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TECHNICAL MEMORANDUM

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EPHEMERIS FOR THE STABLE LIBRATION POINTS OF THE EARTH-MOON SYSTEM FOR OCTOBER 1962 TO DECEMBER 1963

By William R. Wells and William H. Michael, Jr.

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Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

An ephemeris giving the position and mean variation per hour in the position of the two stable libration points L_4 and L_5 of the Earth-Moon system has been constructed for those days of each month for which the illumination is favorable for viewing possible particle clouds in the vicinity of these points. (The libration point L_4 leads the Moon's motion about the Earth by 60°, and L_5 lags by 60°.) The ephemeris covers the last quarter of 1962 and entire calendar year of 1963. Simple construction of the ephemeris was possible because of the assumption of planar motion of the Moon for large parts of the month and the employment of existing ephemerides giving the position of the Moon.

INTRODUCTION

From the literature of celestial mechanics, it is well known that there exist particular solutions (the libration points) of the restricted three-body problem. There are five particular positions relative to the two finite masses such that a small body placed at these positions with the proper initial conditions will remain there indefinitely unless acted upon by forces exterior to the three-body system. These points are called the Lagrangian points, or libration points, and all five are located in the plane of motion of the two finite masses. It can be shown that motion of a small body about any of the three colinear libration points (L_1, L_2, or L_3 of fig. 1) is unstable, but that motion about the equilateral triangle points (L_4 and L_5) is stable with sufficiently small ratios of the secondary to the primary mass.

It turns out that the mass ratios of the Earth-Moon system and the Sun-Jupiter system satisfy the criterion for stable motion about the equilateral triangle libration points. With respect to the Sun-Jupiter system, nature has provided a remarkable example of this stability in the existence of a group of asteroids, known as the Trojan asteroids, situated in the neighborhood of the L_4 and L_5 points and in motion about these points. Thirteen such asteroids have been discovered thus far, eight of which are in motion about L_4 and five about L_5.

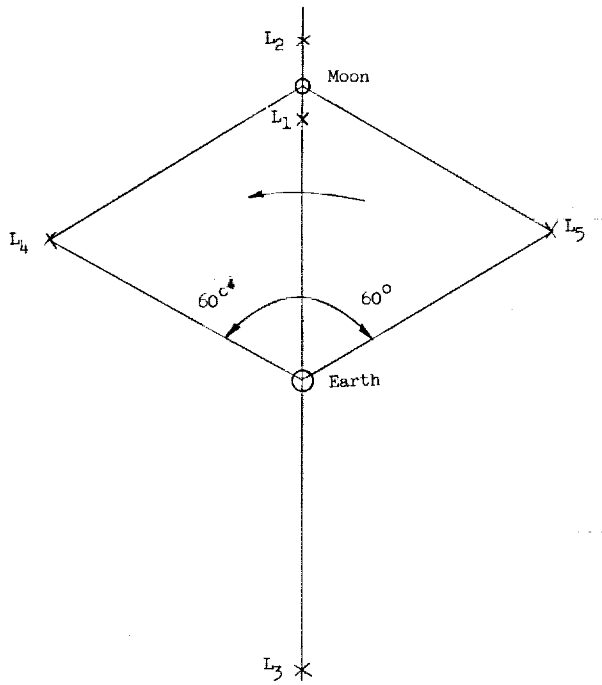


Figure 1.- The libration points of the Earth-Moon system. The line between the Earth and Moon rotates with the angular velocity of the Moon.

The existence of the Trojan asteroids naturally leads to speculation regarding the existence of objects in orbit about points L_4 and L_5 of the Earth-Moon system. A very recent report of observations of objects in orbit about point L_5 has been made by K. Kordylewski of the Crakow Observatory, as reported in reference 1 with a few additional details given in references 2 and 3.

As reported by Kordylewski, the observations were an outcome of many years of searching in the vicinity of the libration points for single meteoroids, none of which as bright as magnitude 12 were found. Instead, two faintly luminous patches near both L_4 and L_5 were discovered, suggesting perhaps a swarm of tiny particles, none of which would be observable individually. The luminous patches are described as being at least 2° in diameter, separated by about 8°, and a magnitude or two fainter than the central part of the Gegenschein. They are reported to be visible with the naked eye under extremely favorable circumstances. This is the only known report of sightings of objects orbiting about the libration points of the Earth-Moon system.

In view of the current interest in this subject, the present report provides an approximate ephemeris for the positions of the stable libration points L_4 and L_5. The data should be useful in attempts to confirm the observations reported in reference 1.

From the standpoint of illumination, the most favorable time of the month for observations of the clouds is near those times at which the libration point and the Sun are in opposition. The most favorable time for observations of L_4 occurs about 4 days before full Moon, and for L_5 about 4 days after full Moon. (The age of Moon at these respective times is approximately 10 and 20 days.) Good conditions for observations exist for at least 2 days before and after the respective oppositions, and possibly for longer periods with less favorability. For conciseness, the data given here are for five dates centered about the favorable oppositions. Other factors, such as sky brightness, interference of the Milky Way and Gegenschein, and altitude of the libration point will also influence the favorability of observation for a given date.

SYMBOLS

i	inclination of the Earth-Moon plane to the equatorial plane, deg
L	libration point
L_1,L_2,L_3	unstable colinear libration points
L_4	stable libration point which leads Moon's motion about Earth by 60°
L_5	stable libration point which lags Moon's motion about Earth by 60°
M	Moon
α_L	right ascension of libration point, hr
α_M	right ascension of Moon, hr
α_o	right ascension of Moon's ascending node, hr
$\Delta\alpha_L$	variation of right ascension of libration point, hr/hr
$\Delta\alpha_M$	variation of right ascension of Moon, hr/hr
γ	vernal equinox
δ_L	declination of libration point, deg
δ_M	declination of Moon, deg
$\Delta\delta_L$	variation of declination of libration point, deg/hr
$\Delta\delta_M$	variation of declination of Moon, deg/hr
Ω_L	location of libration point along Earth-Moon plane, hr
Ω_M	location of Moon along Earth-Moon plane, hr

GENERAL CONSIDERATIONS

Geocentric Equatorial Coordinates of Libration Points

In the construction of the present ephemeris it has been assumed that the Moon exhibits planar motion over large parts of its orbit in order that available information (see refs. 4 and 5) giving the Moon's location could be utilized. The problem then reduces to one of spherical trigonometry as shown in figure 2.

In figure 2 the quantities δ_M, α_M, and α_o are considered known since they can be taken directly from such sources as references 4 and 5. The quantities in figure 2 which are not known and which are desired are δ_L and α_L. These quantities can be obtained by the application of spherical trigonometry to figure 2 as follows:

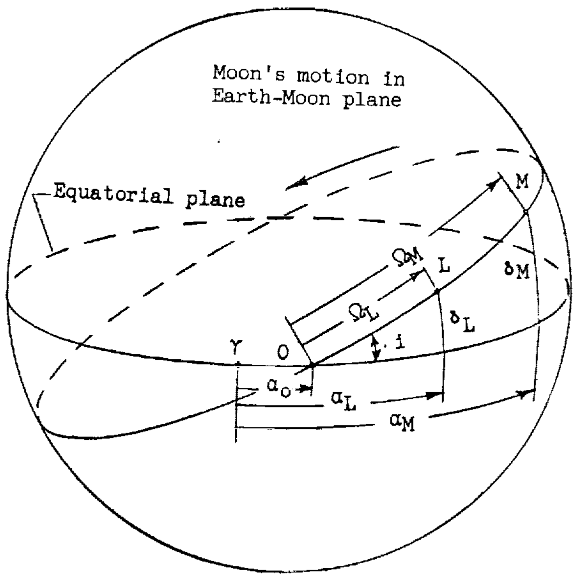


Figure 2.- Geometry describing position and motion of the libration points.

The declination of the libration point δ_L is obtained from

$$\sin \delta_L = \sin i \sin \Omega_L \tag{1}$$

where

$$\sin i = \frac{\sin \delta_M}{\sin \Omega_M} \tag{2}$$

and

$$\Omega_L = \Omega_M \pm 60^\circ \tag{3}$$

Expression (3) is a result of the fact that the stable libration points lead or lag the Moon by 60°. The positive sign is used with L_4 and the negative with L_5.

The angle Ω_M which appears in equations (2) and (3) can be obtained in terms of the knowns of the problem from the expression

$$\cos \Omega_M = \cos \delta_M \cos\left(\alpha_M - \alpha_o\right) \tag{4}$$

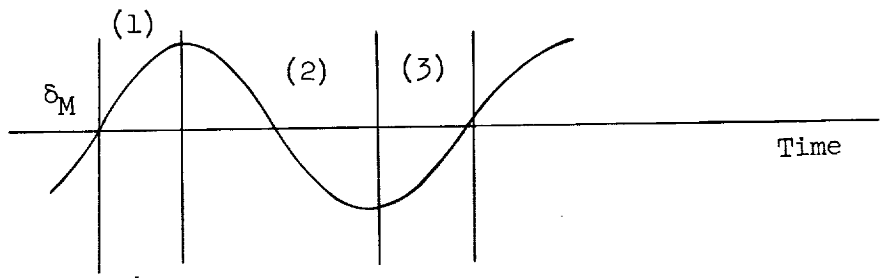
The proper quandrant for Ω_M can be determined by applying the rule: whenever $\delta_M \geqq 0^\circ$, $0 \leqq \Omega_M \leqq 180^\circ$ and whenever $\delta_M \leqq 0^\circ$, $180^\circ \leqq \Omega_M \leqq 360^\circ$. The right ascension of the libration point α_L is obtained from

$$\cos\left(\alpha_L - \alpha_o\right) = \frac{\cos \Omega_L}{\cos \delta_L} \tag{5}$$

and

$$\tan\left(\alpha_L - \alpha_o\right) = \cos i \tan \Omega_L \tag{6}$$

Since the value of α_o varies slightly from month to month throughout the year more accuracy in the calculations can be obtained if the value of α_o is changed for various parts of the month. In the calculations presented here, the following rule regarding α_o was adopted: The lunar month (Moon moves from ascending to descending node) was separated into the three regions shown below:



Sketch 1

For days in which the Moon is at the declinations shown in region (1), the value of α_o that exists when the Moon is at the ascending node for that lunar month is used. The value of α_o used for days which fall in region (2) is that which exists when the Moon is at its descending node. The value of α_o used for days in region (3) is that existing when the Moon is at the ascending node of the next lunar month.

Time Rate of Change of the Coordinates

Since the location of the libration points is to be given only for one time during each day, the hourly variation of the coordinates (right ascension and declination) at this particular time is desirable. The time rate of change of the coordinates can be obtained by differentiating with respect to time of equations (1) to (6). Differentiation of equation (1) gives

$$\Delta\delta_L = \frac{\sin i \cos \Omega_L}{\cos \delta_L} \Delta\Omega_L \tag{7}$$

From equation (3)

$$\Delta\Omega_L = \Delta\Omega_M \tag{8}$$

and from equation (4)

$$\Delta\Omega_M = \csc \Omega_M \left[\sin \delta_M \cos\left(\alpha_M - \alpha_o\right)\Delta\delta_M + \cos \delta_M \sin\left(\alpha_M - \alpha_o\right)\Delta\alpha_M\right] \tag{9}$$

Substitution of equations (8) and (9) into (7) gives

$$\Delta\delta_L = \frac{\sin i \cos \Omega_L}{\cos \delta_L \sin \Omega_M}\left[\cos \delta_M \sin\left(\alpha_M - \alpha_o\right)\Delta\alpha_M + \sin \delta_M \cos\left(\alpha_M - \alpha_o\right)\Delta\delta_M\right] \tag{10}$$

In the same manner differentiation of equation (6) gives

$$\Delta\alpha_L = \frac{\cos i \cos^2\left(\alpha_L - \alpha_o\right)}{\cos^2\Omega_L} \Delta\Omega_M \tag{11}$$

Again substitution of equations (8) and (9) into (11) gives

$$\Delta\alpha_L = \frac{\cos i \cos^2\left(\alpha_L - \alpha_o\right)}{\sin \Omega_M \cos^2\Omega_L}\left[\cos \delta_M \sin\left(\alpha_M - \alpha_o\right)\Delta\alpha_M + \sin \delta_M \cos\left(\alpha_M - \alpha_o\right)\Delta\delta_M\right] \tag{12}$$

The hourly variation of the right ascension and declination of the Moon $\Delta\alpha_M$ and $\Delta\delta_M$ are also considered as known quantities since they too can be obtained from such sources as references 4 and 5.

PRESENTATION OF RESULTS AND DISCUSSION

Given in tables I and II are the geocentric right ascension and declination and variation per hour of these quantities for the stable libration points L_4 and L_5. Results are given for five dates of each lunar month for both L_4 and L_5 for the last quarter of 1962 and the complete year of 1963. These five dates correspond to the most favorable illumination conditions of each month for viewing the points. The results apply for 0 hours ephemeris time. Values of right ascension are given in decimal parts of an hour, and declinations are given in decimal parts of a degree. The variation of right ascension is given as hours per hour and that of declination is given as degrees per hour.

The geocentric equatorial coordinates given in tables I and II can be corrected for parallax; however, the separation of the clouds at L_5 was reported in reference 1 to be about 8° as compared to a parallax less than 1°.

In addition to the condition of favorable illumination there are several other factors that affect attempts at observation of the clouds in the vicinity of the stable libration points. Since the clouds are presumably very faint (a magnitude or two fainter than the central part of the Gegenschein as reported in ref. 1), such factors as sky brightness, possible interference with the Gegenschein, and possible interference with the Milky Way can further limit the observations. Because of sky brightness the observations should be made when the Moon is not above the horizon. This means L_4 will be observable after moonset (morning hours) and L_5 before moonrise (evening hours). Interference of the Milky Way occurs whenever the libration points are near the nodes of the ecliptic and galactic equator. The range of right ascensions for this situation is between about 5.3 and 6.3 hours and between about 16.0 and 18.5 hours.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 25, 1963.

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3. Anon.: More About the Earth's Cloud Satellites. Sky and Telescope, vol. XXII, no. 2, Aug. 1961, pp. 63, 83.

4. Anon.: The American Ephemeris and Nautical Almanac for the Year 1962. Nautical Almanac Office, U.S. Naval Observatory, 1960.

5. Anon.: The American Ephemeris and Nautical Almanac for the Year 1963. Nautical Almanac Office, U.S. Naval Observatory, 1961.

TABLE I.- RIGHT ASCENSION, DECLINATION, AND THEIR HOURLY

VARIATION OF THE STABLE LIBRATION POINTS FOR 1962

L	Date	α_L, hr	δ_L, deg	$\Delta\alpha_L$, hr/hr	$\Delta\delta_L$, deg/hr
4	Oct. 7	23.17	-8.77	0.0351	0.1786
	8	.02	-4.24	.0355	.1978
	9	.88	.65	.0365	.2084
	10	1.77	5.67	.0380	.2077
	11	2.71	10.51	.0400	.1929
5	Oct. 17	0.81	0.27	0.0368	0.2105
	18	1.68	5.19	.0358	.1976
	19	2.53	9.69	.0353	.1759
	20	3.38	13.59	.0352	.1476
	21	4.22	16.76	.0352	.1144
4	Nov. 5	0.62	-0.92	0.0350	0.2022
	6	1.48	3.98	.0361	.2044
	7	2.36	8.80	.0378	.1054
	8	3.29	13.25	.0399	.1731
	9	4.28	17.00	.0421	.1360
5	Nov. 15	2.20	7.91	0.0364	0.1938
	16	3.07	12.24	.0362	.1661
	17	3.94	15.83	.0361	.1320
	18	4.80	18.55	.0361	.0938
	19	5.67	20.32	.0360	.0534
4	Dec. 5	3.03	12.01	0.0380	0.1775
	6	3.96	15.92	.0399	.1464
	7	4.94	18.94	.0418	.1029
	8	5.96	20.78	.0432	.0488
	9	7.01	21.24	.0436	-.0110
5	Dec. 15	4.47	17.60	0.0369	0.1136
	16	5.35	19.83	.0368	.0718
	17	6.23	21.04	.0365	.0289
	18	7.10	21.23	.0358	-.0187
	19	7.95	20.44	.0350	-.0520

TABLE II.- RIGHT ASCENSION, DECLINATION, AND THEIR HOURLY VARIATION OF THE STABLE LIBRATION POINTS FOR 1963

L	Date	α_L, hr	δ_L, deg	$\Delta\alpha_L$, hr/hr	$\Delta\delta_L$, deg/hr
4	Jan. 4	5.69	20.41	0.0417	0.0638
	5	6.70	21.28	.0423	.0081
	6	7.71	20.80	.0419	-.0482
	7	8.70	19.01	.0407	-.0996
	8	9.66	16.10	.0390	-.1418
5	Jan. 14	6.79	21.30	0.0366	0.0031
	15	7.65	20.87	.0357	-.0382
	16	8.50	19.50	.0347	-.0754
	17	9.32	17.29	.0336	-.1078
	18	10.11	14.36	.0326	-.1351
4	Feb. 3	8.46	19.58	0.0403	-0.0857
	4	9.41	17.00	.0386	-.1283
	5	10.31	13.51	.0369	-.1606
	6	11.18	9.37	.0353	-.1821
	7	12.01	4.85	.0341	-.1931
5	Feb. 13	9.05	18.13	0.0341	-0.0984
	14	9.85	15.41	.0329	-.1271
	15	10.63	12.08	.0320	-.1501
	16	11.39	8.25	.0315	-.1678
	17	12.14	4.07	.0314	-.1803
4	Mar. 4	10.11	14.36	0.0370	-0.1545
	5	10.98	10.37	.0352	-.1769
	6	11.80	5.95	.0338	-.1893
	7	12.60	1.34	.0329	-.1928
	8	13.39	-3.25	.0325	-.1882
5	Mar. 14	10.40	13.15	0.0322	-0.1452
	15	11.16	9.43	.0314	-.1638
	16	11.91	5.34	.0311	-.1767
	17	12.65	1.00	.0312	-.1841
	18	13.41	-3.45	.0319	-.1857

TABLE II.- RIGHT ASCENSION, DECLINATION, AND THEIR HOURLY VARIATION OF THE STABLE LIBRATION POINTS FOR 1963 - Continued

L	Date	α_L, hr	δ_L, deg	$\Delta\alpha_L$, hr/hr	$\Delta\delta_L$, deg/hr
4	April 2	11.62	6.98	0.0342	-0.1906
	3	12.42	2.34	.0329	-.1947
	4	13.20	-2.30	.0323	-.1909
	5	13.97	-6.77	.0321	-.1802
	6	14.75	-10.91	.0324	-.1631
5	April 13	12.46	2.20	0.0309	-0.1842
	14	13.20	-2.25	.0313	-.1863
	15	13.96	-6.68	.0322	-.1819
	16	14.75	-10.93	.0336	-.1703
	17	15.58	-14.79	.0356	-.1500
4	May 2	13.79	-5.68	0.0323	-0.1867
	3	14.56	-9.98	.0323	-.1700
	4	15.34	-13.81	.0328	-.1477
	5	16.14	-17.03	.0335	-.1199
	6	16.95	-19.53	.0343	-.0870
5	May 12	13.78	-5.57	0.0318	-0.1857
	13	14.55	-9.91	.0330	-.1751
	14	15.36	-13.90	.0346	-.1563
	15	16.22	-17.34	.0366	-.1283
	16	17.12	-19.98	.0387	-.0905
4	May 31	15.13	-12.77	0.0329	-0.1577
	June 1	15.92	-16.24	.0334	-.1303
	2	16.74	-18.53	.0332	-.0932
	3	17.56	-20.92	.0349	-.0617
	4	18.40	-21.94	.0354	-.0220
5	June 10	15.17	-12.97	0.0344	-0.1640
	11	16.02	-16.60	.0361	-.1374
	12	16.91	-19.48	.0380	-.1012
	13	17.84	-21.38	.0397	-.0559
	14	18.81	-22.11	.0409	-.0036

TABLE II.- RIGHT ASCENSION, DECLINATION, AND THEIR HOURLY VARIATION OF THE STABLE LIBRATION POINTS FOR 1963 - Continued

L	Date	α_L, hr	δ_L, deg	$\Delta\alpha_L$, hr/hr	$\Delta\delta_L$, deg/hr
4	June 30	17.31	-20.44	0.0348	-0.0744
	July 1	18.15	-21.76	.0353	-.0352
	2	19.00	-22.11	.0356	.0059
	3	19.86	-21.47	.0356	.0475
	4	20.71	-19.85	.0353	.0877
5	July 10	17.62	-21.04	0.0397	-0.0686
	11	18.59	-22.07	.0409	-.0166
	12	19.57	-21.80	.0412	.0390
	13	20.56	-20.21	.0408	.0932
	14	21.53	-17.37	.0399	.1416
4	July 29	18.72	-22.12	0.0355	-0.0076
	30	19.58	-21.81	.0355	.0338
	31	20.42	-20.51	.0353	.0743
	Aug. 1	21.27	-18.26	.0349	.1122
	2	22.10	-15.15	.0345	.1461
5	Aug. 8	19.32	-22.02	0.0421	0.0256
	9	20.32	-20.72	.0416	.0825
	10	21.31	-18.12	.0405	.1330
	11	22.26	-14.42	.0390	.1733
	12	23.18	-9.90	.0376	.2016
4	Aug. 27	20.14	-21.10	0.0351	0.0614
	28	20.98	-19.17	.0346	.0995
	29	21.80	-16.36	.0342	.1341
	30	22.62	-12.78	.0338	.1642
	31	23.43	-8.53	.0338	.1887
5	Sept. 6	21.02	-19.06	0.0420	0.1238
	7	22.01	-15.53	.0404	.1695
	8	22.96	-11.05	.0387	.2014
	9	23.87	-5.98	.0372	.2190
	10	.75	-.65	.0363	.2231

TABLE II.- RIGHT ASCENSION, DECLINATION, AND THEIR HOURLY VARIATION OF THE STABLE LIBRATION POINTS FOR 1963 - Concluded

L	Date	α_L, hr	δ_L, deg	$\Delta\alpha_L$, hr/hr	$\Delta\delta_L$, deg/hr
4	Sept. 26	22.33	-14.14	0.0333	0.1535
	27	23.13	-10.15	.0331	.1789
	28	23.92	-5.61	.0333	.1985
	29	.73	-.69	.0341	.2111
	30	1.56	4.44	.0354	.2146
5	Oct. 6	23.58	-7.66	0.0389	0.2246
	7	.50	-2.14	.0376	.2325
	8	1.39	3.38	.0368	.2259
	9	2.27	8.61	.0365	.2072
	10	3.15	13.26	.0367	.1786
4	Oct. 25	23.67	-7.28	0.0323	0.1901
	26	.45	-2.54	.0327	.2039
	27	1.24	2.44	.0338	.2106
	28	2.07	7.49	.0355	.2081
	29	2.95	12.34	.0378	.1937
5	Nov. 5	1.97	6.88	0.0378	0.2254
	6	2.88	11.97	.0378	.1970
	7	3.79	16.26	.0380	.1590
	8	4.70	19.56	.0382	.1144
	9	5.62	21.74	.0382	.0662
4	Nov. 24	1.80	5.79	0.0337	0.2053
	25	2.63	10.62	.0356	.1954
	26	3.51	15.07	.0381	.1736
	27	4.46	18.83	.0409	.1374
	28	5.48	21.54	.0436	.0857
5	Dec. 4	3.43	14.68	0.0392	0.1831
	5	4.38	18.55	.0394	.1373
	6	5.33	21.24	.0394	.0862
	7	6.27	22.68	.0389	.0338
	8	7.19	22.88	.0379	-.0163
4	Dec. 24	4.16	17.74	0.0386	0.1462
	25	5.12	20.75	.0411	.1026
	26	6.13	22.56	.0432	.0466
	27	7.18	22.91	.0444	-.0181
	28	8.25	21.68	.0442	-.0843

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