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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

LUNAR FLIGHT STUDY SERIES: VOLUME 3

EARTH TO MOON TRAJECTORY INVESTIGATION FOR MISSION PROFILES INVOLVING A LUNAR PARKING ORBIT

By

Nolan J. Braud

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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FUTURE PROJECTS BRANCH
AEROBALLISTICS DIVISION

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LIST OF DEFINITIONS

Retrograde Orbit	A satellite path moving in a westward direction, or that motion where the Right Ascension is decreasing.
Coplanar Trajectory	A flight path where all motion is contained in the earth-moon plane.
Ecliptic	The projection of the plane of the earth's orbit on the celestial sphere.
Periselenum	The point in an orbit where closest approach to the moon occurs.
MEP-Coordinate System	Moon-Earth-Plane System where the moon-earth plane and the line joining their centers are employed in defining geometrical parameters, (explained in text).

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SUMMARY

The purpose of this report is to relate injection and periselenum conditions of earth-moon transfer trajectories that are designed with the capability of establishing a retrograde lunar parking orbit. The Jacobian model of the restricted three-body problem is employed in the investigation and the earth-moon plane is utilized as a convenient reference in relating geometrical conditions.

SECTION I. INTRODUCTION

The particular problem to be considered here is the investigation of relationships between injection conditions and conditions at periselenum on earth-moon transfer trajectories. The interpretation of the results is made for applications which include a lunar parking orbit as part of the profile.

Both coplanar and three-dimensional trajectories are investigated by use of the Jacobian model of the restricted three-body problem. The shaping of trajectories was not influenced by powered plane changes; however, certain flight plane bending is present, which is caused by the dynamics of the earth-moon system. In this report it is assumed that the equators of the earth and moon lie within their common plane of motion. This is done to place the analysis on a model where geometrical properties can be more easily understood.

SECTION II. DISCUSSION

A. PARAMETERS OF INTEREST

Sòme of the points which were of utmost interest in this study are the amount of bending, or warping, which the "flight plane" undergoes as the vehicle travels from earth to moon, and the velocity magnitude of the vehicle at the periselenum point of the trajectory. The former is of interest in that it would assist in answering the question of the optimum manner of meeting certain plane change requirements. The latter point feeds into the flight profile's "velocity budget" if a lunar parking orbit is to be established by a braking thrust at periselenum.

The velocity budget referred to here is the amount of incremental velocity required to effect various maneuvers plus the braking velocity needed to establish the lunar parking orbit.

B. GEOMETRICAL CONSIDERATIONS

Before discussing the coordinate system used in the report, a brief description of the geometry involved in earth-moon motion is in order. First it is noted that the plane of the moon's motion about the earth describes a constant angle of about 5.1° with respect to the ecliptic, and has a precessional rate of one cycle per 18.6 years. The earth's equator is inclined by 23.4° to the ecliptic and has such a slow precessional rate that it may be assumed to be space-fixed. This results in a varying inclination of from 18.3° to 28.5° between the earth's equator and the moon's orbit plane. The situation for the limiting inclinations is shown on Figure 1.

According to one of the laws of Cassini, the moon's equator has a constant inclination of about 6.7° to the earth-moon orbit plane and also has a precessional rate of 18.6 years. If one observes the equatorial planar alignments for both earth and moon during a month he sees a behavior similar to that depicted in Figure 2. As noted on the figure the line of nodes formed by the intersection of the lunar equator plane and the lunar plane of motion is not in general parallel with the line of nodes formed by the lunar plane of motion and the earth's equator.

In this report, however, the investigations are not concerned with the geographical net conventionally used for earth and moon. It was felt that the principal characteristics are easier studied and also presented, if they are done for a reference system of coordinates that does not change with time. This viewpoint led to the acceptance of a simple coordinate system that principally is based on the plane in which earth and moon travel and on the connecting line of the two bodies. For future easy reference we call it the Moon-Earth-Plane Referenced System (short: "MEP").

The MEP-equators at earth and moon are then the intersection of the earth-moon plane with the bodies. The MEP-latitudes and MEP-poles are defined with respect to the MEP-equators. The direction of north and correspondingly of positive latitude matches with the positive vector of angular velocity of the earth-moon system. MEP-longitude at the moon counts eastward thru 360° starting at the near earth intersection of the earth-moon line with the moon; MEP-longitude at the earth is also measured eastward thru 360°, but starts at the moon-far-point of the earth-moon line. East at each body is defined as the direction of sunrise.

C. ANALYSIS

The mathematical model used in this investigation has a spacecraft of negligible mass moving within earth-moon space, where the earth and moon maintain a constant distance in describing circular orbits within a common plane about their barycenter. The distance from earth to moon was assumed at 361,400 km which is near the actual mean perigee distance.

In generating earth to moon trajectories on this model, each case is completely defined by a set of six initial conditions and time. They are a three-dimensional position and velocity vector as well as time. In addition, time is instrumental in positioning the moon; however, for all trajectories initial time is assumed to be zero and the moon is assumed to have a constant initial reference position. The initial position vector of the spacecraft is defined by its radial distance and MEP-latitude and MEP-longitude. The initial velocity vector is defined by its magnitude as well as the initial flight path angle (from local vertical) and azimuth. The azimuth is measured positive east from MEP-north.

For this study the injection radius was assumed to be 7120 km (altitude of 750 km) and injection path angle was fixed at 81.2°.* These two parameters are held constant for this study and were arrived at by considering the use of a Saturn C-5 in achieving representative injection conditions for earth-moon transfer trajectories of 60 to 80 hours travel time.

The manner in which the investigation was conducted was to accept the fixed values of radial distance and flight path angle and then stepwise vary the initial MEP-latitude while allowing MEP-longitude, MEP-azimuth and velocity magnitude to be independent variables in isolating on certain conditions at the moon. The conditions that were sought at the moon were the achievement of a certain MEP-latitude, periselenum distance and time to periselenum. The periselenum MEP-latitude was merely used as a control parameter by which periselenum conics of varying inclinations were achieved and has only an indirect bearing on the results presented herein.

The first step in the earth to moon trajectory investigation was to look at transit trajectories completely imbedded in the earth-moon-plane so that a feeling for the behavior of some of the parameters could be had. Then the investigation was carried to three dimensional cases where the more interesting flight mechanical features occur. The results and discussions of these investigations will follow in the ensuing chapters.

1. Coplanar Transits

In the coplanar investigation it is assumed that the transfer trajectory from earth to moon lies entirely within the earth-moon plane of motion. It was further assumed that figure-eight type trajectories would be used because they provide periselenum conditions suited for generating retrograde parking orbits about the moon (orbits where the direction of motion is opposite to that of the moon's rotation).

Using the fixed values of radial distance and flight path angle at injection, a family of coplanar transfer trajectories was generated where all combinations of periselenum altitudes of 140, 185 and 230 km along with transit times of 66, 72 and 78 hours were achieved. The injection MEP-longitude and velocity magnitude were the parameters which were varied to achieve these trajectories.

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* These values were representative for C-5 at the inception of this investigation. The currently accepted values are, injection radius of 6645 km (altitude of 275 km) and injection path angle of 84.3 degrees (100 nautical mile parking orbit assumed in injection phase).

a. Injection Conditions

The behavior of the MEP-longitude and velocity magnitude* at injection is displayed as a function of the time from injection to periselenum on Figures 3 and 4. From these figures it can be seen that both injection MEP-longitude and velocity magnitude are primarily dependent upon the time from injection to periselenum, with periselenum altitude having a second order effect on the MEP-longitude and no apparent effect on velocity magnitude.

b. Periselenum Conditions

Attention is now brought to the moon referenced values of periselenum MEP-longitude and velocity. Referring to Figure 5 one notes the behavior of the periselenum MEP-longitude as a function of transit time from injection to periselenum. The range of periselenum MEP-longitudes for the transit times considered is from 159 to 178 degrees. Again the effect due to altitude variations is very small. The positioning of the periselenum points is very near the center of the backside of the moon and toward its trailing edge.

The periselenum velocity as a function of transit time to periselenum and periselenum altitude is shown on Figure 6. As one would suspect, the lower the periselenum altitude the greater its velocity will be for a fixed transfer time.

c. Impulse Requirement to Establish Lunar Circular Parking Orbit and Effect Plane Changes

It is noted from Figure 6 that for the transit times and periselenum altitudes considered, the periselenum arrival velocity ranges from about 2400 to 2500 meters per second. The range of arrival velocities is very important in that it reflects the amount of periselenum velocity that must be braked in order to establish a low altitude lunar parking orbit. For example, if one wishes to achieve a 185 km circular parking orbit about the moon, he must reduce the periselenum velocity to 1596 meters per second on a trajectory which has the desired periselenum altitude. For the transfer times under consideration the braking incremental velocity would amount to between 825 and 900 meters per second. If one extended this example by assuming a Hohmann transfer down to the moon's surface where a final incremental velocity

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* The space-fixed velocity magnitude is about 25 to 30 meters per second greater than that in the moon-earth rotating system for the initial conditions assumed.

effects the landing with zero velocity, he finds that an additional 1762 meters per second of braking is utilized. This results in from 2587 to 2662 meters per second of braking velocity to achieve a soft landing on the moon by going through a 185 km circular parking orbit.

The example just cited only provides for the establishment of a parking orbit within the plane of the periselenum conic. If any plane change would be required it would place an additional demand on a planned "velocity budget." Figure 7 presents information for the consideration of the plane change problem. Shown is the incremental velocity required to establish circular lunar satellite orbits with an incorporated plane change of up to twenty degrees. The values presented were determined by the cosine relationship and are given for periselenum conditions of 140, 185 and 230 km altitude and 66, 72 and 78 hour transfer time. From the figure it can be seen that a ten degree plane change costs about 65 to 70 meters per second. In the next section three-dimensional trajectories will be discussed where the problem of inclination of the lunar parking orbit will be treated in more detail.

2. Three Dimensional Transits

The analysis of three-dimensional earth to moon transfer trajectories also concerns itself with the relationships between injection and periselenum conditions. Some of the factors under consideration are: position, inclination and line of nodes at injection and periselenum. These points will be discussed later, but first it may be of interest to relate some of the techniques employed in conducting the investigation.

Again the fixed conditions of initial radial distance and flight path angle were assumed. Then for stepwise constant values of injection MEP-latitude, initial velocity, MEP-longitude and azimuth were varied to achieve a set of periselenum conditions; namely, lunar MEP-latitude time and altitude. By utilizing the symmetric features which exist for trajectories run on the restricted three-body model, only cases which had initial positions on one side of the earth-moon plane were needed. Once such a set of trajectories has been generated they can be interpreted to be applicable for departures on the opposite side of the earth-moon plane. Being cognizant of this fact allows one to double the information to be gained from a given trajectory.

At this point it is noted that initially in the three-dimensional study some work was done on 66 and 78 hour trajectories. Early results indicated that the behavior of most parameters would be similar to those for 72 hour transit; therefore, for the sake of expediency, only the 72 hour transit was concentrated upon.

a. Injection Conditions

Since the MEP-latitude at injection and the MEP-latitude at periselenum were used somewhat as control parameters in conducting the investigation, it was convenient to use them in relating results. The mean injection azimuth is presented as a function of the MEP-latitude at injection on Figure 8. The first point to be made in connection with this figure is that for a given reference latitude at injection there was found to be a very small variation required in azimuth to achieve any periselenum MEP-latitude. The second fact to be noted is the symmetric relationship of the azimuth about its 90 degree value. For any absolute MEP-latitude it is noted that the corresponding values of MEP-azimuth are at equal deflection from 90 degrees.

The final point to be made about the azimuth involves a relationship between flight path angle and transit time or energy level. The relationship is a simplifying approximation from two body analysis which is valid for trajectories of a given energy level. In essence it states that the space-fixed geocentric angle from perigee to the moon is equal to the space-fixed angle from injection to the moon plus twice the flight path angle measured from horizontal. This total angle for 72 hour transits is about 174 degrees for coplanar flights and increases slightly as the injection plane is inclined to the moon-earth plane. For the coplanar trajectory the angle from injection to the moon is about 155 degrees. Since we are using the MEP-latitude at injection as a parameter of investigation, it is easy to show that when relatively large values of MEP-latitude are used, the flight would have to pass over a MEP-pole of the earth so that the required central angle of flight would be available. The limiting MEP-latitudes for 72 hour trajectories would be about 25 degrees where a MEP-azimuth of 0 or 180 degrees would be needed.

If one assumes injection with a fixed and constant MEP-latitude and generates transit trajectories that have periselenum conics inclined from plus to minus ninty degrees to the earth-moon plane, he finds that a small belt of MEP-longitudes and a small corridor of MEP-azimuths will be required. It turns out that as injection MEP-latitudes are varied the MEP-injection locations trace out a circular belt on the earth which possesses a certain width. Associated with each small area of injection points on this belt there is a corridor of required MEP-azimuths. These features are shown on Figure 9 for 72 hour transits with the assumed radius and path angle of injection. The width of the belt corresponds to about one degree MEP-longitude at zero degrees MEP-latitude, and the MEP-azimuth corridor is about plus or minus 3 degrees about a mean value.

Figure 10 shows the injection velocity as a function of the MEP-latitude at injection. From this it can be seen that the value changes by only about 4 meters per second in going from the coplanar case to an injection with a MEP-latitude of 20 degrees.

Also of interest may be the inclination of the injection conic to the earth-moon plane and the location of its line of nodes. On Figure 11 is displayed the injection conic inclination with respect to the earth-moon plane, as a function of the MEP-latitude at injection. There is a very small variation in this inclination for a given latitude of injection, which is associated with the small azimuth variation that was referred to previously. This variation is caused by the requirement on initial conditions to achieve periselenum conics of various inclinations. Next on Figure 12 is shown the location of the ascending line of nodes of the injection conic as a function of the MEP-latitude at injection. Appearing on the same figure are parameters of MEP-latitude at periselenum. The figure only represents injections from MEP-northern or positive latitudes. For MEP-southern or negative latitudes of injection the ascending line of nodes would be 180 degrees out of phase.

b. Periselenum Conditions

Looking at the trajectory parameters at periselenum one finds many interesting features. The first that will be discussed is presented on Figure 13. This figure shows the location of periselenum by its MEP-latitude and MEP-longitude. Indicated as a parameter is the MEP-latitude of injection. The periselenums occur in the same general area of lunar longitude as did the coplanar cases. It is noted that the parameters tend to bundle up and cross over in an area concentrated about a periselenum MEP-latitude of two degrees. For negative or southern MEP-latitudes of injection the figure would require a rotation about the longitude axis such that this bundling area would appear at about minus two degrees. The reason for this is also tied in with the symmetry of trajectories.

The behavior of the velocity at periselenum is the next point to be taken up. Figure 14 shows the periselenum velocity as a function of the inclination of the injection conic. From this figure it can be seen that for about a 60 degree change in injection inclination there is a 50 meters per second change in periselenum velocity. This is caused by the manner in which the probes velocity vector and the lunar velocity vector are summed as the probe enters the moon's sphere of influence. This is another factor which must be considered in laying out a "velocity budget" for a mission of this type. For a given injection conic inclination there are small variations of periselenum velocity of about plus or minus 1.5 meters per second as the periselenum conic inclination varies. This would have the effect of widening the parameter as plotted.

Considering the inclination of the periselenum conic, Figure 15 shows what is achievable as a function of latitude of periselenum with injection latitude parameters. It can be seen that for a given latitude of injection there is a minimum achievable periselenum inclination. This could be an important factor when reference would be desired to the realistic lunar equator plane and a certain inclination to it is needed. With reference to this figure it is noted that in some cases the parameters were extended to as much as 90 degree periselenum conic inclinations, but the study was not exhaustive enough for them to be included here. This figure would also have a mirror image about the zero periselenum latitude line for negative injection latitudes.

Figure 16 is a composite picture elaborating on the information presented in the previous figure where the ascending line of nodes of the periselenum conic is also used. From this figure can be read the possible combinations of injection latitude and periselenum latitude which result in a given inclination of the periselenum conic to the earth-moon plane. Associated with these parameters is the location of the ascending line of nodes of the periselenum conic. There is a wealth of information contained here, but rather than expound upon it, one is reminded that it can be very useful in specifying trajectory parameters for missions which involve a lunar parking orbit as part of their profile.

The inclusion of geometric figures which depict such things as the actual bending or warping of the flight plane in earth-moon transits was considered for this report; however, it was felt that these features could be treated with better understanding in a future report of the Lunar Flight Study Series.

SECTION III. CONCLUSIONS

The trajectory features at injection and periselenum for mission profiles which assume the capability of generating lunar parking orbits are presented. It can be concluded from them, that parking orbits about the moon with less than about 20 to 25 degrees inclination to the earth-moon plane are fairly easy to achieve, assuming no restrictions on earth-departure conditions. It is also shown that between 2587 and 2662 meters per second of braking impulse is required to soft land on the moon from transfer trajectories of 66 to 78 hours, where a circular parking orbit of 185 km about the moon is utilized. This consideration neglects any guidance requirements and up to 50 meters per second which may be needed to effect a plane change at periselenum.

Since in general most transfer trajectories do not provide for a free return to reentry conditions, there would be a need for determining the optimum guidance correction which would result in reentry conditions should the establishment of a lunar parking orbit be aborted.

The results of the report are qualified by being generated on the Jacobian model of the restricted three-body problem and by the reference of most measurements to the earth-moon plane, but they should be helpful in design considerations for establishing trajectories for earth to moon missions which involve lunar parking orbits.

ACKNOWLEDGMENT

The author is grateful to Robert J. Hill for his assistance in the preparation of the material which is contained in this report.

SOLAR SYSTEM IN 1960 AND 1969

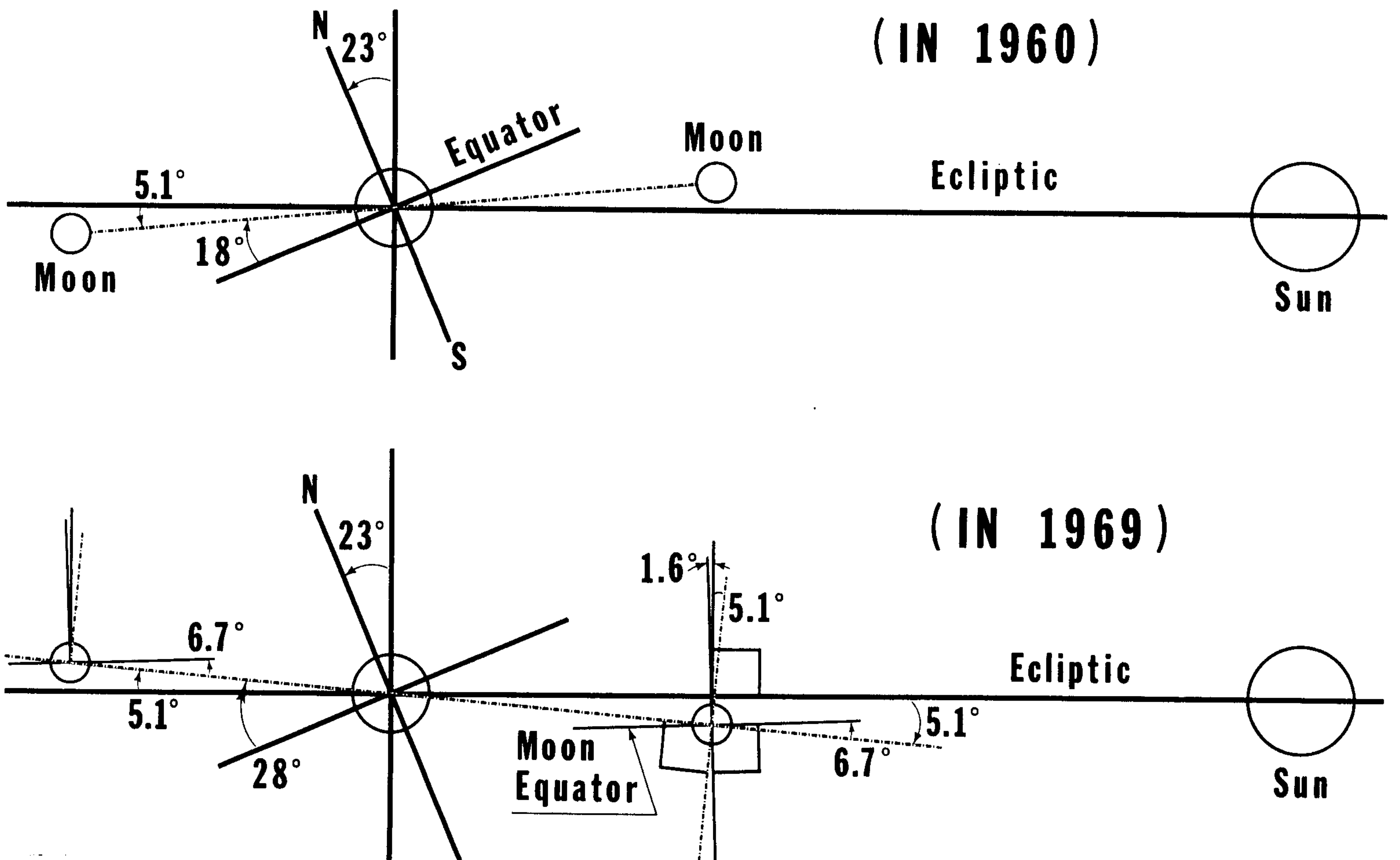


FIG. 1.

EARTH-MOON-GEOMETRY

(DEPICTING A PARTICULAR CASE SERVING AS REFERENCE)

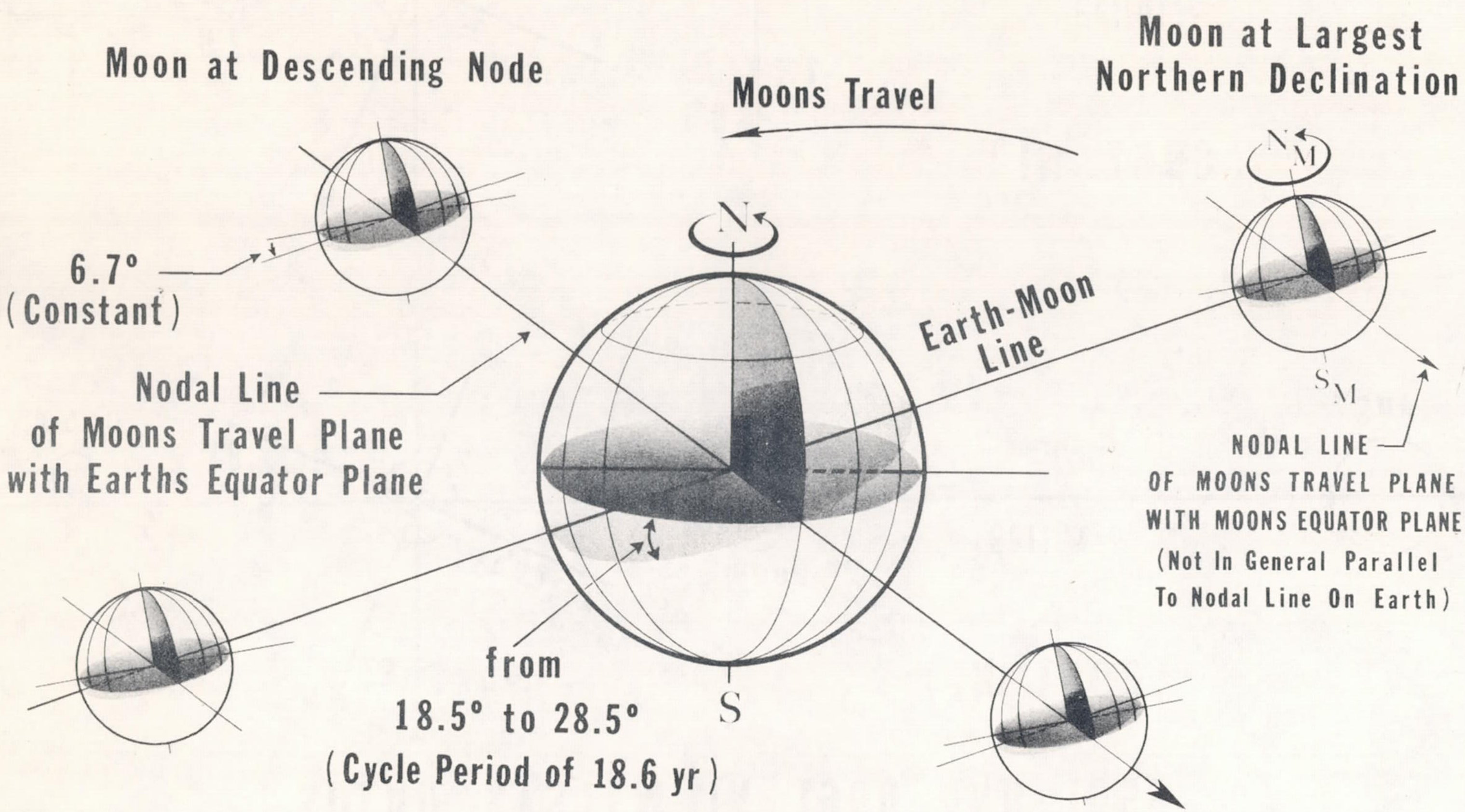


FIG. 2.

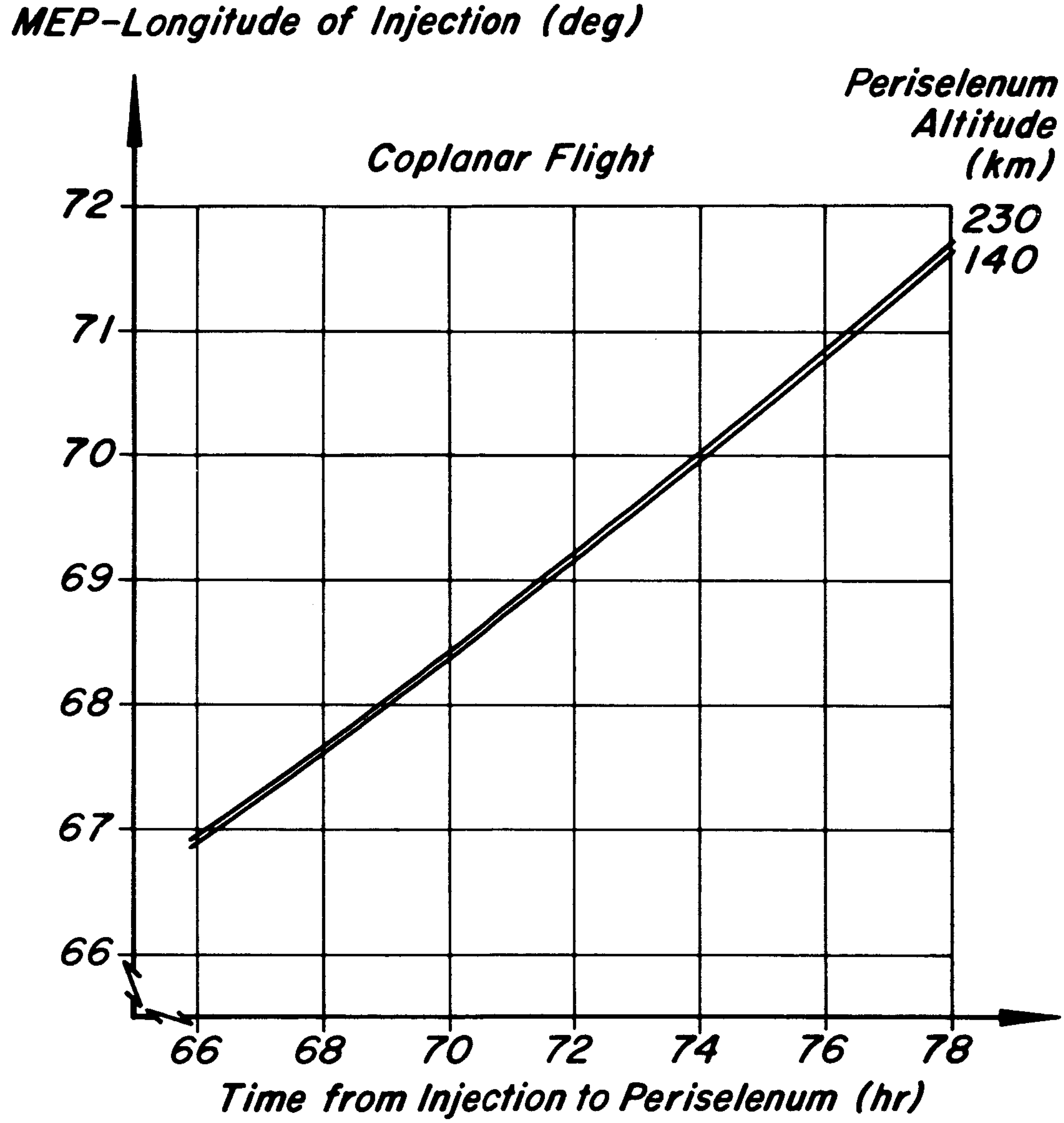
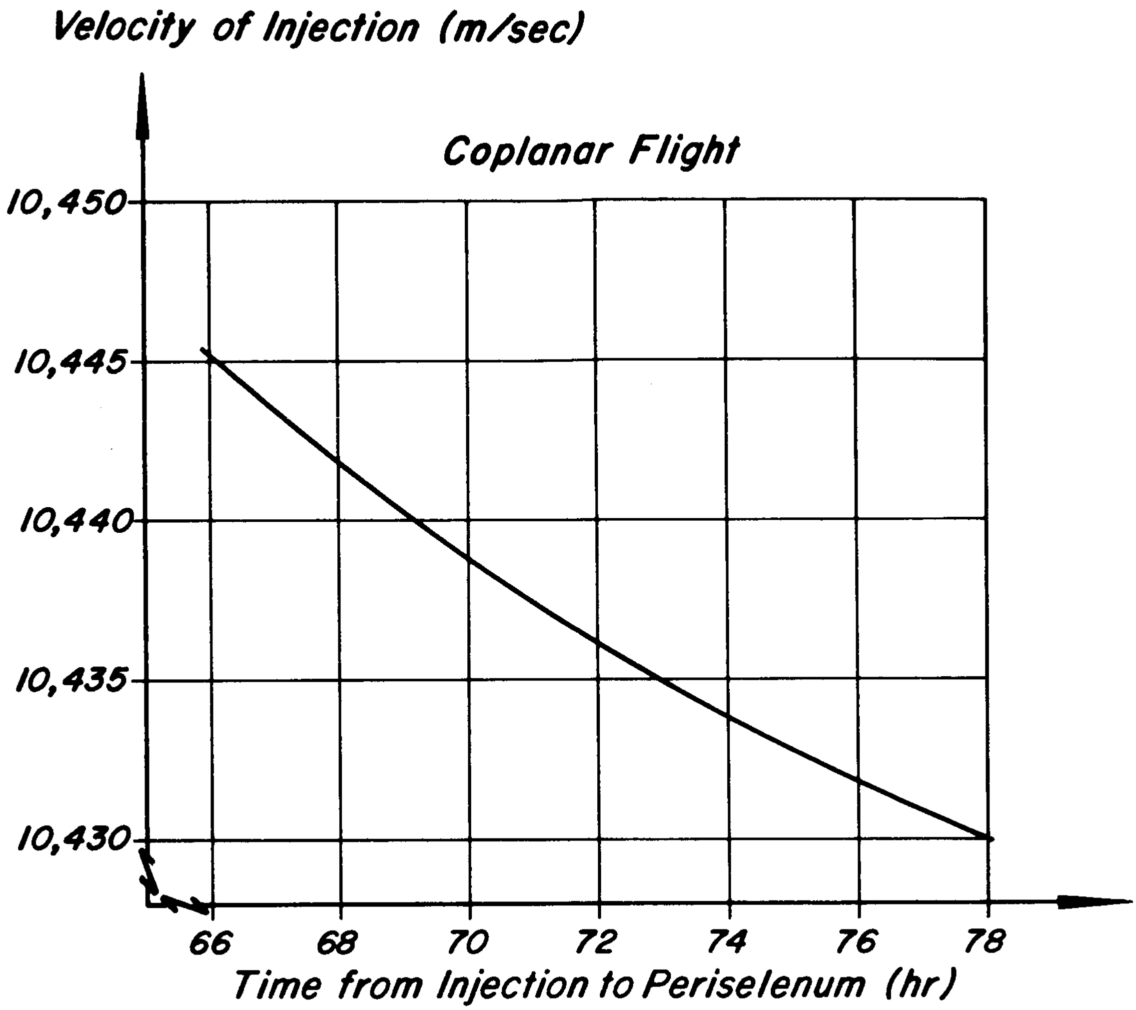


FIG. 3. MEP-LONGITUDE OF INJECTION AS FUNCTION OF TIME FROM INJECTION TO PERISELENUM WITH PERISELENUM ALTITUDE AS A PARAMETER



Applicable for Periselenum Altitude from 140 to 230 km

Velocity Measured in the ME-Rotating System

FIG. 4. VELOCITY OF INJECTION AS FUNCTION OF TIME FROM INJECTION TO PERISELENUM

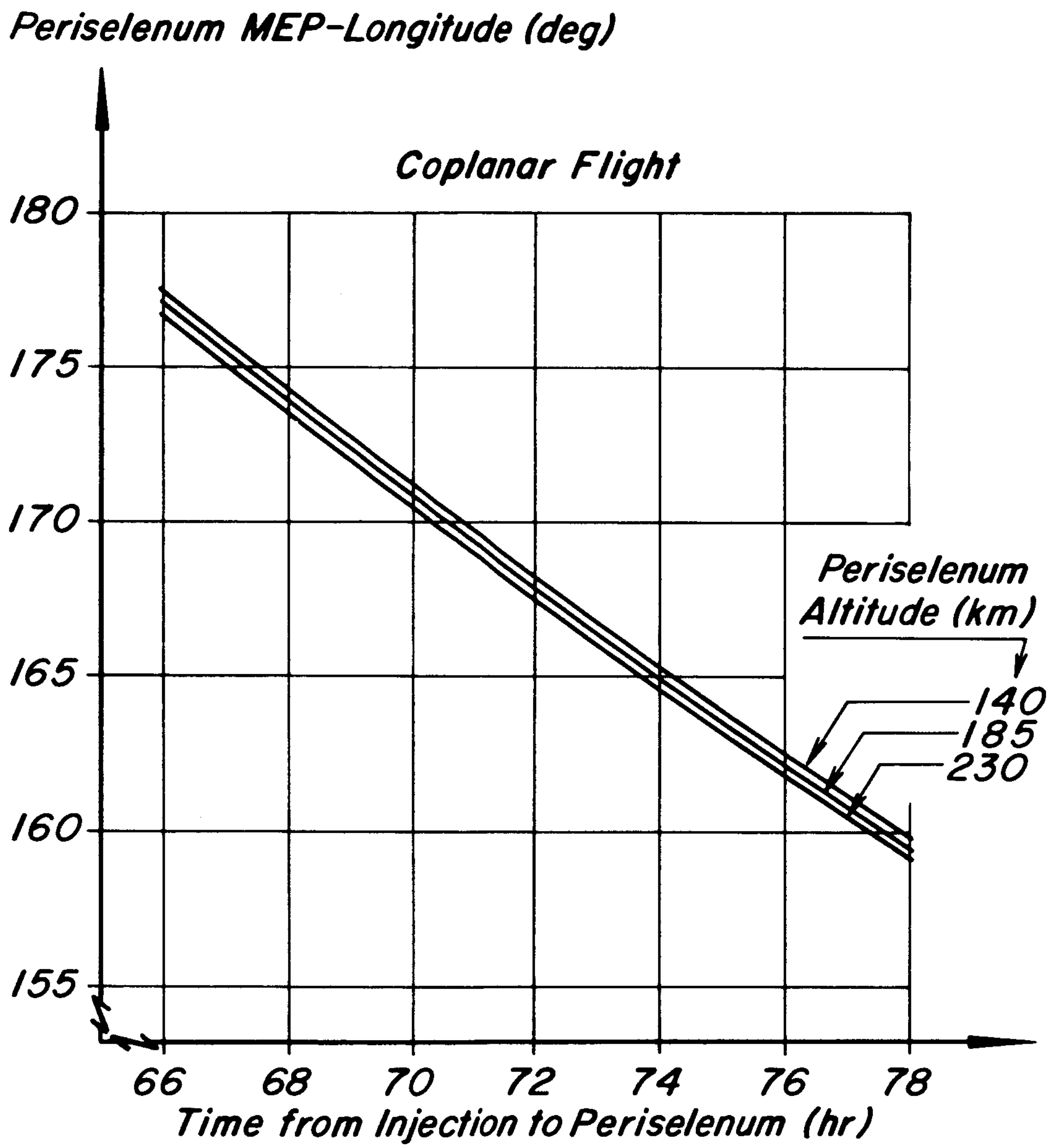


FIG. 5. MEP-LONGITUDE AT PERISELENUM AS FUNCTION OF TIME FROM INJECTION TO PERISELENUM WITH PERISELENUM ALTITUDE AS PARAMETER

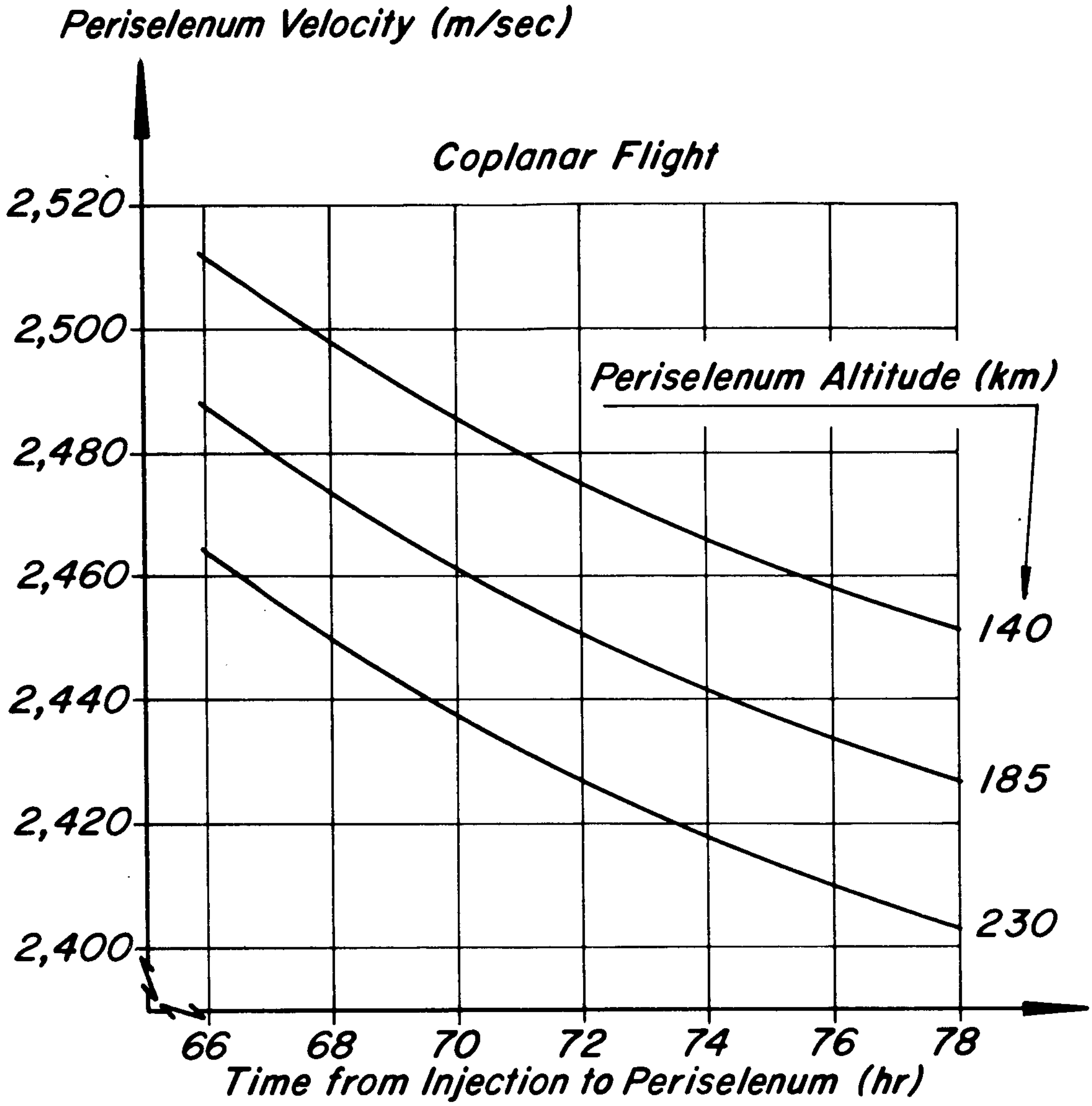


FIG. 6. VELOCITY AT PERISELENUM AS FUNCTION OF TIME FROM INJECTION TO PERISELENUM WITH PERISELENUM ALTITUDE AS A PARAMETER

(Velocity Measured In ME - Rotating System)

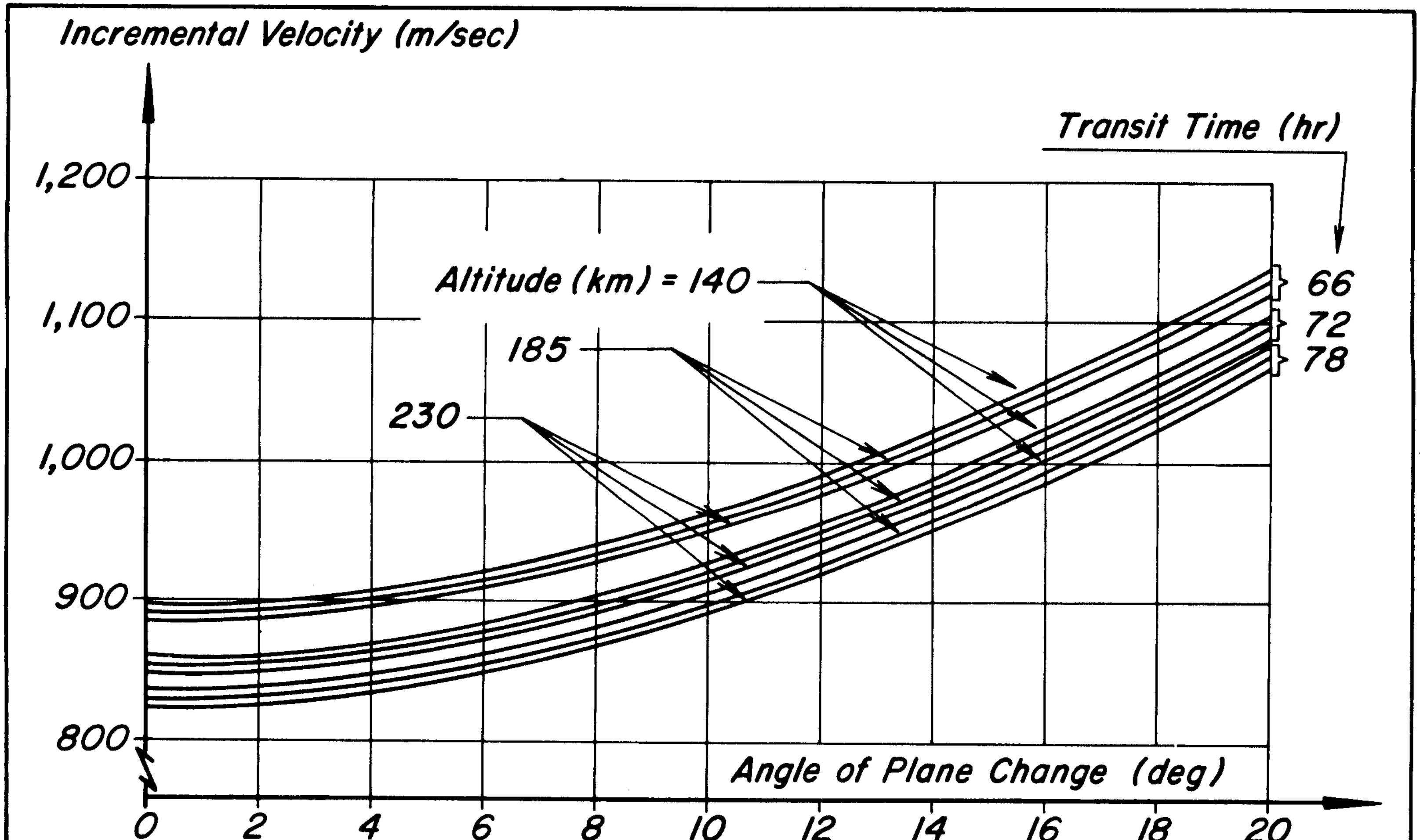


FIG. 7. INCREMENTAL VELOCITY REQUIRED TO ESTABLISH CIRCULAR LUNAR SATELLITE ORBITS OF VARIOUS INCLINATIONS FROM CONDITIONS AT PERISELENUM ON EARTH-MOON TRANSFER TRAJECTORIES

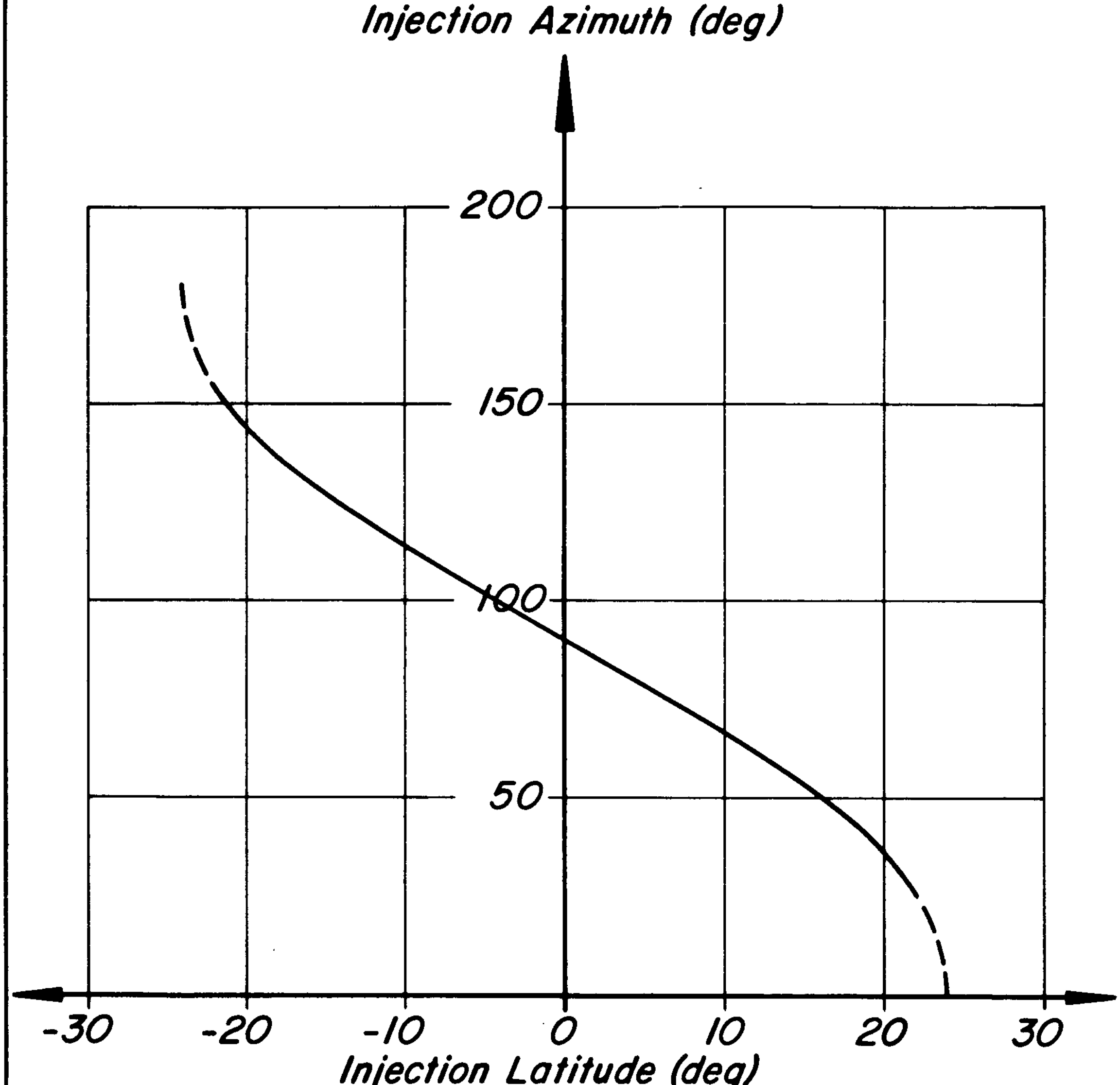
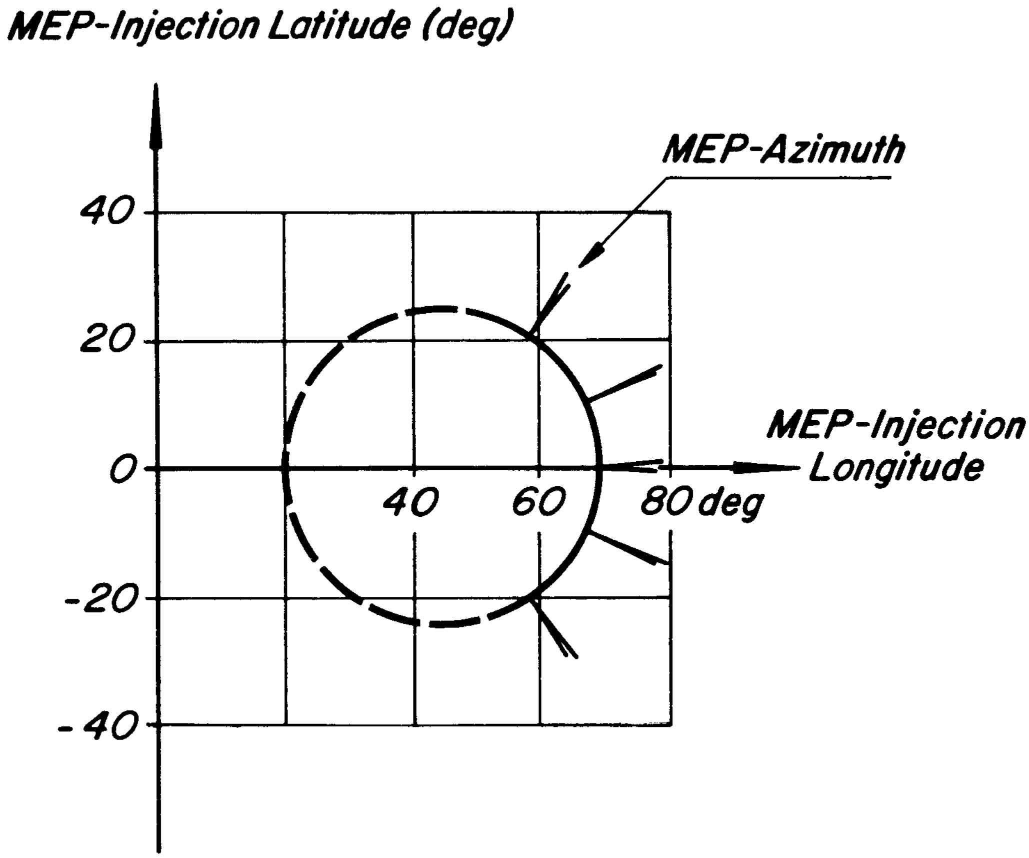


FIG. 8. MEAN INJECTION AZIMUTH AS FUNCTION OF INJECTION LATITUDE

Radius = 7120 km, Flight Path Angle = 81.2 deg



Path Angle = 81.2 deg
Radius = 7,120 km

FIG. 9. MEP-INJECTION LOCATIONS FOR 72 HR TRANSITS TO 185 KM PERISELENUM (RETROGRADE AT MOON) WITH REPRESENTATIVE MEP-AZIMUTHS INDICATED

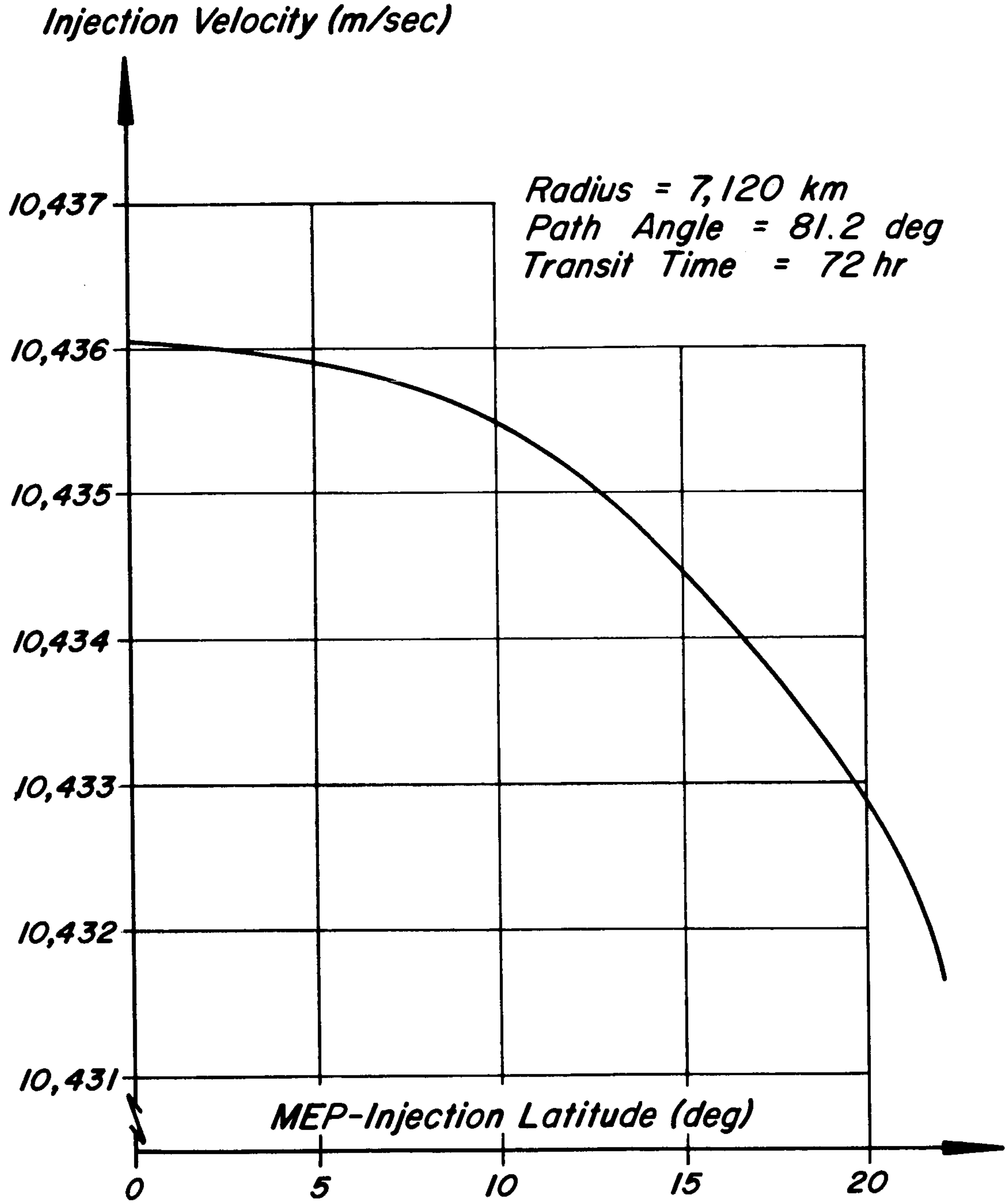


FIG. 10. INJECTION VELOCITY AS FUNCTION OF MEP - LATITUDE AT INJECTION

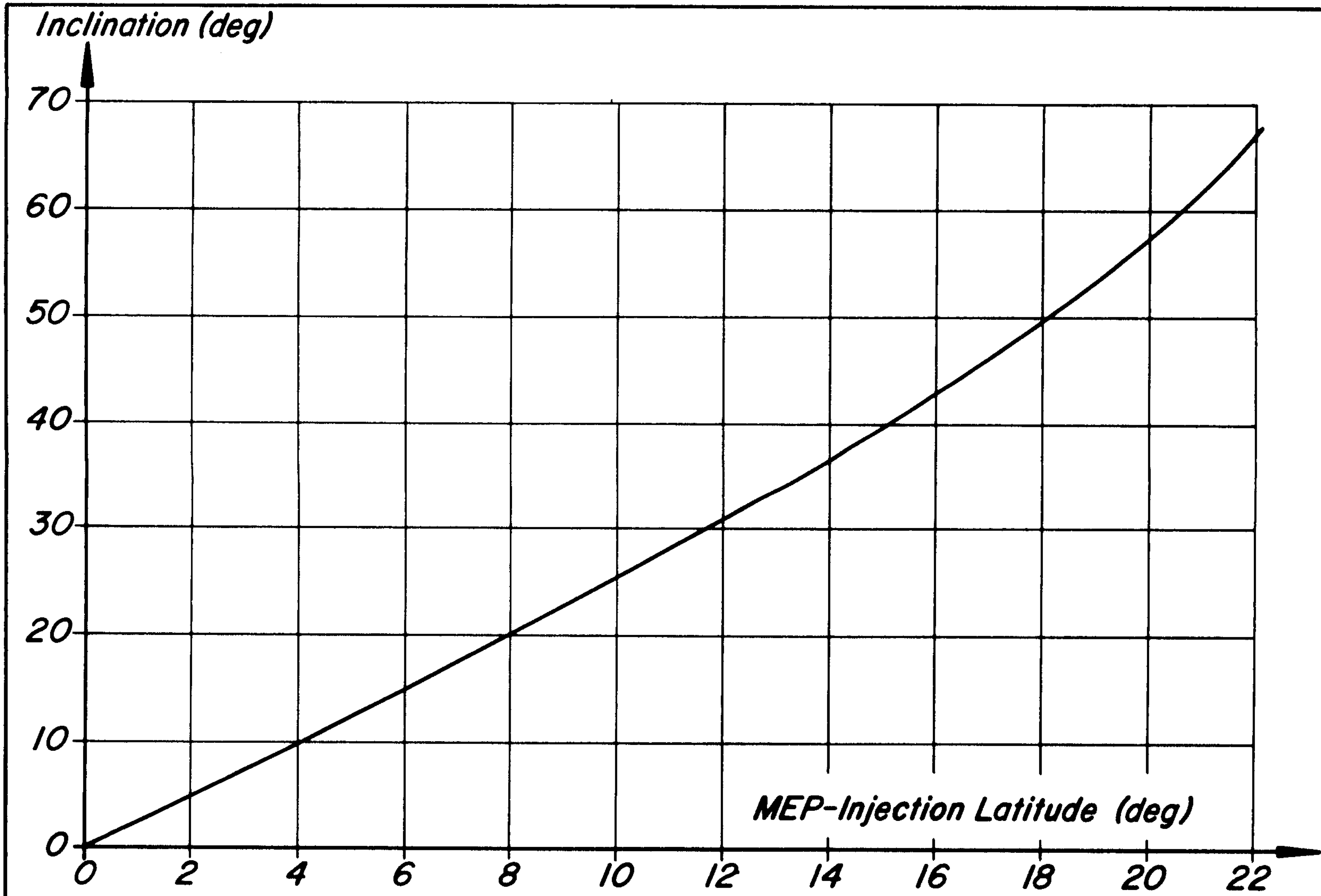


FIG. 11. INCLINATION OF INJECTION CONIC TO ME-PLANE AS FUNCTION OF MEP INJECTION LATITUDE

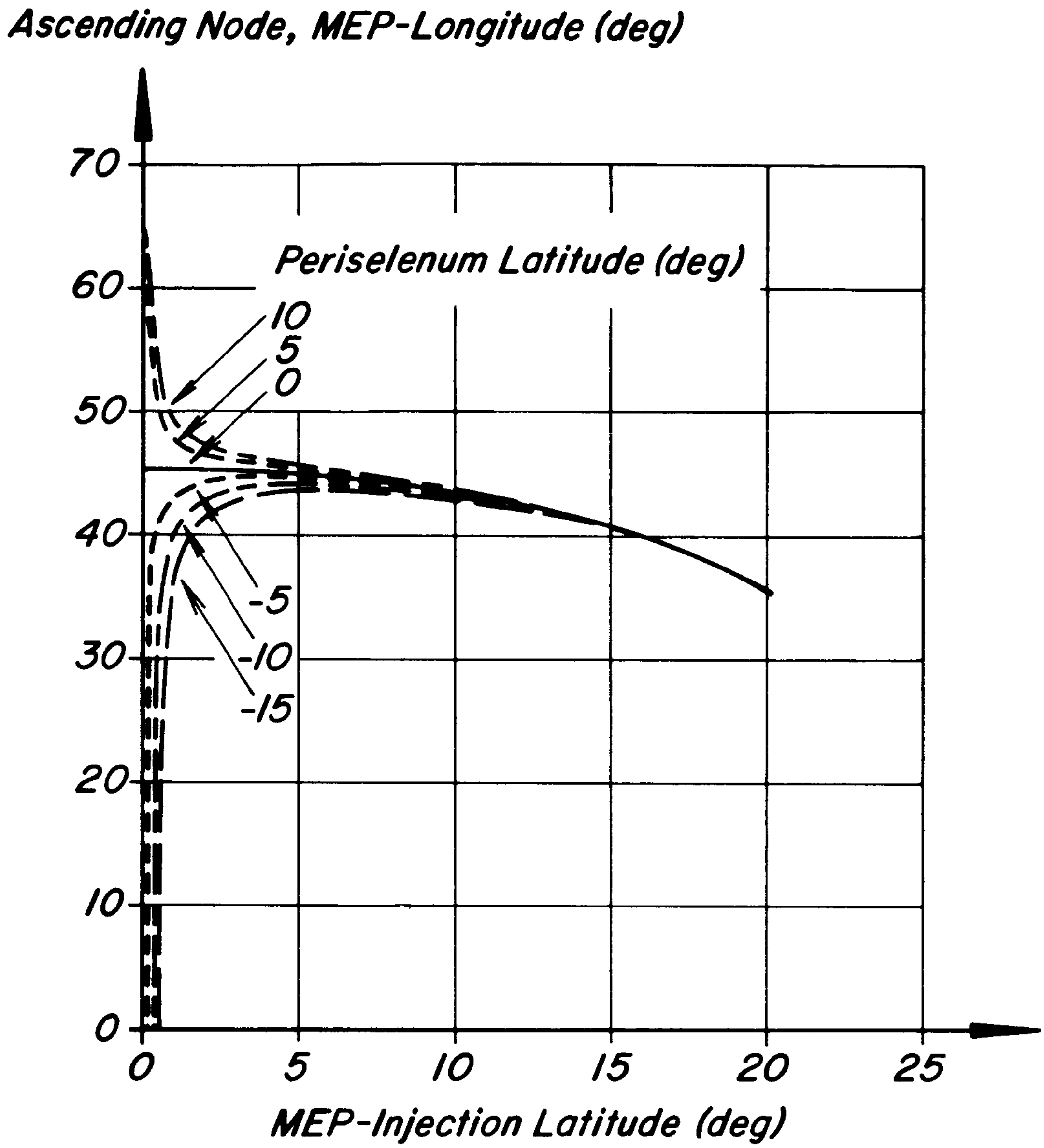


FIG. 12. ASCENDING LINE OF NODES OF INJECTION CONIC AS FUNCTION OF MEP-LATITUDE WITH PERISELENUM LATITUDE AS PARAMETER

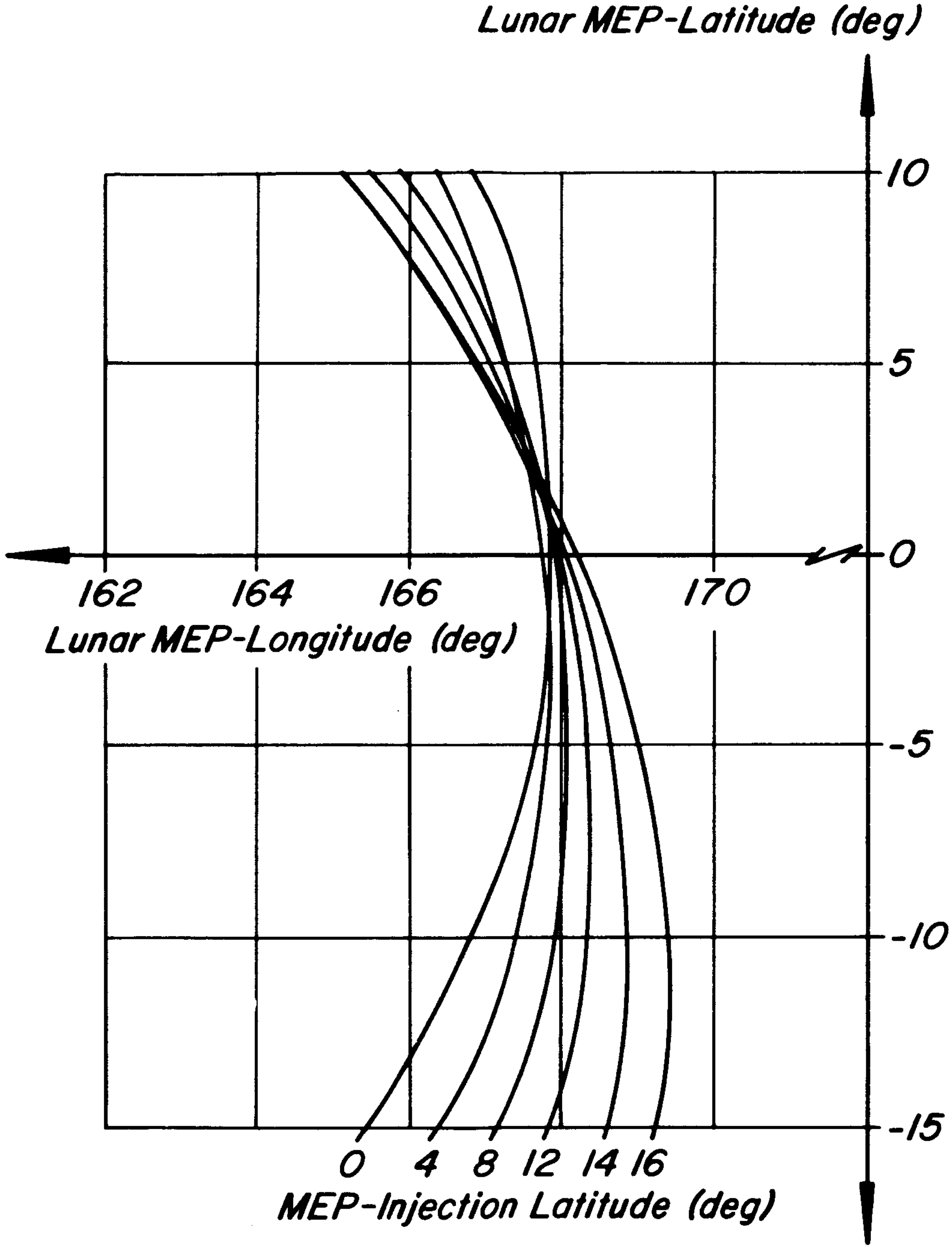


FIG. 13. MEP-LATITUDE AT PERISELENUM VERSUS LONGITUDE

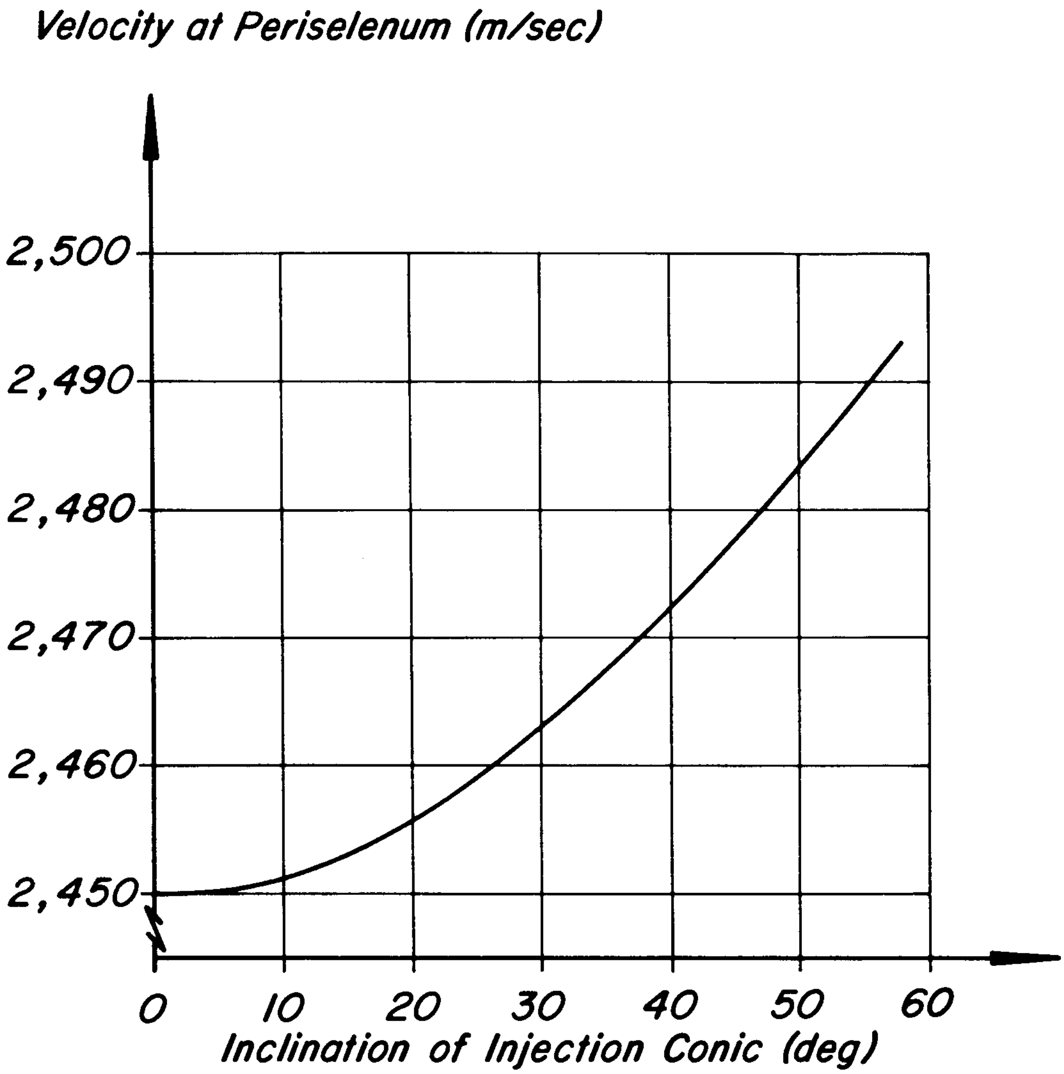


FIG. 14. VELOCITY AT PERISELENUM AS FUNCTION OF INCLINATION OF INJECTION CONIC

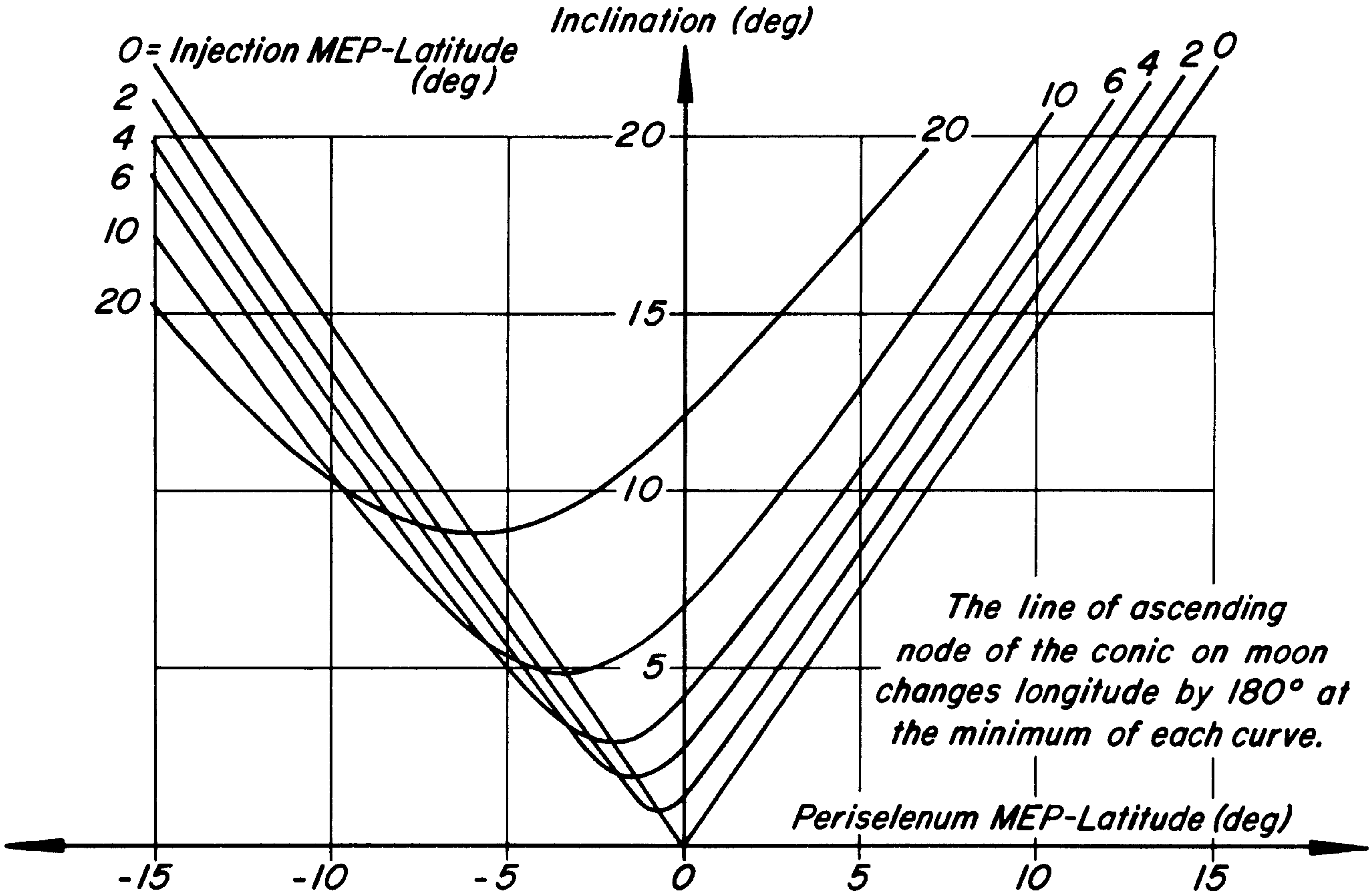


FIG. 15. INCLINATION OF PERISELENUM CONIC TO ME-PLANE AS FUNCTION OF PERISELENUM MEP-LATITUDE

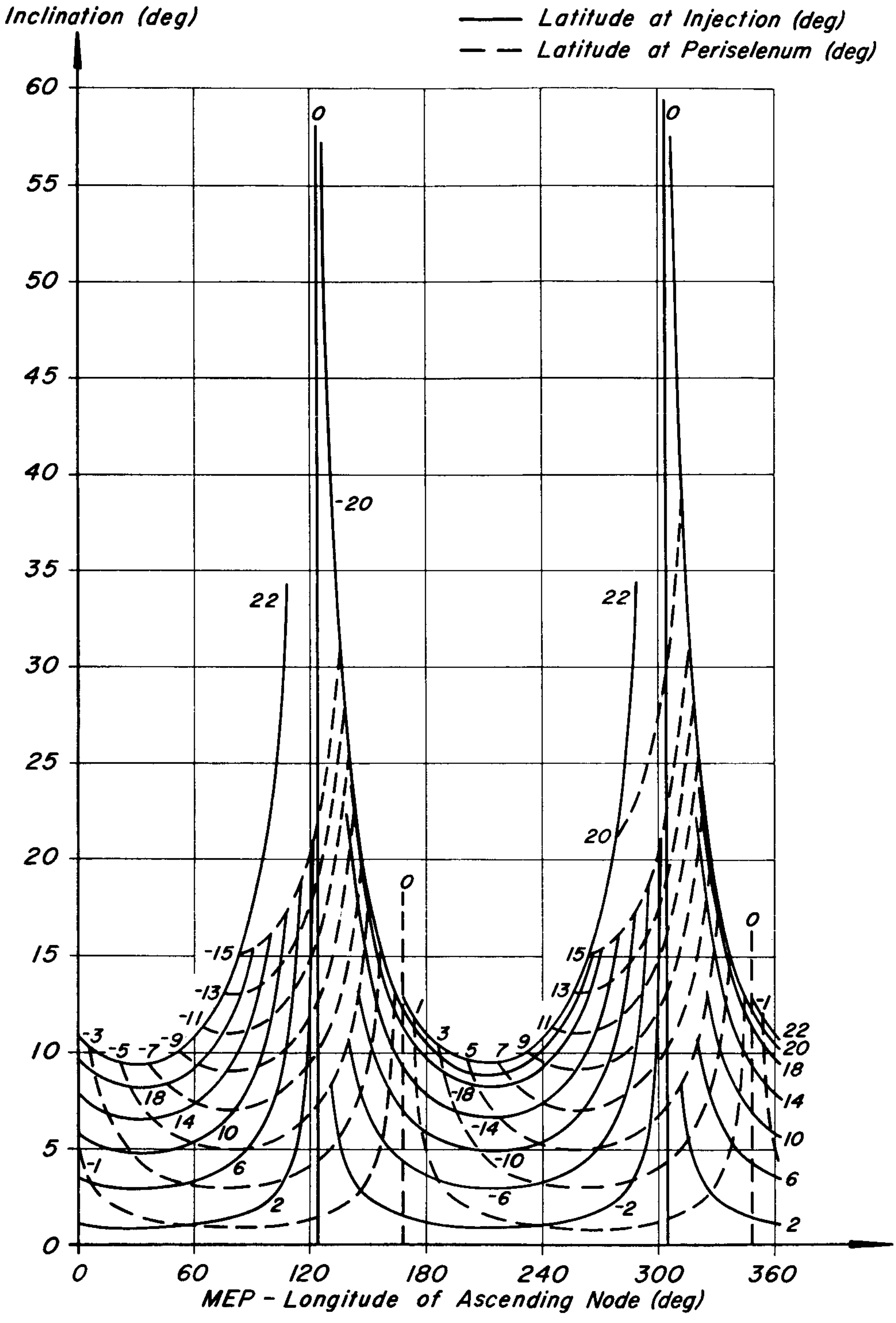


FIG. 16. INCLINATION OF PERISELENUM CONIC TO ME-PLANE AS FUNCTION OF POSITION OF ITS ASCENDING NODE

APPROVAL

EARTH TO MOON TRAJECTORY INVESTIGATION

FOR MISSION PROFILES INVOLVING A LUNAR PARKING ORBIT

By

Nolan J. Braud

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be Unclassified.

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