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THEORETICAL CONSIDERATIONS FOR A PRELIMINARY DESIGN OF A SOLAR CELL GENERATOR ON A SATELLITE

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SUMMARY

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This report presents the preliminary calculations necessary for choosing a solar cell generator from the two design types possible: with solar cells on the satellite skin, or with solar cells on both sides of independent paddles. For each of those two types, numerical results of their aspect ratios are included to permit a direct comparison.

CONTENTS

THEORETICAL CONSIDERATIONS FOR A PRELIMINARY DESIGN OF A SOLAR CELL GENERATOR ON A SATELLITE *

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INTRODUCTION

General Considerations

The primary electrical power generator on a satellite usually consists of a photovoltaic silicon cell array that directly transforms sun-radiated energy into electrical energy and whose series-parallel connections determine the electrical characteristics of the generator.

This primary generator is connected to a sealed secondary battery (sometimes two batteries) so that, during orbital day, the generator supplies the satellite load and charges the secondary battery and, during orbital night, this battery alone supplies the load.

For spin-stabilized satellites, if other parameters are not involved, the preliminary design of the generator may use either solar cells placed directly on the satellite's external surface (skin) or cells placed on both sides of four independent paddles, extended at the injection time.

In these two design cases, the generator's electrical power is proportional to the sum of sunny elementary surfaces, each surface being multiplied by the cosine of the angle it makes with sun direction. This "useful equivalent surface," which represents the sunny-part projection on a perpendicular-to-sun-direction plane, is a function of:

1. Satellite attitude with respect to sun—that is, the angle θ between the spin axis and the sunline.

2. Generator shape—that is, elementary facet position on the satellite skin in the first case or external paddle position in the second case.

We can determine the useful variation gap of the angle θ for launching conditions and the useful lifetime given if the spin axis is assumed to remain steady.

* A related discussion may be found in "A Method to Optimize the Solar Cell Power Supply for Interplanetary Spacecraft," NASA Technical Note D-1846 by Grady B. Nichols.
† Mr. Saint-Jean is a research engineer in the Satellite Division of the Centre National d'Etudes Spatiales, Paris, France, and was assigned as a research associate at Goddard Space Flight Center from July 1962 to February 1963.

Purpose of Study

This study includes the following objectives:

1. Calculation of a theoretical formula for the generator's aspect ratio as a function of the angle θ.

2. Study of some specific problems for both design cases, such as the following:

In the *first case*, determination of the temperature of the generator's various facets and practical calculation of the output power for any given satellite shape.

In the *second case*, determination of the aspect ratio variations during one rotation and calculation of its average value versus sun position and versus paddle extension angle for random variations of the angle θ.

3. The design, for any useful variation gap of the angle θ, of the facet positioning or the paddle extension angle that would give optimum electrical power output.

4. Comparison of the advantages of the two possible design choices.

A GENERATOR WITH SOLAR CELLS ON THE SATELLITE SKIN

Description of Generator

This first design places silicon solar cells directly on various external surfaces of the satellite. The generator thus consists of facets symmetrically placed with respect to the spin axis such that association of each set of facets in the same plane perpendicular to the spin axis may be considered as an *elementary ring*. All of these rings generally do not cover the satellite's total external surface (Figure 1), since it is necessary to leave some surface area free either for fastening other elements (experiments, antennas, de-spin mechanism, separation mechanism) or for thermal control (suitable emissivity and absorptivity paint zones).

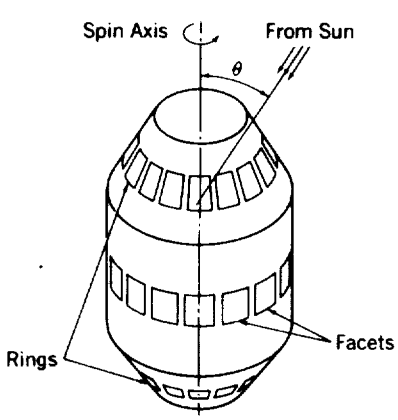


Figure 1—Rings of solar cells on a satellite.

For a spin-stabilized satellite, the resulting efficiency of the generator's various parts is variable: first, because the incident radiation angle depends on the satellite's geometrical shape and on the sun direction and, second, because the generator's different rings reach different temperatures, changing the cells' basic efficiency.

To determine the electrical power output of such a generator, the electrical connections of the cells in each facet, the influence of attitude parameters (rings' aspect ratio), and the temperature effect will be studied herein.

Electrical Series-Parallel Connections of the Solar Cells

Assume that the number of cells connected in series in each facet corresponds to the average operating voltage of the entire generator, that facets of the same ring are connected in parallel, and, finally, that all rings are also connected in parallel (Figure 2).

If the rotation period is assumed to be low with respect to the facets' thermal constant, every facet of the same ring will reach the same temperature and the output current of a ring will be equal to the sum of the output currents of "sunny" facets. Then, since the sun direction changes much more slowly than the rotation angle, the number of sunny facets per ring may be considered as constant. Therefore, if blocking diodes are placed at each facet output to prevent the current from flowing through periodically shadowed facets, the output currents from each ring will be proportional to the aspect ratios of the corresponding rings.

The same reasoning applies to every ring; but, since the temperatures and aspect ratios reached are different, the generator output power will vary. For instance, for the shadowed facets shown in Figure 2 (dashed facets in proper shadow), the first ring could deliver the highest current under the weakest voltage, the second ring could deliver weaker current under higher voltage, and the third ring nothing at all.

The choice of operating conditions must take into consideration these variations: Operating voltage has to be lower than the weakest possible voltage with temperature variations, and operating current has to be higher than the highest load current (to avoid battery discharge into the satellite load during orbital day and, in fact, to charge the battery) (Figure 3).

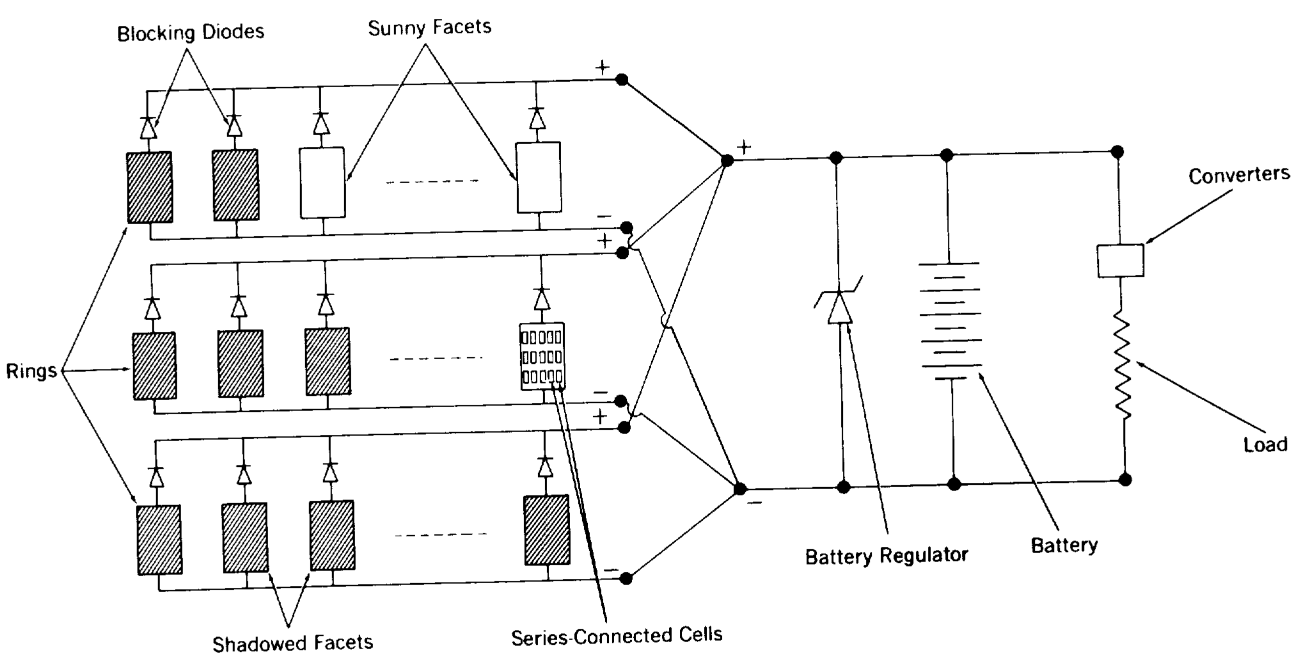


Figure 2—Electrical diagram of a generator.

Aspect Ratio Calculation of the Spherical Spinning Satellite's Rings for a Given Sun Position

We propose to calculate the ratio between a ring's "equivalent useful surface" and its total surface:

$$r = \frac{\sigma}{S}.$$

This aspect ratio r is, first, proportional to the useful output of the ring and, second, involves the ring temperature (ratio between useful absorptive area and emissive area).

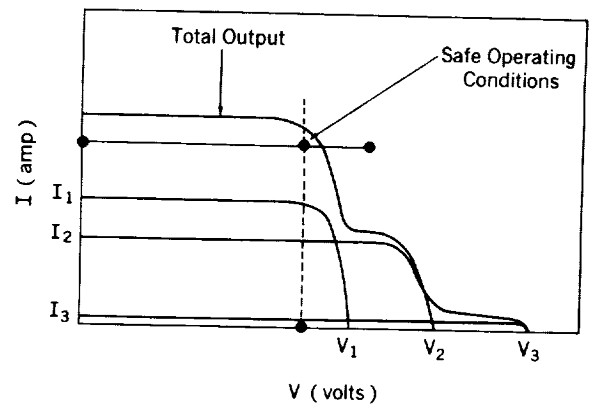


Figure 3—Matching of the rings.

To make a general calculation that will enable the study of every contemplated shape (by approximation), we will consider a spinning sphere that is receiving radiation at angle θ and is divided into elementary rings, each one being defined by the angle ϕ with the satellite equator.

Obviously there will be on the sphere three categories of rings: entirely sunny rings (zone 1), partially sunny rings (zone 2), or non-sunny rings (zone 3), Figure 4.

The Equation

Consider the following reference axis system (Figure 5):

$\vec{Z}_0$ = the satellite spin axis,

$\vec{Y}_0$ = the perpendicular to Z_0 in the plane containing the sun,

$\vec{X}_0$ = the perpendicular to Y_0 and Z_0.

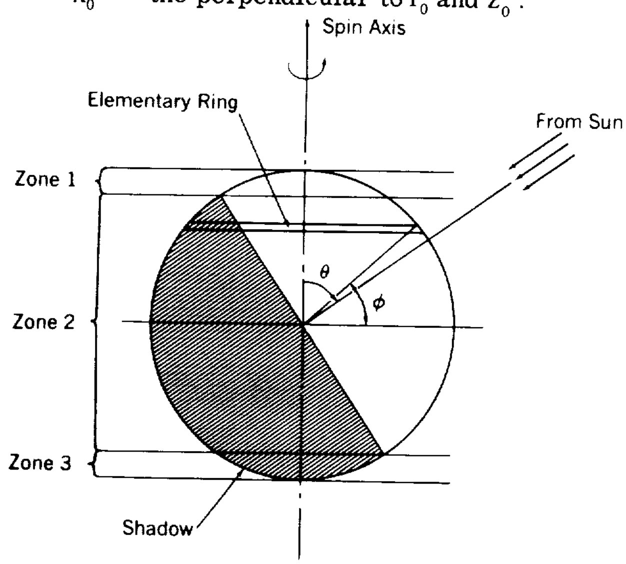


Figure 4—Calculation of the equivalent useful surface of a ring.

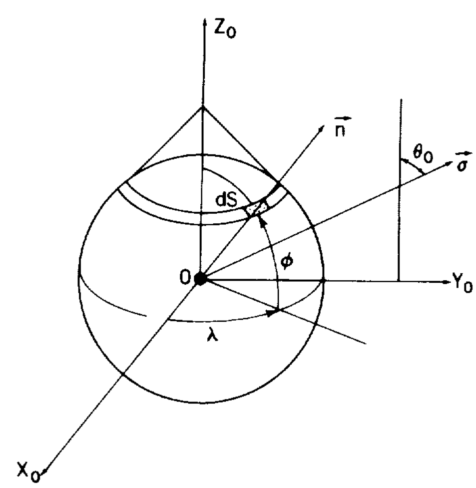


Figure 5—Reference system.

Consider an elementary area dS on one ring, defined by the two angles:

λ = the proper rotation angle,

ϕ = the ring latitude,

and let:

$\vec{n}$ = the axial unit vector of the area dS,

$\vec{\Sigma}$ = the unit vector towards the sun, making the angle θ_0 with $\vec{Z}_0$,

R = the sphere radius.

Components of $\vec{\Sigma}$ and $\vec{n}$ are

$$\vec{\Sigma} \begin{cases} 0 \\ \sin\theta_0 \\ \cos\theta_0 \end{cases} \qquad \vec{n} \begin{cases} \cos\phi \cos\lambda \\ \cos\phi \sin\lambda \\ \sin\phi \end{cases}$$

and the elementary area equals

$$dS = R^2 \cos\phi \, d\lambda \, d\phi \ .$$

Total Area of Any Ring

Integrate the expression dS with respect to λ, within 0 and 2π:

$$S = 2\pi R^2 \cos\phi \, d\phi \ . \tag{1}$$

Useful Equivalent Area of Any Ring

The elementary useful equivalent area of area dS equals its projection on a plane perpendicular to $\vec{\Sigma}$:

$$d\sigma = dS \times \cos\left(\vec{n}, \vec{\Sigma}\right) = dS \times \vec{n} \cdot \vec{S} \ ,$$

$$d\sigma = R^2 \cos\phi \, d\lambda \, d\phi \left(\cos\phi \sin\lambda \sin\theta_0 + \sin\phi \cos\theta_0\right) \ .$$

Removing the integration variable λ, we obtain:

$$d\sigma = R^2 \cos\phi \, d\phi \left(\cos\phi \sin\theta_0 \sin\lambda \, d\lambda + \sin\phi \cos\theta_0 \, d\lambda\right) .$$

Three categories of rings of zones 1, 2, or 3 correspond to different variation limits on ϕ and to different integration limits on λ as follows.

Limits	Zone 1	Zone 2	Zone 3
Variation limits of ϕ	$\theta_0 < \phi < \frac{\pi}{2}$	$-\theta_0 < \phi < +\theta_0$	$-\frac{\pi}{2} < \phi < -\theta_0$
Integration limits of λ	Between 0 and 2π	Between Arc sin $\frac{\tan \phi}{\tan \theta_0}$ and π + Arc sin $\frac{\tan \phi}{\tan \theta_0}$	Between 0 and 0 ($\sigma = 0$)

For Zone 1, then,

$$\sigma = R^2 \cos \phi \, d\phi \left(\cos \phi \sin \theta_0 \int_0^{2\pi} \sin \lambda \, d\lambda + \sin \phi \cos \theta_0 \int_0^{2\pi} d\lambda \right) ,$$

or

$$\sigma = 2\pi R^2 \cos \phi \sin \phi \cos \theta_0 \, d\phi . \tag{2}$$

For Zone 2,

$$\sigma = 2R^2 \cos \phi \, d\phi \left(\cos \phi \sin \theta_0 \int_{-\text{Arc sin} \frac{\tan \phi}{\tan \theta_0}}^{\frac{\pi}{2}} \sin \lambda \, d\lambda + \sin \phi \cos \theta_0 \int_{-\text{Arc sin} \frac{\tan \phi}{\tan \theta_0}}^{\frac{\pi}{2}} d\lambda \right) ,$$

or

$$\sigma = 2R^2 \cos \phi \, d\phi \left[\cos \phi \sin \theta_0 \cos \left(\text{Arc sin} \frac{\tan \phi}{\tan \theta_0} \right) + \sin \phi \cos \theta_0 \left(\frac{\pi}{2} + \text{Arc sin} \frac{\tan \phi}{\tan \theta_0} \right) \right] . \tag{3}$$

For Zone 3, obviously

$$\sigma = 0 . \tag{4}$$

Aspect Ratio Values for the Three Ring Categories

If Equations (2), (3), and (4) are divided by (1), analytical formulas of the aspect ratio are obtained for a given sun position θ_0 versus the angle ϕ, which defines the ring's position:

$$\left.\begin{aligned}
&\theta_0 \le \phi \le \frac{\pi}{2} : \\
&\qquad r = \sin \phi \cos \theta_0 , \\
&-\theta_0 \le \phi \le \theta_0 : \\
&\qquad r = \frac{1}{\pi} \left[\cos \phi \sin \theta_0 \cos \left(\text{Arc sin} \frac{\tan \phi}{\tan \theta_0} \right) + \sin \phi \cos \theta_0 \left(\frac{\pi}{2} + \text{Arc sin} \frac{\tan \phi}{\tan \theta_0} \right) \right] , \\
&-\frac{\pi}{2} \le \phi \le -\theta_0 : \\
&\qquad r = 0 .
\end{aligned}\right\} \tag{5}$$

Since the variable ϕ was not integrated during the calculations, we can consider those formulas as valid not only for elementary rings but also for the *finite* or *infinite* areas that envelop those rings, that is, planes ($\phi = 90°$), cones ($0° < \phi < 90°$), and cylinders ($\phi = 0°$).

Aspect Ratio Calculation of Any Given Satellite's Parts as a Function of Sun Position

We consider now a given satellite shape, for which each part corresponds to a given value of the angle ϕ_0 (Figure 6).

To determine the aspect ratio value of one satellite part versus the sun position, we contemplate ϕ_0 as fixed and θ as variable in Equations (5).

Limiting ϕ_0 to between 0 and 90 degrees, we thus obtain

$$\left.\begin{aligned}
&0 \le \theta \le \phi_0 : \\
&\qquad r = \sin\phi_0 \cos\theta , \\
&\phi_0 \le \theta \le 180° - \phi_0 : \\
&\qquad r = \frac{1}{\pi}\left[\cos\phi_0 \sin\theta \cos\left(\mathrm{Arc}\sin\frac{\tan\phi_0}{\tan\theta}\right) + \sin\phi_0 \cos\theta\left(\frac{\pi}{2} + \mathrm{Arc}\sin\frac{\tan\phi_0}{\tan\theta}\right)\right] , \\
&180° - y_0 \le \theta \le \phi_0 : \\
&\qquad r = 0 .
\end{aligned}\right\} \tag{6}$$

If we analyze the physical phenomena corresponding, for example, to the upper frustum of a cone's *external surface*, the first formula indicates that all this surface is sunny, the second formula that only one part is sunny (the two shadow generatrices move as θ varies), and the third formula that all this surface is shadowed (Figure 7).

Figure 8 presents the results of Equations (6), obtained with the help of the Advanced Orbital Programming Branch of Goddard Space Flight Center (GSFC). This plot gives the aspect ratio values corresponding to ten values of ϕ_0 (0, 10, 20, 30, 40, 50, 60, 70, 80, and 90 degrees), for all sun positions (θ varies between 0 and 180 degrees).

For each curve, the value $\theta = \phi_0$ corresponds to shadow occurrence and the value $\theta = 180° - \phi_0$ corresponds to complete shadow (no aspect ratio). For instance, a truncated conical surface opened at 30

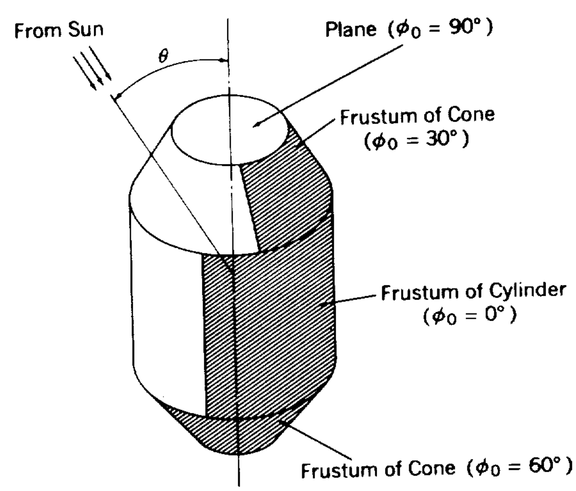


Figure 6—Shadowing of a satellite.

degrees, lighted by rays making an angle of 40 degrees with its axis, will have a theoretical efficiency of r = 41.25 percent.

Influence of Temperature

The facet's elementary cell efficiency is a function of the temperature $\eta(T)$ as approximately shown in Figure 9.

Therefore, we must consider a temperature correction coefficient to obtain the efficiency of an operating ring:

$$r_T = \eta(T) \cdot r .$$

One calculation of ring temperature results from considerations of thermal equilibrium between the energy received and the energy radiated.

This temperature can be determined approximately by considering only the radiations received from the sun and the earth and by neglecting the conduction, convection, and absorptivity variation with incidence:

$$T^4 = \frac{1-f}{\sigma}\left[r_s \frac{\bar{\alpha}_s}{\bar{\epsilon}} C_1 + r_e \left(\frac{\bar{\alpha}_s}{\bar{\epsilon}} C_2 + \frac{\bar{\alpha}_e}{\bar{\epsilon}} C_3\right)\right],$$

with the following notations:

T = the equilibrium temperature,

σ = 5.67×10^{-8} watt/m^2/°C^4 (Stefan-Boltzmann constant),

f = part of incident energy converted to electrical energy by the cells,

r_s = the ring aspect ratio with respect to the sun (angle θ_s between solar radiation and spin axis),

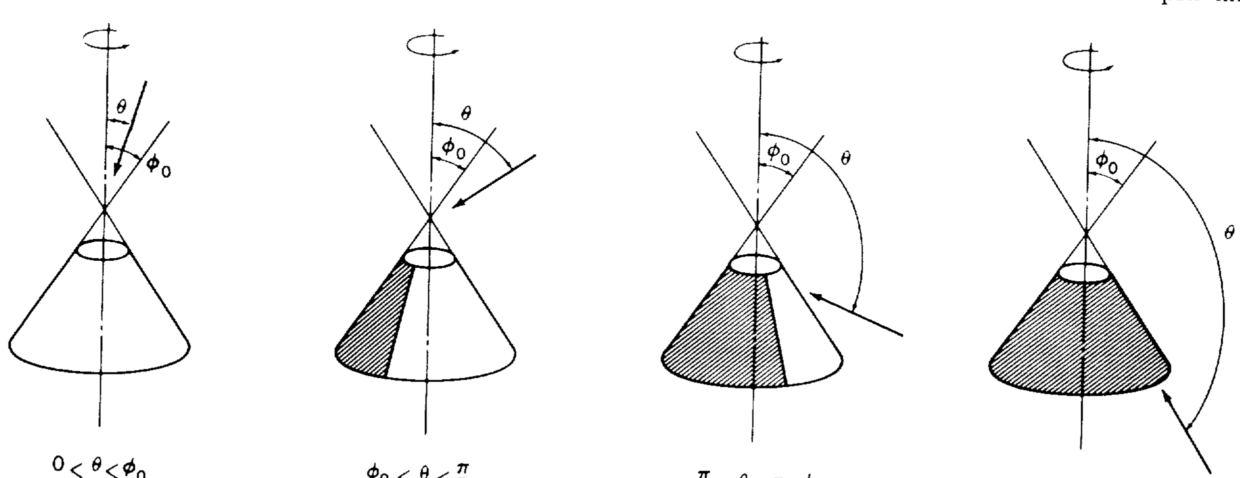


Figure 7—Moving of the shadow generatrices.

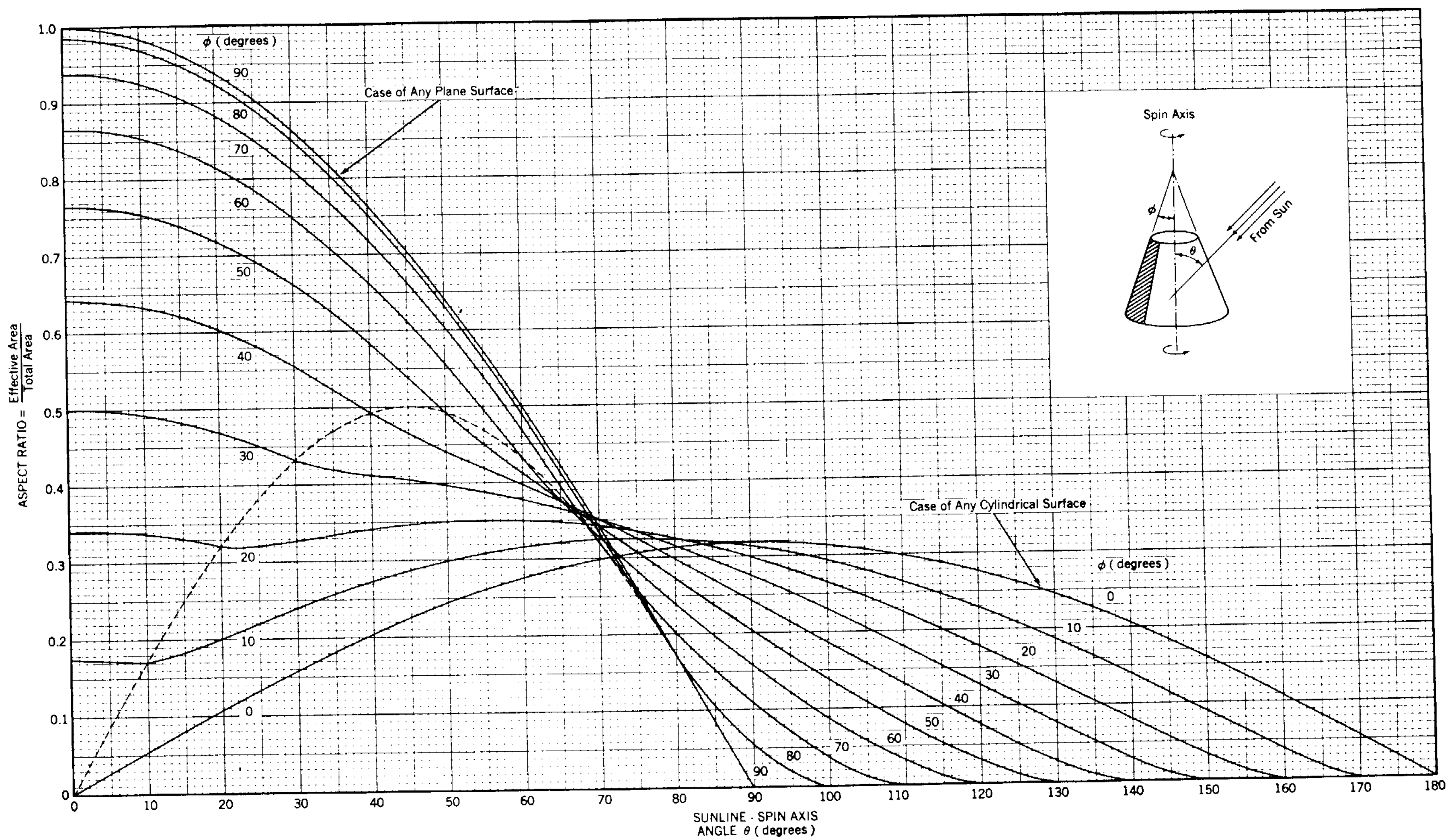
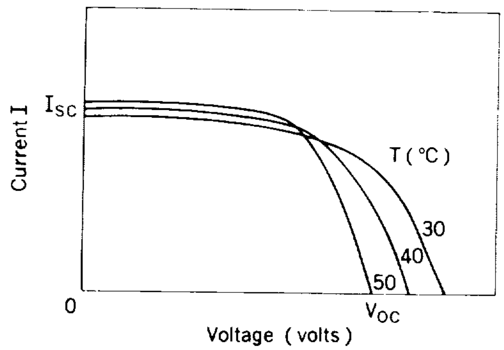


Figure 8—Preliminary design of power supplies, 1st case: solar cells on the skin of a spinning satellite. Study of the aspect ratio for different body shapes. (A large working copy of Figure 8 may be obtained by using the request card bound in the back of this report.)



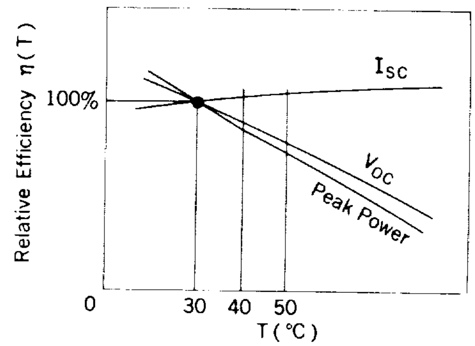


Figure 9—Temperature influence. (I_{sc}, $I_{short\ circuit}$; V_{oc}, $V_{open\ circuit}$.)

r_e = the ring aspect ratio with respect to the earth (angle θ_e between earth radiation and spin axis),

C_1 = 1400 watt/m² = direct solar radiation, integrated as a function of wavelength,

C_2 = 160 watt/m² = solar radiation reflected by the earth (albedo),

C_3 = 140 watt/m² = direct earth radiation, integrated as a function of wavelength,

$\bar{\alpha}_s$ = the ring's average absorptivity for sun radiation,

$\bar{\alpha}_e$ = the ring's average absorptivity for earth radiation,

$\bar{\epsilon}$ = the ring's average emissivity.

In the previous formula, calculation is made step by step since $\bar{\epsilon}$ and f, which are related to the cells' efficiency $\eta(T)$, are variable with the temperature.

Determination of Electrical Power Output

For a given satellite shape (Figure 10), the generator's total output is obtained by adding the outputs delivered by each ring:

$$P_{watts} = \sum_i \left(k_i \times r_i \times \eta_i \text{ watt/cm}^2 \times S_i \text{ cm}^2\right),$$

in which

i = a summation index (for the different rings),

k_i = a coefficient for electrical losses,

r_i = the index i ring aspect ratio,

η_i = the cells' efficiency in space at temperature T,

S_i = the total surface of index i ring cells.

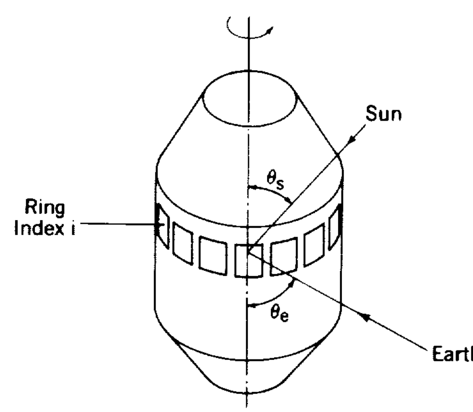


Figure 10—Angles of radiation of ring index i.

So we will obtain the generator's electrical output versus sun position for the sunny part of the orbit. Then, to determine the average output available for the load, we will consider the orbit shadow percentage and the battery charge rate.

A GENERATOR WITH SOLAR CELLS ON FOUR INDEPENDENT PADDLES

Description of Generator

In the second generator design, solar cells are placed on four paddles, independent of the satellite structure (Figure 11). These paddles are attached to the structure by means of coupled shafts and are extended at the injection time with a timer command. They are almost plane, with cells on *both* sides.

Paddle Positioning

The extension of paddles results from the motion of shafts, which may be complicated by the space available under the nose cone; but the final position of each paddle depends, practically, on only one angle, the angle Φ between the axial vector $\vec{n}$ of the paddle and the spin axis. A rotation of each paddle around its axial vector does not involve its aspect ratio, that is, its electrical efficiency.

Generally, the four paddles are placed 90 degrees one from each other, so that their corresponding angles Φ are all equal $(\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4)$. During the spinning motion of the satellite, they will therefore occupy successively coincident positions. From an analytical standpoint, the period of the output function will be divided by the number of paddles; for four paddles, it will equal $\pi/2$.

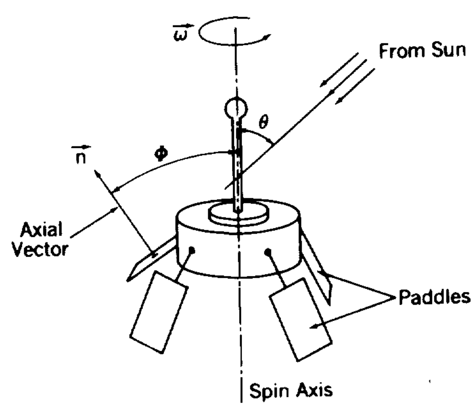


Figure 11—Paddle generator.

Envelope of the Paddles During Spin Motion

During the spin motion of the satellite, the infinite planes that prolong the paddles will remain tangent to a cone having for its axis the spin axis of the satellite.

Let $\vec{Z}$ be the spin axis, (P) the infinite plane that contains one paddle, $\vec{z}$ the projection of $\vec{Z}$ on (P), and ω the point intersected by $\vec{Z}$ on (P), Figure 12.

The angle $\phi = \pi/2 - \Phi$ between $\vec{Z}$ and $\vec{z}$ being constant, the path of $\vec{z}$ will be a cone whose apex is ω, whose axis is $\vec{Z}$, and whose half apex angle is ϕ. This straight line $\vec{z}$ is the contact slant height between (P) and the cone. Since the four planes intersect $\vec{Z}$ at the same point ω—if the centers

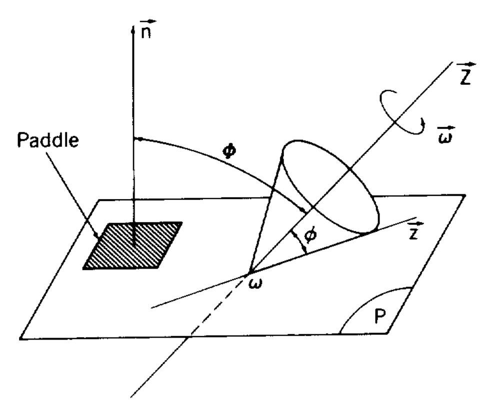


Figure 12—Paddle cone envelope.

of the paddles are in the same plane perpendicular to the spin axis, then their envelope will be this cone.

Figure 13 shows a view of four paddles and their cone envelope in a general case.

General Formula for the Aspect Ratio

The voltage of this generator is determined mainly by the number of cells connected in series and by the operating temperature. Since the four extension angles ϕ are assumed to be equal, we can admit that the temperature of the four paddles is the same. Since the number of cells connected in series on each paddle is chosen to be the same, and since the four paddles are connected in parallel, the current output of the generator will be proportional to the sum of the aspect ratios of the four paddles.

We will first establish a general formula for this aspect ratio as a function of the extension angle ϕ of the paddles, of the position of the sun (angle θ between solar rays and spin axis), and of time, for a spin rate ω (proper rotation angle $\lambda = \omega t$).

Shadow Problems

For certain positions of the sun with respect to the satellite structure, projected shadow problems and self shadow problems occur.

1. <u>Self shadow</u>

Solar cells are placed on both sides of each paddle. Nevertheless, only one side will be sunny at any given time. Over one complete revolution of the satellite, it might be always the same side, or alternately one and then the other (to be discussed on p. 16).

2. <u>Projected shadow (Figure 14)</u>

Depending on the geometrical shapes and the positions of the structure and the paddles, four cones can be defined to study these problems:

For the shadow projected by the structure on the paddles, the two half apex angles are α_1 and α_2:

$0° < \theta < \alpha_1$: no shadow

$\alpha_1 < \theta < \alpha_2$: shadow

$\alpha_2 < \theta < 180°$: no shadow

For the shadow projected by one paddle on another, the two half apex angles are β_1 and β_2:

$0° < \theta < \beta_1$: no shadow

$\beta_1 < \theta < \beta_2$: shadow

$\beta_2 < \theta < 180°$: no shadow

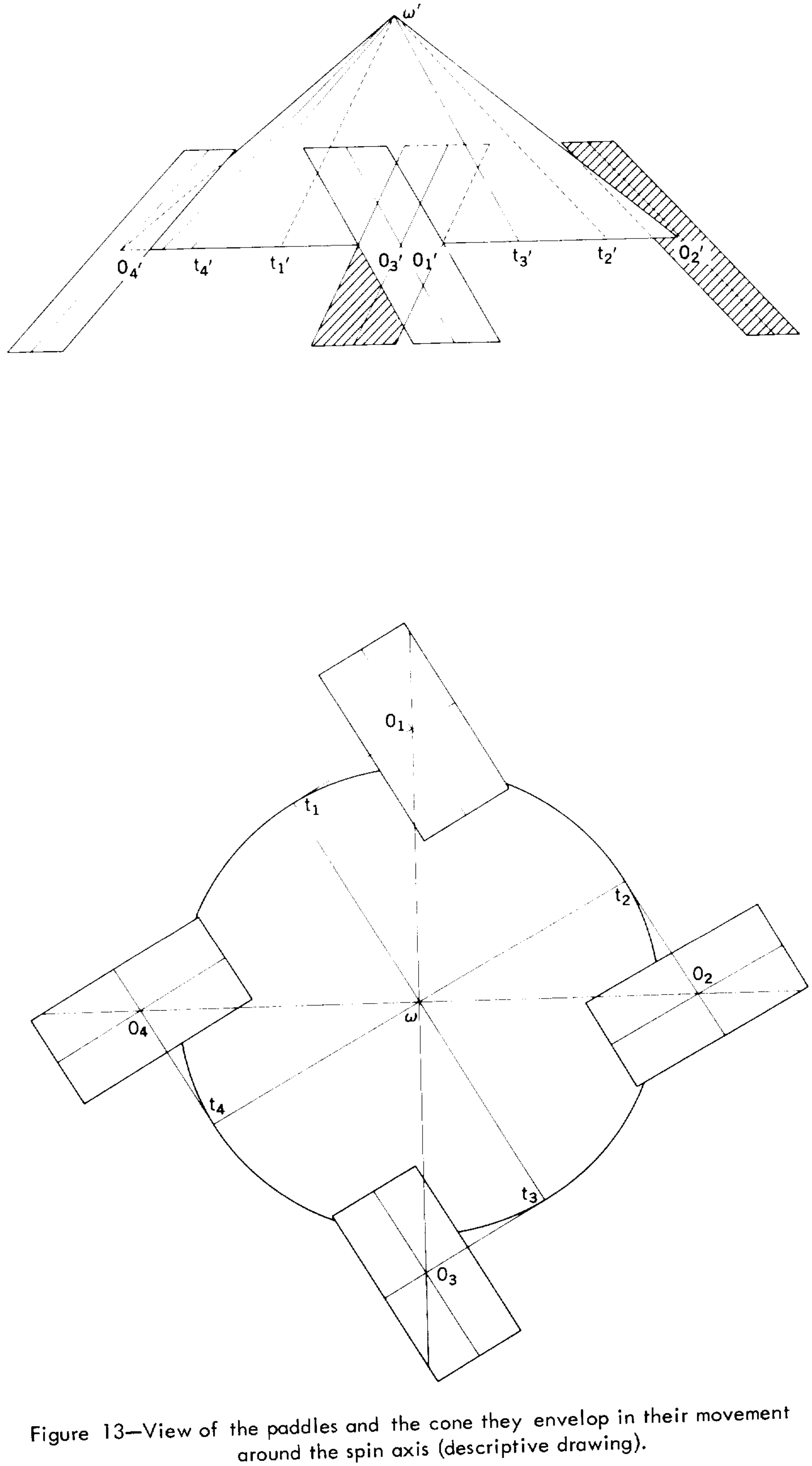


Figure 13—View of the paddles and the cone they envelop in their movement around the spin axis (descriptive drawing).

In the following study, we will consider only the self shadow problems, the others being widely dependent on each particular case and only slightly relevant in a theoretical study.

The Equation

Consider a reference axis system, half-steady with respect to the satellite, so defined (Figure 15):

$\vec{Z}_0$ = satellite spin axis,

$\vec{Y}_0$ = perpendicular to $\vec{Z}_0$ in the plane that contains the sun,

$\vec{X}_0$ = perpendicular to $\vec{Y}_0$ and $\vec{Z}_0$.

Let the paddles be $P_{1,2,3,4}$, and their axial vectors $\vec{n}_{1,2,3,4}$.

Their angular positions are defined by the two angles of proper rotation $\lambda_{1,2,3,4}$ and extension $\phi_{1,2,3,4}$.

We assume the paddles to be 90 degrees from each other so that

$$\lambda_1 = \omega t ,$$
$$\lambda_2 = \omega t + \frac{\pi}{2} ,$$
$$\lambda_3 = \omega t + \pi ,$$
$$\lambda_4 = \omega t + \frac{3\pi}{2} ,$$

and to be extended by the same angle so that

$$\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi .$$

The four axial vectors $\vec{n}_{1,2,3,4}$, having 0 as their origin make with $\vec{Z}_0$ the same angle Φ:

$$\Phi = \frac{\pi}{2} - \phi .$$

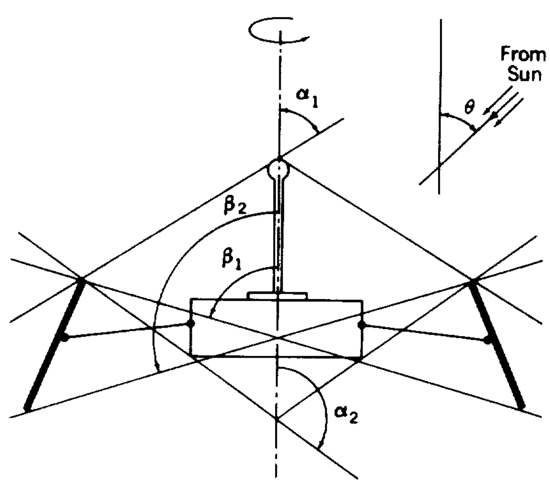


Figure 14—Limits of the projected shadows.

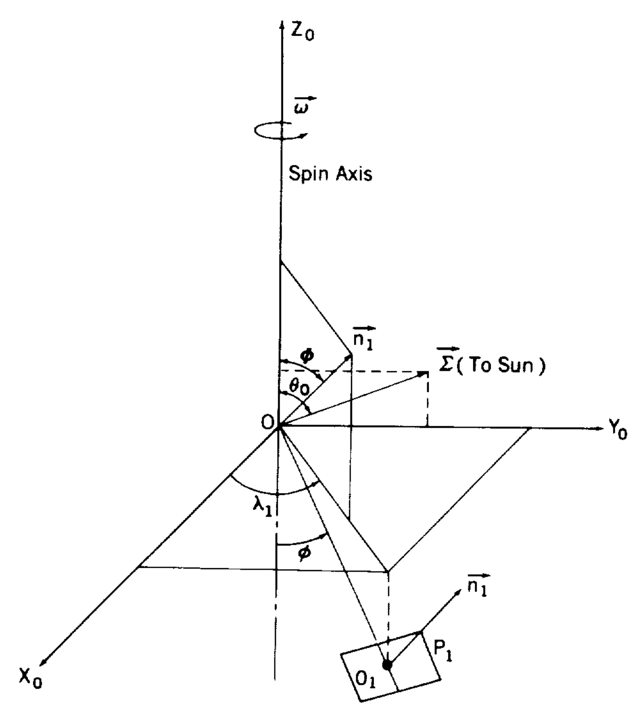


Figure 15—Reference system.

The unit solar vector is $\vec{\Sigma}$, in the plane Y_0Z_0, making with $\vec{Z}_0$ the angle θ_0. Its components are:

$$\vec{\Sigma} \begin{cases} 0 \\ \sin\theta_0 \\ \cos\theta_0 \end{cases}$$

Assume the initial time $t = 0$ to be the time when the axial vector $\vec{n}_1$ of P_1 is parallel to the plane X_0Z_0 (or directly in this plane). At any time t, the axial vector components will therefore be

$$\vec{n}_1 \begin{cases} \cos\phi\cos\omega t \\ \cos\phi\sin\omega t \\ \sin\phi \end{cases} \qquad \vec{n}_2 \begin{cases} -\cos\phi\sin\omega t \\ \cos\phi\cos\omega t \\ \sin\phi \end{cases} \qquad \vec{n}_3 \begin{cases} -\cos\phi\cos\omega t \\ -\cos\phi\sin\omega t \\ \sin\phi \end{cases} \qquad \vec{n}_4 \begin{cases} \cos\phi\sin\omega t \\ -\cos\phi\cos\omega t \\ \sin\phi \end{cases}$$

Let S be the surface of one side of the paddles. The total surface of the cells of the generator is therefore

$$S_{total} = 8S .$$

The "useful equivalent surface" σ of each paddle is equal to the projection of the sunny side on a plane perpendicular to the sun direction:

$$\sigma = S \times \cos\left(\vec{n}, \vec{\Sigma}\right) = S \times \left(\vec{n} \cdot \vec{\Sigma}\right) .$$

In fact we have to contemplate the modulus of the former function, for both sides are involved in furnishing current. A negative value of σ corresponds to a current identical to the one that would be furnished by the other side, but positive.

Figure 16 represents the real value of σ, the value of σ for a paddle having cells only on one side, and the value of σ for a paddle having cells on both sides. Thus we will consider

$$\sigma = S \times \left| \vec{n} \cdot \vec{\Sigma} \right| .$$

For the complete generator, the "useful equivalent area" is

$$\sigma_{total} = S \times \left(\left|\vec{n}_1 \cdot \vec{\Sigma}\right| + \left|\vec{n}_2 \cdot \vec{\Sigma}\right| + \left|\vec{n}_3 \cdot \vec{\Sigma}\right| + \left|\vec{n}_4 \cdot \vec{\Sigma}\right| \right) ,$$

and the aspect ratio is

$$r = \frac{\sigma_{total}}{S_{total}} = \frac{\left|\vec{n}_1 \cdot \vec{\Sigma}\right| + \left|\vec{n}_2 \cdot \vec{\Sigma}\right| + \left|\vec{n}_3 \cdot \vec{\Sigma}\right| + \left|\vec{n}_4 \cdot \vec{\Sigma}\right|}{8} ,$$

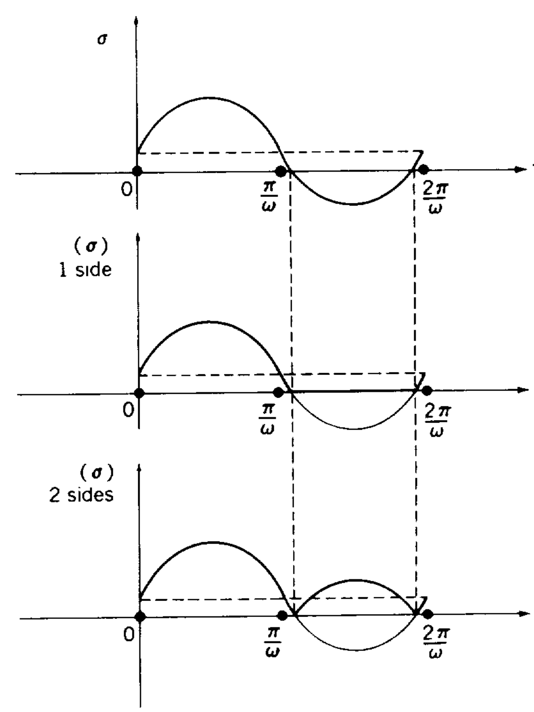


Figure 16—Possible values of σ.

whose analytical expression is

$$r = \frac{1}{8}\Big[\left|\sin\theta\cos\phi\sin\omega t + \cos\theta\sin\phi\right| + \left|\sin\theta\cos\phi\cos\omega t + \cos\theta\sin\phi\right| + \left|-\sin\theta\cos\phi\sin\omega t + \cos\theta\sin\phi\right| + \left|-\sin\theta\cos\phi\cos\omega t + \cos\theta\sin\phi\right|\Big] . \tag{7}$$

Discussion of the Two Operating Cases for a Given Position of the Sun

Each one of the four terms of r is a sine function of time, whose mean value is

$$\cos\theta_0 \sin\phi$$

and whose amplitude is

$$\sin\theta_0 \cos\phi .$$

The analytical corresponding forms of these terms are to be changed by their opposites when the amplitude is superior to the mean value, that is, when

$$\sin\theta_0 \cos\phi \geq \cos\theta_0 \sin\phi .$$

If we limit the variations to

$$0 < \theta_0 < \frac{\pi}{2} ,$$

$$0 < \phi < \frac{\pi}{2} ,$$

the condition may be written

$$\frac{\tan\phi}{\tan\theta_0} \leq 1 ,$$

or

$$0 \leq \phi \leq \theta_0 .$$

Accordingly, there are two operating cases, depending on the value of the extension angle ϕ with respect to the value of the sun ray angle θ_0 (Figure 17).

1st Case, $\theta_0 \leq \phi \leq \pi/2$:

The aspect ratio curve for each paddle does *not* cut the ωt axis, which means that only one side of the paddle is sunny while the other side is always shadowed.

In this case the moduli are *not* involved, and the generator aspect ratio remains constant during the rotation. (The analytical study is given in the next section.)

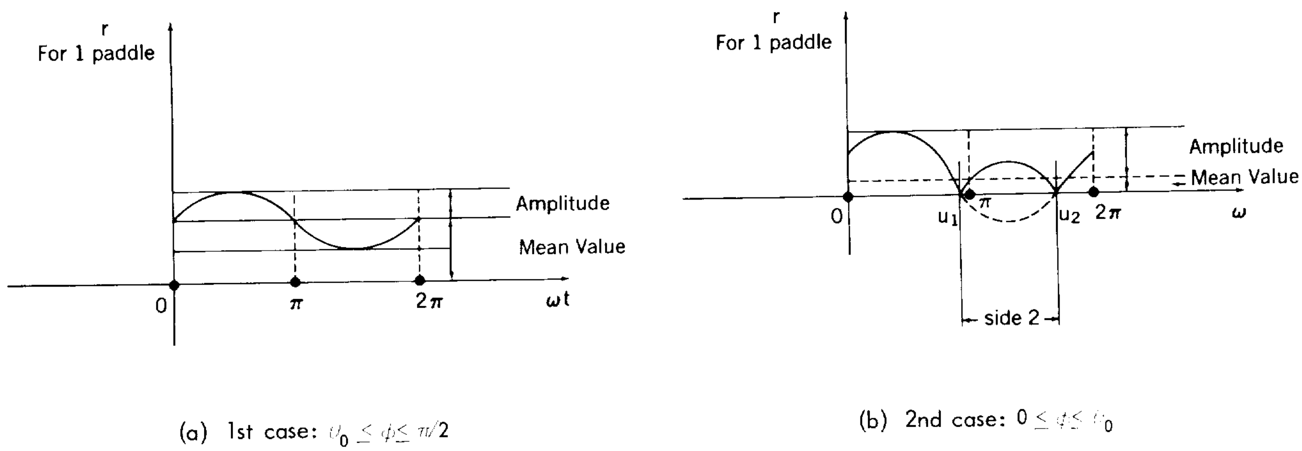


Figure 17—The two operating cases.

2nd Case, $0 \leq \phi \leq \theta_0$:

The aspect ratio curve for each paddle cuts the ωt axis, which means that each side of the paddle is alternately sunny. We then must consider the moduli; and the aspect ratio is not constant during the rotation.

For paddle P_1 , for instance:

If ωt is in the interval u_1, u_2: $\quad |r_{P_1}| = -r_{P_1}$.

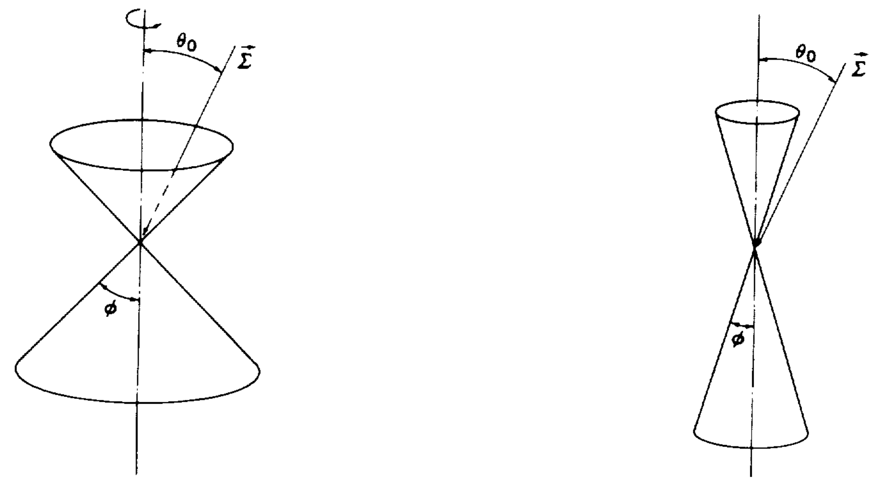
If ωt is out of the interval u_1, u_2: $\quad |r_{P_1}| = r_{P_1}$.

(The analytical study is given below.)

It should be noted that the physical operation of the generator corresponding to these two cases can be explained geometrically (Figure 18). The cone enveloped by the paddles during their movement has a half apex angle ϕ, and the solar vector $\vec{\Sigma}$ makes an angle θ_0 with the spin axis. So the two cases correspond to the fact that the vector $\vec{\Sigma}$ is inside the cone in the first case $\left(\theta_0 \leq \phi \leq \pi/2\right)$ and outside the cone in the second case $\left(0 \leq \phi \leq \theta_0\right)$.

Aspect Ratio Variation During One Satellite Revolution

Consider the angle θ_0 as given, the angle ϕ as a parameter, and the proper rotation angle $\lambda = \omega t$ as the variable. We will study the shapes of the aspect ratios r of the four paddles P_1, P_2, P_3, P_4; their sum two by two $P_1 + P_3$, $P_2 + P_4$; and then their complete sum $P_1 + P_2 + P_3 + P_4$ (Figures 19 and 20).



(a) 1st case: $\theta_0 \leq \phi \leq \pi/2$ (b) 2nd case: $0 \leq \phi \leq \theta_0$

Figure 18—Geometrical explanation of the two cases.

Study of the 1st case, $\theta_0 \leq \phi \leq \pi/2$:

The analytical formula of r is, in this case,

$$r = \frac{1}{8}\left(r_{P_1} + r_{P_2} + r_{P_3} + r_{P_4}\right) ,$$

with

$$r_{P_1} = \sin\theta_0 \cos\phi \sin\omega t + \cos\theta_0 \sin\phi ,$$

$$r_{P_2} = \sin\theta_0 \cos\phi \cos\omega t + \cos\theta_0 \sin\phi ,$$

$$r_{P_3} = -\sin\theta_0 \cos\phi \sin\omega t + \cos\theta_0 \sin\phi ,$$

$$r_{P_4} = -\sin\theta_0 \cos\phi \cos\omega t + \cos\theta_0 \sin\phi .$$

Each term r_{P_1}, r_{P_2}, r_{P_3}, r_{P_4} is represented by a sine curve that does not cut the ωt axis. The partial sums $P_1 + P_3$, $P_2 + P_4$ are represented by straight lines; and the general sum $P_1 + P_2 + P_3 + P_4$ is represented by a straight line:

$$r = \frac{1}{2}\cos\theta_0 \sin\phi .$$

Figure 19(b) shows this general case.

Figure 19(a) represents the particular case $\phi = \pi/2$, which corresponds to paddles perpendicular to the spin axis (the aspect ratios are all constant).

(a) Particular case, $\phi = \pi/2$ (paddles perpendicular to spin axis)

(b) Study of the 1st, or general, case $(\theta_0 \leq \phi \leq \pi/2)$

(c) Particular case, $\phi = \theta_0$

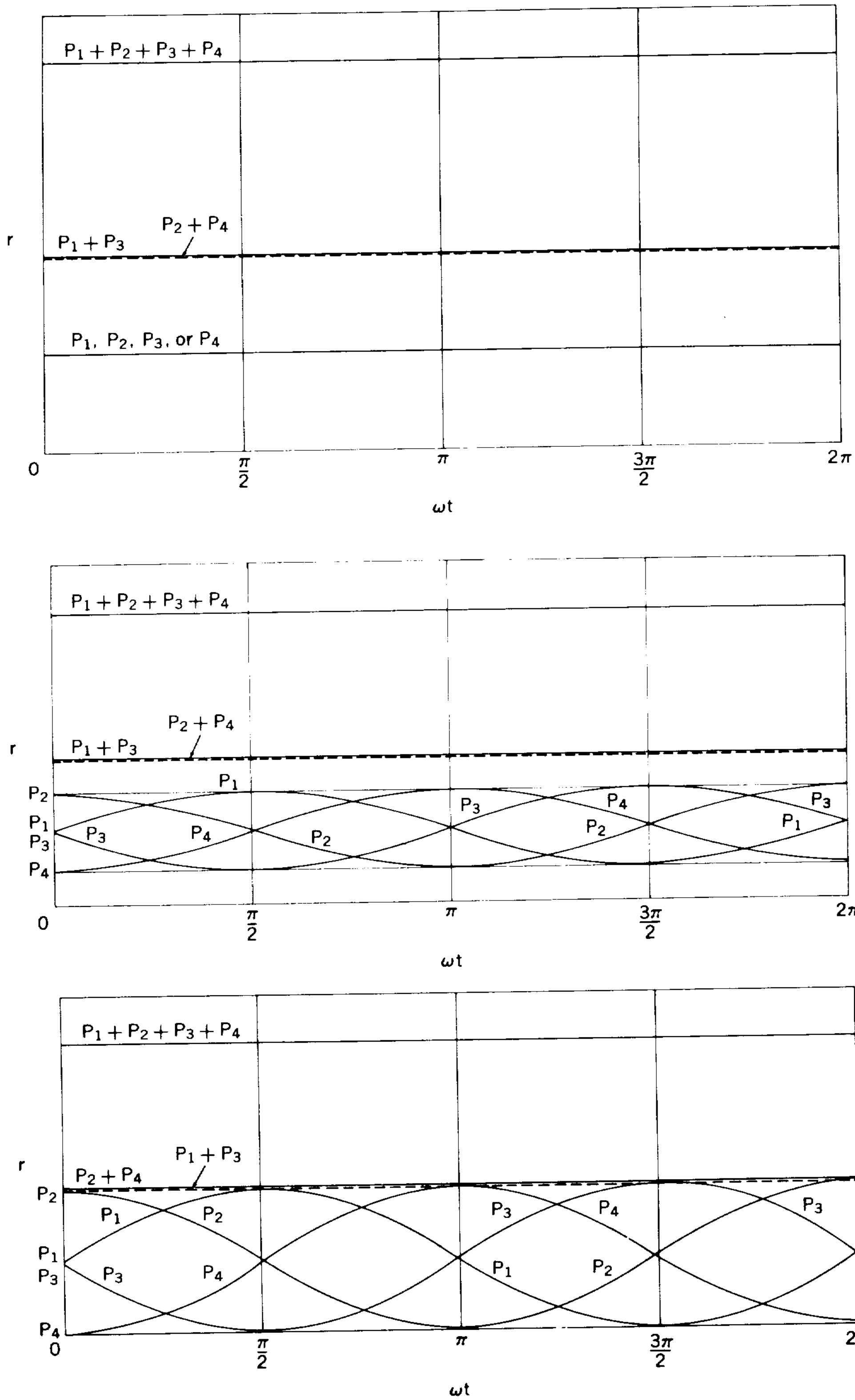
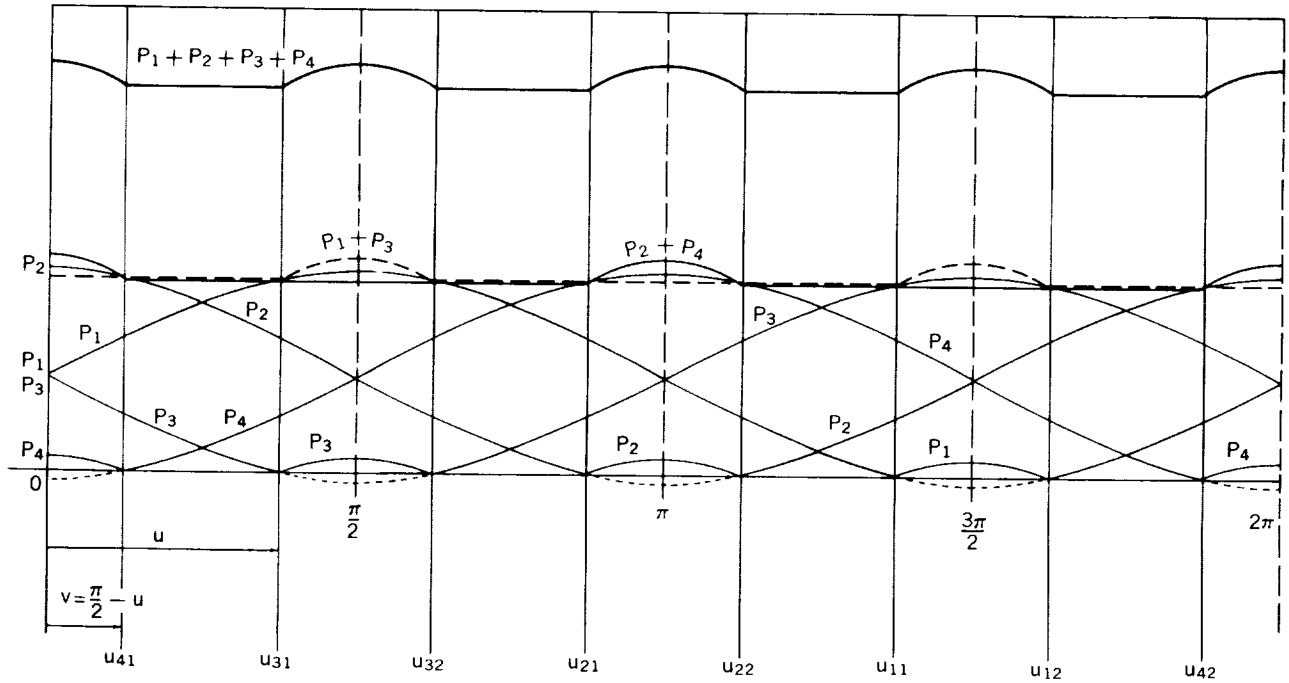
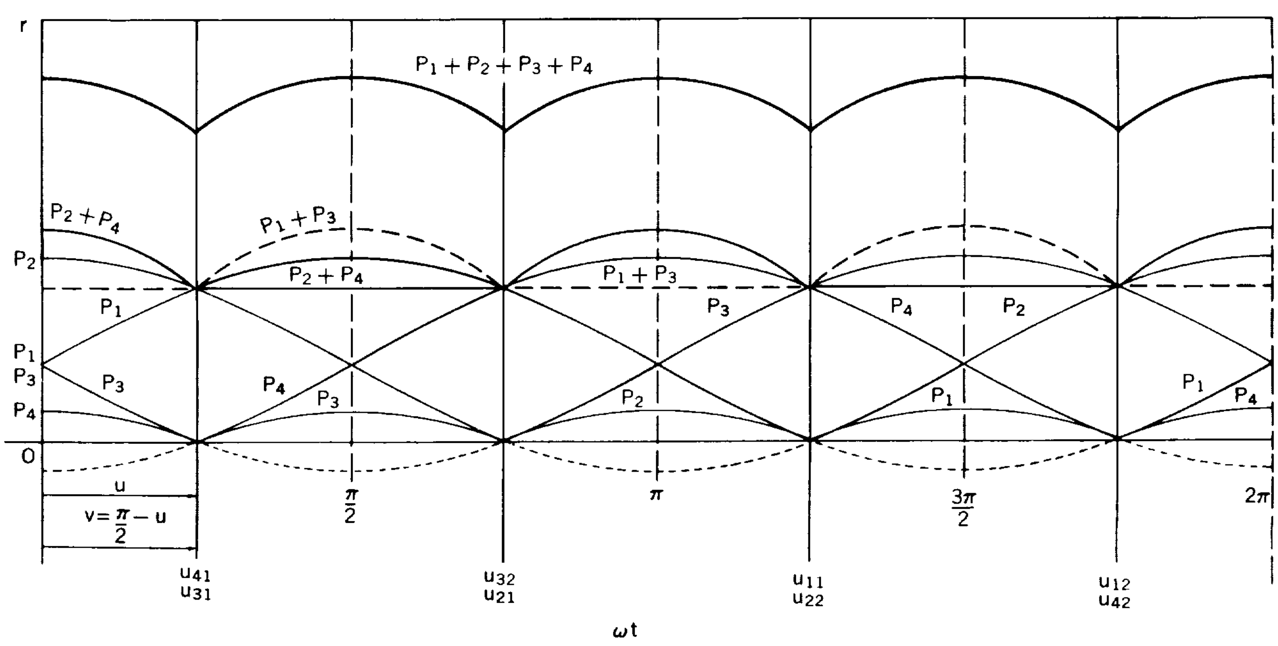


Figure 19—Aspect ratio variations of the four paddles P_1, P_2, P_3, P_4 for one period of revolution: 1st case, $\theta_0 \leq \phi \leq \pi/2$.

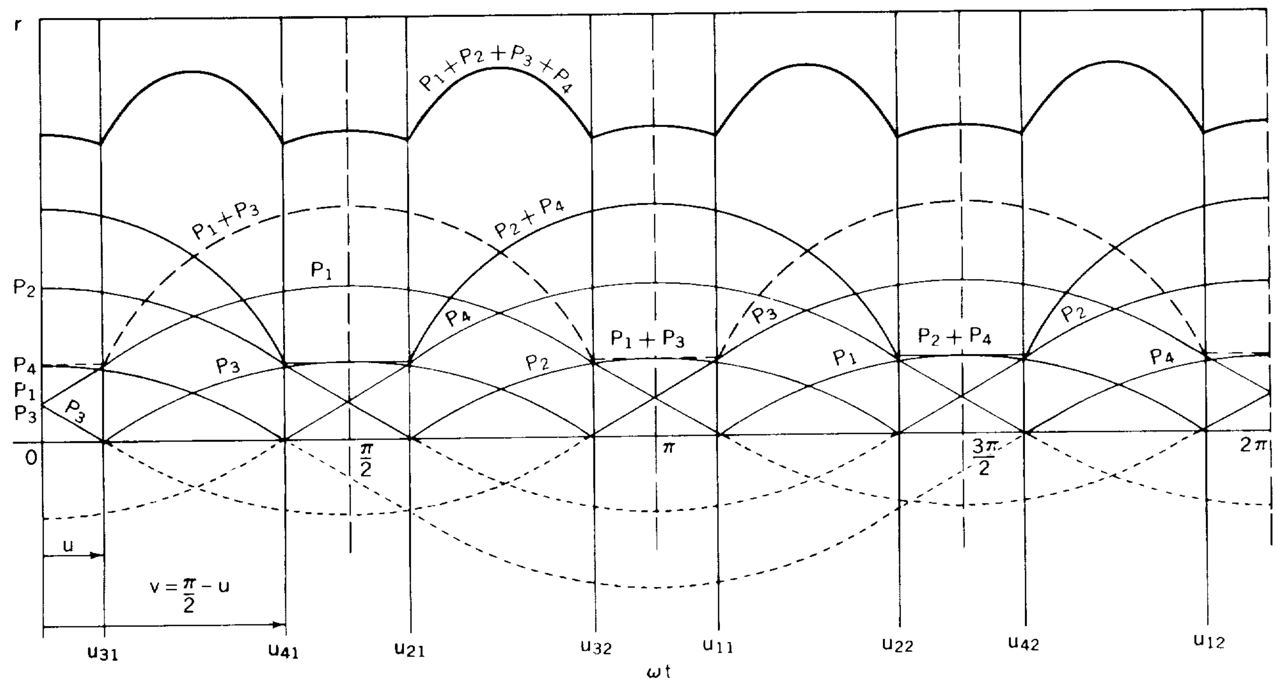


(a) For $\frac{\pi}{4} < u = \text{Arc sin} \frac{\tan \phi}{\tan \theta_0} < \frac{\pi}{2}$

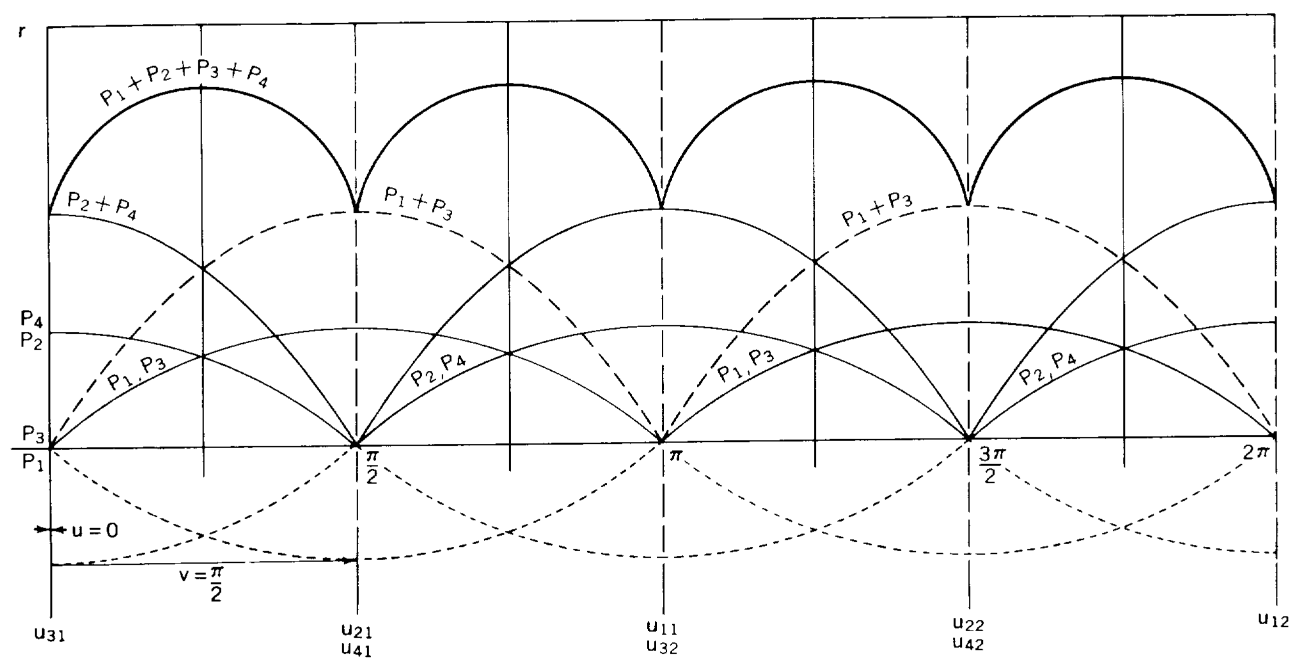


(b) For $u = \text{Arc sin} \frac{\tan \phi}{\tan \theta_0} = \frac{\pi}{4}$

Figure 20—Aspect ratio variations of the four paddles P_1, P_2, P_3, P_4 for one period of revolution: 2nd case, $0 \leq \phi \leq \theta_0$.



(c) For $0 < u = \text{Arc sin}\ \frac{\tan\phi}{\tan\theta_0} < \frac{\pi}{4}$



(d) For $\phi = 0$ ($u = \text{Arc sin}\ \frac{\tan\phi}{\tan\theta_0} = 0$), paddles parallel to spin axis

Figure 20(Concluded)—Aspect ratio variations for the four paddles P_1, P_2, P_3, P_4 for one period of revolution: 2nd case, $0 \le \phi \le \theta_0$.

Figure 19(c) gives the particular case $\phi = \theta_0$, which corresponds to paddles whose cone envelope has *one* slant height parallel to the solar rays. The aspect ratios equal zero for only *one* value of ωt, but the second side still is not sunny.

Study of the 2nd case, $0 \leq \phi \leq \theta_0$:

The moduli must be considered in this case. Let us calculate first the two values of ωt corresponding to the sign change and defining the sunny side change. If we call u and v the main determinations as follows,

$$\left.\begin{aligned} u &= \text{Arc sin}\,\frac{\tan\phi}{\tan\theta_0} \\ v &= \text{Arc cos}\,\frac{\tan\phi}{\tan\theta_0} \end{aligned}\right\} \quad 0 < u,\, v < \frac{\pi}{2},$$

the values are, respectively,

For paddle P_1,

$$\omega t_1 = \text{arc sin}\left(-\frac{\tan\phi}{\tan\theta_0}\right) \quad \begin{cases} 1^{st} \text{ value: } u_{11} = \pi + u, \\ 2^{nd} \text{ value: } u_{12} = 2\pi - u; \end{cases}$$

For paddle P_2,

$$\omega t_2 = \text{arc cos}\left(-\frac{\tan\phi}{\tan\theta_0}\right) \quad \begin{cases} 1^{st} \text{ value: } u_{21} = \frac{\pi}{2} + u = \pi - v, \\ 2^{nd} \text{ value: } u_{22} = \frac{3\pi}{2} - u = \pi - v; \end{cases}$$

For paddle P_3,

$$\omega t_3 = \text{arc sin}\left(\frac{\tan\phi}{\tan\theta_0}\right) \quad \begin{cases} 1^{st} \text{ value: } u_{31} = u, \\ 2^{nd} \text{ value: } u_{32} = \pi - u; \end{cases}$$

For paddle P_4,

$$\omega t_4 = \text{arc cos}\left(\frac{\tan\phi}{\tan\theta_0}\right) \quad \begin{cases} 1^{st} \text{ value: } u_{41} = \frac{\pi}{2} - u = v, \\ 2^{nd} \text{ value: } u_{42} = \frac{3\pi}{2} + u = 2n - v. \end{cases}$$

So, these eight values u_{11}, u_{12}, u_{21}, u_{22}, u_{31}, u_{32}, u_{41}, and u_{42} must be sorted by order of increasing values and the two cases distinguished according to the value of $u = \text{Arc sin}\,\frac{\tan\theta}{\tan\theta_0}$ with respect to $\pi/4$. The shapes of the curves will be different in the two following cases:

1. For $0 < u = \text{Arc sin}\,\frac{\tan\phi}{\tan\theta_0} < \frac{\pi}{4}$ [Figure 20(c)] :

The complete aspect ratio $P_1 + P_2 + P_3 + P_4$ has a wave shape of period $\pi/2$ involving two different parts of sine functions. The corresponding analytical formulas, given in Table 1, change at each of the eight former discontinuity points.

2. For $\frac{\pi}{4} < u = \text{Arc sin}\,\frac{\tan\phi}{\tan\theta_0} < \frac{\pi}{2}$ [Figure 20(a)] :

The complete aspect ratio has a wave shape of period $\pi/2$ but involves only one sine function, the other part of the curve being constant. The corresponding analytical formulas are given in Table 2.

Likewise, two particular cases are given on page 24.

Table 1

$$0 \le u = \text{Arc sin} \frac{\tan \phi}{\tan \theta_0} \le \frac{\pi}{4}$$

r	ωt		
	$0 \le \omega t \le u$	$u \le \omega t \le \frac{\pi}{2} - u$	$\frac{\pi}{2} - u \le \omega t \le u$
r_{p_1}	$\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$
r_{p_2}	$\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$
r_{p_3}	$-\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \sin \omega t - \cos \theta_0 \sin \phi$*	$\sin \theta_0 \cos \phi \sin \omega t - \cos \theta_0 \sin \phi$*
r_{p_4}	$\sin \theta_0 \cos \phi \cos \omega t - \cos \theta \sin \phi$*	$\sin \theta_0 \cos \phi \cos \omega t - \cos \theta_0 \sin \phi$*	$-\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$
r	$\frac{1}{4} \left[\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi\right]$	$\frac{1}{4} \cos \theta_0 \cos \phi \left[\sin \omega t + \cos \omega t\right]$	$\frac{1}{4} \left[\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi\right]$

*The index * means that the sign has been changed.

Table 2

$$\frac{\pi}{4} \le u = \text{Arc sin} \frac{\tan \phi}{\tan \theta_0} \le \frac{\pi}{2}$$

r	ωt		
	$0 \le \omega t \le \frac{\pi}{2} - u$	$\frac{\pi}{2} - u \le \omega t < u$	$u \le \omega t \le \frac{\pi}{2}$
r_{p_1}	$\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$
r_{p_2}	$\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$
r_{p_3}	$-\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$-\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi$	$\sin \theta_0 \cos \phi \sin \omega t - \cos \theta_0 \sin \phi$*
r_{p_4}	$\sin \theta_0 \cos \phi \cos \omega t - \cos \theta_0 \sin \phi$*	$-\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$	$-\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi$
r	$\frac{1}{4} \left[\sin \theta_0 \cos \phi \cos \omega t + \cos \theta_0 \sin \phi\right]$	$\frac{1}{2} \cos \theta_0 \sin \phi$	$\frac{1}{4} \left[\sin \theta_0 \cos \phi \sin \omega t + \cos \theta_0 \sin \phi\right]$

*The index * means that the sign has been changed.

1. For $u = \text{Arc sin}\, \frac{\tan \phi}{\tan \theta_0} = \frac{\pi}{4}$ [Figure 20(b)]:

The aspect ratio curve in this case is symmetrical; this corresponds to the fact that the second side of the paddles is sunny exactly one-fourth the time of revolution.

2. For $u = \text{Arc sin}\, \frac{\tan \phi}{\tan \theta_0} = 0$ [Figure 20(d)]:

The aspect ratio curve still has a symmetrical shape, but the ratio between the maximum and the minimum is higher in this case. The paddles are parallel to the spin axis, and each side is sunny exactly one-half of the time.

Calculation of the Average Value of the Aspect Ratio as a Function of Sun Position and Extension Angle

This study of the paddle generator is chiefly interesting in regard to the ratio between the maximum and the minimum of the generator's current output during one period. This effectively involves the performance of the converters.

Nevertheless, *the average power output* over a long period of time is the very significant parameter of any preliminary design.

We will calculate this average value of the aspect ratio as a function of the two angles ϕ (paddle extension) and θ_0 (sun position), assuming that the spin rate ω is constant. The average value of r is given by the formula

$$r_m = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} r(\lambda)\, d\lambda, \qquad \text{with } \lambda = \omega t .$$

<u>1st Case, $\theta_0 \le \phi \le \frac{\pi}{2}$:</u>

We previously noted that the aspect ratio was constant in this case, so its average value is equal to its value

$$r_m = \frac{1}{2} \cos \theta_0 \sin \phi .$$

<u>2nd Case, $0 \le \phi \le \theta_0$:</u>

We have to consider the following two cases:

1. For $0 \le u \le \frac{\pi}{2}$, from Table 1:

$$r_m = \frac{1}{\pi} \left[\int_0^u \left(\sin \theta_0 \cos \phi \cos \lambda + \cos \theta_0 \sin \phi \right) d\lambda + \int_u^{\frac{\pi}{4}} \sin \theta_0 \cos \phi \left(\sin \lambda + \cos \lambda \right) d\lambda \right] ,$$

or

$$r_m = \frac{1}{\pi} \left[\sin \theta_0 \cos \phi \cos u + u \cos \theta_0 \sin \phi \right] .$$

2. For $\frac{\pi}{4} \leq u \leq \frac{\pi}{2}$, from Table 2:

$$r_m = \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}-u}\left(\sin\theta_0\cos\phi\cos\lambda + \cos\theta_0\sin\phi\right)d\lambda + 2\int_{\frac{\pi}{2}-u}^{u}\cos\theta_0\sin\phi\, d\lambda\right],$$

or

$$r_m = \frac{1}{\pi}\left[\sin\theta_0\cos\phi\cos u + u\cos\theta_0\sin\phi\right].$$

So, if we consider the sun position as steady with respect to the spin axis (angle θ_0), the average value of the aspect ratio as a function of extension angle ϕ is given by

$$\left.\begin{aligned}
&0 \leq \phi \leq \theta_0: \\
&\qquad r_m = \frac{1}{\pi}\left[\cos\phi\sin\theta_0\cos\left(\operatorname{Arc}\sin\frac{\tan\phi}{\tan\theta_0}\right) + \sin\phi\cos\theta_0\operatorname{Arc}\sin\frac{\tan\phi}{\tan\theta_0}\right], \\
&\theta_0 \leq \phi \leq \frac{\pi}{2}: \\
&\qquad r_m = \frac{1}{2}\sin\phi\cos\theta_0 .
\end{aligned}\right\} \quad (8)$$

Otherwise, if we consider the paddle position as steady with respect to the spin axis (angle ϕ_0), the aspect ratio average value as a function of the sun's position is given by

$$\left.\begin{aligned}
&0 \leq \theta \leq \phi_0: \\
&\qquad r_{m_1} = \frac{1}{2}\sin\phi_0\cos\theta , \\
&\phi_0 \leq \theta \leq \frac{\pi}{2}: \\
&\qquad r_{m_2} = \frac{1}{\pi}\left[\cos\phi_0\sin\theta\cos\left(\operatorname{Arc}\sin\frac{\tan\phi_0}{\tan\theta}\right) + \sin\phi_0\cos\theta\operatorname{Arc}\sin\frac{\tan\phi_0}{\tan\theta}\right].
\end{aligned}\right\} \quad (9)$$

It should be noted that in Equations (8) and (9) the two variables ϕ and θ play the same part. Indeed, if we change ϕ by $\pi/2 - \theta$, they do not change. So, we make a plot of just one of them (and it will be useful for the other group).

Figure 21 gives the computed results of Equations (9), obtained with the help of the Advanced Orbital Programming Branch, GSFC. This plot gives the aspect ratio average value r_m corresponding to ten values of $\Phi = \pi/2 - \phi$ (0, 10, 20, 30, 40, 50, 60, 70, 80, and 90 degrees), for all the positions of the sun between 0 and 180 degrees.

For each curve, at the value $\Phi = 90° - \theta$, the generator starts to behave as in the second case, with two sides alternately sunny.

For instance, a generator whose paddles make a 40 degree angle with the spin axis (that is, $\Phi = 50$ degrees), and sunlit by rays making an 80 degree angle with the spin axis, will have an average efficiency of $r_m = 24.3$ percent. This plot may be useful when the useful interval of θ during the lifetime of the satellite (which can be deduced from the launching conditions) is known, in order to get the variation of the power output as a function of time.

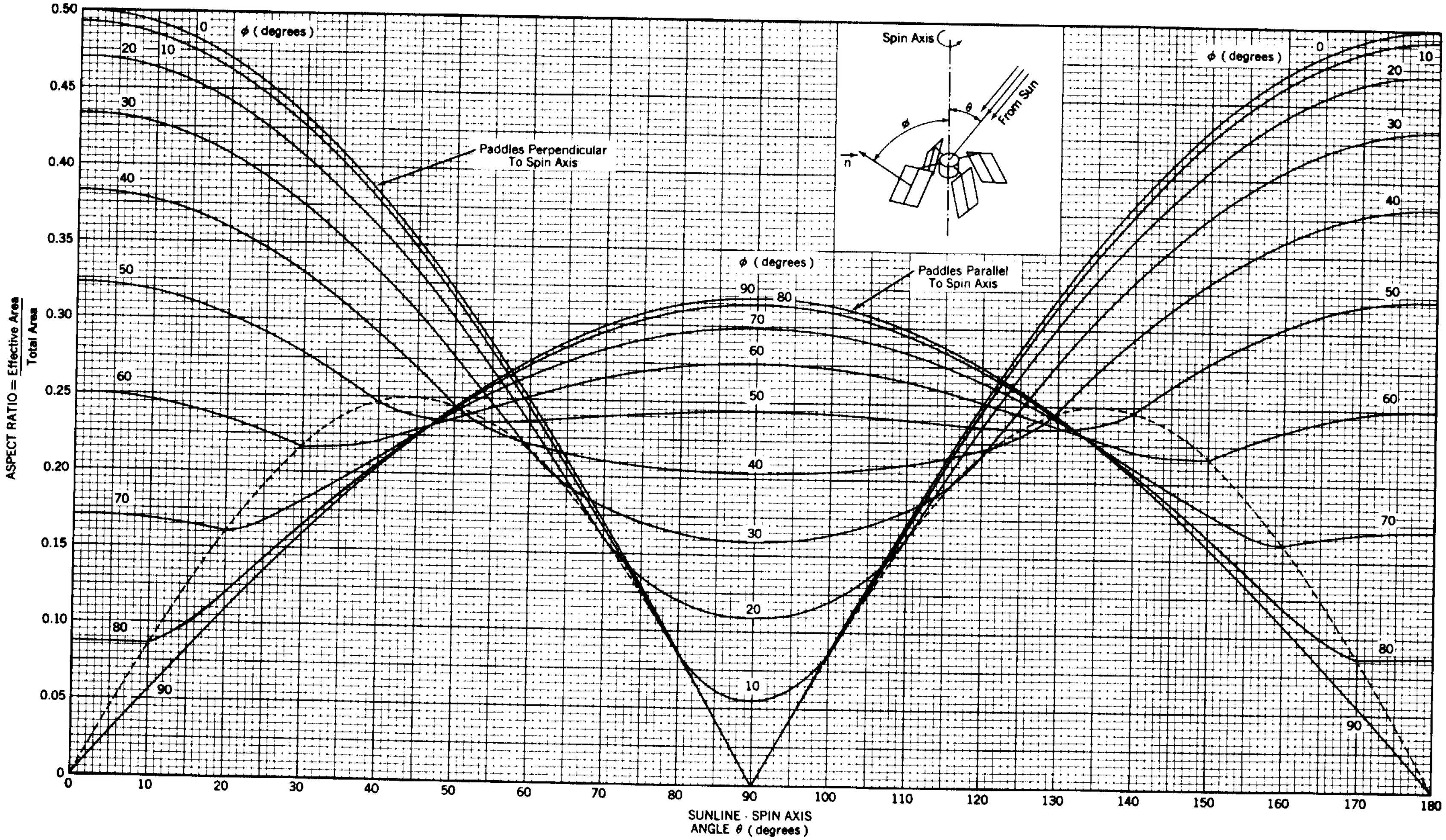


Figure 21—Preliminary design of power supplies, 2nd case: solar cells on both sides of independent spinning paddles. Study of the average aspect ratio for different positions of the paddles (without shadow effects). (ϕ is the angle between the axial vector of the paddles and the spin axis.) (A large working copy of Figure 21 may be obtained by using the request card bound in the back of this report.)

Calculation of the Average Value of the Aspect Ratio Versus Paddle Extension Angle for Random Variations of Satellite Position with Respect to the Sun

The variations of the aspect ratio's average value when the satellite is *not* well stabilized (for example, if a precession occurs, or if the de-spin mechanism does not work correctly) are of interest. Assume the different angles θ representing the sun rays to be equally distributed, which means they take all the possible values between 0 and 180 degrees with equal probabilities. We will analytically study the mean value $\bar{r}_m$ as a function of ϕ only and then determine whether a value of ϕ exists for which $\bar{r}_m$ has a maximum. For all the possible values of θ, the generator will work partly in the first case and partly in the second case; as an average, we will have:

$$\bar{r}_m = \frac{2}{\pi}\left(\int_0^{\phi_0} r_{m_1}\, d\theta + \int_{\phi_0}^{\frac{\pi}{2}} r_{m_2}\, d\theta\right),$$

$$\bar{r}_m = \frac{2}{\pi}\left[\frac{1}{\pi}\int_{\phi_0}^{\frac{\pi}{2}} \left(\sin\theta\cos\phi_0\cos u + u\cos\theta\sin\phi_0\right) d\theta + \frac{1}{2}\int_0^{\phi_0} \cos\theta\sin\phi_0\, ds\right],$$

$$\bar{r}_m = \frac{\sin^2\phi_0}{\pi} + \frac{2}{\pi^2}\int_{\phi_0}^{\frac{\pi}{2}}\left[\cos\phi_0\sin\theta\cos\left(\text{Arc}\sin\frac{\tan\phi_0}{\tan\theta}\right) + \sin\phi_0\cos\theta\,\text{Arc}\sin\frac{\tan\phi_0}{\tan\theta}\right] d\theta\,.$$

This mean value $\bar{r}_m$, which is a function only of the extension angle ϕ_0, has been computed with the help of the Advanced Orbital Programming Branch, GSFC.

The results are shown in Figure 22. It appears that $\bar{r}_m$ does not have an optimum other than 90 degrees between 0 and 90 degrees; it is an increasing function of ϕ_0. For a badly stabilized satellite,

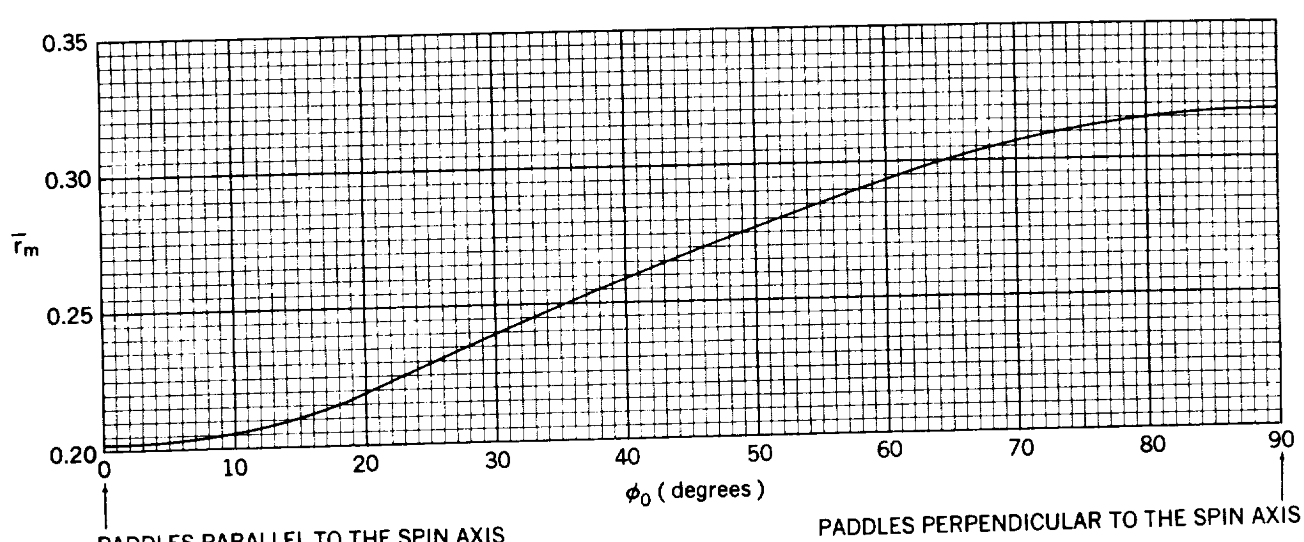


Figure 22—Average aspect ratio of a paddle generator for random variations of the spin axis with respect to the sun.

and if we forget the "maxi-mini" problems, the best design is to place the paddles perpendicular to the spin axis. By so doing, the average efficiency is $\bar{r}_m = 1/\pi \approx 31.8$ percent.

CONCLUSIONS

Comparison of the Two Generator Types

The differences between the two possible designs are the following:

1. Free room inside the nose cone: For a given size of satellite, the first design (cells on skin) takes less room and might even weigh less than the second design.

2. Electrical power output: In both cases, the aspect ratio is a function of time. The first design (cells on skin) is better suited to the constancy of the output, as far as we can choose the shape of the satellite. The second design (cells on paddles) leads to bigger maxi-mini ratios but has the advantage of being more adaptable to design changes. An exact comparison can be made on the two plots shown in Figures 8 and 21.

3. Shadow: For projected shadows, the first design (cells on skin) is better. For the self shadow phenomenon, the two designs are comparable.

4. Temperature: The second design (cells on paddles) is better provided the paddles are extended at the same angle.

5. Handling, tests, countdown: The second design (cells on paddles) is better.

6. Free windows on the structure of the satellite: If free surface areas are needed on the faces of the structure, the second design (cells on paddles) seems better. It is difficult in this case to design the generator symmetrically with cells on skin, and even so this might result in an inadequate power supply.

Compensating the Bombardment Effects

After launch, the solar cells' electrical characteristics are affected by particle bombardment (protons, electrons, micrometeorites). If we can deduce the damage time rate (from the particular orbit, the intensity of the beams, and the particle energy) and if we know the aspect ratio variations (from angle θ variations), it is possible to choose the launching conditions (time, azimuthal speed) to compensate this damage, that is, to start the lifetime at a point for which the aspect ratio would increase at the right speed.

NASA-Langley, 1963 G-443