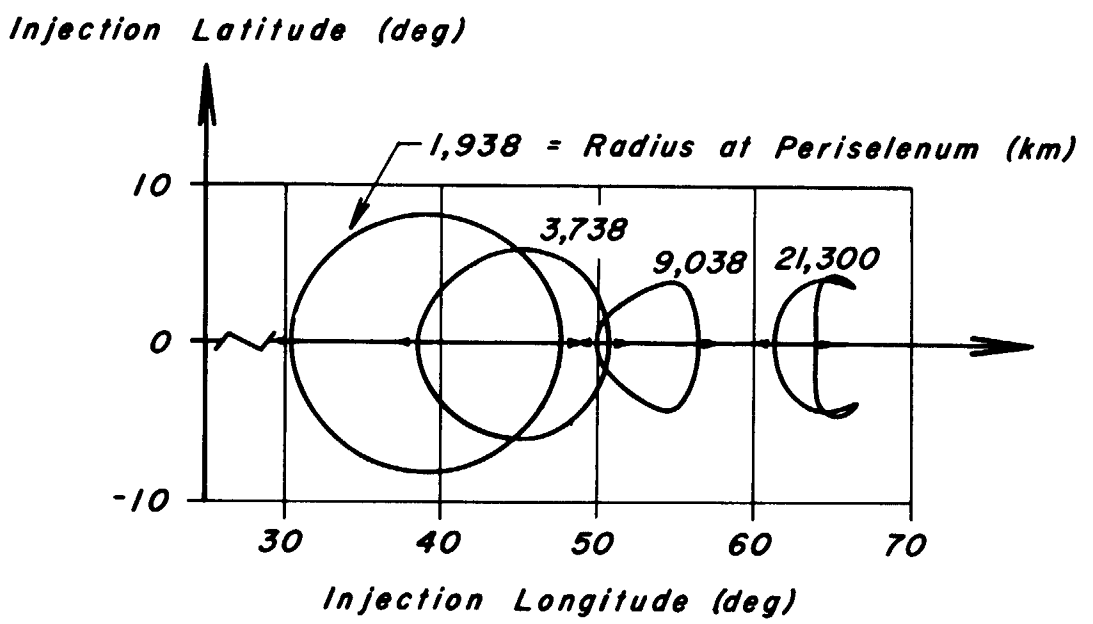


FIG. 14. INJECTION LOCI FOR CIRCUMLUNAR FREE RETURN OF THE SECOND KIND

Injection at Perigee Radius = 6,555 km

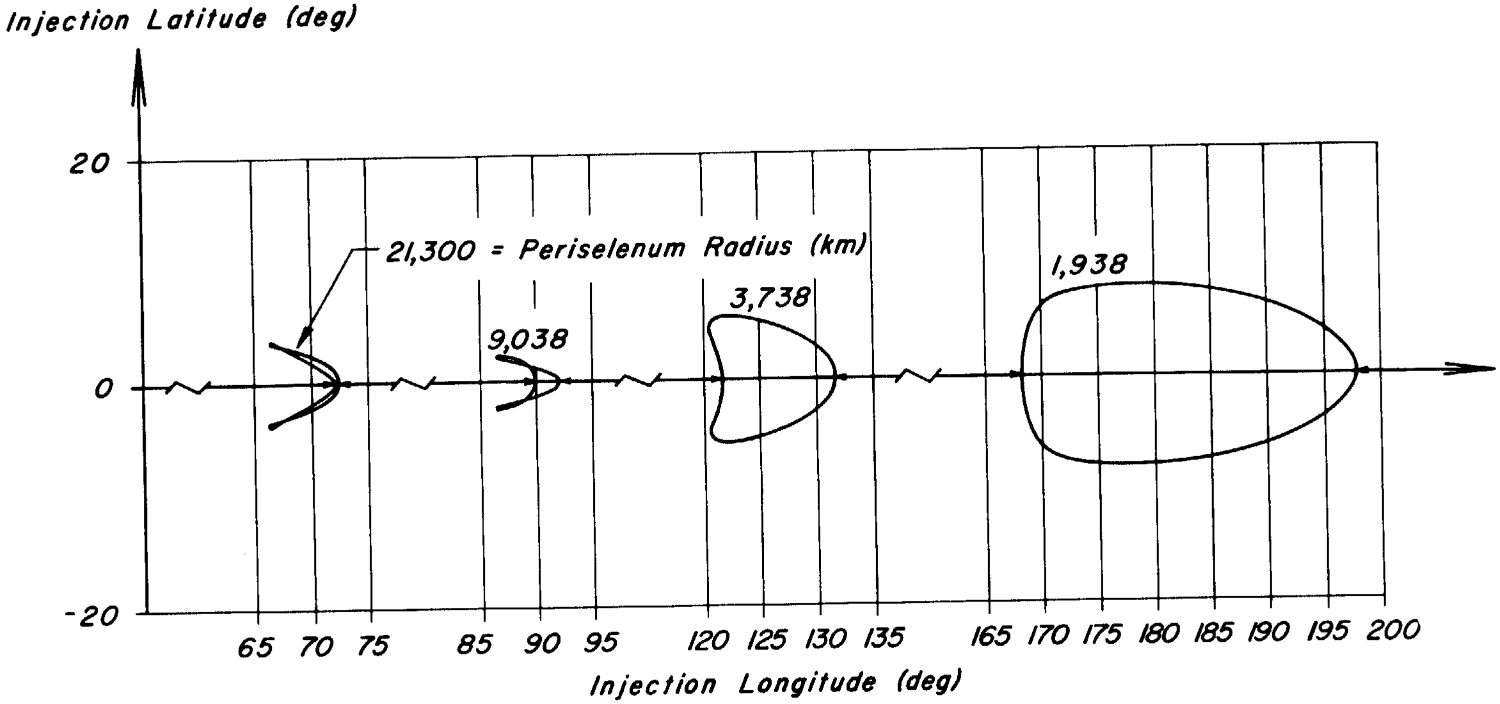


FIG. 15. INJECTION LOCI FOR CISLUNAR FREE RETURN OF THE SECOND KIND

Injection at Perigee Radius = 6,555 km

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