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Code 1

TECHNICAL NOTE

D-1085

EQUATIONS FOR THE NEWTONIAN STATIC AND DYNAMIC AERODYNAMIC COEFFICIENTS FOR A BODY OF REVOLUTION WITH AN OFFSET CENTER-OF-GRAVITY LOCATION

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON June 1963

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SUMMARY

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Equations which are easily adaptable to machine computation are presented for the preliminary prediction of the supersonic and hypersonic stability characteristics of bodies of revolution with an arbitrary center-of-gravity location. The equations have been programed and an example of their application to a blunt reentry configuration is presented.

INTRODUCTION

The Newtonian theory has proven to be a useful tool for the preliminary prediction of supersonic and hypersonic stability characteristics. A general background in the application of the theory may be obtained from references 1 to 4. The machine computation of the Newtonian static stability characteristics of bluff entry bodies with various heat-shield curvatures, afterbody angles, and corner edge radii is in agreement with experiments on Apollo type reentry configurations. (For example, see ref. 5.) Since entry lift-to-drag ratio may be obtained by offsetting the center of gravity, the program was extended to the computation of both static and dynamic coefficients for bodies of revolution with an offset center of gravity at angles of attack, angles of sideslip, and angles of roll. The purpose of this paper is to present equations which are readily machine adaptable for the computation of the Newtonian static and dynamic aerodynamic coefficients for a body of revolution with an offset center of gravity at angles of attack, sideslip, or roll, or any combination of these angles.

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SYMBOLS

A,B,C,E,F,G	defined algebraic expressions
C_C	axial-force coefficient, $\frac{4F_X}{\pi d^2 q_\infty}$
C_D	drag coefficient, $\frac{(4)(\text{Drag})}{\pi d^2 q_\infty}$
C_L	lift coefficient, $\frac{(4)(\text{Lift})}{\pi d^2 q_\infty}$
C_l	rolling-moment coefficient, $\frac{-4M_X}{\pi d^3 q_\infty}$
C_m	pitching-moment coefficient, $\frac{4M_Y}{\pi d^3 q_\infty}$
C_N	normal-force coefficient, $\frac{4F_Z}{\pi d^2 q_\infty}$
C_n	yawing-moment coefficient, $\frac{-4M_Z}{\pi d^3 q_\infty}$
C_p	Newtonian pressure coefficient, $2\left(\frac{V_N}{V_\infty}\right)^2$
C_Y	lateral-force coefficient, $\frac{4F_Y}{\pi d^2 q_\infty}$
C_{l_p},C_{n_r},C_{m_q}	roll, yaw, and pitch damping coefficients
d	characteristic diameter
F_X	axial force
F_Y	lateral force

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F_Z	normal force
$\vec{i},\vec{j},\vec{k}$	unit vectors for body coordinates
L/D	lift-to-drag ratio
M	Mach number
M_X	rolling moment
M_Y	pitching moment
M_Z	yawing moment
$\frac{\vec{n}}{\lvert n \rvert}$	unit vector normal to surface, positive inward
$\vec{P},\vec{Q},\vec{R}$	surface vectors in roll, pitch, and yaw planes
p,q,r	roll, pitch, and yaw rates about body-axis system
q_∞	free-stream dynamic pressure
R	body radius
V_N	relative velocity, normal to surface
V_∞	free-stream velocity
X,Y,Z	body coordinates
x,y,z	distance along body axes
x_O,z_O	center-of-gravity coordinates
$\left(\frac{x}{d}\right)_L,\left(\frac{x}{d}\right)_M$	lower and maximum integration limits for $\frac{x}{d}$
α	angle of attack
β	angle of sideslip
δ	surface slope from body axis, $\delta\left(\frac{x}{d}\right)$
θ	cylindrical coordinate angle

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θ_1, θ_2	flow-see boundary limits
θ_a	body afterbody angle
ρ	radius vector for cylindrical coordinates, $\rho\left(\frac{x}{d}\right)$
ϕ	roll angle

Subscripts (see fig. 4):

a,v	afterbody - vertex juncture
c	corner edge
c,a	corner edge - afterbody juncture
n	nose
n,c	nose - corner edge juncture
v	vertex
vr	vertex radius

DEVELOPMENT OF EQUATIONS

The Conventions and Basic Relations

The vector directions used in this paper are found in the diagram of figure 1. The respective yaw, pitch, and roll rate vectors (see fig. 2) are

$$\vec{r} = -r\vec{k} \tag{1a}$$

$$\vec{q} = q\vec{j} \tag{1b}$$

$$\vec{p} = -p\vec{i} \tag{1c}$$

The surface vectors normal to their respective rotation axes (fig. 2) are

$$\vec{R} = \left(x - x_0\right)\vec{i} - \rho \cos \theta\vec{j} \tag{2a}$$

$$\vec{Q} = \left(x - x_0\right)\vec{i} + \left(\rho \sin \theta - z_0\right)\vec{k} \tag{2b}$$

$$\vec{P} = - \rho \cos \theta\vec{j} + \left(\rho \sin \theta - z_0\right)\vec{k} \tag{2c}$$

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The inwardly directed unit vector, normal to the surface, is

$$\frac{\vec{n}}{|n|} = \sin \delta \vec{i} + \cos \delta \cos \theta \vec{j} - \cos \delta \sin \theta \vec{k} \tag{3}$$

The unit free-stream velocity vector from figure 3 is

$$\begin{aligned} \frac{\vec{V}_\infty}{|V_\infty|} = {} & \cos \alpha \cos \beta \vec{i} - (\sin \alpha \cos \beta \sin \phi + \sin \beta \cos \phi)\vec{j} \\ & + (\sin \alpha \cos \beta \cos \phi - \sin \beta \sin \phi)\vec{k} \end{aligned} \tag{4}$$

The relative velocity, normal to the surface, may be expressed as

$$V_N = \vec{V}_\infty \cdot \frac{\vec{n}}{|n|} - \vec{p} \times \vec{P} \cdot \frac{\vec{n}}{|n|} - \vec{q} \times \vec{Q} \cdot \frac{\vec{n}}{|n|} - \vec{r} \times \vec{R} \cdot \frac{\vec{n}}{|n|} \tag{5}$$

The local surface pressure coefficient, according to the Newtonian theory, is

$$C_p = 2\left(\frac{V_N}{V_\infty}\right)^2 = 2(A \cos \theta - B \sin \theta + C)^2 \tag{6}$$

where

$$A = -\cos \delta\left[\sin \alpha \cos \beta \sin \phi + \sin \beta \cos \phi - \frac{rd}{V_\infty}\left(\frac{\rho}{d} \tan \delta + \frac{x - x_0}{d}\right) - \frac{pd}{V_\infty} \frac{z_0}{d}\right]$$

$$B = \cos \delta\left[\sin \alpha \cos \beta \cos \phi - \sin \beta \sin \phi + \frac{qd}{V_\infty}\left(\frac{\rho}{d} \tan \delta + \frac{x - x_0}{d}\right)\right]$$

$$C = \sin \delta\left(\cos \alpha \cos \beta + \frac{qd}{V_\infty} \frac{z_0}{d}\right)$$

The coefficients may be expressed as

$$C_C = \frac{4}{\pi} \int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \int_{\theta_1}^{\theta_2} \frac{\rho}{d} C_p \tan \delta \, d\theta \, d\left(\frac{x}{d}\right) \tag{7a}$$

$$C_Y = \frac{4}{\pi} \int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \int_{\theta_1}^{\theta_2} \frac{\rho}{d} C_p \cos \theta \, d\theta \, d\left(\frac{x}{d}\right) \tag{7b}$$

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$$C_N = -\frac{4}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M}\int_{\theta_1}^{\theta_2} \frac{\rho}{d}\, C_p \sin\theta \, d\theta \, d\left(\frac{x}{d}\right) \tag{7c}$$

$$C_l = -\frac{4}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M}\int_{\theta_1}^{\theta_2} \frac{\rho}{d}\,\frac{z_0}{d}\, C_p \cos\theta \, d\theta \, d\left(\frac{x}{d}\right) \tag{7d}$$

$$C_m = \frac{4}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M}\int_{\theta_1}^{\theta_2} \frac{\rho}{d}\, C_p\left[\left(\frac{x-x_0}{d} + \frac{\rho}{d}\tan\delta\right)\sin\theta - \frac{z_0}{d}\tan\delta\right] d\theta \, d\left(\frac{x}{d}\right) \tag{7e}$$

$$C_n = -\frac{4}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M}\int_{\theta_1}^{\theta_2} \frac{\rho}{d}\, C_p\left(\frac{x-x_0}{d} + \frac{\rho}{d}\tan\delta\right)\cos\theta \, d\theta \, d\left(\frac{x}{d}\right) \tag{7f}$$

$$C_{l_p} = \frac{V_\infty}{d}\left(\frac{\partial C_l}{\partial p}\right) \tag{7g}$$

$$C_{m_q} = \frac{V_\infty}{d}\left(\frac{\partial C_m}{\partial q}\right) \tag{7h}$$

$$C_{n_r} = \frac{V_\infty}{d}\left(\frac{\partial C_n}{\partial r}\right) \tag{7i}$$

Flow-See Boundary

The flow-see boundary is given by the integration limits θ_1 and θ_2 established at each $\frac{x}{d}$ station.

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For $0 \leq \cos^{-1}(\cos \alpha \cos \beta) \leq \frac{\pi}{2}$ and all values of $\frac{x}{d}$ where $\delta - \cos^{-1}(\cos \alpha \cos \beta) \geq 0$,

$$\theta_1 = -\frac{\pi}{2}$$

and

$$\theta_2 = \frac{3\pi}{2}$$

The upper limit of the integration with respect to $\frac{x}{d}$ is given by the condition where $\delta + \cos^{-1}(\cos \alpha \cos \beta) = 0$.

For $\frac{\pi}{2} < \cos^{-1}(\cos \alpha \cos \beta) \leq \pi$ and all values of $\frac{x}{d}$ where $\delta + \left[\pi - \cos^{-1}(\cos \alpha \cos \beta)\right] \leq 0$,

$$\theta_1 = -\frac{\pi}{2}$$

and

$$\theta_2 = \frac{3\pi}{2}$$

The lower limit of the integration with respect to $\frac{x}{d}$ is given by the condition where $\delta - \left[\pi - \cos^{-1}(\cos \alpha \cos \beta)\right] = 0$.

For values of $\frac{x}{d}$ not included in the above two paragraphs, θ_1 and θ_2 are found by setting the pressure coefficient equal to zero.

$$\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{-AC}{A^2 + B^2} - \sqrt{\frac{A^2C^2}{(A^2 + B^2)^2} + \frac{B^2 - C^2}{A^2 + B^2}}\right) \\ &= \sin^{-1}\left(\frac{BC}{A^2 + B^2} + \sqrt{\frac{B^2C^2}{(A^2 + B^2)^2} + \frac{A^2 - C^2}{A^2 + B^2}}\right) \end{aligned} \tag{8}$$

and

$$\begin{aligned} \theta_2 &= \cos^{-1}\left(\frac{-AC}{A^2 + B^2} + \sqrt{\frac{A^2C^2}{(A^2 + B^2)^2} + \frac{B^2 - C^2}{A^2 + B^2}}\right) \\ &= \sin^{-1}\left(\frac{BC}{A^2 + B^2} - \sqrt{\frac{B^2C^2}{(A^2 + B^2)^2} + \frac{A^2 - C^2}{A^2 + B^2}}\right) \end{aligned} \tag{9}$$

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Coefficients

Substitution of the pressure coefficient (eq. 6) into the coefficient equations (eqs. (7a) to (7i)) enables an integration of the coefficients with respect to the coordinate θ. The resulting coefficients expressed below require only one integration with respect to the axial coordinate of the body selected. For other then simple bodies the coefficients may be obtained by a machine integration for each combination of α, β, ϕ, p, q, and r.

$$C_C = \frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d} \tan \delta\left[\frac{1}{2}\left(A^2 + B^2 + 2C^2\right)\left(\theta_2 - \theta_1\right)\right.$$

$$+ \frac{1}{4}\left(A^2 - B^2\right)\left(\sin 2\theta_2 - \sin 2\theta_1\right) + 2AC\left(\sin \theta_2 - \sin \theta_1\right)$$

$$\left. + 2BC\left(\cos \theta_2 - \cos \theta_1\right) - AB\left(\sin^2 \theta_2 - \sin^2 \theta_1\right)\right] d\left(\frac{x}{d}\right) \tag{10a}$$

$$C_Y = \frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\left[\frac{1}{3}\left(B^2 - A^2\right)\left(\sin^3 \theta_2 - \sin^3 \theta_1\right)\right.$$

$$+ \left(A^2 + C^2\right)\left(\sin \theta_2 - \sin \theta_1\right) + AC\left(\theta_2 - \theta_1 + \frac{1}{2} \sin 2\theta_2 - \frac{1}{2} \sin 2\theta_1\right)$$

$$\left. - BC\left(\sin^2 \theta_2 - \sin^2 \theta_1\right) + \frac{2}{3} AB\left(\cos^3 \theta_2 - \cos^3 \theta_1\right)\right] d\left(\frac{x}{d}\right) \tag{10b}$$

$$C_N = -\frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\left[\frac{1}{3}\left(B^2 - A^2\right)\left(\cos^3 \theta_2 - \cos^3 \theta_1\right) - \left(C^2 + B^2\right)\left(\cos \theta_2 - \cos \theta_1\right)\right.$$

$$+ AC\left(\sin^2 \theta_2 - \sin^2 \theta_1\right) - BC\left(\theta_2 - \theta_1 - \frac{1}{2} \sin 2\theta_2 + \frac{1}{2} \sin 2\theta_1\right)$$

$$\left. - \frac{2}{3} AB\left(\sin^3 \theta_2 - \sin^3 \theta_1\right)\right] d\left(\frac{x}{d}\right) \tag{10c}$$

$$C_l = -\frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\,\frac{z_0}{d}\left[\frac{1}{3}\left(B^2 - A^2\right)\left(\sin^3\theta_2 - \sin^3\theta_1\right)\right.$$

$$+ \left(A^2 + C^2\right)\left(\sin\theta_2 - \sin\theta_1\right) + AC\left(\theta_2 - \theta_1 + \frac{1}{2}\sin 2\theta_2 - \frac{1}{2}\sin 2\theta_1\right)$$

$$\left. - BC\left(\sin^2\theta_2 - \sin^2\theta_1\right) + \frac{2}{3}AB\left(\cos^3\theta_2 - \cos^3\theta_1\right)\right]d\left(\frac{x}{d}\right) \tag{10d}$$

$$C_m = \frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\left(\frac{x-x_0}{d} + \frac{\rho}{d}\tan\delta\right)\left[\frac{1}{3}\left(B^2 - A^2\right)\left(\cos^3\theta_2 - \cos^3\theta_1\right)\right.$$

$$-\left(C^2 + B^2\right)\left(\cos\theta_2 - \cos\theta_1\right) + AC\left(\sin^2\theta_2 - \sin^2\theta_1\right)$$

$$- BC\left(\theta_2 - \theta_1 - \frac{1}{2}\sin 2\theta_2 + \frac{1}{2}\sin 2\theta_1\right) \tag{10e}$$

$$\left. - \frac{2}{3}AB\left(\sin^3\theta_2 - \sin^3\theta_1\right)\right]d\left(\frac{x}{d}\right) - \frac{z_0}{d}C_C$$

$$C_n = -\frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\left(\frac{x-x_0}{d} + \frac{\rho}{d}\tan\delta\right)\left[\frac{1}{3}\left(B^2 - A^2\right)\left(\sin^3\theta_2 - \sin^3\theta_1\right)\right.$$

$$+ \left(A^2 + C^2\right)\left(\sin\theta_2 - \sin\theta_1\right) + AC\left(\theta_2 - \theta_1 + \frac{1}{2}\sin 2\theta_2 - \frac{1}{2}\sin 2\theta_1\right)$$

$$\left. - BC\left(\sin^2\theta_2 - \sin^2\theta_1\right) + \frac{2}{3}AB\left(\cos^3\theta_2 - \cos^3\theta_1\right)\right]d\left(\frac{x}{d}\right) \tag{10f}$$

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$$
\begin{aligned}
C_{l_p} = -\frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\left(\frac{z_0}{d}\right)^2 \cos^2\delta\bigg[2\left(F + E\frac{rd}{V_\infty} + \frac{pd}{V_\infty}\frac{z_0}{d}\right)\bigg(\sin\theta_2 - \sin\theta_1 \\
- \frac{1}{3}\sin^3\theta_2 + \frac{1}{3}\sin^3\theta_1\bigg) + \tan\delta\left(\frac{qd}{V_\infty}\frac{z_0}{d} + \cos\alpha\cos\beta\right)\bigg(\theta_2 - \theta_1 \\
+ \frac{1}{2}\sin 2\theta_2 - \frac{1}{2}\sin 2\theta_1\bigg) + \frac{2}{3}\left(E\frac{qd}{V_\infty} + G\right)\left(\cos^3\theta_2 - \cos^3\theta_1\right)\bigg] d\left(\frac{x}{d}\right)
\end{aligned}
\tag{10g}
$$

$$
\begin{aligned}
C_{m_q} = \frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d} E\cos^2\delta\bigg\{\frac{2}{3} E\left(\frac{qd}{V_\infty}E + G\right)\left(\cos^3\theta_2 - \cos^3\theta_1\right) \\
- 2\left[\tan^2\delta\,\frac{z_0}{d}\left(\frac{qd}{V_\infty}\frac{z_0}{d} + \cos\alpha\cos\beta\right) + \frac{qd}{V_\infty}E^2 + EG\right]\left(\cos\theta_2 - \cos\theta_1\right) \\
+ \frac{z_0}{d}\tan\delta\left(\frac{pd}{V_\infty}\frac{z_0}{d} + F + E\frac{rd}{V_\infty}\right)\left(\sin^2\theta_2 - \sin^2\theta_1\right) \\
- \tan\delta\left(2\frac{qd}{V_\infty}\frac{z_0}{d}E + \frac{z_0}{d}G + E\cos\alpha\cos\beta\right)\bigg(\theta_2 - \theta_1 - \frac{1}{2}\sin 2\theta_2 \\
+ \frac{1}{2}\sin 2\theta_1\bigg) - \frac{2}{3}E\left(\frac{pd}{V_\infty}\frac{z_0}{d} + F + \frac{rd}{V_\infty}E\right)\left(\sin^3\theta_2 - \sin^3\theta_1\right)\bigg\} d\left(\frac{x}{d}\right) \\
+ \frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d}\frac{z_0}{d}\sin\delta\cos\delta\bigg\{\bigg[\tan^2\delta\left(\frac{qd}{V_\infty}E^2 + EG + 2\frac{z_0}{d}\right)\bigg(\frac{qd}{V_\infty}\frac{z_0}{d} \\
+ \cos\alpha\cos\beta\bigg)\bigg]\left(\theta_2 - \theta_1\right) - \frac{1}{2}E\left(\frac{qd}{V_\infty}E + G\right)\left(\sin 2\theta_2 - \sin 2\theta_1\right) \\
+ 2\frac{z_0}{d}\tan\delta\left(\frac{pd}{V_\infty}\frac{z_0}{d} + F + E\frac{rd}{V_\infty}\right)\left(\sin\theta_2 - \sin\theta_1\right) \\
+ 2\tan\delta\left(2\frac{qd}{V_\infty}\frac{z_0}{d} + E + E\cos\alpha\cos\beta + \frac{z_0}{d}G\right)\left(\cos\theta_2 - \cos\theta_1\right) \\
- E\left(F + \frac{rd}{V_\infty}E + \frac{pd}{V_\infty}\frac{z_0}{d}\right)\left(\sin^2\theta_2 - \sin^2\theta_1\right)\bigg\} d\left(\frac{x}{d}\right)
\end{aligned}
\tag{10h}
$$

$$C_{n_r} = -\frac{8}{\pi}\int_{\left(\frac{x}{d}\right)_L}^{\left(\frac{x}{d}\right)_M} \frac{\rho}{d} E^2 \cos^2 \delta\left[2\left(\sin \theta_2 - \sin \theta_1 - \frac{1}{3} \sin^3 \theta_2\right.\right.$$

$$\left. + \frac{1}{3} \sin^3 \theta_1\right)\left(E \frac{rd}{V_\infty} + F + \frac{pd}{V_\infty} \frac{z_0}{d}\right) + \tan \delta\left(\theta_2 - \theta_1 + \frac{1}{2} \sin 2\theta_1\right.$$

$$\left. - \frac{1}{2} \sin 2\theta_1\right)\left(\cos \alpha \cos \beta + \frac{qd}{V_\infty} \frac{z_0}{d}\right) + \frac{2}{3}\left(\cos^3 \theta_2\right.$$

$$\left.\left. - \cos^3 \theta_1\right)\left(G + E \frac{qd}{V_\infty}\right)\right] d\left(\frac{x}{d}\right) \tag{10i}$$

where

$$E = \frac{x-x_0}{d} + \frac{\rho}{d} \tan \delta$$

$$F = -(\sin \alpha \cos \beta \sin \phi + \sin \beta \cos \phi)$$

$$G = \sin \alpha \cos \beta \cos \phi - \sin \beta \sin \phi$$

TYPICAL APPLICATIONS OF EQUATIONS

The body force, moment, and damping coefficient equations have been programed on an IBM 7090 computer. The typical blunt reentry configuration geometry, as presented in figure 4, has been programed into the body coefficient expressions. The body equations for the configuration shown in figure 4 are:

For

$$0 \leq \frac{x}{d} \leq \frac{x_{n,c}}{d}$$

then

$$\frac{\rho}{d} = \sqrt{\left(\frac{R_n}{d}\right)^2 - \left(\frac{R_n}{d} - \frac{x}{d}\right)^2} \tag{11a}$$

and

$$\delta = \tan^{-1}\left(\frac{\frac{R_n}{d} - \frac{x}{d}}{\frac{\rho}{d}}\right) \tag{11b}$$

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For

$$\frac{x_{n,c}}{d} \leq \frac{x}{d} \leq \frac{x_{c,a}}{d}$$

then

$$\frac{\rho}{d} = \frac{1}{2} - \frac{R_c}{d} + \sqrt{\left(\frac{R_c}{d}\right)^2 - \left(\frac{x}{d} - \frac{x_c}{d}\right)^2} \tag{12a}$$

and

$$\delta = \tan^{-1} \frac{\frac{x_c}{d} - \frac{x}{d}}{\sqrt{\left(\frac{R_c}{d}\right)^2 - \left(\frac{x}{d} - \frac{x_c}{d}\right)^2}} \tag{12b}$$

For

$$\frac{x_{c,a}}{d} \leq \frac{x}{d} \leq \frac{x_{a,v}}{d}$$

then

$$\frac{\rho}{d} = \frac{R_{c,a}}{d} - \left(\frac{x}{d} - \frac{x_{c,a}}{d}\right)\tan\theta_a \tag{13a}$$

and

$$\delta = -\theta_a \tag{13b}$$

For

$$\frac{x_{a,v}}{d} \leq \frac{x}{d} \leq \frac{x_v}{d}$$

then

$$\frac{\rho}{d} = \sqrt{\left(\frac{R_v}{d}\right)^2 - \left(\frac{x}{d} - \frac{x_{v,r}}{d}\right)^2} \tag{14a}$$

and

$$\delta = \tan^{-1}\left(\frac{\frac{x_{v,r}}{d} - \frac{x}{d}}{\frac{\rho}{d}}\right) \tag{14b}$$

A computed case for a typical blunt reentry configuration with $\frac{R_n}{d} = 1.2$, $\frac{R_c}{d} = 0.05$, $\theta_a = 33°$, and $\frac{R_v}{d} = 0.1$ is compared with theory in figure 5. The constants for the configuration shown in figure 4 for this case are:

$$\frac{R_n}{d} = 1.2 \qquad \frac{R_c}{d} = 0.05$$

$$\theta_a = 33° \qquad \frac{R_v}{d} = 0.1$$

$$\frac{x_{n,c}}{d} = 0.09568 \qquad \frac{x_c}{d} = 0.1417$$

$$\frac{x_{c,a}}{d} = 0.1689 \qquad \frac{R_{c,a}}{d} = 0.4919$$

$$\frac{x_{a,v}}{d} = 0.7973 \qquad \frac{x_v}{d} = 0.8428$$

$$\frac{x_{v,r}}{d} = 0.7428$$

DISCUSSION

Typical Application

Results. - Generally good agreement with experiment was obtained for both the static and dynamic coefficients, as shown in figure 5. The experimental results for the static coefficients were extracted from unpublished data obtained in the 21-inch hypersonic tunnel of Jet Propulsion Laboratory, California Institute of Technology. The experimental results for the dynamic coefficients were taken from unpublished data obtained in the Langley Unitary Plan wind tunnel.

Approach to indeterminateness. - Some of the expressions, as presented, are invalid where the body surface is normal to the body axis of revolution because of the appearance of the indeterminate form $\frac{\rho}{d} \tan \delta$. These points may be singularly evaluated or values may be obtained by extrapolation. For the example configuration as $\frac{x}{d} \rightarrow 0\left(\frac{\rho}{d} \rightarrow 0,\ \tan \delta \rightarrow \infty\right)$, the value of $\frac{\rho}{d} \tan \delta$ is

$\frac{R_n}{d}$ which may be substituted into the expressions. However, for the computations in figure 5, when applicable the integration was started and stopped 0.0001 $\frac{x}{d}$ interior of the points where the body slope is normal to the body of revolution; hence, avoiding the indeterminate points.

General Application

Choice of stagnation point coefficient.- All the coefficients computed from the presented equations may be modified to correspond to the actual stagnation point pressure coefficient by multiplying the computed coefficients by the ratio of the actual stagnation point pressure coefficient to the Newtonian value (2.0).

Extension of application.- It may be noted that the expressions in this paper are only as valid as the assumptions of Newtonian or modified Newtonian Flow Theory. The accuracy of this theory depends on the particular configuration under consideration.

CONCLUDING REMARKS

Equations which are easily adaptable to machine computation are presented for the preliminary prediction of the supersonic and hypersonic stability characteristics of bodies of revolution. The equations have been programed and an example of their application to a blunt entry configuration is given.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, January 30, 1963

REFERENCES

1. Ivey, H. Reese, Klunker, E. Bernard, and Bowen, Edward N.: A Method for Determining the Aerodynamic Characteristics of Two-and Three-Dimensional Shapes at Hypersonic Speeds. NACA TN 1613, 1948.

2. Grimminger, G., Williams, E. P., and Young, G. B. W.: Lift on Inclined Bodies of Revolution in Hypersonic Flow. Jour. Aero. Sci., vol. 17, no. 11, Nov. 1950, pp. 675-690.

3. Tobak, Murray, and Wehrend, William R.: Stability Derivatives on Cones at Supersonic Speeds. NACA TN 3788, 1956.

4. Fisher, Lewis R.: Equations and Charts for Determining the Hypersonic Stability Derivatives of Combinations of Cone Frustrums Computed by Newtonian Impact Theory. NASA TN D-149, 1959.

5. Johnston, Patrick J., and Snyder, Curtis D.: Static Longitudinal Stability and Performance of Several Ballistic Spacecraft Configurations in Helium at a Mach Number of 24.5. NASA TN D-1379, 1962.

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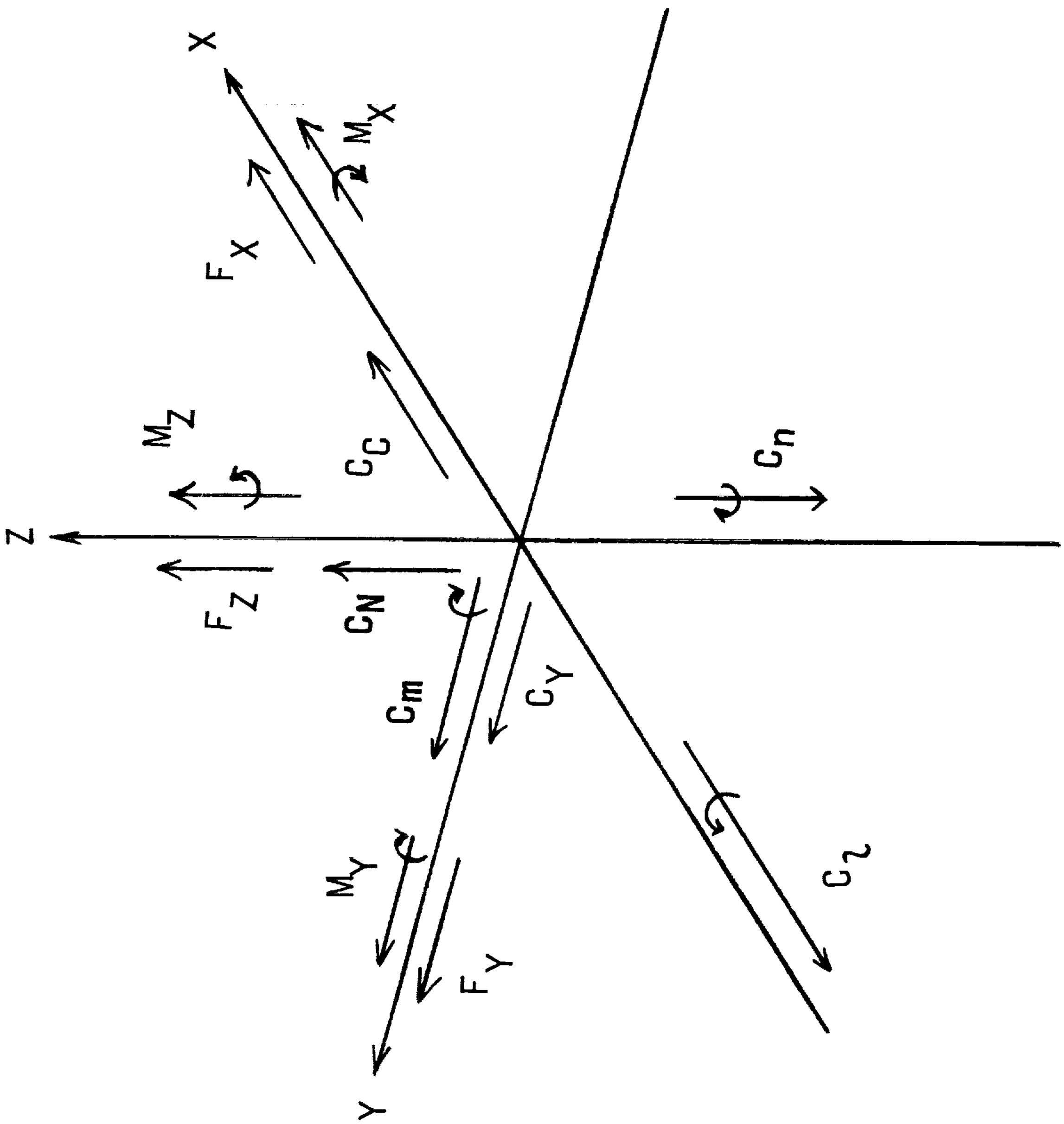


Figure 1.- Vector directions.

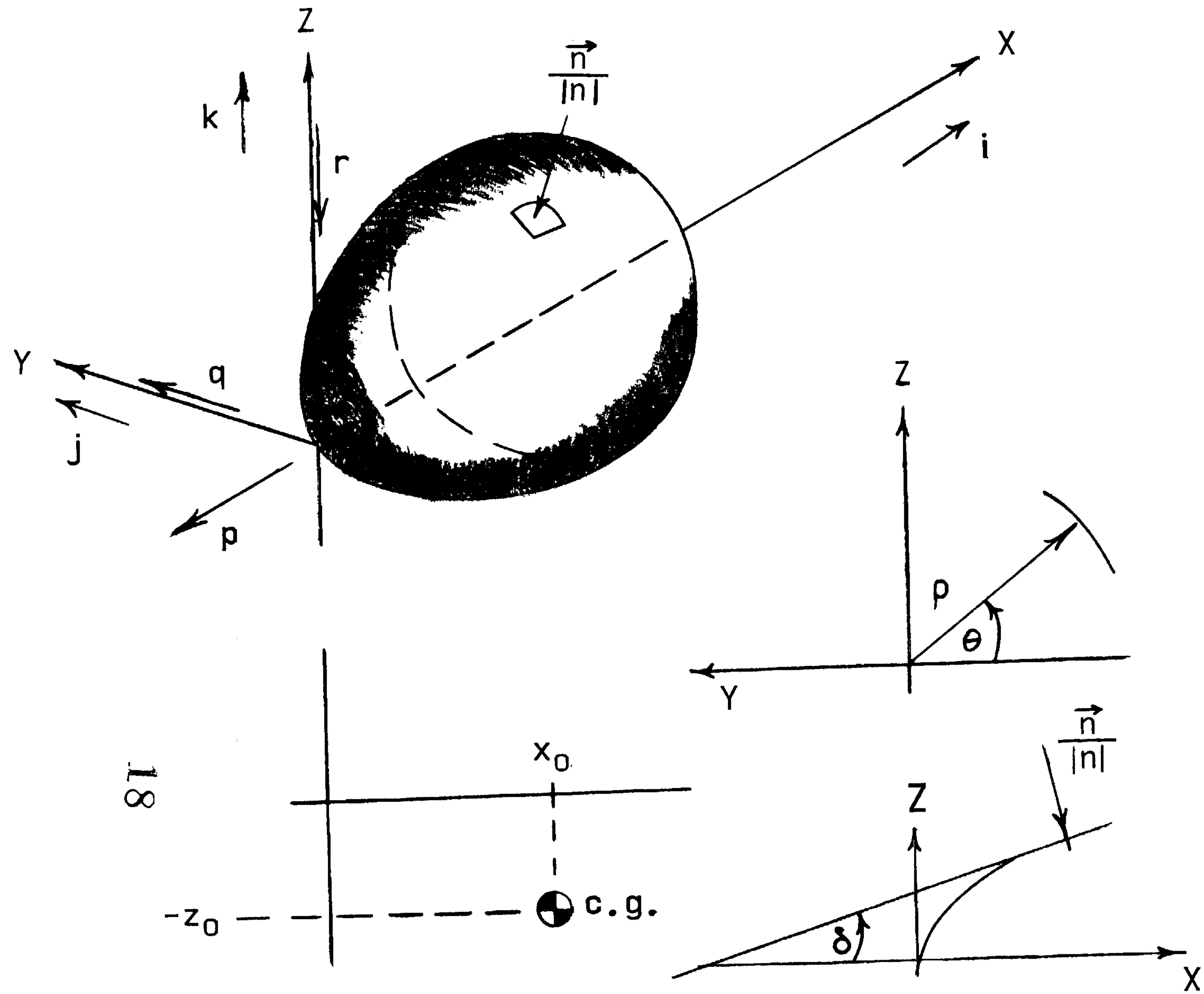


Figure 2.- Body coordinates.

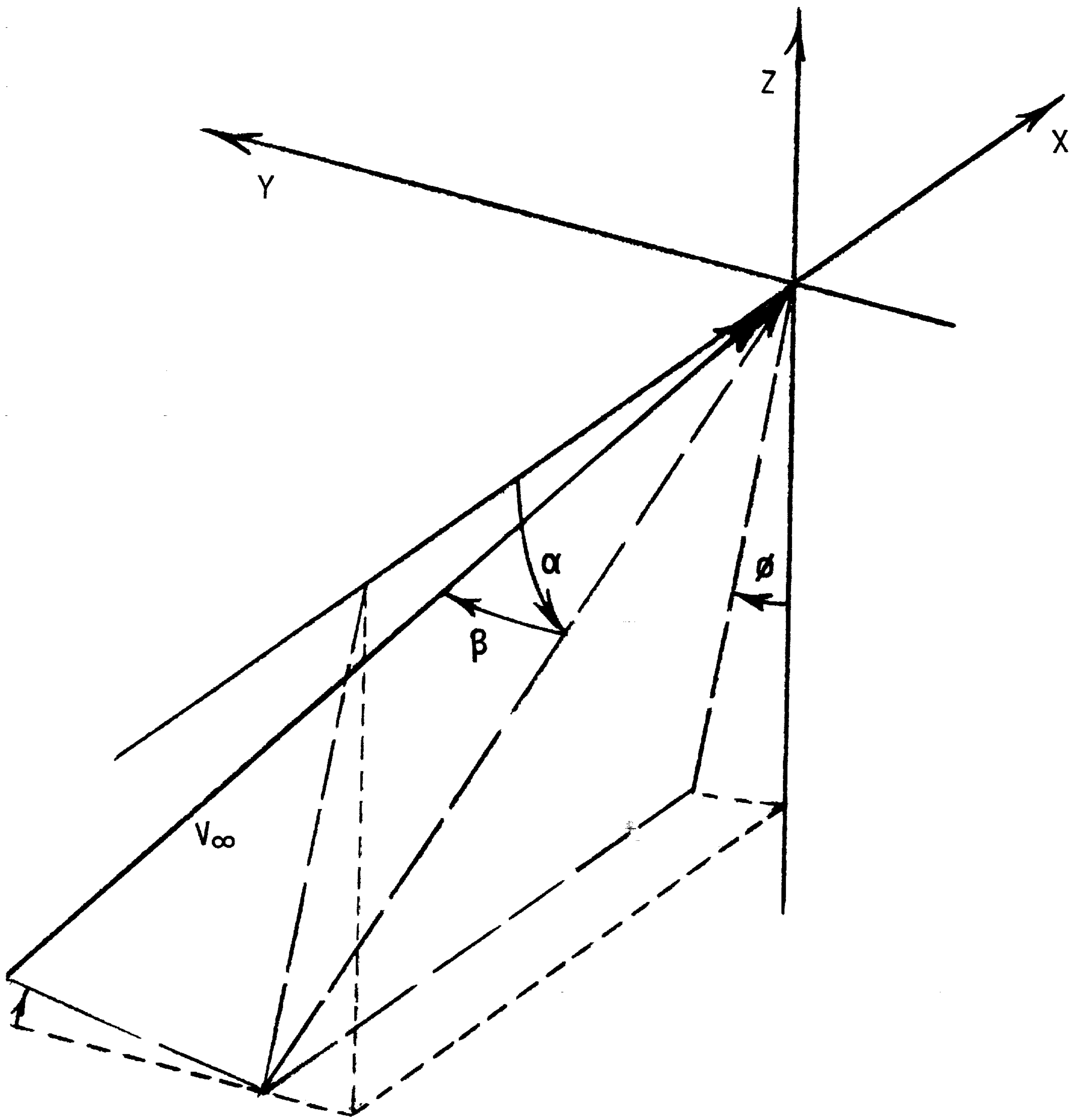


Figure 3.- Free-stream velocity vector.

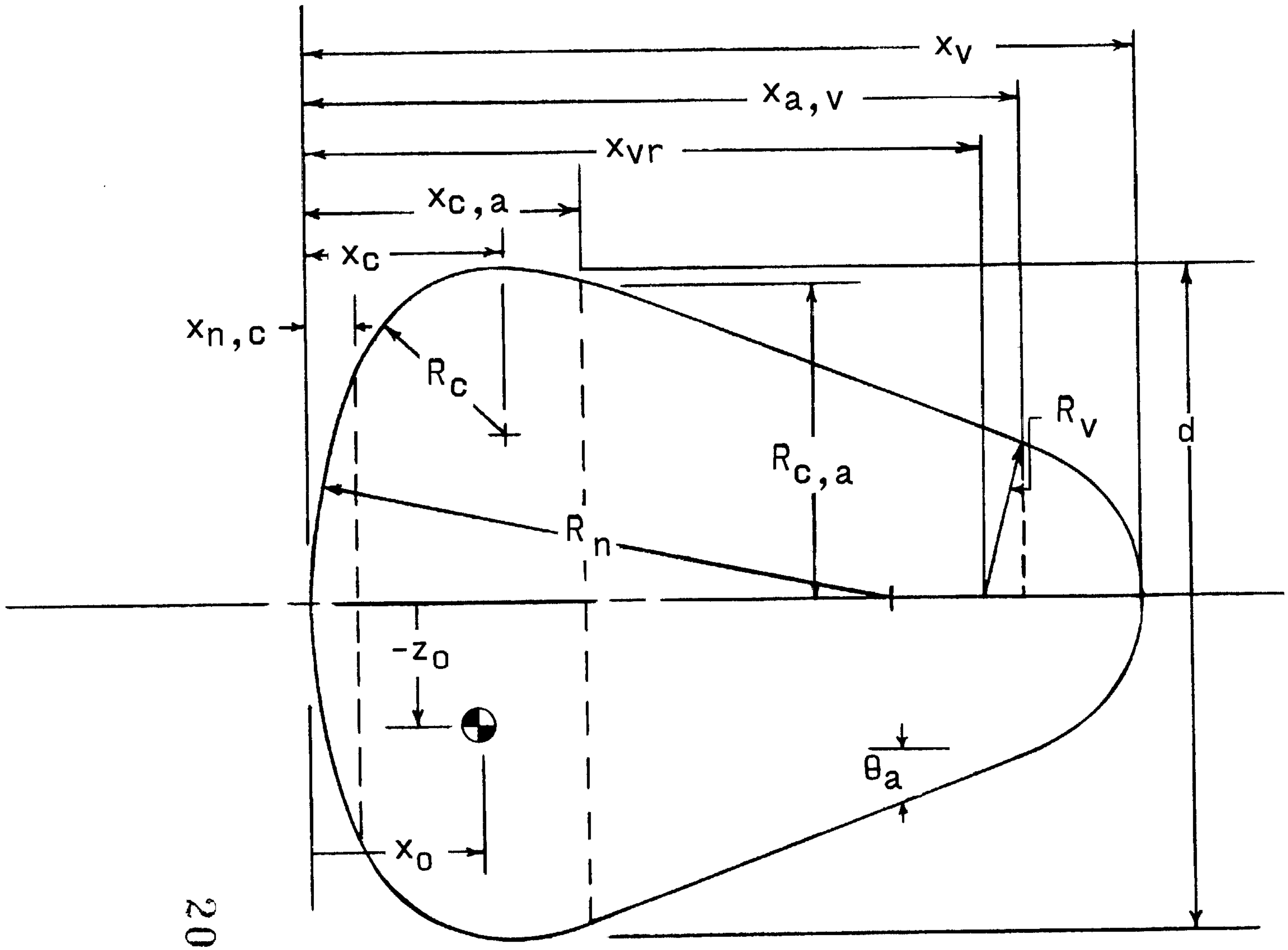
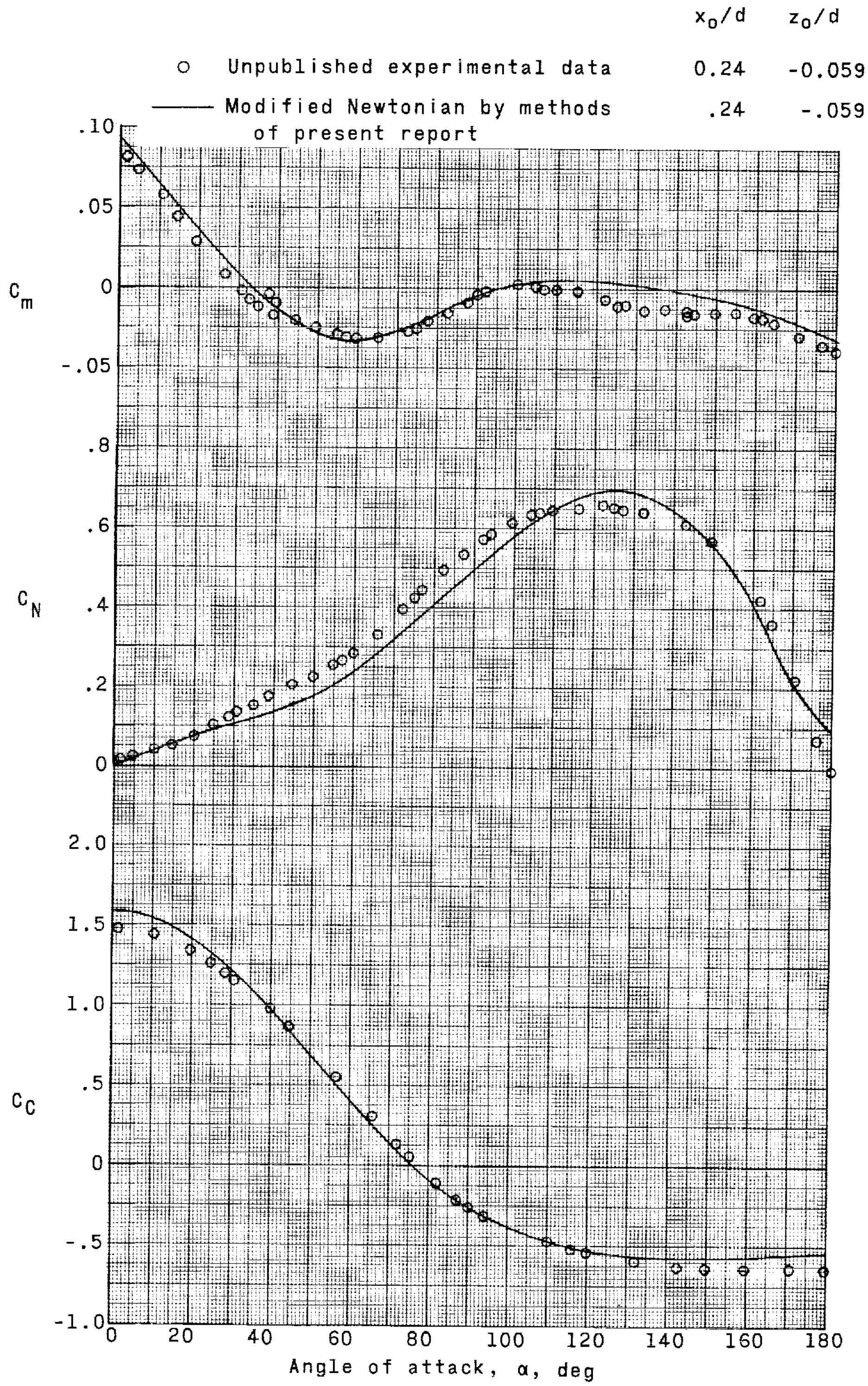
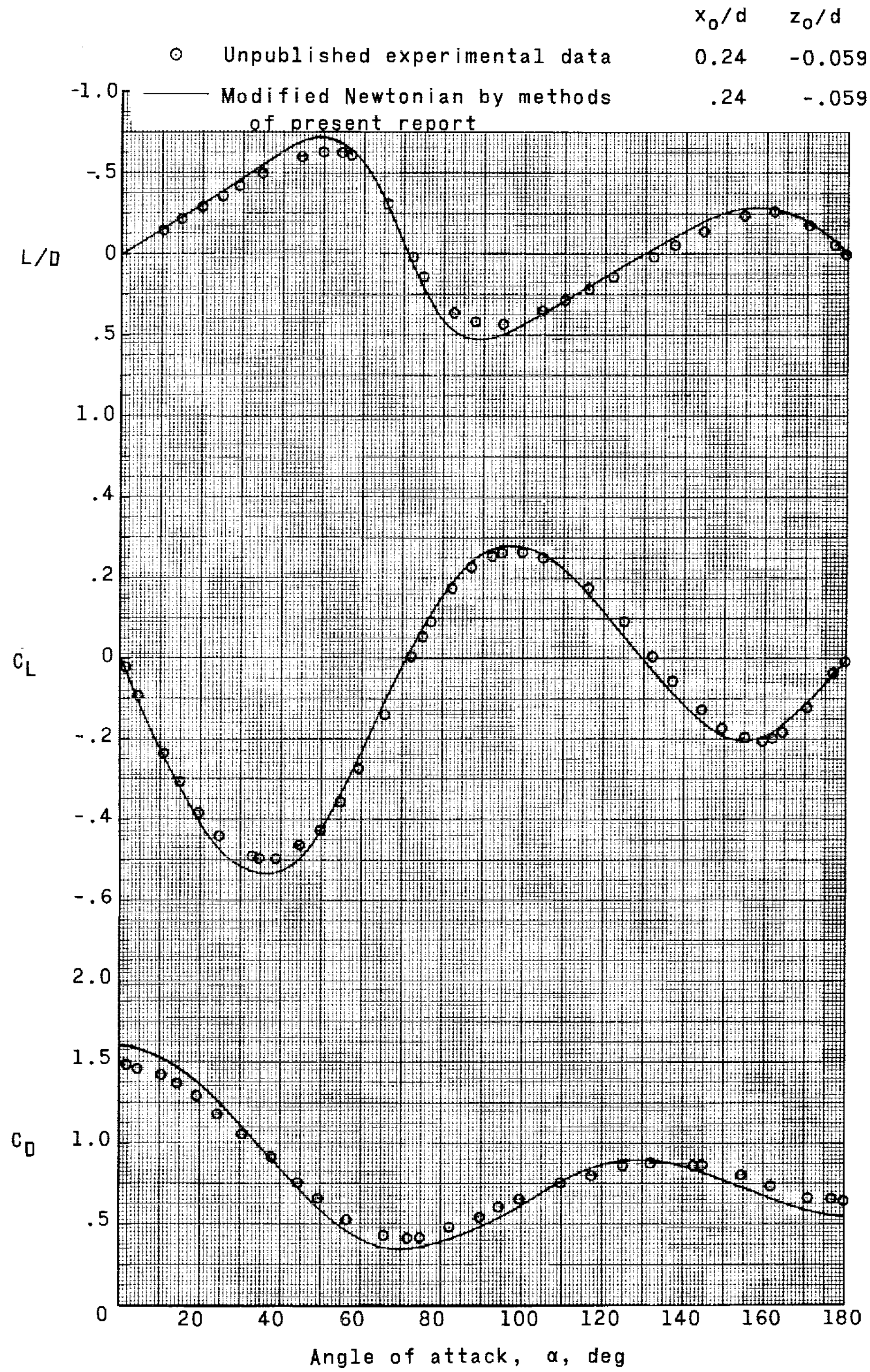


Figure 4.- Configuration setup for programing into body coefficient expressions.



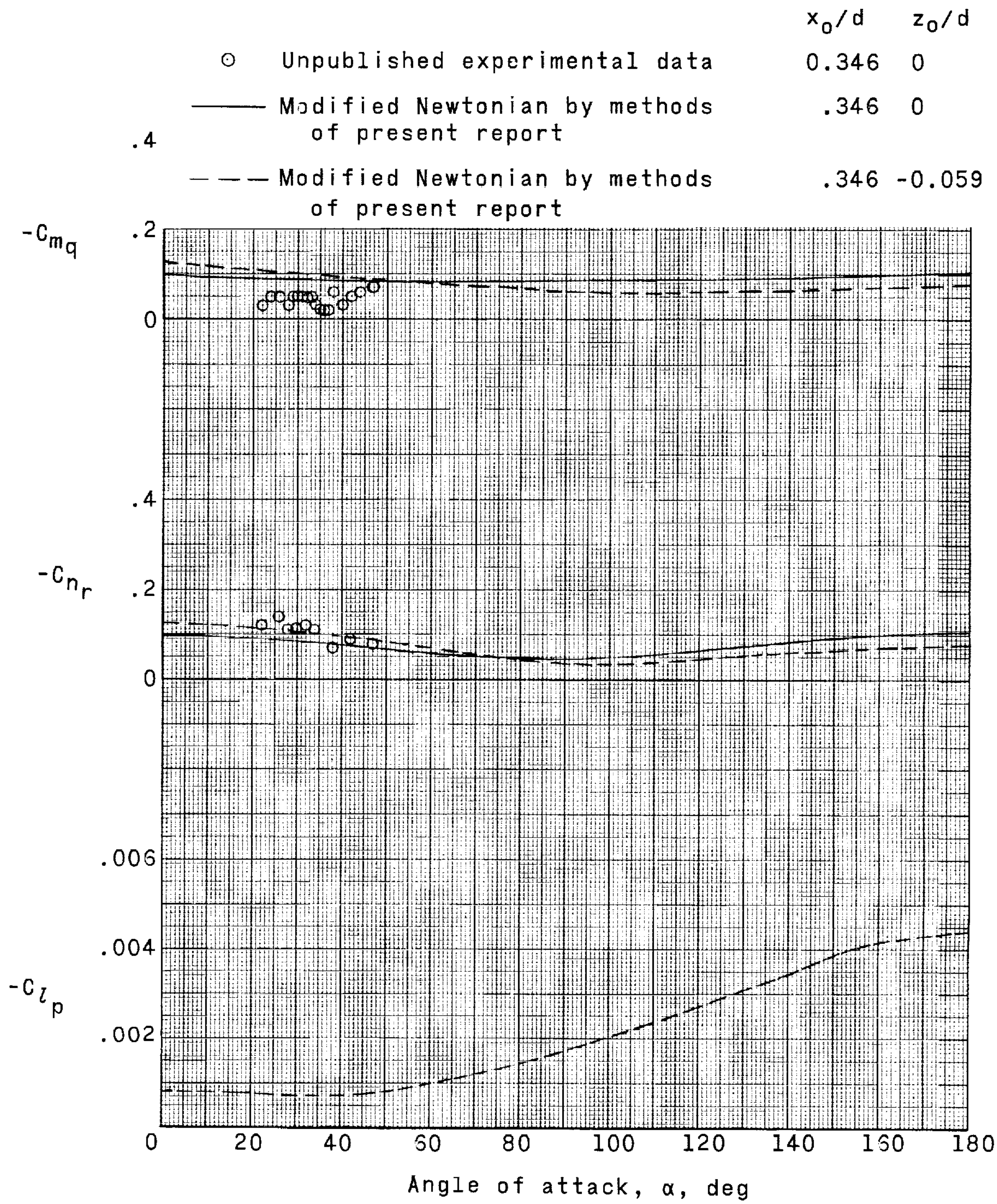
(a) Static characteristics for M = 9.

Figure 5.- Aerodynamic characteristic of $R_n/d = 1.2$; $R_c/d = 0.05$; $\theta_a = 33°$; $R_v/d = 0.1$; entry configuration.



(a) Concluded.

Figure 5.- Continued.



(b) Dynamic characteristics for M = 4.65.

Figure 5.- Concluded.

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