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CONFERENCE ON LUNAR EXPLORATION

PART A

VIRGINIA POLYTECHNIC INSTITUTE

BLACKSBURG, VIRGINIA

AUGUST, 1962

VIRGINIA ENGINEERING EXPERIMENT STATION

The Virginia Engineering Experiment Station was established in 1921 by action of the Board of Visitors of the Virginia Polytechnic Institute. It was established to stimulate and advance engineering education and to investigate problems of special importance to engineering, manufacturing, mining, transportation, and other industrial interests of the state and of the nation. Under the original arrangement, it was necessary for the departments of the school to carry on research within their normal operating budgets. This meant that monies used by each department for research under the program were used at the discretion of the departmental staffs and it also meant, therefore, that at most only meager amounts were available for research expenditures. Consequently, during the period between 1921 and 1950, research did not receive the impetus that it should have.

In 1950, the General Assembly of Virginia established the Virginia Engineering Experiment Station as a separate division of the Virginia Polytechnic Institute. Creation of the station as a distinct research organization within the school established the Station as an integral unit with its own budget. Benefits both to the institution and to industry have been multiplied many-fold since the 1950 legislation. The chief benefit, of course, has been to fulfill the chief aims, that is, to solve technical problems of industry and to extend knowledge; this has been accomplished through research carried on by staff members of the Engineering Experiment Station and the VPI School of Engineering and Architecture and also through short courses and "clinics" administered by members of these staffs. Of almost equal importance to the college as an educational institution, of course, has been the very real stimulation given to faculty and students through direct contact with and investigation of live problems in industry. Also noteworthy is the experience afforded research fellows who each year receive post graduate training and research know-how while carrying on investigations for the Station. These men gain experience that fits them especially well for important positions in industrial research organizations.

The Station Director is always happy to receive suggestions from industries regarding research and investigation. Where possible, means will be found for cooperation between the Station's investigators and individuals or organizations in conducting research. This is of special value to smaller industries which are individually unable to finance needed research. Commonly they will find that it is possible by group organization to support such work by the Experiment Station personnel at a nominal expense to each member of the group.

When research is undertaken for the private benefit of an industry, the industry will be expected to finance the work and reports will be made to those concerned. Results of investigations made at public expense and papers presented at the "clinics" will be published as bulletins of the Virginia Polytechnic Institute, Engineering Experiment Station Series, if they are considered to be of wide interest. Results of commercial value will be patented in accordance with contracts between the station and industries sponsoring the research. Typically, the proceeds are applied to promotion of the research work of the Station, with suitable recognition as to the rights of industries that may have assisted in financing the work.

The effects upon industrial development in Virginia of investigations now being carried on by staff members of the Virginia Engineering Experiment Station and VPI School of Engineering and Architecture will undoubtedly become readily apparent during the years ahead. It is hoped that as time goes on industry will realize more and more the value of this research. This will lead necessarily to more contributions by industry to the Station and to the School and will assure the full utilization of our research personnel and equipment.

A partial list of the Bulletins of the Virginia Polytechnic Institute, Engineering Experiment Station Series, which are still available, appears on the inside of the back cover. A complete list of available bulletins, including those of the Wood Research Laboratory Special Reports Series, can be obtained from the Director upon request.

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BULLETIN

OF THE

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Engineering Experiment Station Series No. 152

(In three parts: A, B, C)

PART A

PROCEEDINGS OF THE CONFERENCE ON LUNAR EXPLORATION

August 12 through August 17, 1962

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ACKNOWLEDGMENTS

The Virginia Polytechnic Institute wishes to acknowledge its indebtedness to the National Science Foundation for providing the funds in support of this Conference. Without this support the one hundred faculty members and more than two hundred guests could not have been brought together for the lively discussions and enlightening lectures which prevailed throughout the meetings.

The assistance and guidance of the Steering Committee, and the valuable assistance from staff members of the Langley Research Center and others of the National Aeronautics and Space Administration, is also gratefully acknowledged. Their efforts and their counsel, in a large measure, made the conference the success that it was.

The Poly-Scientific Corporation, of Blacksburg, is recognized for its help in providing a tour of the manufacturing facilities and for entertainment for the group.

Lastly, the conference committee members wish to express their personal thanks to those mentioned above and to members of the VPI staff and administration for assistance, cooperation and encouragement throughout the entire conference operation.

The Conference Committee
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T. E. Gilmer, Professor, Physics
J. B. Eades, Jr., Head, Aerospace Engineering
and Chairman

PREFACE

The year 1961 will go down in history as the year man first embarked on a space journey. Though these first steps toward the exploration of the new frontier, space, have been relatively small, nevertheless we now know that the journey into the cosmos is feasible and realistic.

With the present day state of the art such as it is, man's first extra-terrestrial journey will necessarily be to our closest neighbor, the Earth's own satellite, the moon. It is quite apparent that this impending journey has long been contemplated when one realizes the effort and energy expended by the scientists and engineers of the world on study of the characteristics of the moon and the means of carrying out its exploration. It is equally apparent that much more needs to be known of our celestial neighbor before a base of operation for scientific exploration can be established here. The need for a greater number of qualified and trained investigators to carry out the many facets of such a study is apparent; and equally important is the need to transmit the presently known facts and data to as many scientists and engineers as possible. These needs can best be met by bringing together the people who know with those who need to know so that accurate and timely communication can be established.

The purpose of the conference was to meet this need of communication - to bring together those interested and engaged in space exploration so that they could benefit by the knowledge gained from other investigators. It is hoped that the information gathered here will serve the purposes of the researcher and the teacher since both are able to provide the stimulus and the work needed for a successful lunar exploration.

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THE MOON AS AN EARTH SATELLITE

THE ORBIT OF THE MOON, PERTURBATION DUE TO THE SUN, ETC.

BY

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THE ORBIT OF THE MOON, PERTURBATION DUE TO THE SUN, ETC.

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Introduction

The problem of constructing a gravitational theory of the moon's motion was considered by Sir Isaac Newton in his Principia, and held the attention of many illustrious mathematicians during the eighteenth and nineteenth centuries. Some of the most important advances in celestial mechanics were made in connection with the development of new methods for treating the lunar problem. Particularly significant were the contributions by C. E. Delaunay and G. W. Hill, both during the second half of the nineteenth century.

The Equations of Motion

If the mass of the moon is neglected, the equations of motion for the moon about the center of the earth can be written down at once. Let E and m' be the masses of the earth and the sun, respectively, G the gravitational constant. Let the earth be at the origin of a Cartesian coordinate system, $\vec{r}$ the position vector of the moon, $\vec{r}'$ the position vector of the sun, and let r, r', Δ be the distances earth-moon, earth-sun, sun-moon. The equations of motion for the moon are then

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GE\vec{r}}{r^3} + Gm'\left[\frac{\vec{r}' - \vec{r}}{\Delta^3} - \frac{\vec{r}'}{r'^3}\right].$$

The last term represents the negative of the acceleration exerted by the sun on the earth; it must be included because the coordinate system is earth centered. The terms having Gm' as a factor can be written as the derivation of a disturbing function. Hence

$$\frac{d^2\vec{r}}{dt^2} + \frac{GE\vec{r}}{r^3} = \nabla R$$

$$R = Gm'\left[\frac{1}{\Delta} - \frac{\vec{r}\cdot\vec{r}'}{r'^3}\right]$$

$$= Gm'\left[\frac{1}{\Delta} - \frac{r\cos S}{r'^2}\right]$$

in which S is the angle between the directions EM and Em'.

In any satellite system the ratio r/r' is small compared with unity. For the earth-moon system it is about 1/400. Owing to this circumstance the reciprocal of Δ can be expanded in a rapidly converging series in powers of r/r'. From the geometry of the triangle EMm' it follows that

$$\Delta^2 = r'^2\left[1 - 2\frac{r}{r'}\cos S + \left(\frac{r}{r'}\right)^2\right]$$

Hence

$$\frac{1}{\Delta} = \frac{1}{r'}\left[1 + \frac{r}{r'}P_1(\cos S) + \left(\frac{r}{r'}\right)^2 P_2(\cos S) + \ldots\right],$$

in which P_1, P_2. . . are Legendre polynomials,

$$P_1(\cos S) = \cos S$$

$$P_2(\cos S) = -1/2 + 3/2\cos^2 S$$

$$P_3(\cos S) = -3/2\cos S + 5/2\cos^3 S$$

$$P_4(\cos S) = +3/8 - 15/4\cos^2 S + 35/8\cos^4 S$$

$$P_5(\cos S) = +15/8\cos S - 35/4\cos^3 S + 63/8\cos^5 S$$

An important property of all Legendre polynomials is that for all real values of S, i.e. for $-1 \leqslant \cos S \leqslant +1$, $-1 \leqslant P_k (\cos S) \leqslant +1$.

If the expression for Δ^{-1} is substituted in R, it is seen that the complementary part is canceled by the $P_1(\cos S)$ part of Δ^{-1}. The term $1/r'$ in Δ^{-1} may be omitted because it is independent of the coordinates of the moon. Finally, therefore, the disturbing function becomes

$$R = \frac{Gm' r^2}{r'^3} \left[P_2 (\cos S) + \frac{r}{r'} P_3 (\cos S) + \dots \right].$$

This was derived on the assumption that the mass of the moon may be neglected. A fairly involved derivation shows that the moon's mass may be taken into account by a slight modification of the equation, provided that the coordinates of the sun are reckoned from the center of mass of the earth-moon system. It can be shown that the motion of the sun relative to this center of mass is so very closely represented by an elliptic orbit that the solar coordinates may be treated as known functions of the time. This circumstance greatly simplifies the problem: instead of having to treat six simultaneous equations each of the second order, the problem of the moon's motion requires solution of three equations, each of the second order,

$$\frac{d^2\vec{r}}{dt^2} + \frac{G (E + M)\vec{r}}{r^3} = \nabla R$$

$$R = \frac{Gm' r^2}{r'^3} \left[P_2 (\cos S) + \frac{E - M}{E + M} \frac{r}{r'} P_3 (\cos S) + \frac{E^2 - EM + M^2}{(E + M)^2} \left(\frac{r}{r'} \right)^2 P_4 (\cos S) \right.$$

$$\left. + \frac{E^3 - E^2 M + EM^2 - M^3}{(E + M)^3} \left(\frac{r}{r'} \right)^3 P_5 (\cos S) + \dots \right]$$

One further modification is desirable. Kepler's third law for the motion of the earth-moon system about the sun is

$$G(m' + E + M) = n'^2 a'^3 ,$$

if n' is the sun's mean motion, a' the semi-major axis of the earth's orbit. Hence

$$\frac{Gm'}{r'^3} = \frac{m'}{m' + E + M} n'^2 \frac{a'^3}{r'^3} .$$

The mass factor m'/(m' + E + M) is 1 - 0.000003. It is customary to replace this mass factor by unity in treating the solar perturbations in the moon's motion, and to make allowance for the modification afterwards.

With this understanding the principal part of the disturbing function is written

$$R = n'^2 \frac{a'^3}{r'^3} . r^2 P_2 (\cos S)$$

$$= n'^2 \frac{a'^3}{r'^3} (- 1/2 r^2 + 3/2 r^2 \cos^2 S) .$$

The series inside the square brackets converges rapidly because of smallness of the ratio r/r'. Some idea of the magnitude of solar perturbations in the moon's motion is obtained by the following consideration.

The ratio between the solar perturbing acceleration and the direct acceleration produced by the earth's attraction on the moon is

$$\frac{Gm' r^2}{r'^3} \Big/ \frac{G(E + M)}{r} = \frac{m'}{E + M} . \frac{r^3}{r'^3}$$

$$\sim (1/3) \times 10^6 . (1/400)^3$$

$$\sim 1/200$$

The largest periodic perturbation in the moon's longitude by the P_2 part of the disturbing function has the coefficient 4586", by the P_3 part 125". From these it may be concluded that the largest terms arising from the P_4 and P_5 parts are $0\overset{''}{.}03$ and $0\overset{''}{.}001$, respectively. No parts beyond P_5 in the expansion of R will be needed in order to satisfy modern observational requirements.

Non-Solar Perturbations

The problem so stated is the so-called "main problem" of the lunar theory. All three bodies concerned are treated as point masses; and the motion of the center of mass of the earth-moon system about the sun is treated as a fixed ellipse. After this problem is solved, it will be necessary to evaluate the direct and the indirect planetary perturbations, and the effects on the motion produced by the non-sphericity of the earth and of the moon.

Especially the calculation of the planetary perturbations constitutes a problem of considerable difficulty. The direct perturbations arise from the addition of parts to the disturbing function each having the mass of a planet as a factor; the indirect planetary perturbations arise from the changes in the motion of the center of mass of the earth-moon system produced by the attraction of the planets. This is accomplished by introducing into the equations of motion for the main problem the solar coordinates affected by the planetary perturbations.

One of the important aspects of the indirect planetary perturbations is the secular acceleration in the moon's mean longitude. It arises from the fact that the eccentricity of the earth's orbit, owing to planetary perturbations changes with a very long period. At the present time it is diminishing at the rate of 0.000041 per century and will continue to diminish for the next 25,000 years -- at which time the eccentricity of the earth's orbit will have only about one-third its present value. Such long-period effects are most conveniently introduced in the lunar theory in the form of a power series in the time. Hence in the development of the solar perturbations in the moon's motion the eccentricity of the sun's orbit is to be replaced by $e' = e_o' - \alpha T$ (T being counted in centuries). As a consequence, terms having T as a factor appear in the disturbing function. The most important result is an insignificant negative term proportional to T in the moon's mean distance from the earth and a corresponding significant term $+ 5''.8T^2$ in the moon's mean longitude.

The solution of the main problem of the lunar theory is found to depend on four fundamental arguments that are linear functions of the time:

D = the mean elongation of the moon from the sun
ℓ = the moon's mean anomaly
ℓ'= the sun's mean anomaly
F = the moon's mean argument of the latitude

These arguments appear in linear combinations with integral coefficients in arguments of sine terms present in the expression for the moon's longitude and latitude, and in the arguments of cosine terms in the moon's parallax (which is usually tabulated, rather than the radius vector).

The coefficient of t in D is simply n - n', the difference between the mean motions of the moon and the sun; that of t in ℓ' is n', the sun's mean motion. The coefficients of t in ℓ and F must be determined from the theory, they are usually put equal to cn and gn, respectively. It is thus easily seen that (1 - c)n is the motion of the perigee, (1 - g)n the motion of the node.

The calculation of these motions is among the most important parts of the theory. Especially that of the perigee is notoriously difficulty. After the solution of the main problem has been obtained, the various contributions by other perturbing causes are added. The results obtained in E. W. Brown's theory are

Mean Annual Motions:	Perigee	Node
Principal solar action	+ 146426".92	- 69672".04
Mass of the earth	- .68	+ .19
Planetary (direct)	+ 2.69	- 1.42
Planetary (indirect)	- .16	+ .05
Figure of the earth	+ 6.41	- 6.00
Figure of the moon	+ .03	- .14
	+ 146435.21	- 69679.36

A relativity effect + 0".02 per annum is to be added to both.

The "mass of the earth" part is the correction to be made on account of the slight modification of the mass factor by 3 parts in 10^6; the principal uncertainties at the present time are in the small contributions due to the figure of the moon. They are so small compared with the total that it would be bold to attempt to derive any conclusions concerning the figure of the moon from the comparison between observed and theoretical values of the motions of perigee and node. Nor do these data provide a significant observational test of the theory of relativity.

The contributions to the motions of perigee and node caused by non-solar attractions are minute fractions of the parts caused by the sun's attraction. This feature illustrates how overwhelmingly the sun's attraction determines the character of the moon's motion. In the motions of artificial satellites and of some natural satellites in the solar system the oblateness of the primary is the principal cause of the perturbations, and the non-oblateness perturbations are of minor significance.

Delaunay's Method

If the disturbing function is ignored the equations are simply those of unperturbed elliptic motion. The solution of these equations is well known. The coordinates can be expressed as functions of the time and six constants of integration, the latter are known as the six elliptic elements of the orbit. Various sets of elliptic elements may be used, the choice depending on the particular problem to be solved, on personal preference, or on the method of solution selected.

In the method of the variation of elements (or variation of arbitrary constants) the coordinates and velocity components in perturbed motion are represented by the same functions of the time and the orbital elements as in elliptic motion, but the elements themselves become functions of the time. It actually amounts to a transformation from the coordinates and velocity components as dependent variables, to the orbital elements as variables, the transformation equations being the expressions for the coordinates and velocity components in terms of the elements and the time. The new equations are of the first order.

A particularly eligant set is that chosen by Delaunay. The variables are

$L = \sqrt{\mu a}$ $\qquad$ ℓ = mean anomaly

$G = L\sqrt{1 - e^2}$ $\qquad$ g = argument of perigee

$H = G \cos I$ $\qquad$ h = longitude of ascending node in which $\mu = G(E + M)$, a the semi-major axis, e the eccentricity, I the inclination of the orbit relative to the chosen plane of reference. In the main problem this plane of reference is the plane of the orbit of the center of mass of the earth-moon system about the sun, assumed to be fixed.

It should be noted that ℓ is a pseudo element; in elliptic motion it is not a constant but a linear function of the time.

With the choice of variables indicated the equations assume the canonical form:

$$\frac{dL}{dt} = \frac{\partial F}{\partial \ell} \qquad \frac{d\ell}{dt} = -\frac{\partial F}{\partial L}$$

$$\frac{dG}{dt} = \frac{\partial F}{\partial g} \qquad \frac{dg}{dt} = -\frac{\partial F}{\partial G}$$

$$\frac{dH}{dt} = \frac{\partial F}{\partial h} \qquad \frac{dh}{dt} = -\frac{\partial F}{\partial H}$$

$$F = \frac{\mu^2}{2L^2} + R \quad .$$

The function F is called the Hamiltonian of the system.

The disturbing function must be expressed as a function of these variables. It turns out to be of the form of an infinite cosine series with arguments

$$p_1 \ell + p_2 g + p_3 (h - \varpi') + p_4 \ell' \quad ,$$

p_1 to p_4 being integers, in which ϖ' is the longitude of the sun's perigee, ℓ' the mean anemaly of the sun. The coefficients of the cosine terms are functions of L, G, H, and such parameters as e', the eccentricity of the sun's orbit. Call any such argument θ. Delaunay selects a single periodic term $C \cos \theta$, and considers the equations with F replaced by

$$\overline{F} = \frac{\mu^2}{2L^2} + C_o + C \cos \theta \ ,$$

C_o being the "constant" part of R, i.e. the part independent of the angular variables. He then shows how to obtain a canonical transformation to new variables L', G', H', ℓ', g', h', such that in the new variables, the new Hamiltonian, $\overline{F}$', is a function of L', G', H', and no longer of θ. This transformation is now applied to the complete equations. The disturbing function must be expressed in terms of the new variables, but it is clear that the term C cos θ will have disappeared. The equations have the same form as before. The next step is to select a new argument θ' in the new Hamiltonian F', and to proceed to a new transformation to variables L", G", H", ℓ", g", h", so selected that this new argument disappears. By a sequence of transformations all of the significant terms in the disturbing function can be made to disappear. In the end, the Hamiltonian F* will be a function of the final variables L*, G*, H* only, and the solution of the equations is

$$\begin{aligned} L^* &= \text{const.} & \ell^* &= \nu_1 t + \text{const.} \\ G^* &= \text{const.} & g^* &= \nu_2 t + \text{const.} \\ H^* &= \text{const.} & h^* &= \nu_3 t + \text{const.} \ , \end{aligned}$$

ν_1, ν_2, ν_3 being functions of the constants L*, G*, H*.

Since the relation between the original variables and the final "variables" is known through the chain of transformations, the problem is completely solved. Delaunay performed this solution for the main problem; others added the non-solar perturbations.

A closer look at the method shows that it leads to a development in powers of n'/n, the ratio of the mean motions of the sun and the moon. In principle the method is beautiful; it was called by Poincaré the most perfect analytical solution of the problem of the moon's motion. In practice it suffers from slow convergence according to powers of n'/n, at least if n'/n is as large as it is in the moon's motion, about 0.075. It is even worse in cases such as Jupiter's eighth and ninth satellites, for which the ratio is 0.17. Also, the procedure used by Delaunay is extremely laborious.

A modification of Delaunay's method, due to von Zeipel, permits including in a single transformation a whole class of terms, so that the number of transformations necessary is greatly reduced. Although von Zeipel's procedure follows a totally different route, it leads to the same end result as Delaunay's method. Finally, it may be possible with this modified approach to get rid of most of the slow convergence in powers of n'/n. This has not yet been fully established. If it should be so, it would enhance the value of the method significantly.

The Hill-Brown Method.

The slow convergence of the solution according to powers of n'/n caused various investigators to choose methods in which this ratio is adopted from the very beginning as a fixed numerical quantity. The entire theory is developed without ever permitting it to appear otherwise. A fortunate circumstance is that the mean motions of the sun and moon are known from observation with all the accuracy desired; even if a small correction were needed at the end of the work, the necessary modifications in the theory could be introduced with little effort. Of course, a lunar theory so constructed will apply to the earth-moon system only. In that respect it is less general than a completely literal theory such as Delaunay's, which can be used for any satellite for which the solar perturbations are the principal perturbations in the motion, provided the ratio n'/n is not much larger than for the moon.

The theory of the moon's motion that has been used since 1923 for the calculation of the moon's ephemeris is the result of many years of concentrated effort by Ernest W. Brown. It is based upon the principles developed by G. W. Hill in two celebrated papers that appeared in 1877-1878. For this reason the method is usually called the Hill-Brown method.

The foundation of the theory is a periodic orbit for the moon in a coordinate system rotating with uniform angular velocity equal to the sun's mean motion. In obtaining this periodic orbit the ratio n'/n is adopted as fixed; the inclination of the moon's orbit with reference to the elliptic is ignored, as well as the eccentricity of the moon's orbit and of the sun's orbit. Also, the disturbing function is limited to its principal part, which amounts to ignoring the ratio a/a'. The periodic orbit has the period of one half the synodic month. It contains the important perturbation historically called the variation. All the remaining structure of the theory is built upon this "variation orbit". A sequence of calculations of considerable magnitude yields successively the contributions to the theory that have e, e', sin I, a/a' as factors, called the terms of the first order. The terms factored by e produce the principal part of the motion of the moon's perigee, which Hill obtained to fifteen decimal places; it also yields the evection, the largest perturbation in the moon's longitude. Similarly, the first-order terms factored by sin I yield the principal part of the motion of the node as well as the largest perturbation in the latitude.

The next phase is to evaluate the terms of the second order, having e^2, ee', e'^2, etc. as factors. In the course of the calculation of the terms of the third order small contributions to the motions of perigee and node, having e^2. e'^2, $\sin^2 I$ as factors, are obtained. Further contributions arise from the terms of the fifth order.

Brown carried the solution forward until a solution of the main problem had been obtained to a degree of accuracy adequate to satisfy the foreseeable observational requirements. Subsequently he added the non-solar perturbations, and constructed his monumental Tables of the Motion of the Moon, published in 1919 by the Yale University Press.

Brown's lunar theory has been tested by comparison with other theories and by comparison with observations during almost forty years. In addition, a numerical test by Dr. W. J. Eckert, not yet completed in every detail, has confirmed the excellence of the theory. Briefly stated this test consists of substituting Brown's solution in numerical form into the right-hand members of the differential equations and ascertaining what corrections to the coefficients and to the motions of the perigee and node are needed in order to satisfy the equations.

In recent years Brown's Tables are no longer used to calculate the lunar ephemeris. Instead, the calculation, performed on high-speed calculators, is based directly on the evaluation of the individual terms that constitute the lunar theory. This procedure permits the calculation of the moon's position to one decimal place beyond that obtained from the Tables, an increase in accuracy required by modern observational needs. The next section will deal with an important modification introduced in the Improved Lunar Ephemeris, published separately for the years 1952-1959, and in the annual ephemeris volumes since 1960.

Ephemeris Time and Irregularities in the Earth's Rotation

One reason for the increased interest in the lunar theory in the second half of the nineteenth century was the fact that the observed path of the moon began to deviate seriously from Hansen's Tables of the Moon soon after they began to be used for the construction of the moon's ephemeris in 1862. The cause for this discordance was first sought in the possible omission of some important planetary perturbations. Simon Newcomb undertook a lengthy investigation of the problem both by comparing Hansen's Tables with observations, as far back as available records permitted, and by a new calculation of the planetary perturbations. Newcomb's efforts and those of others gradually led to the suspicion that irregularities in the rate of rotation of the earth were the primary cause of the failure of the lunar tables to represent the observations, but it took more than fifty years before this was established beyond doubt. What was needed was observational evidence that deviations from the ephemerides corresponding to those observed in the moon's motion are present in the motions of the sun and the inner planets. The difficulty is, of course, that sun and planets move so much more slowly among the stars than the moon. In a second of time the moon moves among the stars on the average $0''.55$, but the sun moves only $0''.041$. All of the accuracy of modern observations was needed in order to prove the correspondence beyond question, but after a comprehensive discussion of all the evidence by Sir Harold Spencer Jones in 1939 the issue was no longer in doubt. The earth's rate of rotation changes in three different ways: superposed on a gradual lengthening of the day. ascribed to tidal friction, there are irregular changes from year to year by amounts up to a few thousands of a second and finally minute periodic changes with a period of a year ascribed to meteorological causes. With such an unpredictable behavior, the earth's rotation is not suitable for measuring time; the orbital motions of the moon and the planets furnish a more reliable clock. This led in 1950 to the introduction of Ephemeris Time, defined with the aid of the motion of the sun relative to the vernal equinox. The unit of ephemeris time is defined as the Ephemeris Second, the fraction 1/31,556,925.9747 of the length of the tropical year at the beginning of the year 1900. In practice the difference Δt between Ephemeris Time and Universal Time (the latter based upon the rotation of the earth) is obtained by comparing observations of the moon with the Improved Lunar Ephemeris.

In the construction of the Improved Lunar Ephemeris two empirical features of Brown's Tables were eliminated: a quadratic expression in the time and the "great empirical term", the latter with coefficient $10''.71$ and a period of 257 years. Brown had in this respect followed Hansen's example. With these modifications the lunar ephemeris was made to correspond as closely as was possible to the solar ephemeris so that, on the basis of past performance, observations of the two bodies furnish the same Ephemeris Time.

Although the changes in the length of the day from year to year, and even from century to century, are small, the difference in time between Ephemeris Time and time based on the earth's rotation builds up to significant amounts. Between 1750 and 1900 the difference oscillated between + 8 seconds and - 8 seconds; since 1902 it has been positive and has now reached the value + 35 seconds.

The small range in the 150 years before 1900 is no accident: it is very nearly the period covered by the observations of the sun used to obtain the constants of the solar tables in current use for ephemeris calculations.

Since the concept of ephemeris time did not exist when the solution was made, the least squares solution produced a minimum fluctuation during this period. The rapid departure of Δt after 1900 is of the same character as the experience with the moon's motion after the introduction of Hansen's tables in 1862.

The implication of using the lunar ephemeris for the practical determination of ephemeris time is, of course, that the lunar ephemeris is considered to be errorless. No other problem in celestial mechanics has been treated as exhaustively as that of the moon's motion and Brown's theory sets beyond question a standard of completeness and accuracy that is not easily surpassed. Yet it would be highly desirable to have a more accurate comparison with another celestial clock than the comparison provided by the motions of the sun and the inner planets. Atomic clocks have become reliable enough to compete with the moon's motion for the purpose of time determination, and eventually an interesting comparison will be available between Atomic Time and Ephemeris Time. A question of basic importance is whether the two times are the same or whether one might be accelerated with respect to the other. Perhaps eventually suitable artificial satellites will be available to provide a check on the lunar theory.

THE DETERMINATION OF THE MASS, SHAPE, MOMENTS OF INERTIA, AND GRAVITATIONAL FIELD OF THE MOON

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THE DETERMINATION OF THE MASS, SHAPE, MOMENTS OF INERTIA, AND GRAVITATIONAL FIELD OF THE MOON

1. The Mass of the Moon.

The only means we have at present for the determination of the mass of the Moon is by the measurement of the Lunar Equation, the displacement in the apparent position of any other member of the solar system, as observed from the Earth, arising from the Earth's monthly revolution about the center of mass, G, of the Earth and Moon. The position vector of that center of mass relative to the Sun, $\underset{\sim}{R}$ say, is the quantity studied in the treatment of the Earth's motion relative to the Sun, and the position vector of the Moon relative to the Earth, $\underset{\sim}{r}_{☽}$ say, is the quantity studied in the theory of the Moon's motion. Then if $\underset{\sim}{r}_P$ is the position vector of any other body P, relative to the Sun, the apparent position of P, as observed from the Earth, is given by the relative position vector $\underset{\sim}{\rho}_P$, where

$$\underset{\sim}{\rho}_P = \underset{\sim}{r}_P - \underset{\sim}{R} + \frac{m_{☽}}{m_{\oplus} + m_{☽}} \underset{\sim}{r}_{☽} , \qquad (1)$$

$m_{\oplus}$ and $m_{☽}$ being the masses of the Earth and Moon respectively.

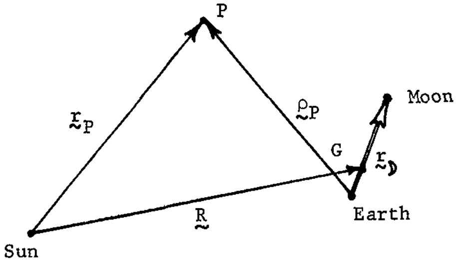

FIGURE 1

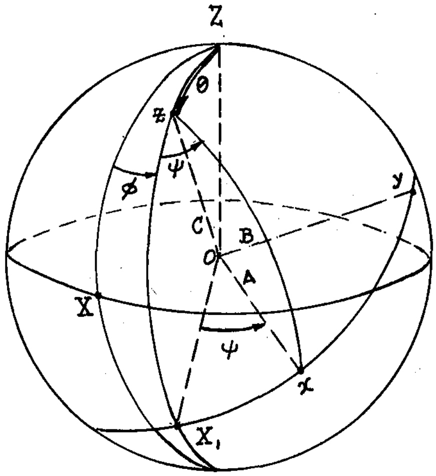

FIGURE 2

then

$$\omega_1 = \dot{\theta} \sin \psi - \dot{\phi} \sin \theta \cos \psi$$

$$\omega_2 = \dot{\theta} \cos \psi + \dot{\phi} \sin \theta \sin \psi$$

$$\omega_3 = \dot{\phi} \cos \theta + \dot{\psi} \quad . \qquad (5)$$

Now in the case of the Earth, there is symmetry about an axis to a very high degree of accuracy, and so if we choose this axis to be Oz we have $A = B$ and then from the third equations of (3) and (4) we see that

$$\omega_3 = \omega, \text{ a constant} \quad . \qquad (6)$$

The complementary function of the first two equations of (3) is from the same series of observations, so this corresponds to $6''.4305 \pm 0''.0031$ for the coefficient $L = \frac{m_{☽}}{m_{\oplus} + m_{☽}} \cdot \frac{\pi_{\odot}}{\sin \pi_{☽}}$. Hinks' value was used in Brown's lunar theory. Eros again approached close to the Earth in 1931, to 16.2 million miles, and from observations made then Spencer Jones (1942) derived $L = 6''.4390 \pm 0''.0015$, at the same time as he derived $\pi_{\odot} = 8''.790 \pm 0''.001$ from the same data. By grouping the observations in periods of two weeks, instead of one month, to increase the degrees of freedom, Jeffreys (1942) obtained $L = 6''.4378 \pm 0''.0018$. Repeating the reductions, and improving the estimates of the orbital elements of Eros at the same time, Delano (1950) obtained $L = 6''.4429 \pm 0''.0015$. Finally Rabe (1950) using improved calculations of the perturbations on Eros, derived $L = 6''.437 \pm 0''.002$ and $\pi_{\odot} = 8''.79835 \pm 0''.00039$ from the same data. Brouwer and Clemence (1961), by weighting the various estimates appropriately, conclude that $L = 6''.4378 \pm 0''.0023$. Using their similarly determined $\pi_{\odot} = 8''.7984 \pm 0''.0006$ leads to $m_{\oplus}/m_{☽} = 81.366 \pm 0.029$, probably the best value at present available.

2. The Precession and Nutation.

It used to be thought that the rigid body motion of the Earth provided information about the mass of the Moon. Euler's equations for any rigid body are, (A, B, C) being its principal moments of inertia at the center of mass, 0, and $(\omega_1, \omega_2, \omega_3)$ the components of its angular velocity about the principal axes,

$$A\,\dot{\omega}_1 - (B - C)\,\omega_2\,\omega_3 = L \quad ,$$
$$B\,\dot{\omega}_2 - (C - A)\,\omega_3\,\omega_1 = M \quad ,$$
$$C\,\dot{\omega}_3 - (A - B)\,\omega_1\,\omega_2 = N \quad , \qquad (3)$$

where L, M, N are the components of the moment of the total force about the center of mass, 0. Due to a point mass m' at position (x', y', z') referred to the principal axes, Oxyz, these are

$$L = \frac{3Gm'y'z'}{r'^5}(C - B) + 0\left(\frac{1}{r'^4}\right) ,$$

$$M = \frac{3Gm'z'x'}{r'^5}(A - C) + 0\left(\frac{1}{r'^4}\right) ,$$

$$N = \frac{3Gm'x'y'}{r'^5}(B - A) + 0\left(\frac{1}{r'^4}\right) , \qquad (4)$$

where r' is the distance of m' from the center of mass.

Now let (θ, ϕ , ψ) be the Euler angles specifying the orientation of the body, taking the ecliptic and vernal equinox direction as reference. Thus if 0 Z is the pole of the ecliptic, let $\angle Z0z = \theta$, and if 0X is the direction of the first point of Aries, let $\angle XZz = \phi$, and then let $\angle Zzx = 180^0 - \psi$.

If P is the Sun itself, then $\underset{\sim}{r}_P = \underset{\sim}{0}$, and the last term leads to the following contributions to the observed longitude, $\psi_{\odot}$, and latitude, $\beta_{\odot}$, of the Sun, measured on the ecliptic:

$$\delta\psi_{\odot} = \frac{m_{☽}}{m_{\oplus} + m_{☽}} \cdot \frac{\pi_{\odot}}{\sin \pi_{☽}} \cdot \cos \beta_{☽} \sin (\psi_{☽} - \psi_{\odot}) ,$$

$$\delta\beta_{\odot} = \frac{m_{☽}}{m_{\oplus} + m_{☽}} \cdot \frac{\pi_{\odot}}{\sin \pi_{☽}} \cdot \sin \beta_{☽} , \qquad (2)$$

where $\pi_{\odot}$ and $\pi_{☽}$ are the equatorial horizontal parallaxes of the Sun and Moon respectively, and $\psi_{☽}$, $\beta_{☽}$, are the geocentric ecliptic longitude and latitude of the Moon.

From positional observations of the Sun made at seven observatories between 1822 and 1864, Newcomb (1895) derived the value $6''.485 \pm 0''.015$ for the coefficient, L_s, of $\sin (\lambda_{☽} - \lambda_{\odot})$ in $\delta\psi_{\odot}$. ($\lambda_{☽}$ and $\lambda_{\odot}$ are the mean longitudes of the Moon and Sun respectively.) From observations made at Washington and Greenwich between 1900 and 1937, Morgan and Scott (1939) obtain $L_s = 6''.479 \pm 0''.009$.

Now for a body approaching the Earth closer than the Sun does, the angular displacement is correspondingly larger, and consequently is observable to a greater proportional accuracy. Thus Gill (1894) from observations of the minor planet Victoria, obtained $L_s = 6''.443 \pm 0''.009$. In 1901 the minor planet Eros approached to within 30 million miles of the Earth, and from observations secured at that time Hinks (1909) derived the value 81.53 ± 0.047 for the ratio $m_{\oplus}/m_{☽}$. He used the value $\pi_{\odot} = 8''.806 \pm 0''.0026$ which he derived

$$\omega_1 = K \cos \left(\frac{C - A}{A} \omega t + k \right) , \quad \omega_2 = K \sin \left(\frac{C - A}{A} \omega t + k \right) , \tag{7}$$

where K and k are arbitrary constants. This represents a free motion of the axis of rotation in a circular cone about Oz, with period $\frac{C - A}{A}$ days, or 305 days approximately. The "variation of latitude" is an observed, somewhat irregular motion of the Earth's axis, with a period of 440 ± 15 days.

The first two equations of (3), (4) and (5) may be rearranged to give

$$A(\ddot{\theta} - \dot{\phi}^2 \sin \theta \cos \theta) + C\omega \dot{\phi} \sin \theta$$

$$= - 3(C - A \sum_{m'} \frac{Gm'}{r'^5} X_1' z' ,$$

and

$$A(\ddot{\phi} \sin \theta + 2\dot{\phi} \dot{\theta} \cos \theta) - C\omega \dot{\theta}$$

$$= - 3(C - A) \sum_{m'} \frac{Gm'}{r'^5} Y_1' z' \tag{8}$$

where the summation is over all the external bodies m', and the axes OX_1Y_1 lie in the plane Oxy, OX_1 being its intersection with OzZ. The direction of the Earth's axis is observed to remain sensibly fixed for periods much longer than one day, so we may neglect $\dot{\phi}$ and $\dot{\theta}$ in comparison with ω, and $\ddot{\phi}$ and $\ddot{\theta}$ in comparison with $\omega\dot{\phi}$ and $\omega\dot{\theta}$. So the terms factored by A in (8) may be neglected. If we neglect the eccentricity of the Earth's orbit, we have for the Sun

$$X_1 = a_{\odot} \cos \theta \cos (\lambda_{\odot} - \phi)$$

$$Y_1 = a_{\odot} \sin (\lambda_{\odot} - \phi)$$

$$z = a_{\odot} \sin \theta \cos (\lambda_{\odot} - \phi) \tag{9}$$

where $a_{\odot}$ is the major semi-axis of the Earth's orbit, and $\lambda_{\odot}$ the mean longitude of the Sun. If $m_{\odot}$ is its mass, and $n_{\odot}$ its mean motion, then

$$Gm_{\odot} = n_{\odot}^{2} \quad a_{\odot}^{3}$$

very closely. In the case of the Moon, assuming its orbit to be circular,

$$\begin{aligned}
X_1 &= a_{\rightmoon}\Big\{\cos\theta\cos(\lambda_{\rightmoon} - \phi) \\
&\qquad - I_{\rightmoon}\sin\theta\sin(\lambda_{\rightmoon} - \Omega_{\rightmoon}) + 0(I_{\rightmoon}^{2})\Big\} \\
Y_1 &= a_{\rightmoon}\Big\{\sin(\lambda_{\rightmoon} - \phi) + 0(I_{\rightmoon}^{2})\Big\} \\
z &= a_{\rightmoon}\Big\{\sin\theta\cos(\lambda_{\rightmoon} - \phi) \\
&\qquad + I_{\rightmoon}\cos\theta\sin(\lambda_{\rightmoon} - \Omega_{\rightmoon}) + 0(I_{\rightmoon}^{2})\Big\} \qquad (10)
\end{aligned}$$

$I_{\rightmoon}$ being the inclination of its orbit to the ecliptic, about 5°8', and $\Omega_{\rightmoon}$ the longitude of its node. Now

$$G(m_{\oplus} + m_{\rightmoon}) = n_{\rightmoon}^{2}\, a_{\rightmoon}^{3} \quad .$$

The equations (8) then become, omitting periodic terms with $\lambda_{\odot}$ or $\lambda_{\rightmoon}$ appearing in the arguments (i.e. with period one year or less),

$$\begin{aligned}
\dot{\phi} = -\frac{3}{\omega\sin\theta}\cdot\frac{C - A}{C}\Bigg[\frac{1}{2}\, n_{\odot}^{2}\sin\theta\cos\theta \\
+ n_{\rightmoon}^{2}\cdot\frac{m_{\rightmoon}}{m_{\oplus} + m_{\rightmoon}}\left\{\frac{1}{4}\sin 2\theta - \frac{1}{2} I_{\rightmoon}\cos 2\theta\sin(\Omega_{\rightmoon} - \phi)\right\}\Bigg]
\end{aligned}$$

and

$$\dot{\theta} = -\frac{3}{2\omega}\cdot\frac{C - A}{C}\cdot n_{\rightmoon}^{2}\,\frac{m_{\rightmoon}}{m_{\oplus} + m_{\rightmoon}}\, I_{\rightmoon}\cos\theta\cos(\Omega_{\rightmoon} - \phi) \qquad (11)$$

measuring devices, and the results of this work are awaited with interest. To fit heliometer observations of the positions of surface features relative to the limb, it has sometimes been found necessary to use a model for the limb profile consisting of a semi-circular arc in the Northern hemisphere, and an elliptic one in the Southern. Koziel (1949) finds, however, that if observations of both limbs are treated together, no departure from a circular profile for the whole limb is called for.

4. The Gravitational Field of the Moon.

This can be expressed in terms of the potential V, which is given, at an exterior point P, by

$$V(P) = -\frac{Gm_{\rightmoon}}{r}\left\{1 - \frac{C - \frac{1}{2}(A + B)}{m_{\rightmoon}\, R_{\rightmoon}^{2}} \cdot \left(\frac{R_{\rightmoon}}{r}\right)^{2} \cdot \frac{(3\sin^{2}\beta - 1)}{2} + \frac{3}{4}\,\frac{(A - B)}{m_{\rightmoon}\, R_{\rightmoon}^{2}} \left(\frac{R_{\rightmoon}}{r}\right)^{2} \cos 2\lambda \cos^{2}\beta + 0\left[\left(\frac{R_{\rightmoon}}{r}\right)^{3}\right]\right\} \qquad (16)$$

where A, B, C are the principal moments of inertia of the Moon, $R_{\rightmoon}$ its mean radius, r is the distance of P from the center of mass, and λ and β the longitude and latitude of P referred to the Moon's equator (Oxy in the notations of section 2, λ being measured from Ox). Consequently the knowledge of the field depends on knowledge of the mass and moments of inertia.

P and N, defining the movements of the equator plane, which is the reference plane used in all modern positional astronomy, are verified by many methods, and scarcely likely to be subject to systematic error. This discrepancy was accordingly one of the puzzles of dynamical astronomy until Jeffreys (1948) showed that, if the Earth's core were treated as liquid, then the predicted value of N could be as low as 9".191, according to the model used for the density distributions. Consequently the rigid body motion of the Earth cannot give any information about the mass of the Moon. The theory, although well known, has been indicated briefly here, however, to provide comparison with that of the rigid body motion of the Moon, which is relevant to the determination of the moments of inertia.

3. The Shape of the Moon.

The shape of the surface of the Moon can be determined by measurement. From the apparent movements of objects on the surface during the librations (see section 5), the geometrical shape of the surface can be deduced. The most extensive study of this kind (prior to that of Baldwin, described at this conference) is that of Franz (1899) who obtains, from measures of 55 craters, for the radius of the Moon directed towards the Earth (1.00114 ± 0.00039) $\bar{a}$, where $\bar{a}$ is the radius of the sphere of equal volume. Saunder (1905) using 38 points on the central meridian, obtained (1.00052 ± 0.00027) $\bar{a}$. Watts, at Washington, is carrying out an extensive study of the limb profile, using photographs, which are traced automatically, using photo-electric

Now if we disregard the term in $\Omega_{\rightmoon}$, the solution is

$$\phi = - Pt + \text{constant} ,$$

$$\theta = \epsilon \qquad (12)$$

where ϵ is a constant (the "obliquity of the ecliptic"), and

$$P = \frac{3}{2\omega} \cdot \frac{C - A}{C} \cdot \cos\epsilon \left\{ n_{\odot}^{2} + n_{\rightmoon}^{2} \cdot \frac{m_{\rightmoon}}{m_{\odot} + m_{\rightmoon}} \right\} \qquad (13)$$

The terms in $\Omega_{\rightmoon}$ give rise to the additions

$$\delta\phi = + \frac{N \cos 2\epsilon}{\cos \epsilon} \cos \Omega_{\rightmoon}$$

$$\delta\theta = + N \sin \Omega_{\rightmoon} \qquad (14)$$

where

$$N = \frac{3}{2} \left(\frac{C - A}{C} \right) \left(\frac{m_{\rightmoon}}{m_{\oplus} + m_{\rightmoon}} \right) \frac{n_{\rightmoon}}{\omega g_{\rightmoon}} \cos \epsilon \qquad (15)$$

with $g = - \dfrac{1}{n_{\rightmoon}} \overline{\dfrac{d\Omega_{\rightmoon}}{dt}}$, (where the bar indicates the mean value)

The solution (12) corresponds to a uniform motion of the Earth's axis on a circular cone of semi vertical angle ϵ , with period $2\pi/P \approx 26{,}000$ years. This is the precession. The additions (14) represent a motion in an ellipse (the nutation) superimposed on this, with the period of $\Omega_{\rightmoon}$, that is 18.600 years. The observed value of $P \sec\epsilon$ (54".93129 ± 0".00175 per year, Brouwer and Clemence, 1961) corresponds, by means of Eq. (13), to the value 0.0032728 ± 0.0000006 for $\frac{C - A}{C}$. This corresponds, using Eq. (15) to N = 9".2266. But the observed value of N is 9".210 ± 0".003 (loc. cit). The constants

5. The Moments of Inertia of the Moon.

The ratios of the moments of inertia of the Moon can be determined from the rigid body motions. In 1693 Cassini gave the following empirical laws, which describe the orientation of the Moon to within about 2 minutes of arc:

(1) The rotation is about an axis fixed in the Moon, of uniform magnitude, with the period of the Moon's revolution about the Earth. That is, $\omega_1 = \omega_2 = 0$, $\omega_3 = \dot{\psi} + \dot{\phi} \cos \theta = n_{☽}$.

(2) The inclination of the axis to the ecliptic is constant, about $1^\circ\ 35'$. That is, θ is constant $= J$, say.

(3) The Moon's equator and orbit plane have a common line of intersection with the ecliptic. That is, the axis of rotation and pole of the orbit are coplanar with the pole of the ecliptic, being in fact on opposite sides of it. That is, $\phi = 90^\circ + \Omega$.

The departures of the orientation of the Moon from the configuration defined by these laws constitute what is known as the "dynamical libration". This results in the Moon's not keeping exactly the same face directed towards the Earth at all times, though this is far exceeded by the "optical librations", which arises from the following causes.

(a) The inclination of the Moon's axis to the orbit plane, giving rise to a monthly apparent oscillation about an axis perpendicular to the line of sight, in the plane of the orbit, of amplitude $I_{☽} + J \approx 7^{\circ}\ 43'$. This is the "optical libration in latitude".

(b) the departure of the Earth's orbit from a circle, giving rise to the "optical librations in longitude", and

(c) the diurnal libration, arising from the translation of the observer as the Earth rotates.

The dynamical libration is governed by equations of the form (3). By far the most important contribution to the couple (4) arises from the Earth. Now θ is small, so for an approximate treatment we put $\sin\theta = \theta, \cos\theta = 1$. (For more detailed treatments see, e.g., Plummer 1918, Hayn 1923, Jeffreys 1955, or Koziel 1962.)

In the equations (3) we make the definitions

$$\alpha = \frac{C - B}{A},\ \beta = \frac{A - C}{B},\ \gamma = \frac{B - A}{C}\ . \tag{17}$$

The third equation becomes

$$\ddot{\psi} + \ddot{\phi} = \frac{3\gamma\ G m_{\oplus} xy}{r^5} \tag{18}$$

For the Earth we have

$$x = -\ r\cos(V_{☽} - \phi - \psi) + 0(I_{☽}^{2},\ \theta^2)$$

$$y = -\ r\sin(V_{☽} - \phi - \psi) + 0(I_{☽}^{2},\ \theta^2)$$

$$z = -\ r\,\theta\cos(V_{☽} - \phi) - r\,I_{☽}\ \sin(V_{☽} - \Omega_{☽}) + 0(I_{☽}^{2},\ \theta^2) \tag{19}$$

where $V_{☽}$ is the true longitude of the Moon. Then (18) becomes

$$\ddot{\psi} + \ddot{\phi} = \frac{3\gamma\, n_{\rightmoon}^{2}}{(1 + m_{\rightmoon}/m_{\oplus})} \left(\frac{a_{\rightmoon}}{r} \right)^{3} \frac{1}{2} \sin 2(V_{\rightmoon} - \phi - \psi) \qquad (18')$$

Now from the theory of the Moon's motion, we may write

$$\left(\frac{a_{\rightmoon}}{r} \right)^{3} \sin 2\,(V_{\rightmoon} - \phi - \psi) = c \sin 2(\lambda_{\rightmoon} - \phi - \psi) + 2 \sum H \sin\,(ht + h')$$

where c = 0.9852, approximately, and H, h, h' are constants, the summation being over all terms arising from the Lunar theory.

Thus, if we put $\phi + \psi = \lambda_{\rightmoon} + 180^{\circ} + \tau$,

$$\ddot{\tau} = - \frac{3\gamma\, n^{2}}{1 + m_{\rightmoon}/m_{\oplus}} \left\{ \frac{1}{2}\, c \sin 2\tau + \sum H \sin\,(ht + h') \right\} \qquad (18'')$$

Free oscillations about $\tau = 0$ are possible if $\gamma > 0$, that is if $B > A$. Then, for small oscillations,

$$\tau = A \sin\,(\nu\, t + \delta) + \sum \frac{\nu^{2}}{h^{2} - \nu^{2}} H \sin\,(ht + h'), \qquad (21)$$

where A and δ are free constants, and $\frac{\nu}{n} = \sqrt{\frac{3\gamma\, c}{1 + m_{\rightmoon}/m_{\oplus}}}$. The two largest forced terms are those in $\ell_{\rightmoon}$ and $\lambda_{\odot}$. That in $\ell_{\rightmoon}$ has $H = 377'$, and $h = n_{\rightmoon}$, and that in $\lambda_{\odot}$ has $H = -\,11'.15$ and $h = n_{\odot}$. The latter dominates, since $n_{\odot} \ll n_{\rightmoon}$.

If we put

$$\xi = \theta \cos \psi$$
$$\eta = \theta \sin \psi \qquad (22)$$

the first two equations of (3) become, using (4),

$$\ddot{\xi} + n\,(1 + \beta)\,\dot{\eta} - n^{2} \beta\, \xi = \frac{3\beta\, Gm_{\oplus}\, zx}{r^{5}}$$
$$\ddot{\eta} - n\,(1 - \alpha)\dot{\xi} - n^{2} \alpha\, \eta = \frac{3\alpha\, Gm_{\oplus}\, yz}{r^{5}} \qquad (23)$$

The right hand sides are, from (19)

$$3\beta\ Gm_{\oplus}\ zx/r^5 = 3\ \beta n_{\leftmoon}^{2}\Big\{ -I_{\leftmoon}\ \sin(\lambda_{\leftmoon} - \Omega_{\leftmoon}) + \frac{5}{2}\,e_{\leftmoon}\,\theta\cos(\ell_{\leftmoon} + \psi)$$

$$+ \frac{1}{2}\,e_{\leftmoon}\,\theta\cos(\ell_{\leftmoon} - \psi) - \frac{5}{2}\,e_{\leftmoon}\ I_{\leftmoon}\sin(2\lambda_{\leftmoon} - \varpi_{\leftmoon} - \Omega_{\leftmoon})$$

$$- \frac{1}{2}\,e_{\leftmoon}\,I_{\leftmoon}\ \sin(\varpi_{\leftmoon} - \Omega_{\leftmoon}) + 0(e_{\leftmoon}^{2}\,I_{\leftmoon},\ e_{\leftmoon}^{2}\,\theta)\Big\}\ ,$$

and

$$3\alpha\ Gm_{\oplus}\ yz/r^5 = 3\alpha\ n_{\leftmoon}^{2}\Big\{ e_{\leftmoon}\,\theta\sin(\ell_{\leftmoon} + \psi) + e_{\leftmoon}\,\theta\sin(\ell_{\leftmoon} - \psi)$$

$$+ e_{\leftmoon}\ I_{\leftmoon}\ \cos(2\lambda_{\leftmoon} - \varpi_{\leftmoon} - \Omega_{\leftmoon}) - e_{\leftmoon}\ I_{\leftmoon}\ \cos(\varpi_{\leftmoon} - \Omega_{\leftmoon})$$

$$+ 0(e_{\leftmoon}^{2}\,\theta,\ e_{\leftmoon}^{2}\,I_{\leftmoon})\Big\}^{*}\ ,$$

The complementary functions of the equations are given by

$$\xi = X_1\cos(\gamma_1 t + \delta_1) + X_2\cos(\gamma_2 t + \delta_2)$$

$$\eta = Y_1\sin(\gamma_1 t + \delta_1) + Y_2\sin(\gamma_2 t + \delta_2) \qquad (24)$$

where X_1, X_2, δ_1 and δ_2 are free constants,

$$\gamma_1 \approx n\ (1 - \frac{3}{2}\beta)\ ,$$

$$\gamma_2 \approx n\ (2\sqrt{-\alpha\beta})\ ,$$

$$Y_1 \approx X_1$$

and $$Y_2 \approx 2\sqrt{-\beta/\alpha}\ X_2\ .$$

These free motions appear from the observations to be quite negligible. Stability requires that α & β have opposite signs.

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* $e_{\leftmoon}$ is the eccentricity of the Moon's orbit, $\varpi_{\leftmoon}$ the longitude of its perigee, and $\ell_{\leftmoon}$ its mean anomaly.

The dominant forced term arises from $-3\beta n_{\rightmoon}^{2} I_{\rightmoon} \sin(\lambda_{\rightmoon} - \Omega_{\rightmoon})$ in the first equation, and gives

$$\theta \cos\psi = \xi = \frac{3\beta I_{\rightmoon}}{3\beta + 2g_{\rightmoon}} \sin(\lambda_{\rightmoon} - \Omega_{\rightmoon})$$

$$\theta \sin\psi = \eta = \frac{-3\beta I_{\rightmoon}}{3\beta + 2g_{\rightmoon}} \cos(\lambda_{\rightmoon} - \Omega_{\rightmoon}) \qquad (25)$$

Now Cassini's third law shows that in the configuration actually existing, using (20),

$$\begin{aligned} \psi &\approx \lambda_{\rightmoon} + 180^{\circ} - \phi \\ &\approx \lambda_{\rightmoon} + 90^{\circ} - \Omega \;, \end{aligned}$$

so that

$$\theta \approx J = \frac{-\,3\beta I_{\rightmoon}}{3\beta + 2g_{\rightmoon}} \qquad (26)$$

This requires that $\beta < 0$, if $\beta > -\frac{2}{3} g_{\rightmoon}$, which is so. Therefore $C > A$, and so the axes of the Moon are oriented so that $A < B < C$, with that of least moment of inertia directed approximately towards the Earth. Jeffreys (1955) shows that this configuration is necessary for stability under dissipative forces.

Quantitative measures of the libration were begun with the heliometer by Bessel in 1837, and continued by his pupils Schlüter and Wichmann. Harting made observations at Dorpat from 1877 to 1922, and a continuous series of observations has been made at Kazan since 1895. The method is usually to measure the position of the crater Mösting A, near the center of the disc, relative to the limb. (For a more complete list of observers and reductions, and bibliography, see Koziel (1962).) The results for J vary from $1^\circ 31'32'' \pm 14''$ to $1^\circ 33'48'' \pm 17''$, and Watts (1955) from some of the results of his survey of the limb, obtains $1^\circ 33'58'' \pm 12''$. Hayn (1923) got $1^\circ 32'13''$, from which he deduced (Eqs. 26) that $\beta = -0.000\,6300$. Jeffreys (1961) concludes that $J = 1^\circ 32'39'' \pm 17''$, combining all determinations. Now β is slightly sensitive to the value of α, from the higher terms, and taking $f = -\alpha/\beta = \frac{2}{3}$, Jeffreys' value gives $\beta = -0.000\,6279 \pm 0.000\,0015$.

The libration in longitude leads to the value of γ. This is usually expressed in terms of $f = -\alpha/\beta$, since $\alpha + \beta + \gamma = -\alpha\beta\gamma \approx 0$, so that if β is known, $\gamma = -\beta(1 - f)$. The term in $\sin \lambda_\odot$ in τ has led to values of f from 0.5 to 0.84. Jeffreys (1961) concludes that $f = 0.639 \pm 0.014$, corresponding to $\gamma = 0.000\,2274 \pm 0.000\,0088$. Yakovkin (1952) however claimed to have determined the coefficient of the free libration, and obtained $\gamma = 0.000\,2098 \pm 0.000\,0022$. But its period is very close to the forced term in $\sin 2(\varpi_{☽} - \Omega_{☽})$, so, the amplitude, $\frac{\nu^2 H}{h^2 - \nu^2}$ is large, in fact the theory breaks down if the free speed ν is equal to h - this happens if $f = 0.662$. Jeffreys (1962) remarks that Yakovkin's determinations of phase are quite consistent with his having in fact found the coefficient of the forced term. In this case his determination leads to $\gamma = 0.000\,2049 \pm 0.000\,0009$. Banachiewicz and Koziel (1949) have pointed out that the existence of this near resonance has the effect that if the least squares method is applied in a straightforward fashion, then a trial value of f on one side of 0.662 will always lead to a corrected value on the same side - the sum of squares of residuals has infinities as $f \rightarrow \pm \infty$, and at $f = 0.662$ and, being always positive, must have two minima, one on each side of 0.662. (Yakovkin asks if it is accidental that f has a value near to that giving resonance, or whether there is a dynamical explanation.) Habibulin (1955 and 1958) shows that the ambiguity can be resolved in two ways. The coefficient of the term in $\sin 2(\varpi_{\rightmoon} - \Omega_{\rightmoon})$ would be negative if $f < 0.662$, and positive if $f > 0.662$. He finds it to be - 17" ± 8". Also the coefficient of the $\sin \lambda_{\oplus}$ term would be greater than 84" if $f < 0.662$, and less otherwise. He finds it to be 100" ± 10". Both very strongly favor the former case, and his result is $f = 0.60 \pm 0.02$, which is probably the best value available. Now on the hypothesis that the Moon is in a hydrostatic state, Jeffreys (1958) shows that $\frac{C - A}{C} = 5 \frac{m_{\oplus}}{m_{\rightmoon}} (\frac{R_{\rightmoon}}{a_{\rightmoon}})^3$, and $\frac{C - B}{C} = \frac{5}{4} \frac{m_{\oplus}}{m_{\rightmoon}} (\frac{R_{\rightmoon}}{a_{\rightmoon}})^3$, leading to $f = 0.25$. This is certainly not the case.

The figure of the Moon leads to perturbations of its orbit about the Earth. The potential energy of the Earth and Moon in each other's gravitational fields is, from MacCullugh's formula,

$$G \left\{ \frac{m_{\oplus} m_{\rightmoon}}{r} + m_{\rightmoon} \frac{A' + B' + C' - 3I'}{r^3} + m_{\oplus} \frac{A + B + C - 3I}{r^3} + 0 \left(\frac{1}{r^5} \right) \right\}$$

where A', B', C' and I' are here moments of inertia of the Earth. Thus the disturbing function for the Moon contains the terms

$$\frac{Gm_{\oplus}}{r} \left\{ \left(\frac{R_{\oplus}}{r} \right)^2 \frac{A' + B' + C' - 3I'}{m_{\oplus} R_{\oplus}^2} + \left(\frac{R_{\leftmoon}}{r} \right)^2 \frac{A + B + C - 3I}{m_{\leftmoon} R_{\leftmoon}^2} \right\}$$

The first leads to perturbations involving the figure of the Earth, and the second involving the figure of the Moon. From the secular motions of the apse and node Jeffreys (1961) derives the equations (using a value for $\frac{C' - A'}{m_{\oplus} R_{\oplus}^2}$ derived from artificial satellite motions)

$$380'' L' - 1192'' K' = + 0''.078 \pm 0''.056$$

$$\text{and} \quad - 460'' L' = - 0''.251 \pm 0''.052$$

respectively, where

$$K' = \frac{3(B - A)}{2m_{\leftmoon} R_{\leftmoon}^2}$$

and

$$L' = \frac{3(C - A)}{2m_{\leftmoon} R_{\leftmoon}^2} .$$

From these

$$K' = (1.07 \pm 0.59) \times 10^{-4}$$

$$\text{and} \quad L' = (5.46 \pm 1.13) \times 10^{-4} ,$$

the uncertainities being mainly due to the incompleteness of the calculation of the solar contributions to $\dot{\varpi}_{\leftmoon}$ and $\dot{\Omega}_{\leftmoon}$. From these values,

$$\frac{B}{m_{\leftmoon} R_{\leftmoon}^2} = - \frac{2L'}{3\beta} = 0.58 \pm 0.12 ,$$

and

$$\frac{C}{m_{\leftmoon} R_{\leftmoon}^2} = \frac{2K'}{3\gamma} = 0.35 \pm 0.21 .$$

These quantities would be 0.4 if the Moon were homogeneous, and could only be greater if the density decreased towards the center. Until better values are available, it is perhaps best to take A, B and C equal to 0.4 $m_{☽} R_{☽}^{2}$, or slightly less. Their ratios are of course very much better determined from the values of β and γ derived from the physical librations, as discussed above, from which

$$C/A \approx 1 - \beta = 1.000\ 628 \pm 0.000\ 002 \quad ,$$

and

$$B/A \approx 1 - \beta + \beta f = 1.000\ 25 \pm 0.000\ 01 \quad .$$

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AGE OF THE MOON, CHEMICAL COMPOSITION, GEOLOGICAL ASPECTS, STRESS AND COOLING HISTORY

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AGE OF THE MOON, CHEMICAL COMPOSITION, GEOLOGICAL ASPECTS, STRESS AND COOLING HISTORY

by

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The topics to be discussed here are indicated by the title of this presentation. All of these with the exception of the geology of the moon will be included, lunar geology is excluded since this material will be covered by others at this meeting. It should be mentioned that much of what is to be discussed here can be found in the author's chapter of the book "Physics and Astronomy of the Moon," edited by Professor Kopal. Several items of interest, not presently in print, will be added at this presentation; it is expected that these will appear in the literature sometime in the near future.

The first subject to be considered is the age of the moon. The moon is a satellite of the earth; and in estimating its age one must be concerned, to some extent, about the process of origin. How did it get where it is and in particular what is the age of the surface? In attempting to answer these questions it would be most gratifying to have a sample of the lunar surface so that its age might be estimated by potassium argon dating. It is quite possible that argon would be absent in the equatorial regions though it is felt that it would be found in the polar regions. Hence, we should be able to determine when the major features of the lunar surface were formed. It has been my contention for some time that most of the surface features of the moon are ancient; and it is expected that when dating of the kind referred to here is done, these will prove to be from 4.5 to 4.7 billion years old. It has been my view that the surface features were acquired primarily during the period of origin of the moon and that this occurred at about the time when the meteorites were formed.

Incidentally, the meteorites have been found to be rather complicated objects. Their age has been estimated, from radioactive dating methods, to be about 4.5 to 4.7 billion years. It is quite likely that these objects were formed with the solar system and acquired their physical and chemical properties at that time.

In Figure I is shown a segment of the lunar surface. This particular photograph was made some five years ago. It was produced by a technique which eliminates foreshortening of the image. The method employed is to project pictures of the moon onto a white sphere and to photograph this projection from directly above the spherical surface. The region shown here is Mare Imbrium; it is that region first discussed by Gilbert in his work published in 1893.

According to the formation shown, an object, coming from the northeast, collided with the lunar surface and plowed out the feature known as Sinus Iridum. This is outlined by the smaller circle in Figure I. This object probably produced a large bulge on the moon and scattered material over the surface, producing a great fan-shaped pattern of ridges and grooves extending all the way to the center of the lunar disk. The edge of the maria was probably produced when the shock wave subsided and material settled back, breaking off, producing this structural feature. There is a good reason to believe that the material thrown out by this collision will be found in the older, pre-Imbrian craters of the moon; some craters, formed later, quite naturally will not have Imbrian material in their interiors. It is most reasonable to believe that all these features were produced by a single event, that collision did occur and produced the total structural feature shown in the Figure.*

There are other circular craters and maria on the moon. For instance Mare Serenitatis, Mare Humorum, Mare Nectaris and Mare Crisium, all of these are likely to be the result of large objects colliding with the moon. Other maria on the moon, such as Oceanus Procellarum, Mare Nubium and Tranquillitatis look as though they might be the result of a lava flooding and solidification. There is a difference of opinion concerning lunar lava flow; some students of the moon contend that the lava came from the interior, much like it does on earth, while others subscribe to Gilbert's suggestion that the lava is a consequence of the heating due to collision.

In regard to the collision producing Sinus Iridum, one might ask the question, where did the object come from that produced this large effect? It should be noticed that it is not possible to maintain a large object, crossing the Earth-Moon orbit, for any appreciable time without a collision occuring. Such a collision must have occurred in a relatively short time, say a hundred million years or so, after it first appeared crossing the Earth's orbital path.

There are other reasons for believing that such an event occurred early in time. Mare Serenitatis is a circular mare of a gray color while Mare Tranquillitatis is a very irregular mare of a darker color bordering Serenitatis on the west. The question arising here is which occurred first, Mare Serenitatis or Mare Imbrium? Had the first formation been Mare Imbrium then it would be expected that the radiating ridges south of Mare Serenitatis would have undergone a great deal of

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* Recently Hartmann and Kuiper have published much more beautiful pictures than that produced here, but I believe that their interpretation is quite incorrect and hence this older and less elegant picture is used to illustrate these features of the moon.

destruction. Had Mare Serenitatis been formed first, then why are mountain ridges on this mare not also radiating from the collision area on Mare Imbrium? One suggestion is that this all came about because Mare Serenitatis was still fluid, or melted, at the time of the collision which produced Mare Imbrium. Fluidity in the sense of a dust gas mixture should not be excluded as possible. Then there is the question of how long did it stay melted? Some few years ago the author made some calculations on this and concluded that the time was rather short - some few thousand years or so. The two events producing these maria must have occurred near to each other in time. As a matter of fact, the large number and distribution of craters on the moon suggests that events occurred somewhat in the following order: first craters were formed, then maria, then other craters, etc. All of this occurred in a short span of time; then the bombardment stopped and nothing else on this scale has occurred since. It is the contention that the surface features of the moon were fashioned toward the terminal stage of the Earth-Moon formation and that little crater formation has occurred since then. An exception to this could be the formation of the great ray craters of the Moon; all others are probably very ancient for the most part.

One other striking feature in regard to Mare Serenitatis is the dark line in its western area, and the darker area in and around this region. This looks to be the result of a lava flow up over the whole of the area. It is quite possible that this was produced when the Imbrian collision occurred, spreading material over much of Mare Serenitatis, which in its solidification produced the line mentioned above. It may be possible to fit some similar reasoning to the hypothesis of deep dust in the lunar maria.

One should not believe that these statements prove the point. They are merely reasonable arguments; no one really knows anything certain about these subjects at all. All that is being done here is to put forward what look to be reasonable arguments with the hope that in time the facts will become known and the hypotheses verified or refuted. If one does not like or agree with such statements then he should not be too concerned since there is no way of absolutely ascertaining many of these things at present.

For reasons of this kind I believe the moon acquired its surface features in a relatively short period of time, and did so a very long time ago. Particularly, it did so during the origin of the solar system.

Toward the end of this discussion some ideas regarding the origin of the moon will be mentioned; that the rather short time scale for the surface features of the moon may have resulted during its capture, while the earth still had some smaller objects moving about it, objects which were the residue of its own accumulation. In a relatively short span of time the satellite swept up this terrestrial trash; aside from some few meteorite impacts since that time, and possibly some small amounts of volcanic activity in its early history, the moon's features were thus formed.

Chemical Composition of the Moon

Next we should look at the chemical composition of the moon. What is meant by this is the chemical composition of the object as a whole. Unfortunately we can only estimate the composition, and this from an interpretation of its low mean density.

The density of the moon is well established from a reasonable knowledge of its mass and size; the value of density is currently taken as 3.34 grams/cm^3. This applies to the moon at a considerable internal pressure and at elevated internal temperatures; temperatures larger than surface levels. The pressure, internal to the moon, can be rather adequately estimated from the well known hydrostatic expression

$$p = \frac{2}{3} \pi G \rho^2 (a^2 - r^2)$$

where G is the gravitational constant, ρ is the density (assumed uniform for the moon) and a is the major radius of the body. This expression should give the internal pressure quite accurately; there will be some small errors but these are not important in our present discussion.

The temperature of the moon, and particularly its internal temperature, is considerably more difficult to determine though several estimates have been made. Some years ago I estimated the temperature of the moon from a study of its thermal history. And with a knowledge of the coefficients of compressibility and thermal expansion for materials which might make up the moon, calculations of the lunar density were also made. Assuming the compressibility of the moon to be about the same as that of olivine or pyroxene, the estimate of density as a round value is 3.40 grams/cm^3. This value is the average of estimates at low pressures and terrestrial surface temperature from approximate values for the temperature distribution and calculated pressures; the range of calculated values varied from 3.38 to 3.41 grams/cm^3. It is possible to estimate the composition of the moon by comparing the values here with those of materials which are close to the make-up of the meteorites. This choice of comparison is made on the basis that the meteorites will provide material samples which more nearly approximate primordial material than will the earth.

Now, starting with the chondritic meteorites, of which there are two distinct groups - one having a density of 3[illegible] grams/cm^3 (calculated from the observed composition) the other having a density of 3.76 grams/cm^3 - it is apparent that the moon is less dense than these objects. Incidentally, the calculated densities for these meteorites is slightly larger than the observed values; probably this can be attributed to the fact that the observed values are an average taken by many persons over a rather long period of time. Too, it is reasonable to expect that the meteorites are porous which would suggest a lower density. Now suppose that in the low-iron chondrites the albite is converted to jadeite and the SiO_2 is taken up as magnesium metasilicates ($MgSiO_3$); the density is then calculated to be 3.65 grams/cm^3 instead of the 3.57 grams/cm^3 as calculated from the observed composition.

With these various calculated values we next ask how much iron must be removed from the meteorites to make their residue equal the calculated density for the moon. This has been done, and it is found that if the iron in the residue appears as FeS and Fe then the required content is 10.78 per cent. If the iron is to be present as FeO then the per cent present in the chondrite residue should be 11.52. These calculations are based on the assumption that the basic physical characteristics are the same as those of olivine, and that the mean temperature of the moon is 1100°C. If the constants for the calculations are altered, say choose the physical characteristics of enstatite, then the iron content of the residue is altered and the average density of the moon is slightly changed. It is these variations of probable material makeup which gives the range of values for density as stated previously.*

It is interesting to note that the abundance of iron in the moon is probably below the level of that found in the chondritic meteorites. To some extent this lends support to the idea that the moon is a primitive body having the same proportion of iron as the sun. Some investigators have calculated the solar abundance, and that for the origin of the elements, to be [illegible]iderably below the value obtained

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* See H. C. Urey, J. Geophys. Res. 64, 1721 (1959).

from a study of the chondrites. Suess and Urey have made calculations of this value and obtained 6 x 10^5, relative to silicon taken at 10^6. The Aller value for solar abundance of iron is 1.4 x 10^6. It is apparent that there is a discrepancy here by a factor of about 3 or 4. In this respect it may be concluded that the earth has a material makeup much like that of the chondrites, but the sun and moon contain much less iron. Of course in the final analysis this question, too, will be answered when a sample of the lunar surface becomes available for study. Such a sample alone will be of great importance since its analysis has a direct bearing on the problem of the origin of the solar system and the synthesis of elements present there.

All in all we might conclude that the moon has a bulk composition more nearly like that of the sun. It appears to be of a material like that of the sun with the gaseous components eliminated, and containing some small fraction of high density material accumulated in the processes which produced the high density earth.

Of course it is likely that the discrepancy in density between the moon and the meteorites can be accounted for by assuming other models. One such case is to assume that the lunar rocks have sufficient water content to reduce the meteorite density to that of the moon. One would only have to assume about a two per cent water content to provide the necessary reduction. Of course such a content of water would cause a lowering of the silicates melting point which would in turn suggest a greater volcanic activity and more evidence of lava flow than is presently observed. This does not suggest that the lunar surface material does not contain water; it does provide a reasonable doubt that the moon as a whole would have this large a percentage.

Another substance which could be present, in large enough quantities to properly reduce the chondritic density to that of the moon, is graphite. Graphite is present in the meteorites, though there would have to be about ten per cent (by weight) of graphite to provide a proper density. Graphite might react with iron oxides to give CO and CO_2 and if these escaped the density of the moon would increase. Of course this too remains as a possibility since the internal temperature of the moon is largely unknown and the chemical reactions may have been sufficiently retarded to allow high percentages of carbon to remain intact.

The curious bulk composition of the moon has led to speculations regarding its origin, beginning in the last century with Sir George Darwin who proposed that the moon escaped from the earth. His reason for doing so was to account for the low density of the moon; his suggestion was that the moon was formed by the throwing off of the lower density earth surface material due to tidal action. A repetition of his calculations by Jeffries, and by Moulton, during the present century have led to a different conclusion. Also, according to Lyttleton, in his book, "The Stability of Rotating Liquid Masses", this could not have occurred by any reasonable physical process. Since this should occur due to an instability, as the body increased its angular momentum, the velocity imparted to the detached mass would have been sufficient to cause it to leave the earth-moon system. In fact it is likely to have been given a velocity large enough to carry it out of the solar system entirely.

Chandrasekhar has expressed the view that Lyttleton's calculations are probably correct and that the only reasonable answer is that the moon is a little chunk of material left behind when a larger mass was thrown off the earth by some cataclysmic action.

All things considered, if we eliminate this last concept, then the evidence favors the idea that the moon is more nearly like the sun in composition, certainly more so than like the composition of the groups of meteorites which have been used in recent years to estimate the so-called cosmic abundance of the elements.

Thermal History

Next we will discuss the thermal history of the moon. In this discussion we will limit the basic premise to that of an originally melted moon; actually it is presumed that the whole of the solar system was initially in a melted state. Since 1900, when radioactivity was discovered, such an assumption is unnecessary since adequate radioactive heat is known to be available in order to account for the observed high temperature processes.

If the moon as melted during its formation 4.5 billion years ago, the solidification which follows requires that the radioactive elements must be concentrated near the surface. Had the moon been formed cold then it may be necessary that the interior be melted now. As a matter of fact if this same hypothesis is applied to the earth, then assuming a uniform distribution or radioactive elements, equivalent to the surface concentration, the earth would have to be melted at present and from the time of its formation.

In regard to the heat generated by radioactivity some consideration of the more important nuclides is in order. Essentially these elements are potassium - 40, thorium and uranium, and are the components found in the chondritic meteorites. At the present time the total energy generated by these components is 1.6 ergs per gram per year. Assuming a specific heat of 1.25 joules per gram, the total heat generated by these nuclides in the chondrites in 4.5 billion years is 2330 joules per gram. This would provide a temperature of 1864°C, which is a considerable rise in temperature during this time period. Such a rise in temperature, of course, presumes no heat loss from the material. To account for this loss in heat one calculates the change in temperature due to surface cooling (for constant heat conduction and uniform heating by the radioactive components) from

$$\frac{\partial T}{\partial t} = K \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} \right) + \sum_i Q_i \, e^{-\alpha_i t}$$

where T is the temperature, t is time, K $(=k/\rho c)$ is the thermal diffusivity. Here, too, k is the (constant) thermal conductivity, ρ is the mean density, and c is the *specific heat of the material. The summation is taken with respect to the "i"* nuclides present; Q_i is the temperature rise rate of the i^{th} nuclide at t=o, and α_i represents its decay constant.

In order to solve thisequation one must assume an initial temperature. It will be assumed that the moon accumulated in a cold state, and an initial temperature of 0°C will be chosen. Having ascertained that heating occurs due to radioactivity a brief look at the conditions of melting is in order.

The silicate materials making up the moon do not all melt at the same temperature. That is, some constituents begin to melt at one temperature with final melting occuring at some higher temperature. On the assumption that initial melting begins at 1100°C and final melting occurs at about 1600°C (at a standard temperature and pressure), and that the melting temperature varies with depth of material (that is, with pressure), then the following formulae are proposed

$$T_i(^oC) = 1100 + 500\ (1 - \frac{r^2}{a^2})$$

and

$$T_f(^oC) = 1600 + 200\ (1 - \frac{r^2}{a^2})$$

where T_i and T_f are the initial and final melting temperatures respectively. According to these expressions initial melting begins at about 1600°C at the center of the moon (r=0) and final melting occurs there at about 1800°C. The above formulae are to be used for estimating the melting within the moon due to radioactive heating.

Now let it be supposed that the moon formed as a completely melted body. In this case solidification occurred from the center outward and the more dense materials, having the higher melting point, would crystallize and sink to the center. Presumably this would be olivine and metallic iron. As solidification continued the lighter components would rise toward the surface and finally solidify at about 1100°C. For the case of a recently melted moon which solidified from the center outward it is assumed that the temperature through the moon follows the formula

$$T\ (^oC) = 1100 + 900\ (1 - \frac{r^2}{a^2})$$

Hence the temperature at the center of the moon would be about 2000°C while the surface temperature would be at 1100°C.

Returning now to the cooling history differential equation it can be shown that a solution is obtained in three terms, each of which is itself an infinite series. The first part of the solution (T_1) shows the influence of the radioactive heating. The second term (T_2) is the cooling from an initial thermal state, at temperature (T_o) throughout; and the last term (T_3) is a consequence of the quadratic temperature variation through the moon itself. The three expressions referred to are:

$$T_1 = \frac{2a}{r} \sum_{i} \sum_{n=1} \frac{Q_i}{\alpha_i - K \left(\frac{n\pi}{a} \right)^2} \frac{(-1)^n}{n\pi} \sin \left(\frac{n\pi r}{a} \right) \exp\,(-\alpha_i t) - \exp \left(- \frac{K n^2 \pi^2 t}{a^2} \right)$$

$$T_2 = - T_o \frac{2a}{r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi} \sin \left(\frac{n\pi r}{a} \right) \exp \left(- \frac{K n^2 \pi^2 t}{a^2} \right)$$

$$T_3 = - \Delta T \frac{12a}{r} \sum_{n=1}^{\infty} \frac{(-1)^n}{(n\pi)^3} \sin \left(\frac{n\pi r}{a} \right) \exp \left(- \frac{K n^2 \pi^2 t}{a^2} \right)$$

The value of Q_i in T_1 can be calculated from the equation

$$Q_i = \frac{E_i}{c} \alpha_i m_{ip} \exp\,(\alpha_i t)$$

where E_i is the specific energy of the i^{th} nuclide, m_{ip} is the mass of the i^{th} nuclide per gram of meteorite material at the time in question (say 4.5 billion years) and c is the specific heat of the meteorite material, assumed constant at 1.25 joules per gram. All other terms in these expressions have been defined previously. Values for the Q_i have been calculated and are available in the literature.

The calculations on the temperatures within the moon depend markedly on the concentrations of potassium, uranium and thorium in lunar material. As explained in the footnote, this problem has caused great confusion during the last ten years or more. At the time that Chapter 13 of "Physics and Astronomy of the Moon" was written, i.e., 1959, no very reasonable estimate of these concentrations different from that of the chondritic meteorites was recognized. Since then, it has become evident from studies on the concentrations of rare elements in the carbonaceous chondrites that these objects contain the rare elements in more nearly the concentrations expected from studies of atomic abundances and the theories of nucleogenesis than do the ordinary chondritic meteorites. (See Goles and Anders (1962) and references there cited.)

The concentrations of potassium in the carbonaceous chondrites is not constant and varies from 380 to 750 ppm (Edwards and Urey, 1955; Edwards, 1955) with an average of 528 ppm instead of 863 ppm which is the average of the chondrites. A small systematic error is probably present in the analytical data and the latter average was lowered to 823 ppm, which seems to be a reasonable correction. For the same reason the average of 528 ppm may be too high and the true value may be considerably less. We will use 500 ppm and there is considerable probability that the true average will not be lower than 400 ppm for this quantity. Adoption of this latter value would lower temperatures by a maximum of 178°C beyond those given below. The concentrations of uranium and thorium are much the same in the carbonaceous and other chondrites. Values secured recently are about 25 per cent higher than those used in the calculations in 1959 as given in the above paper. This increase would raise the maximum temperatures due to uranium and thorium by about 100°C. Such a revision in view of the many uncertainties in concentrations, specific heat, thermal conductivity, initial temperatures, etc., is not justified.

Table I and Figures 2 and 3 give the results of revised calculations for the cases of the intially hot and cold moons using 500 ppm for the abundance of potassium.

The present estimates of lead-lead ages for the meteorites run somewhat higher than the 4.5 AE used in the calculations, and revision of the curves for this effect would raise the curves slightly and extend them beyond 4.5 AE ordinate as given in the figures.

It is evident that lower lunar temperatures than have been calculated previously are possible and even probable, and in this case a more rigid moon at a temperature below the melting point of meteoritic iron-nickel is possible, providing that the initial temperature was not much above 0°C.

From the curves presented it is apparent that in the deeper interior the temperature has accumulated in time, while nearer to the surface the temperature has risen to a peak value and subsequently cooling has produced a small decrement over the last 2.5 billion years (approximately).

MacDonald has carried out machine calculations similar to those shown here and has results which have the same graphical shape. His results differ a bit from those shown here in that his initial lunar temperature was 600°C, and he has included a radiative transfer term in his calculations. Too, MacDonald has calculated the change in radius with time, due to heating, which has not been done in the present studies.

Next consider the question, could the moon have been completely melted from the time of its formation? In this case, the surface temperature would have started at about 1100°C and cooled throughout its history. Of course the same situation exists for all radial stations within the moon, but with more rapid and a greater degree of cooling being apparent near to the surface. At the center of the moon very little cooling will have occurred during the 4.5 billion years of its existence through diffusivity (See Figures 4 and 5). The variation of temperature with depth (or radius) follows the quadratic variation given previously. These graphs do not consider any radioactive effects whatsoever. Should there be any radioactive heating in the interior, then the temperature will necessarily be increased. One might expect to find temperatures of the order of 2500°C to 2800°C, as a maximum, dependent on the choice of potassium abundance.

Now a discussion of how this applies to the irregular shape of the moon is in order. The moon's irregular shape is due to bulges toward and away from the earth. These bulges are estimated to be about one kilometer in height,whereas the equilibrium height of the moon would be about 60 meters. (The 1 km datum comes from the dynamical characteristics of the moon's orbit and is expected to be a rather reliable figure). Hence, the bulge could be accounted for by assuming the moon has had a certain rigidity for quite some time and could support this irregular feature from a time in its early history.

Now to define this situation, I suggested to Professor Elsasser that probably in its accumulation the moon acquired a variation, with angle, in the concentration of high density material. If this occurred the highest density material would be located in the polar regions of the moon and the lowest density located in the direction of the earth-moon radius vector.

For this case we are able to account for the irregular shape of the moon regardless of whether the lunar interior is rigid or not. Of course it should be recognized that any angular density variation such as this could not exist unless the moon was formed as a low temperature body; certainly this could not be so if it has been melted at any time in its history.

There is a new suggestion, recently put forward by Dr. Runcorn, namely, that the moon is a convecting moon. Runcorn has suggested that convection in two cells occurs with rising currents toward and away from the earth and sinking currents at the limb. The rising currents should be hotter than the sinking currents and hence there should be a difference in density. In this way the explanation of Urey, Elsasser and Rochester (1959) for the dynamical bulge of the moon is accounted for without postulating differences in chemical composition between regions along the axis toward the earth and the limb regions. This is a most intriguing suggestion.

Much evidence exists for convection within the mantle of the earth. Possibly the most convincing of such evidence is that from the magnetic anomalies in the Pacific Ocean off the coast of California as investigated by Vacquier, Raff and Warren[8] (1961). Great blocks of the earth's crust, of the order of a thousand kilometers in dimensions, have moved relative to each other by hundreds of kilometers. No explanation for this seems to be reasonable except that the crust floats on and moves with great currents in the mantle below. Much older evidence for the phenomenon exists as well.

There appears to be no definite evidence for such convection cells in the lunar surface features. It is true that there are easily recognized fissures in the central regions of the visible hemisphere which are not of a concentric character about the center of the lunar disk as might be expected on this hypothesis and, in fact, the present writer has maintained that this argues against lunar convection cells. If such cells exist, the lunar surface must be sufficiently rigid now and must have been so since the surface features were formed to resist effective deformation by the convection cells, such as is now occurring in the earth's surface. As argued previously, there is other evidence for a very rigid and thick outer region of the moon. This outer region is much thicker and more rigid than the outer regions of the earth.

Chandrasekhar (1953) has studied convection in spheres and has discussed convection in the earth. He found that the single cell convection should be stable in a sphere without a core and as the core formed and grew that two, then three etc., cell convections should become stable. The two cell convection should become stable when the radius of the core was slightly less than 0.2 of the radius of the sphere, i.e., just about that for the model discussed there. However, Chandrasekhar assumed uniform physical properties throughout, i.e., viscosity; hence his quantitative conclusions will not apply to a convecting moon. It is also very difficult to estimate what the proper boundary conditions for this object with a rigid outer shell are.

In spite of the difficulties and uncertainties present in this model, the history of the moon should be reconsidered in accordance with this explanation of the non-equilibrium lunar shape. It is still possible that the moon formed at low temperatures, that melted iron formed on the interior and formed a core during geologic times, and that now there exist two convection cells as discussed above. This model would account for the rigid outer shell without difficulty and no new problems would arise. On the other hand, we may consider the possibility of a completely melted moon initially since the cool moon hypothesis was proposed only in order to account for its shape in some physically reasonable way.

If the moon were originally melted, it is necessary to consider the source of energy which melted it. Some years ago, the writer suggested that Al^{26} may have been present during the time of accumulation of the solid bodies of the solar system. This idea was not pursued and developed partly because of the difficulties in understanding the structure of the moon. Runcorn's suggestion removes this difficulty. It is immediately obvious from the calculations on the thermal history discussed above that the moon would melt throughout if sufficient Al^{26} or other short-lived radioactive nuclides were present in sufficient amount to produce melting, since no effective loss of heat by conduction would occur, for example, within some ten million years which is about 13 half-lives of Al^{26}. After it decayed, the moon would solidify from the center outward and cooling would proceed along the lines discussed in this paper.* In this case a core would form if elemental iron and nickel were present in the primitive material or if oxides of these elements were reduced by appropriate reducing agents, e.g., graphite or carbonaceous compounds.

There are some incidental advantages to this model. Kozyrev has observed the spectrum of C_2 in an emanation originating near the center peak of Alphonsus.

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* Fish, Goles, and Anders (1961) have discussed the cooling of small bodies of the size of the asteroids after being melted by short-lived radioactive nuclides. In the case of such small bodies, cooling during the heating process will be important and a simple discussion of the course of events is not possible.

Carbon vapor as such can be produced only at very high temperatures far beyond those required to melt silicate rocks. Also, it reduces the oxidized iron of silicate rocks at moderate temperatures and is oxidized to carbon monoxide and dioxide. Carbon vapor is not a constituent of volcanic gases for very well understood physical-chemical reasons. However, if the surface rocks of the moon were once heated at high temperatures (1800°C or higher) in the presence of graphite, with the possibility for carbon monoxide to escape, carbides, such as CaC_2, would be formed. Such compounds react with water to form acetylene, C_2H_2, which would be converted to C_2 in the presence of the ultraviolet light of the sun. In this way Kozyrev's observations become understandable (Urey, 1961). But this process required that at some time near surface materials were heated to high temperatures with the escape of carbon monoxide. This might be supplied by radioactive heating or possibly by heating due to an adiabatically compressed gas sphere (Brainbridge, 1962).

The high temperatures model is appealing due to this possibility. Of course, water must have been stored below the surface and must have been kept there throughout the history of the moon. The amounts required would be comparatively small and possibly it is unnecessary and not possible to specify the mechanism of retention in detail.

It is interesting that some meteorites contain elemental silicon or iron silicide and these compounds are produced from siliceous material and graphite under conditions almost identical to those required for the production of calcium carbide. It is not unreasonable to suppose that the moon and meteorites may have been subjected to similar conditions.

Now we will return to the question of the origin of the moon. In connection with this subject the best basic answer is that nothing is known on how the moon originated, nor do we have any knowledge of how the solar system, as a whole, acquired its present configuration. However, evidence in regard to these questions does exist and it is quite possible to gain knowledge from a study of the available facts. Because of this it is interesting to review the problem of the origin and history of the moon, and in particular to look into the relationship it has with the earth and the solar system as a whole. Any information or conclusions reached on the basis of present information is subject to modification or change, as knowledge is gained from direct exploration of the moon in the future. One should always keep in mind that we do not know how the moon originated.

With this as an introduction, one of almost complete ignorance on the part of everyone, I will now tell you something about the origin of the moon.

All explanations for the origin of the moon are improbable. To mention one, consider the explanation as given by George Darwin; this, incidentally, has been disregarded by astronomers for quite some time. Lyttleton, from his studies, has reached the conclusion that there is no feasible way by which material could be gotten off the earth to form the moon.

Recently Dr. Donald Wise visited La Jolla and made a suggestion on the subject of lunar formation. He proposed that during the formation of the earth iron was present and became distributed throughout the body. As the body warmed up the iron sank to the center and, as a result, the angular velocity of the earth increased. This increase in angular velocity caused material to come off the earth which accumulated to form the moon.

This is an interesting idea but is contrary to the findings of Lyttleton. His finding is that if the angular velocity is increased, for a liquid body, the body flattens from an oblate spheroid to a disk, with no material being spun off its edge. Hence the suggestion of Dr. Wise does not seem to suffice.

Another idea for formation is that the earth and moon accumulated in the same gas cloud. For this case it is difficult to understand why they have different chemical compositions.

The author has suggested another method for the accumulation of the moon. Suppose at some time there was a solar nebula; how it got there will not be discussed. We merely assume it was there at one time. Now the question is asked, "What instabilities might be expected?" Conveniently a formula for such has been derived, by Dr. Schatzman, Dr. Chandrasekhar, and others. In the present case one would expect that the vast nebular disk of material would break up by gravitational instabilities. Next we ask, how large the objects formed would be. This all depends on the density and temperature of the material. The formula for the masses of objects formed is

$$m = \left(\frac{RT \gamma \pi}{\mu G} \right)^{3/2} \frac{1}{\rho^{1/2}}$$

where γ is the ratio of heat capacities of the gas, μ is the molecular weight, G the gravitational constant and ρ a density of the gas. Regarding the temperature, it was assumed to be rather low; as for density, let it be assumed to be 2×10^{-6} divided by the third power of the distance from the sun (in astronomical units). This is the density at which a gaseous object would stay together under its own gravitational attraction in the gravitational field of the sun. For temperature the author has used a temperature low enough to keep the nebula at a relative small size. Since the temperature enters into the formula raised to the three halves power, then as the temperature increases the mass of the nebula also increases and its size does likewise. A result of the calculation, with these assumptions, is that one obtains lunar sized objects in the neighborhood of the terrestrial planets, or objects slightly less than lunar size,i.e. of lunar mass plus the proper proportion of gases. As one moves outward in the system the object sizes increase until they are about ten times as large as the moon at a distance equivalent to the position of Neptune. This suggests a mass of gas and dust of about the right size to produce lunar objects. Of course the material should collect inside the gaseous mass to form a silicate object near its middle. Thus we now have a way for material of cosmic composition to accumulate.

Now, if a moon was captured by the earth, then this is a most improbable capture. It should have come into position along a hyperbolic orbit, had collisions to dissipate some of its energy and then not quite escape. This all is highly improbable; however, if a large number of moons existed at one time in the solar system then the improbability of a capture is not so remote.

If the moon was captured by some such scheme, then it is possible that there could have been a large number of moons in the solar system at one time. I have calculated how long it would take such objects to collide with one another after losing this gas mass which surrounds them. On the basis of equal partition of energy these bodies would be moving in and out and up and down relative to the sun, and they would collide. It is supposed that they would interfere with one another by various processes. Part of the silicate material associated with such bodies would be blown into space along with the gases, leaving behind a residue of more dense materials such as iron and nickel; these residues would then accumulate to form the terrestrial planets. The moon is an object which by accident escaped destruction and was captured by one of the terrestrial planets.

Of course this whole scheme of origin is also improbable and as stated previously, all proposed schemes of origin are likewise improbable. If one does not care for any one or more of these suggestions, he should recognize that they are all improbable and equally subject to supposition and fallacy.

Very extensive areas of disagreement in regard to the structure and history of the moon exist in the minds of many thoughtful students of the subject. Only more detailed evidence can clear up these areas and lead to general agreement.

Specifically, the lunar exploration program should give us further data in regard to the distribution of mass within the body of the moon, the number and intensities of seismic events, and the chemical composition of the surface regions. The concentrations of potassium, uranium and thorium can be determined by radioactive detection devices flown above the surface. If these concentrations are low as they are in the meteorites, then extensive differentiation by melting processes as is characteristic of the earth has not occurred. Such investigations should be interpreted with caution because the surface may be somewhat different from regions immediately below. As Shoemaker (1962) has shown, many blocks of material have been thrown about over the surface by the great collisions, and if men visit the moon and are sufficiently experienced in hard rock geology, they will be able to pick up many varieties of such rocks, if indeed these varieties exist; much valuable information in regard to the surface composition will be secured in this way. We need to have such scientific people among the astronauts who first land on the moon if the Apollo project is to be of maximum scientific value.

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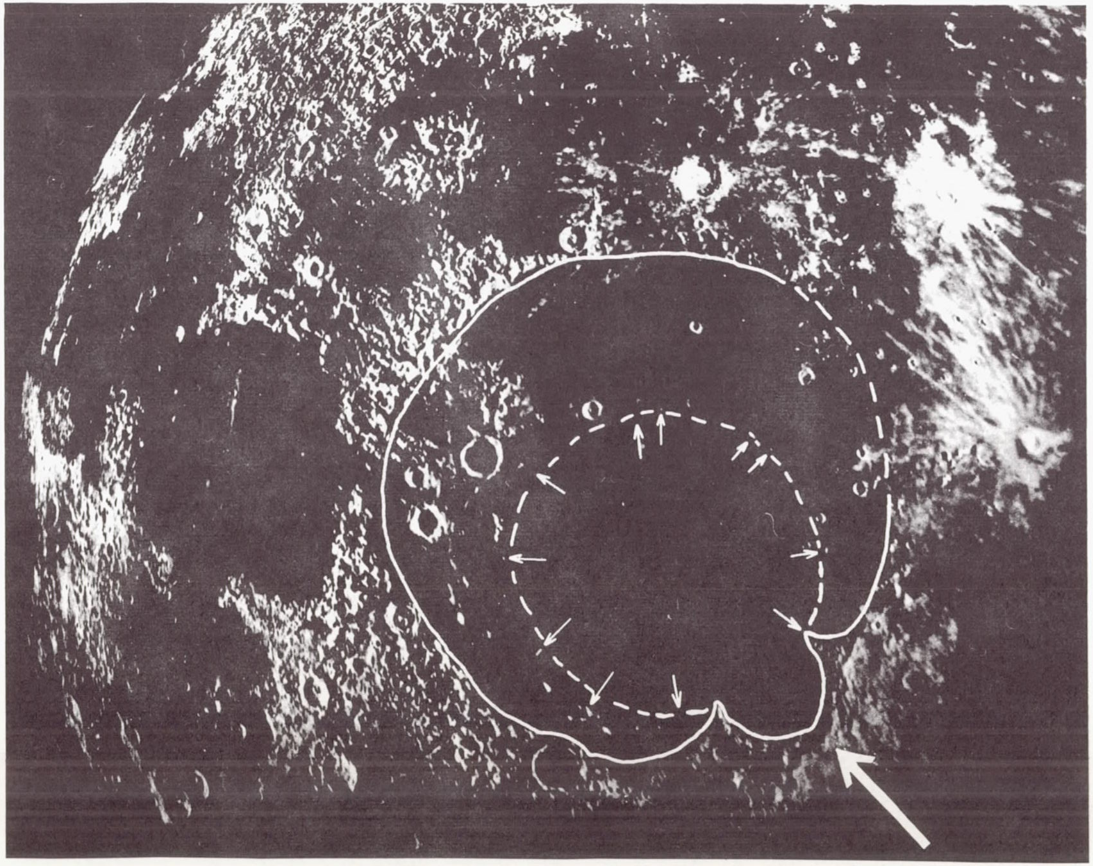

Figure 1
Mare Imbrium, with foreshortening eliminated. Mare Imbrium is outlined by the solid curve. Sinus Iridum is the bay at the lower right. The arrows indicate mountainous masses just outside the collision area. Three of these arrows point to masses not visible in this picture but easily seen on others. (University of Chicago photograph.)

TABLE I

Present Temperature (^{o}C) of a Cold Moon at $0^{o}C$ 4.5 AE Ago

		K = 0.005					K = 0.010				
$\frac{r}{a}$		0.0	0.5	0.8	0.9	0.95	0.0	0.5	0.8	0.9	0.95
Initial Melting Temperature		1600	1475	1280	1195	1145	1600	1475	1280	1195	1145
Complete Melting Temperature		1800	1750	1672	1638	1621	1800	1750	1672	1638	1621
Temp. rise in 4.5 AE due to individual nuclide	K^{40}	888	874	640	365	190	888	806	449	238	119
	Th^{232}	132	131	101	63	36	132	123	79	45	24
	U^{235}	96	88	61	33	17	96	83	42	21	10
	U^{238}	174	170	133	85	47	174	160	99	57	29
	Total	1290	1263	945	546	290	1290	1172	669	361	182

Present Temperatures (^{o}C) of a Moon Solidified 4.5 AE Ago

	K = 0.005					K = 0.010				
No radioactive elements in the interior										
Initial Temperature	2000	1775	1424	1271	1187	2000	1775	1424	1271	1187
Residual Temperature T_2	1098	1052	608	311	153	1071	872	392	189	91
Residual Temperature T_3	772	549	222	108	52	645	437	169	80	38
Total Temperature	1870	1601	830	419	205	1716	1309	561	269	129
With 1/2 of radioactive elements in the interior										
	2515	2232	1302	692	350	2361	1895	895	449	220

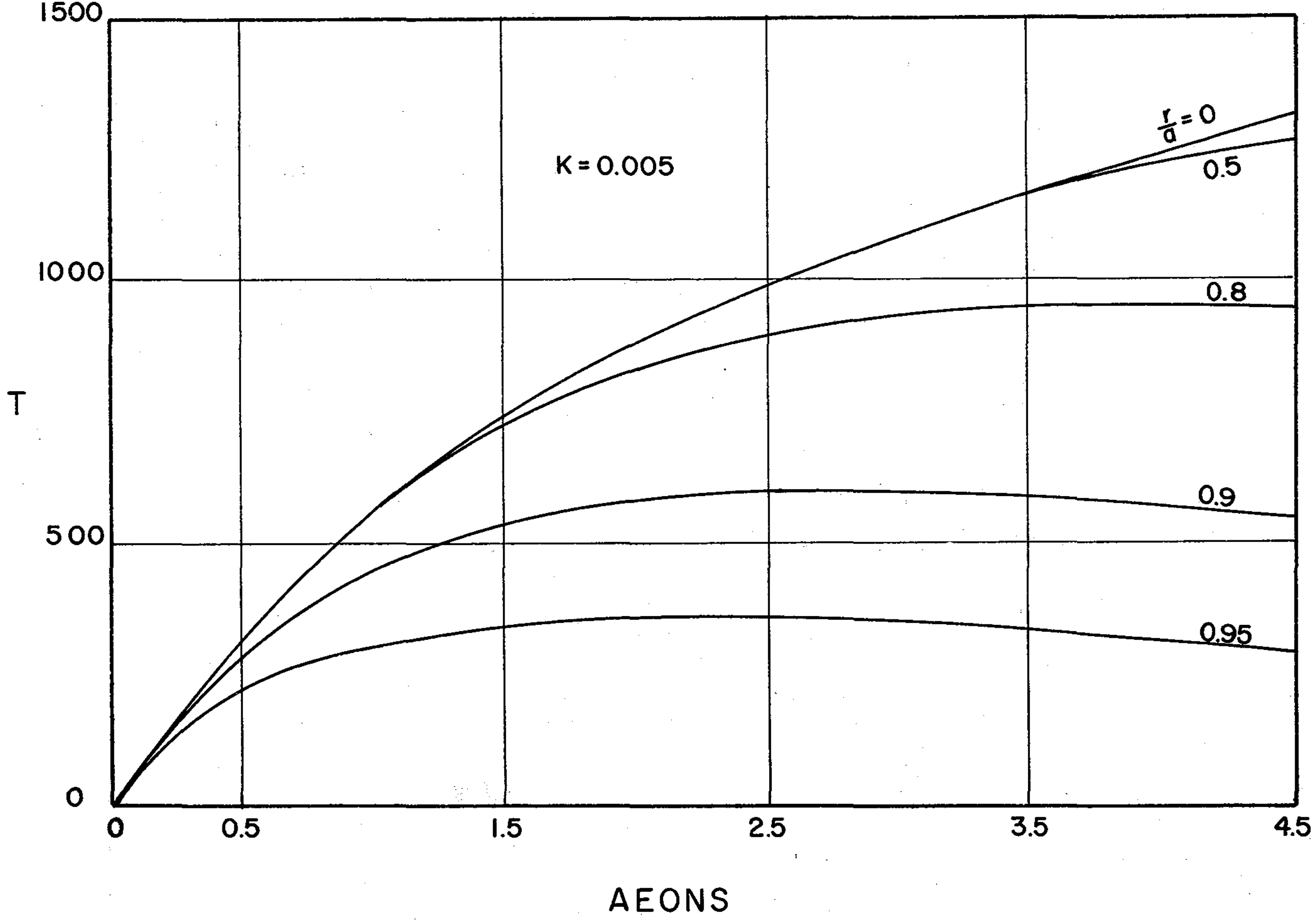

Figure 2

Temperature variation within the Moon, with time, for an initial temperature of 0°C, and for K = 0.005.

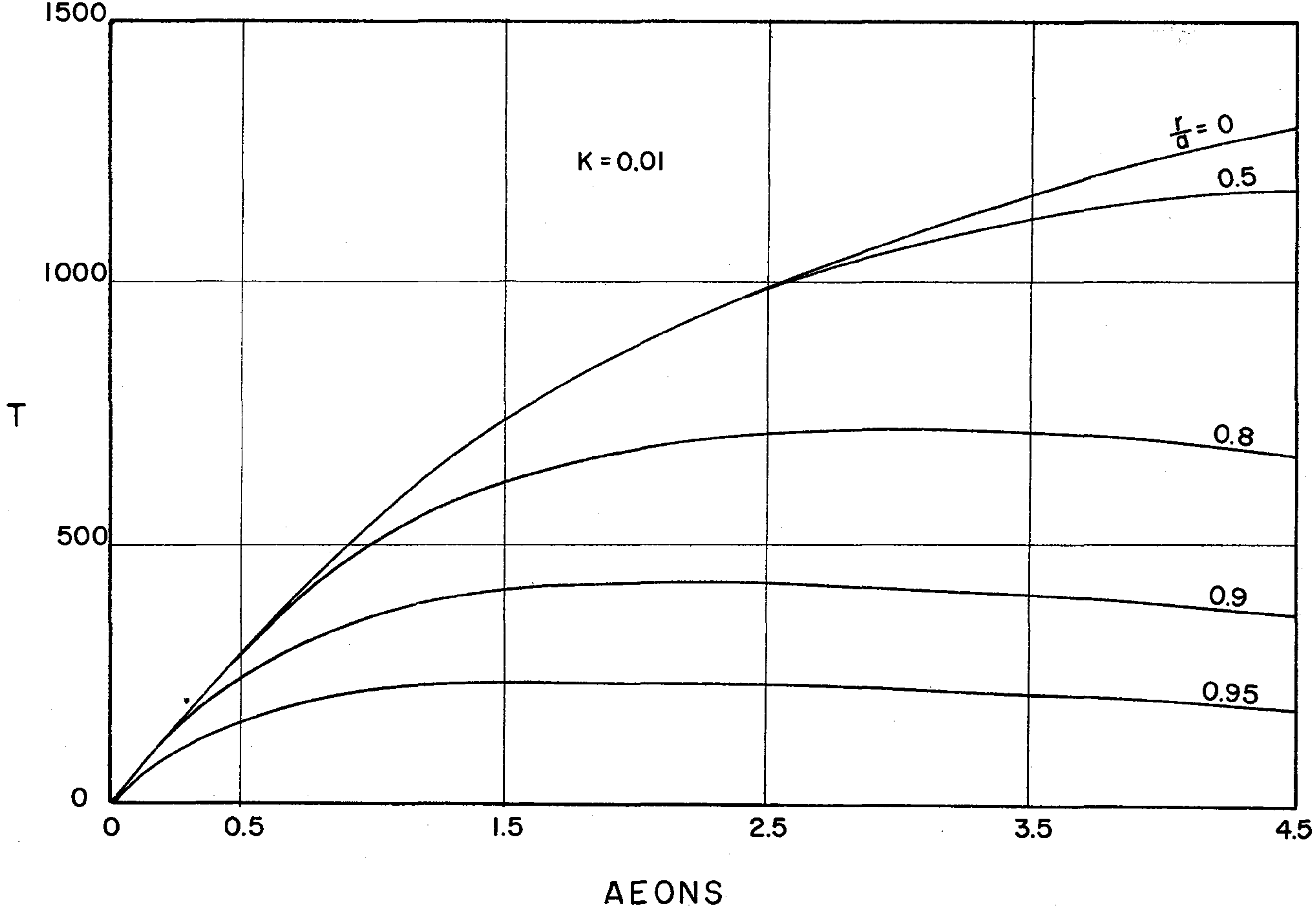

Figure 3
Temperature variation within the Moon, with time, for an initial temperature of 0°C, and for K = 0.01.

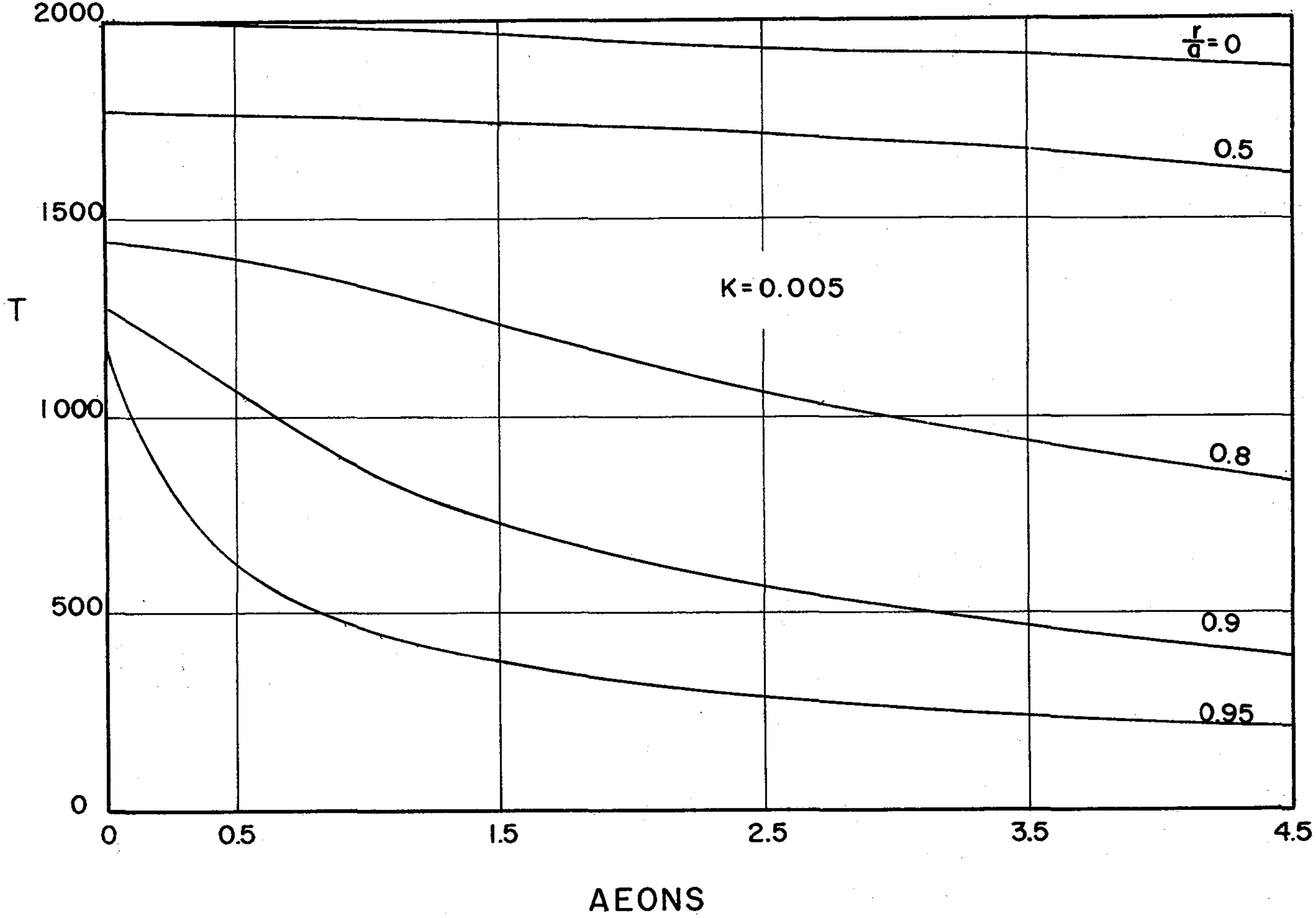

Figure 4
Temperature variation within the moon, as a function of time, for a moon completely melted at formation.

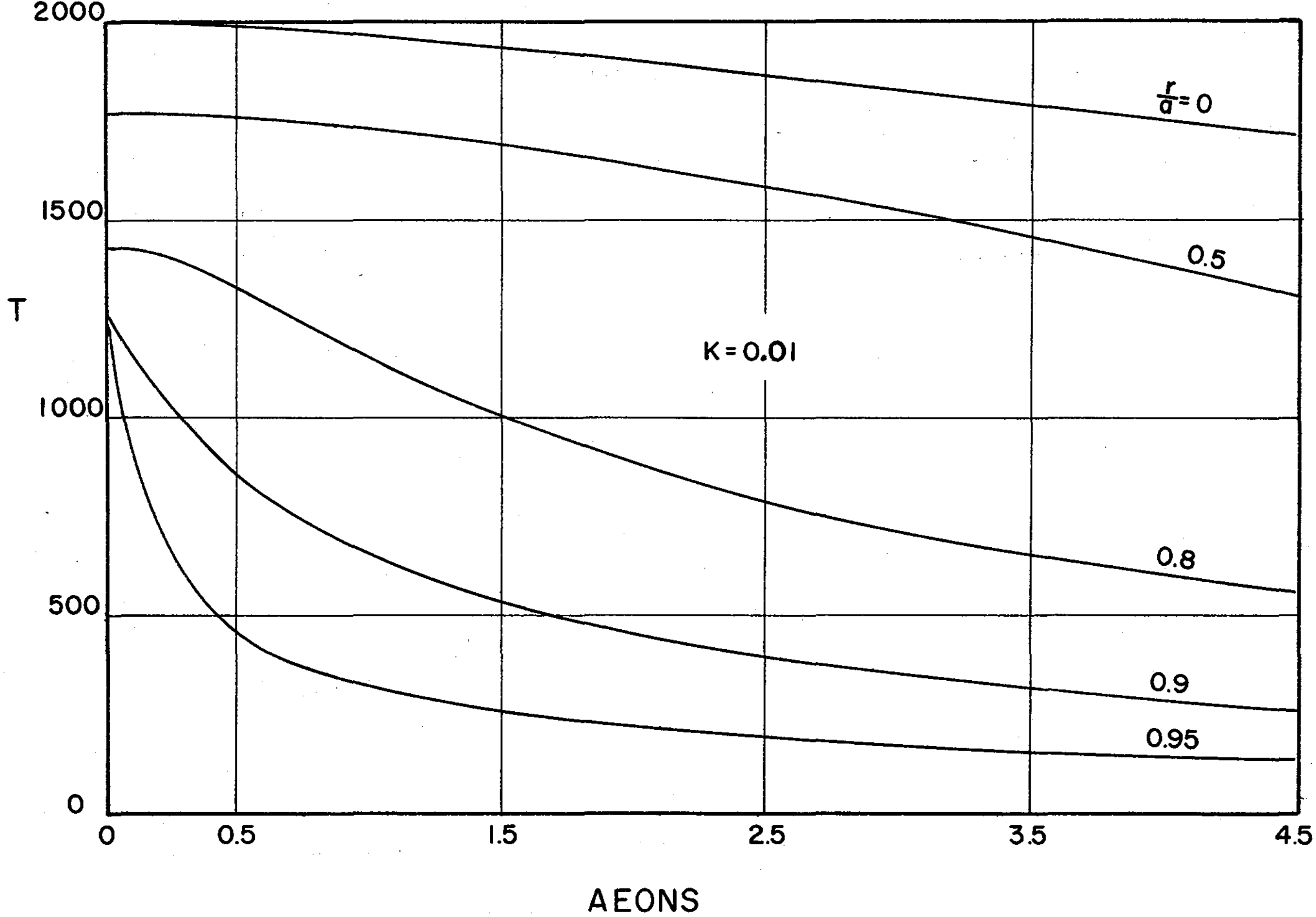

Figure 5
Same as Figure 4 except for the thermal diffusivity, K.

STUDIES OF THE LUNAR SURFACE I

ATMOSPHERE NEAR THE MOON

BY

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Atmosphere Near the Moon*

by

S. F. Singer

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Any reasonably anticipated steady evolution of gas from the moon is insufficient for the formation of an atmosphere. Contrary to expectations from the classical theory of an exosphere, we find that even the heaviest gases, for example the noble gases krypton and xenon, cannot be retained by the moon. Because of the ionizing effects of solar ultraviolet radiation, electric forces now become more important than gravitational forces. The lifetime of an atmosphere containing all the krypton and xenon believed to have evolved since the origin of the moon is of the order of 800 and 50 years respectively. After the atmosphere has thinned to less than a mean-free path (i.e., an exosphere), yet another mechanism of escape of heavy gases exists. It seems likely that the moon as a whole is positively charged to a potential of about +20 volts, owing to the great intensity of solar ultraviolet radiation. Hence, in the vicinity of the moon there will exist a strong electrostatic field. Whenever a krypton or xenon atom is ionized while it is in flight within the screening length near the moon, the ion will be expelled by the electrostatic field. The lifetime turns out to be ~ 1000 years and is somewhat increased by the effect of solar corpuscular

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[2] Director, National Weather Satellite Center

streams. Hence we conclude that there is not much likelihood that light or heavy gases evolving from the moon will be retained by the moon. The only gas in the vicinity of the moon can come from interplanetary space itself.

Accreted, accommodated and reemitted solar coronal gas yields an average permament density of about 80 hydrogen atoms per cm^3; this density may rise to about 10^5 per cm^3 following solar eruptions.

I. Summary of Observations

One of the intriguing problems in current space research activities relates to the presence of a lunar atmosphere. Some upper limits have been obtained by various observations; for example, in France, Dollfus (1) has made polarimetric observations which lead to a density of 10^{-9} of the density of the terrestrial atmosphere. Detailed investigations by J. N. Lipskij (2) gave a density of 10^{-4} of the terrestrial; however, his analysis has been questioned effectively by E. J. Öpik (3) who concludes that the absence of observable optical effects near the moon's cusps give an upper limit of one millionth (and probably less) of the terrestrial.

Quite a different set of observations has been obtained by Elsmore (4); it is concerned with refraction of a radio star as it is obscured by the lunar disk. From such observations a pressure of the order of 10^{-13} atmospheres can be deduced as an upper limit.

II. Theory as Tool for Exploration

In the absence of direct measurements our information on the lunar atmosphere must be obtained by other methods. Theory can be very helpful in anticipating experiments, as well as for interpreting any eventual discoveries with lunar probes. The problem resolves down to finding possible sources for an atmosphere, and comparing the strength to these sources with mechanisms of dissipation of an atmosphere, chiefly escape into interplanetary space. If the sources are weaker than the rate of escape, then an atmosphere cannot build up. We will therefore examine the cases of internal sources (from the moon's interior) and external sources.

III. Atmosphere from Internal Sources

Let us consider first the escape problem of a lunar atmosphere. The outstanding difference between the earth and the moon is the small mass of the moon and therefore its low value of gravitational field. This is reflected in the fact that atoms will escape quite rapidly from the surface of the moon. To examine the rate of escape, one may use an approximate formula derived by Spitzer (5) which gives the time t_1 during which the particle density would decrease by a factor e (for an isothermal atmosphere)

$$t_1 = (2 \sqrt{6\ c^3/9\ g\ v_\infty^2})\ \exp\ (3\ v_\infty^2/2\ c^2) \qquad (1)$$

Here v_∞ is the velocity of escape, g the acceleration of gravity, and $c^2 = 3$ kT/m. If we assume an average lunar surface temperature T of 400°K, the lifetime of a hydrogen atom would by 1600 sec, and of an oxygen atom 1.4 years.

This result shows how sensitive the escape time is to the atomic mass. For example, a straightforward application of Eq. (1) to heavy gases such as xenon and krypton gives lifetimes of the order of 10^{41} and 10^{24} years, respectfully, i.e. times that are large compared to the age of the moon. (See Tables

1, 2 and 3.) Hence, it might be surmised that xenon and krypton evolving from the moon would in fact accumulate and remain in the lunar atmosphere.

Table 1. *Data on the Moon*

Radius km	Area, cm^2	Mass, g	Surface Gravity cm/sec^2	Escape velocity v_∞, km/sec	Escape Energy, ev	
					Kr	Xe
1738	3.8×10^{17}	7.35×10^{25}	162	2.38	2.4	3.7

Table 2. Data on Krypton and Xenon

	Mean Atomic Weight	Mass, g	Gas Kinetic Cross Section, cm^2	Escape Level Load, atoms/cm^2 column	Ionization Potential	
					ev	σ_n, cm^2
Kr	83.8	1.39×10^{-22}	7.8×10^{-15}	1.3×10^{14}	14.0	1.8×10^{-15}
Xe	131.3	2.18×10^{-22}	9.4×10^{-15}	1.1×10^{14}	12.1	2×10^{-15}

Table 3. Escape Times of Neutral Atoms from a Thick Atmosphere

	$T = 1500^\circ K$			$T = 400^\circ K$	
	$v_\infty{}^2/c^2$	t_1, sec	t_1, years	$v_\infty{}^2/c^2$	t_1, years
Kr	12.6	5.1×10^{9}	170	47.2	10^{24}
Xe	19.8	1.3×10^{14}	4×10^{6}	74.3	10^{41}

Edwards and Borst (6) have worked out in some detail the possible sources of xenon and krypton evolving from the moon. They have considered such mechanisms as spontaneous fission of uranium 238, fission from thermal neutrons produced in the lunar rock by α particles, possible effects from cosmic-ray bombardment, and also the decay of iodine 129. The first and the last of these mechanisms are particularly important for the production of krypton and xenon. Finally, of course, the possibility cannot be excluded that appreciable amounts of xenon and krypton were contained in some primeval gas of the moon. Edwards and Borst conclude that the amount of gas evolved from a 1-km thick layer of rocks might be of the order of 2 X 10^{16} krypton atoms and 1.8 X 10^{15} xenon atoms per cm^2 column, more than sufficient to produce a pressure of the order 10^{-13} atmosphere. (This amount is considered by Elsmore (4) to be in accord with radio observations of the occultation of two radio sources.)

We may accept the production rates calculated by Edwards and Borst, but we must modify the conventional calculation for rate of escape.

First we shall consider the presence of a thick atmosphere and show that it reduces very quickly to an exosphere. Next we shall show that even an exosphere cannot be maintained but becomes thinner and thinner.

The treatment of the escape of gases follows the recent paper of Öpik and Singer (7). If we consider first of all a thick atmosphere of the moon, then the escape layer of this atmosphere would be separated from the surface of the moon and might have a temperature of some 1500°K, similar to the temperature of the base of the exosphere of the earth. We can then show that the escape time for krypton is of the order of 170 years and for xenon 4 X 10^6 years; in other words very short compared to the lifetime of the moon. Now as the atmosphere thins down sufficiently, this escape level is still above the surface of the moon but the atmosphere will be sufficiently cooled by contact with the lunar surface so that the temperature is low throughout. Then the atoms will escape with a temperature corresponding to about

400°K. In that case the lifetime would be 10^{24} years and 10^{41} years for krypton and xenon, respectively, and it would seem therefore that such an atmosphere could survive. However, now we must consider the effect of solar radiation. A detailed examination of the photoionization produced by photons in the far ultraviolet shows that the rate of escape of the krypton and xenon ions would be of the order of 10^6 per cm^2 per sec. Thus in the lifetime of the moon, the total loss would have been 1.5×10^{23} atoms per cm^2 column, or more than the probable content of the heavy gases in the entire lunar mass.

(Contrary to certain treatments, for example, by Herring and Licht (8), it can be shown that the thermal corpuscular stream from the solar corona cannot efficiently eject atoms from the lunar atmosphere. It will only cause additional ionization by charge exchange or by impact ionization, but not necessarily increase the rate of escape.)

We now finally consider the situation when the atmosphere of the moon is a true exosphere, i.e. when the atoms do not make collisions with each other but describe free orbits under the influence of gravitational and other forces. We have shown (7) that the degree of ionization must be determined by the ratio of flight time for a particle traveling in a ballistic orbit to the photoionization lifetime. From this we derive a degree of ionization of 1 in 10^5. Now these ions will be affected by any electrical potential of the moon. We have roughly calculated that the lunar potential should be of the order of 20 - 25 volts (postive). Therefore any (positive) krypton or xenon ion that happens to be formed within the region where the potential is not yet screened will be expelled. The screening distance has been calculated (7) and is of the order of 20 meters. This length may be compared with a characteristic scale height for krypton and xenon which is of the order of 20 km. The lifetime of the krypton and xenon ions can now be calculated and it turns out to be about 1000 years; hence, the exosphere cannot survive.

We have considered (7), in addition, the effect of a solar corpuscular stream on such a krypton and xenon exosphere. We find that the solar corpuscular stream causes additional ionization of the xenon and krypton atoms in their ballistic orbits, and would increase the degree of ionization by about a factor 2.5. This in turn increases the electron density in the vicinity of the moon and reduces the screening length to about 4 meters. Hence the lifetime of the krypton-xenon exosphere is increased to about 3000 years.

Our final conclusion is that an independent lunar atmosphere, i.e. one that evolves from the moon, cannot exist, even if it consists of heavy gases such as krypton and xenon. Hence a lunar gaseous envelope is entirely determined by the surrounding interplanetary medium.

IV. External Sources of a Lunar Atmosphere

The presence of a lunar atmosphere as a gravitational condensation of interplanetary gas have been discussed by Firsoff (9) and by Brandt (10). The two authors are in disagreement on many points and a detailed account of their controversy has been published. However, both treatments are fundamentally wrong on theoretical grounds. Consider the idealized case in which magnetic and electric fields are considered absent; both Firsoff and Brandt give the distribution of density of a gravitationally accreted atmosphere around a center of force by a barometric formula of the type

$$N(r)/N_\infty = \exp(Y) \qquad (2)$$

where $Y = GMm/(rkT)$. However, the correct distribution for a pure exosphere (where the particles do not collide with each other, i.e. where the mean length of path exceeds the radius of the planet) is calculated as follows:

$$N(r)/N_\infty = 2\pi^{-1/2}\left[Y^{-1/2} + (\exp Y) \cdot \int_Y^\infty \exp(-x^2)\,dx\right] \qquad (3)$$

When the temperature T is high (and Y is small), the difference between Eq. (2) and our Eq. (3) is not significant. However, for large values of Y, i.e. for low temperatures or for larger values of the molecular mass, Eq. (2) yields the spurious high concentration of atmospheric density in the vicinity of the moon (9).

Actually, the contribution to the density by the gravitational accreted incoming component of interplanetary gas may be at times negligible in comparison to the outgoing component. We may expect that the fast particles, after striking the surface of the moon, will be accommodated. They will lose their charge, if any, and when leaving the surface as neutral atoms or molecules will have a much lower temperature, greater molecular weight and hence a larger value of Y than the incoming gas. (In this respect at least, Brandt's more recent criticism of Firsoff's work is unjustified.) As has been pointed out by Gold (11), it is the component accommodated to the lunar surface temperature which contributes chiefly to the atmospheric density near the surface.

It is tempting to make an estimate of the atmospheric density near the moon based on our best present knowledge of the properties of the interplanetary gas (12), (13). We may view it as the exosphere of the solar corona and adopt values $N_\infty \sim 10/\text{cm}^3$ for the density (14) and $u \sim 40$ km/sec for the outward velocity of the solar protons[1]. These are approximate quiescent values and probably increase by a couple orders of magnitude during solar eruptions.

The accretion rate is therefore given approximately by

$$N_\infty \cdot u\pi R_M^2 (1 + v_\infty^2/u^2) \sim 4 \times 10^7 \pi R_M^2 \quad .$$

Because of the rough nature of the lunar surface we may consider the accommodation coefficient to be of the order of unity. In any case, any inaccuracy in this coefficient is far outweighed by the uncertainty in N_∞.

The escape flux from the sunlit hemisphere is given by (13)

$$2\pi R_M^2 N_o(kT/2\pi m)^{1/2} (1 + Y_o) \exp(-Y_o)$$

where $Y_o = GMm/(R_m kT)$ and turns out to be 0.85. The value of N_o found by equating these two rates turns out to be 80 H-atoms/cm^3. The sealevel density at the earth, for comparison, is 2.69×10^{19} molecules per cm^3.

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[1] The reason for the low velocity of 40 km/sec stems from the fact that the base of the solar exosphere is located at a fairly high level where the temperature of the solar corona may have dropped considerably below the value of one to two million degrees in the inner corona. Actually, the problem is complicated by the fact that the collision cross section of the ions depends on their velocity so that different parts of the Maxwellian distribution have essentially different bases of an exosphere. The problem has been considered in greater detail elsewhere.

V. Conclusion

We have calculated the average permanent gas density in the vicinity of the lunar surface as about 80 hydrogen atoms per cm^3. During eruptions of solar gas the density may rise to 4—40 X 10^4 per cm^3 for a few hours, assuming a solar stream of density 10^2—10^3 per cm^3 and velocity 2000 km/sec.

Of course, any release of gas, whether volcanic or by meteoritic impact, or perhaps manmade, will temporarily enhance the atmospheric density near the moon.

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TOPOLOGY OF THE LUNAR SURFACE

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ABSTRACT

Since the shape and topology of the moon depend upon the physical state of its interior, one prime question to be answered is whether or not the interior is plastic, elastic or viscoelastic. One part of this presentation is given to a discussion of the possible internal processes, the structure of the interior and how these will then influence the shape and topology of the surface. Much of the present evidence points to viscoelastic model for the lunar interior.

The determination of topographical features, the accuracy of measurements and methods of obtaining these are discussed. The influence of factors such as shape, optics, and position - as related to the accuracy of photographic interpretation - is presented. A description is given of the work currently being carried out by various groups to produce lunar charts; the need for additional photographic stations and very high altitude photography is mentioned.

TOPOLOGY OF THE LUNAR SURFACE

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MEASUREMENTS OF LUNAR TEMPERATURE VARIATIONS DURING AN ECLIPSE AND THROUGHOUT A LUNATION

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MEASUREMENTS OF LUNAR TEMPERATURE VARIATIONS DURING AN ECLIPSE AND THROUGHOUT A LUNATION

by

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Most of what we know about the moon comes from observations of its radiation either by reflection or by a process of absorption and re-radiation. Lunar thermal radiation falls into the latter classification and can be detected in the far infrared and at certain radio wave lengths.

The first lunar infrared temperature measurements were made by Lord Rosse in 1868.[1] The method consisted of measuring the total radiation from the lunar disk with a thermopile, using a glass plate to separate planetary heat. This work was followed by S. P. Langley[2] and others.[3] The surface temperature was estimated to be about that of boiling water. Some, however, held that the surface temperature was very low.

The first eclipse temperature measurements were made in 1884.[4] Pettit and Nicholson[5] in 1927 made the first reliable measurements of surface temperatures both on the illuminated moon and during a total lunar eclipse. Their results have been reviewed in several books.[6,7]

Infrared Measurements: During a Lunation

The variation of temperature for a point near the center of the lunar disk is shown in Fig. 1.[*] This curve is for a $(k\rho c)^{-1/2} = 435$ and a subsolar temperature of 389°K.[8] The open circles are from Sinton's measurements and the other points are from Shorthill and Saari.[9] These latter points have been adjusted to the 389°K subsolar

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[*] All Figures will be at the end of text. References will appear as superscripts.

point temperature and for the roughness factor. These points were measured at full moon, however; they were shifted an amount in phase angle determined by the angle of illumination and position relative to the center of the disk at the time their temperatures were measured. Sinton found that the temperature of the subsolar point at full moon was 389°K for a mean spherically emitting surface at 1.000 AU.[10] The temperature of the anitsolar point was measured to be 122°K ± 5°[5] and 120°K ± 3°[8]

Distribution of planetary heat across the disk of the full moon is shown in Fig. 2.[5] Theory would predict an energy variation according to

$$E = A \cos \theta$$

where θ = angular distance from the subsolar point and A the radius of the moon for this plot. The observed radiation, however, follows more closely

$$E = A \cos^{2/3} \theta$$

which has been accounted for by the roughness of the lunar surface. As one moves away from the center of the lunar disk the actual surface temperature decreases as $\cos^{1/4} \theta$, but this variation as seen from the earth is altered by the fact that only illuminated surfaces are seen at full moon. The valleys, slopes and peaks are illuminated near the center of the disk while the valleys on the limb (Fig. 3) are not completely illuminated and therefore a higher temperature is observed. $T_{observed} = T_{ssp} \cos^{1/6}\theta$ applies to the observed full moon temperatures.

Some published results of Sinton [10] are shown in Fig. 4. A Golay cell was used to make these series of temperature surveys. The moon was scanned like a TV picture at the rate of three minutes per single scan. The resolution was 25" of arc (47 km at center of disk). Fig. 5 [10] shows an overlay for about a ten day old moon. The subsolar point is indicated by (+) sign. At this phase the observed subsolar point temperature is $373^\circ < T_{ssp} < 383^\circ K$. It is noted that Mare Crisium seems to appear as a warmer region. Sinton has related the thermal contours to general features. The south-polar regions appear somewhat cooler than the north-polar regions. North is at the top as in Fig. 6 [10] for about a fourteen day old moon. The right ascension and declination axes are shown by the (^{N}✗) sign. In Fig. 7 [10] the isothermal contours over the seventeen day old moon are seen. The (✝) sign is used to indicate the origin of the selenographic coordinate system. Horizontal structure or abnormal excursions parallel to the right ascension motion are seen in all these contours. (This noise may be caused by sudden shifts in the sky background radiation or scanning errors). Fig. 8 [10] shows another isothermal contour over the entire disk of the moon with 25" of arc resolution indicated by the circle labled 3mm.

The decrease of the observed subsolar point temperature at large phase angles is a result of the surface roughness. Valleys should be warmer since they receive radiation from more than a hemisphere. A lower temperature is observed because these valleys are not seen at large lunar phase angles. Fig. 9 illustrates this effect of roughness.

Infrared Measurements: During an Eclipse

In 1927 Pettit and Nicholson measured the temperature variation in the 10-14 micron band at a point near the southern limb. Pettit in 1939 measured a point near the center of the lunar disk. Their results are shown in Fig. 10. Strong and Sinton [11] observed an eclipse in 1953. They found that the limb areas, cooler to start with, lost a larger fraction of their heat.. They explained this in terms of surface roughness. The depressions on the limb areas are not observed, they cool less rapidly because they cannot radiate to a full hemisphere. The cooling of the limb peaks is observed and since they can radiate more nearly to a hemisphere an apparent larger fraction of heat loss is noted.

Radio Measurements: During a Lunation

It is not intended to give a complete review of radio thermal emission but only an indication of some of the results. See (reference 6) and (reference 8) for a more complete review. When lunar thermal radiation is measured at relatively short wave lengths two features are noticed. First, the average temperature is lower than that measured in the 8 to 14 micron band and second, there is a phase lag relative to the optical phase of the moon. Gibson's measurements were made at the 8.6mm wave length [12] during a lunation and an eclipse. The results are shown in Fig. 11 and Fig. 12. This radiation comes from below the lunar surface, that is, the lunar surface is somewhat transparent at these wave lengths. As the probing wave length increases the depth of the layers contributing to this radiation therefore increases. The problem is further complicated by the fact that all the upper layers contribute to the observed temperatures as well. Another very interesting set of contour maps was made by R. J. Coates.[13.] They are included here as an example of how radio temperature measurements may be correlated with visible features. These were made at 4.3mm with 6!7 of arc resolution (~ 1/5 lunar diameter). The brightness temperature for the center of the disk (Fig. 13) as given by the author is 182°K. This map was made near first quarter. The contours over Tranquillitatis, Foecunditatis and Nectaris follow the general shape of this region. These regions seem to heat up more rapidly than the surrounding mountain areas. It is also noticed that Procellarum is slightly brighter than the surrounding mountains. At 126° lunar phase (Fig. 14) the brightness temperature of the center of the disk is now 243°K. Maria Tranquillitatis, Foecunditatis and Nectaris are now more uniformly bright. Mare Imbrium, however, remained cooler - just opposite the effect of the other maria with the same illumination. At third quarter (Fig. 15) the brightness temperature of the center of the disk is 254°K. Mare Tranquillitatis is now cooler than the surrounding mountains. Maria Nubium and Humorum follow the brightness contours. Imbrium is now cooler than its surroundings. Coates concludes by stating that at 4.3mm the maria heat up more rapidly and cool off more rapidly than the mountain regions. Imbrium is an exception and remains cooler throughout the lunar cycle. Since the characteristics of the moon at millimeter wave lengths are not the same for all regions it would be useful to repeat certain longer wave length measurements with higher resolution in order to study these characteristics.

Measurements have been made at radio wave lengths from 0.15cm to 168cm.[14] Variations with lunar phase have been detected at the shorter wave lengths with a lag as much as 50° lunar phase angle. At the longer wave lengths little or no variation is found. Also at the longer wave lengths only the average disk temperature is measured and is averaged over a certain depth as well.

Radio Measurements: During an Eclipse

Eclipse measurements at 8.6mm were shown in Fig. 12. At this wave length no variation was observed. Sinton[15] did report a variation at 1.5mm wave length with about an hour phase lag. Later measurements did not show any variation; however, conditions were not the most favorable for observations.

Recent Measurements: Infrared

An eclipse of the moon is exciting to observe both visually and with a radiometer - if you choose the proper wave length! Fig. 16 is a reproduction of a color negative taken during totality of the lunar eclipse of September 5, 1960. In an attempt to detect variations from the previously described cooling characteristics Shorthill, Borough and Conley[16] measured the infrared lunar thermal radiation with improved spatial resolution. With a Barnes' thermistor bolometer mounted at the Newtonian focus (Fig. 17) the lunar surface was scanned with 7" of arc resolution (12 km at center of disk). The KRS-5 window limited the spectral bandpass to the 0.5-40 micron band. A germanium filter was used to further limit the bandpass to the infrared region. The rms noise level corresponded to a radiation flux of 3×10^{-9} watts for a 10 cps bandwidth. The radiation was mechanical chopped at 80 cps and synchronously rectified. The scanning cycle is shown in Fig. 18 which was designed to include many different types of regions such as mountains, bright and rayed areas and features reported to have shown activity of some kind. Fig. 19 shows an example of one scan path. A filter wheel with six positions rotated in front of the KRS-5 window. Three positions were open filter (no filter), two positions were microscope cover glass and one position was germanium filter. Note that in several areas the reflected light, passed by the microscope cover glass decreases, while the infrared deflection, passed by the germanium filter, remains at the same level. Clouds prevented the carrying out of the complete scanning program during the penumbral phase. During the umbral phase however, the seeing was good and the sky background produced a constant deflection. The lunar surface was now somewhat below 200°K. It was discovered that Aristarchus and Copernicus produced deflections greater than the general lunar background. No other deflections were noted in the area covered by the scanning cycle. The crater Alphonsus was scanned several times and no deflections above its' environs were observed.

When the crater Tycho was scanned a deflection above the local background by at least a factor of two was observed. The actual chart recording is shown in Fig. 20. The scan rate for Tycho was 30" of arc per minute of time. On each trace a sharp rise occurred near the south rim, gradually decreasing toward the north rim, however, due to collimation uncertainty the rise could not be definitely ascribed to the rim of the crater. Tycho is about 40° above its environs.

The three lunar features found to exhibit this enhanced radiation during an eclipse were all rayed craters. The most immediate interpretation of these observations was that the rayed craters are covered by a thinner dust layer. Other interpretations are open, such as vulcanism, radioactivity and emissivity variations. Further, the data do not disagree with the assumption that these craters are among the younger features on the moon.

The eclipse of September 5, 1960 provided an opportunity to make another series of measurements to verify this anomalous cooling and map the extent of this effect both spatially and in time. The instrumentation was the same except the signal was converted to FM (1.2 ± 0.5 kc) and recorded on magnetic tape. Since this was recorded with a short time constant the signal could be demodulated, smoothed with different filters and expanded or compressed in time at the laboratory to obtain optimum reduction of the data.

With the telescope drive near lunar rate the slow setting motion was used to drive east and west at about 6" of arc per second of time over the lunar surface. Combined with the normal drift in declination a sawtooth like scan path was traced over a localized region, as shown in Fig. 21 for Tycho. Corrections were made for parallax and refraction for each scan in order to calculate the motion of the detector relative to the lunar disk.

Since we were primarily concerned with relative temperatures it was felt sufficient to calibrate on the subsolar point and the sky periodically. For the purposes of our calculations it was assumed that the subsolar point temperature was 374°K. The atmospheric transmission curve used in our calculations is shown in Fig. 22. Using the black body curve for a given temperature T_i the amount of energy $E\ (T_i)$ getting through the atmospheric window was determined. The ratios $E\ (T_i)/E\ (T_{374}) = R_i$ were formed. It was found that the equation

$$\log_{10} R_i = A - B/T_i + C/T_i^{\,2}$$

represented the relationship between R_i and T_i. Fig. 23 shows the calibration data from the sky and the subsolar point for one night. The energy ratios were formed such as

$$R_{moon} = \frac{E\ (T_{moon}) - E\ (T_{sky})}{E\ (T_{ssp}) - E\ (T_{sky})}$$

for each datum reading. Then R_{moon} was related to the corresponding R_i and the observed lunar surface temperature T_{moon} was found.

The error due to detector noise alone is very small, 0.25° at 374°K increasing to 2.1° at 200°K. Errors due to sky background fluctuations were measured to be higher in some cases; however, they were less than a degree at the higher temperatures, increasing toward the lower temperatures.

Crater Surveys*

About thirty isothermal maps of eleven crater regions were constructed for September 4, 5 and 6, 1960 including some during the totality of the lunar eclipse. All the original maps were made to the scale of Kuiper's Photographic Lunar Atlas. Fig. 24 shows the regions that have been mapped. The most important results are related to the prominent rayed craters Tycho, Aristarchus, Copernicus, Proclus and Kepler. The region of Aristarchus has some interesting features associated with it. In Fig. 25 the region [17] is seen in the afternoon. Herodotus, an older crater, is to the southeast, and Schröter's Valley is to the northeast. Notice the white area near the apparent end of Schröter's Valley.

Isotherms in the region of Aristarchus September 6, 1960, one day after full moon are represented in Fig. 26. This scan covered 100" of arc right ascension and 80" of arc declination. The scale is represented by the 50" of arc on the overlay. Sensor size is represented as a circle on each thermal map and amounts to 8" of arc. The hachures indicate cooler regions. Recall that this scan was made one day after full moon; the sun is still rising. The actual photograph however represents the area in the afternoon and

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*These measurements were made on the 60-inch telescope at The Mt. Wilson Observatory in California.

the shadow configuration is not correct for the time when the scan was made. Several things are noticed:

1) The contours are related to the feature Aristarchus.
2) The crater is cooler than its general surroundings.
3) Herodotus does not seem to affect the shape of the contours directly nor does Schröter's Valley.
4) There is a general gradient in the direction of the subsolar point. This last characteristic was to be expected.

Other important features can be seen. The cooler region not only is centered on the crater but extends beyond and somewhat into Herodotus. There is a correlation with the general white area near the end of Schröter's Valley. In general the contours seem affected by the brighter areas! It is known that Aristarchus has a high albedo. Therefore it is reasonable then that more of the sun's incident radiation would be reflected, less absorbed and less re-radiated as planetary heat accounting for this lower temperature. The effect of geometry may also alter the distribution of temperature in and around a crater. This is due to the elevation angle of the sun with respect to the local surface and should change during the three day observation period. The same region about twenty-four hours before shows a 2° difference between the crater and its environs (Fig. 27). There is considerable noise or horizontal structure in the direction of scan (right ascension). If the gradient in the direction of the subsolar point (due to curvature of the lunar surface) were removed the local character of the thermal contours can be observed. In Fig. 28 this has been accomplished, leaving the temperature of the crater center the same. As a by-product of this method some of the horizontal structure has also been removed.

Copernicus is another rayed crater over which thermal mapping was performed. The sensor diameter was between 1/5 and 1/6 crater diameters. The effect of albedo is observed here (Fig. 29), in that, the whole area in and around Copernicus is cooler than its surroundings, corresponding to the bright area around the crater. The center of the crater is 369°K and the western interior is cooler by 2.5°. The only other correlation that can be seen is the sharp gradient outside the west rim into the darker area. The night before during full moon (Fig. 30) the effect of geometry is seen in this large scan (200" of arc RA by 225" of arc declination). The observed temperature differential between the east and west interior slopes is 4° to 5°. Note again the general gradient toward the subsolar point, 358°K near Mayer to 370°K southwest of Copernicus. If the crater were not present the contours would be more parallel to each other such as the 368°K and 367°K southwest of Copernicus. The elevation angle of the sun is less for the interior southwest slope than for the interior northeast slope. The opposite is true for the exterior slopes. This gives rise to the difference in the observed temperatures and will be called the "geometry effect." It must be mentioned that variations in local albedo are superimposed and the two effects cannot be completely separated.

The effect of geometry is not apparent in the thermal map over the region of Menelaus (Fig. 31) at full moon. The crater is more uniformly illuminated and the shape of the contours around the crater reflect the effect of albedo. The cool region centered on the crater extends well beyond and follows somewhat the bright environs. The interior is perhaps cooler by 4° to 5°. There is some horizontal structure in evidence as well as an overall upward shift in temperature above the assumed subsolar point temperature. This shift is caused by an increase in sky background radiation which was not taken into account because the sky was monitored when the background was normal some time before and after this scan.

In the next two figures the region of Tycho is seen on the nights September 5 and 6, 1960, (Fig. 32 and 33). The direction of the gradient which is to the northwest on September 5 shifts to north on the 6th consistent with the motion of the subsolar point. The cooler and warmer regions in the crater move counter-clock-wise. In the larger scan on September 5 the contours follow the general outline of several other craters, Pictet, Street and Longomontanus.

One of the more complicated contours is that of the region around Proclus (Fig. 34). The gradient between Paulus Somnii and Proclus drops 6° in 16" of arc. Proclus is not much greater than the sensor size, however, due to many overlapping scans, detail in the crater can be obtained. The effect of geometry is indicated by the cooler interior slope nearest the subsolar point. There is also a cool region around Promontorium Olivium. The last example of contours on the illuminated moon is that over Alphonsus (Fig. 35). The general effect of geometry causes the northern interior to be cooler, less than 367°K while the outside northern slopes are 369°K.

The most exciting results were obtained during the penumbral and umbral phases of the total lunar eclipse of September 5, 1960. The circumstances were as follows:

Entered penumbra	8:37 UT
Entered Umbra	9:36
Totality started	10:38
Totality ended	12:06
Left umbra	13:08
Left penumbra	14:07

Fig. 36 represents temperature traces over Aristarchus during the penumbral and part of the umbral phases. These traces were obtained by centering first on the feature moving off in right ascension for a few seconds of time then reversing in right ascension and scanning back through and well beyond the feature. The small dashed lines below the crater deflections show one of the means by which the so called interpolated environs were found. The chart deflection is proportional to $E(T_{moon}) + E(T_{sky})$ and has not been reduced to the calculated temperature values. Aristarchus entered penumbra at 8:51 UT and entered umbra at 9:56 UT. Several things are noticed:

1) The crater, cooler during the full moon, is warmer than its environs during the umbral phase.
2) The warmest point is centered on the crater and the anomaly extends beyond the crater.
3) The gradient toward the subsolar point grows less during the penumbral and into the umbral phase as expected.

Similar results were obtained on Copernicus and Kepler. Sinton also obtained comparable results for Tycho.[18] The eclipse cooling curve for the crater Aristarchus and its environs is shown in Fig. 37. The cooling curve of a point 166" of arc west of Aristarchus is almost parallel to the environ cooling curve and serves as a rough check on the method of interpolation.

In Fig. 38 the difference $R_{Crater} - R_{Environ} = \Delta R$ is shown. This parameter ΔR has been plotted because the effect of sky background is reduced. In this graph observe the following:

1) The cross over point where the temperatures of the crater and the environs are equal is $t/t_o \sim 0.2$.
2) A constant ΔR is reached at $t/t_o \sim 0.8$.
3) ΔR before eclipse is -0.044 and during totality 0.048. That is, the absolute value of the energy difference between the crater and its environs is almost constant before eclipse and during totality. This effect will require investigation during some future eclipse.

One of the so called "hot spots" [19] is shown in Fig. 39, temperature contours over Aristarchus during the eclipse. The crater is 228°K, the environs 203°, giving a ΔT of 25°. Little is left of the gradient toward the subsolar point. The "hot spot" is centered on the crater and extends beyond as already described. There is no correlation with other features such as Herodotus or Schröter's Valley. There does, however, seem to be a significant relationship with the optically bright regions around Aristarchus shown as an unlabeled dashed line.

Hottest of the "hot spots" is the crater Tycho (Fig. 40) with an observed temperature difference of 31°. It is again maximum at the crater center falling to the background level somewhat beyond the crater. Correlations with other features are left to the reader's imagination!

The normalized cooling curves (Fig. 41) for Aristarchus and its environs are plotted with the theoretical homogeneous surface model of Jaeger.[20] The values of $(K\rho c)^{-1/2}$ depend on the initial value of temperature T_o and the duration of the penumbra t_o. The parameter $(K\rho c)^{-1/2}$ describes the family of cooling curves. Here K is thermal conductivity, ρ is the density and c is the specific heat. For terrestrial rock $(K\rho c)^{-1/2}$ is 20, for gravel or pumice it is 100, and for fine dust under the lunar environment it may be 1000. The data for the area 166" of arc west of Anstarchus fall on the interpolated environ curve in this normalized plot. This theoretical homogeneous model does not fit the experimental results from this eclipse cooling data.

If the cooling data are plotted along with the two layer model of Jaeger and Harper [21] there is better agreement between the theory and experiment. This two layer model has a constraint determined by microwave results which is $d = 610\ K\ (K'\ \rho'\ c')^{-1/2}$ where d is the surface thickness in cm, $K = 2.8 \times 10^{-6}$ cal $cm^{-1} sec^{-1} deg^{-1}$, and $(K'\ \rho'\ c')^{-1/2}$ represents the substratum the values of which are shown on Fig. 42.

Summary of Crater Surveys:

Illuminated Moon:

1) Geometry effect - the crater interior slopes nearest the subsolar point are cooler than the interior slope farthest away. This effect is caused by the variation in inclination of the local surface.

2) Albedo effect - the bright rayed craters are cooler than their environs. In most cases the cooler area is centered on the feature and the temperature increases as one moves away from the center to the environ temperature somewhat beyond the crater. This effect is a result of the variation in local albedo. The higher the albedo the greater is the amount of the incident sunlight that is reflected resulting in a corresponding decrease in planetary heat.

3) Variation in the albedo effect for the three day period was noticed and may be related to directional effects. Many other details must be traced through a lunation before the contours around full moon can be completely explained.

Eclipsed Moon:

1) "Hot Spots" - the bright rayed craters cool less rapidly than their environs during penumbral phase. The crater is warmer than its environs during the umbral phase.

2) Extent of warm area - the warm area is centered on the crater and extends beyond in some cases, decreasing to the environ level there.

3) Only the five rayed craters observed were found to exhibit this effect. Many other features should be surveyed during another eclipse. Measurement during the lunar night may provide similar data.

4) These results could be explained as a variation in dust thickness. This interpretation will be published later.[22] Other interpretations are possible such as permeability to infrared emission, suggested by Buettner.[23]

Review of Calculated Results

Energy Ratio

Crater	Tycho	Aristarchus	Copernicus	Proclus	Kepler
$R_{initial}$	0.751	0.751	0.862	0.924	0.842
$R_{eclipse}$	0.116	0.079	0.072	0.066	0.037
Environs					
$R_{initial}$	0.771	0.791	0.905	0.936	0.842
$R_{eclipse}$	0.049	0.037	0.046	0.046	0.027

Temperature °K

Crater	Tycho	Aristarchus	Copernicus	Proclus	Kepler
$T_{initial}$	350	350	361	367	359
T_{final}	243	228	225	222	203
Environs					
$T_{initial}$	352	354	365	368	359
T_{final}	212	203	210	210	194
ΔT_{final} (Crater-Environ)	31	25	15	12	9

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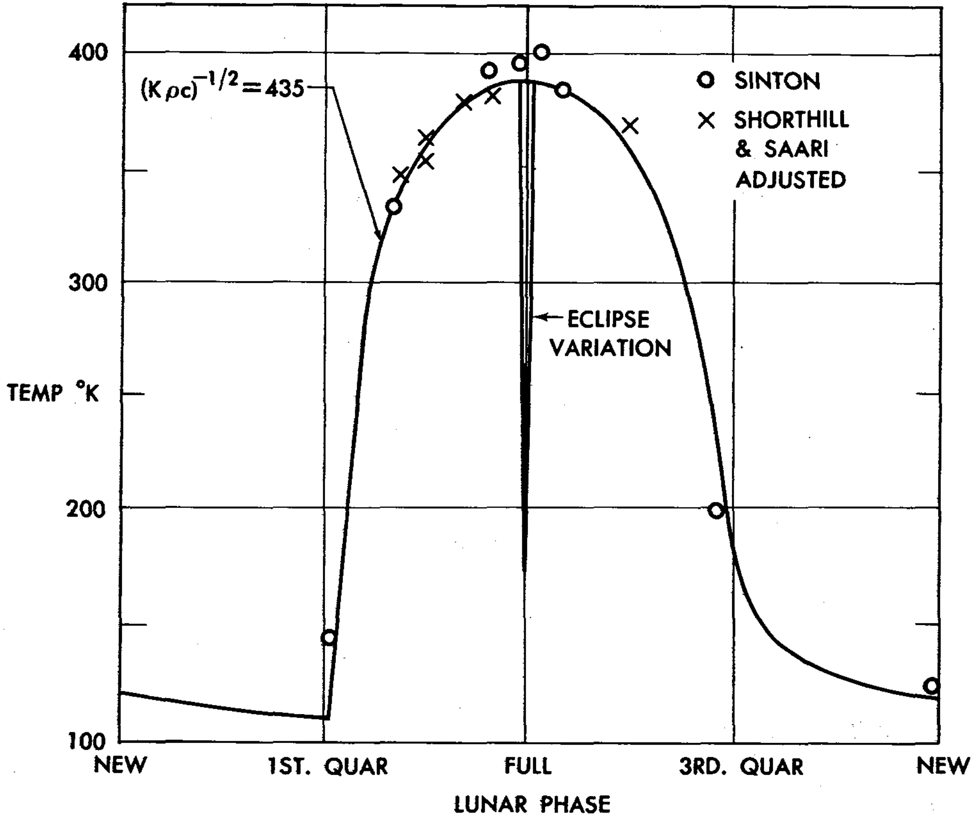
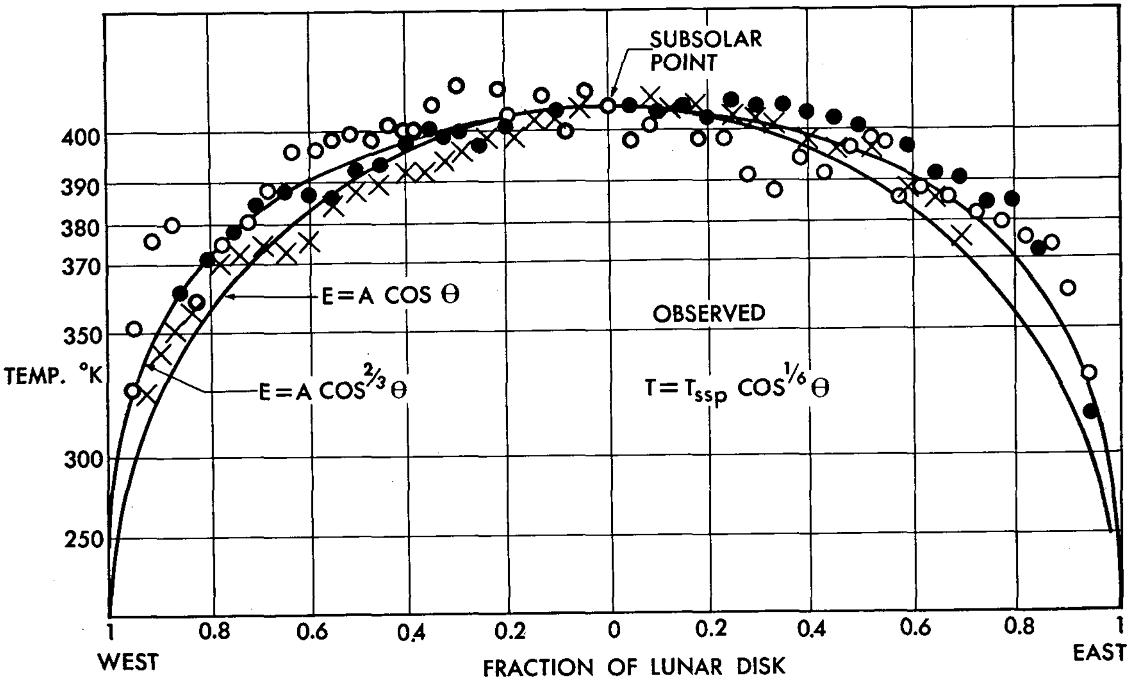


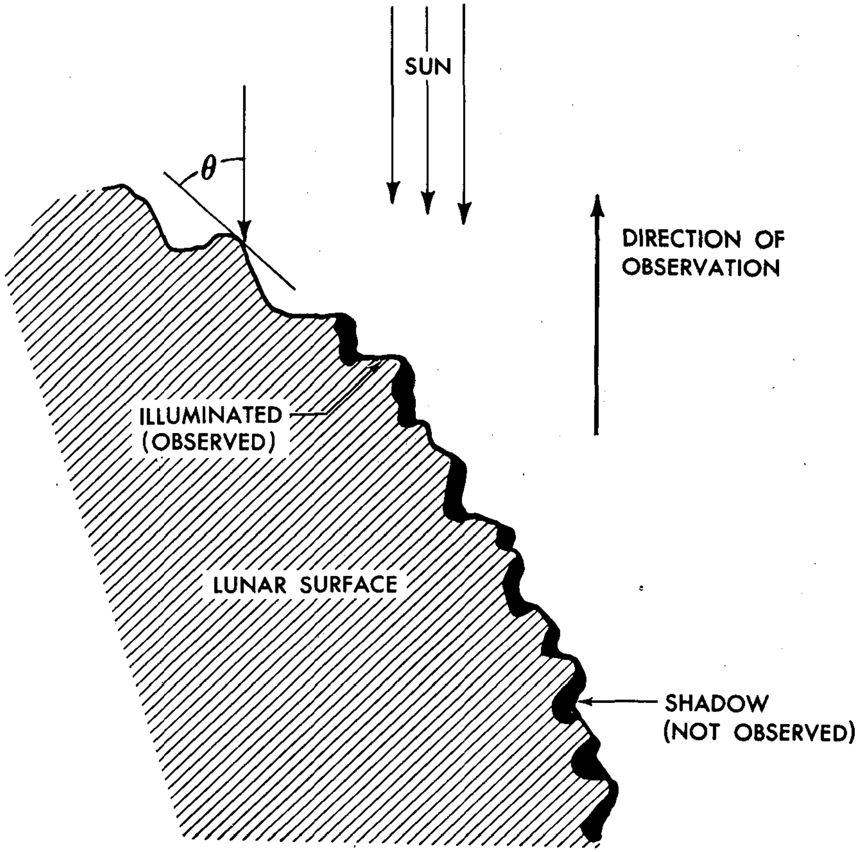
TEMPERATURE VARIATION DURING A LUNATION

Fig. 1



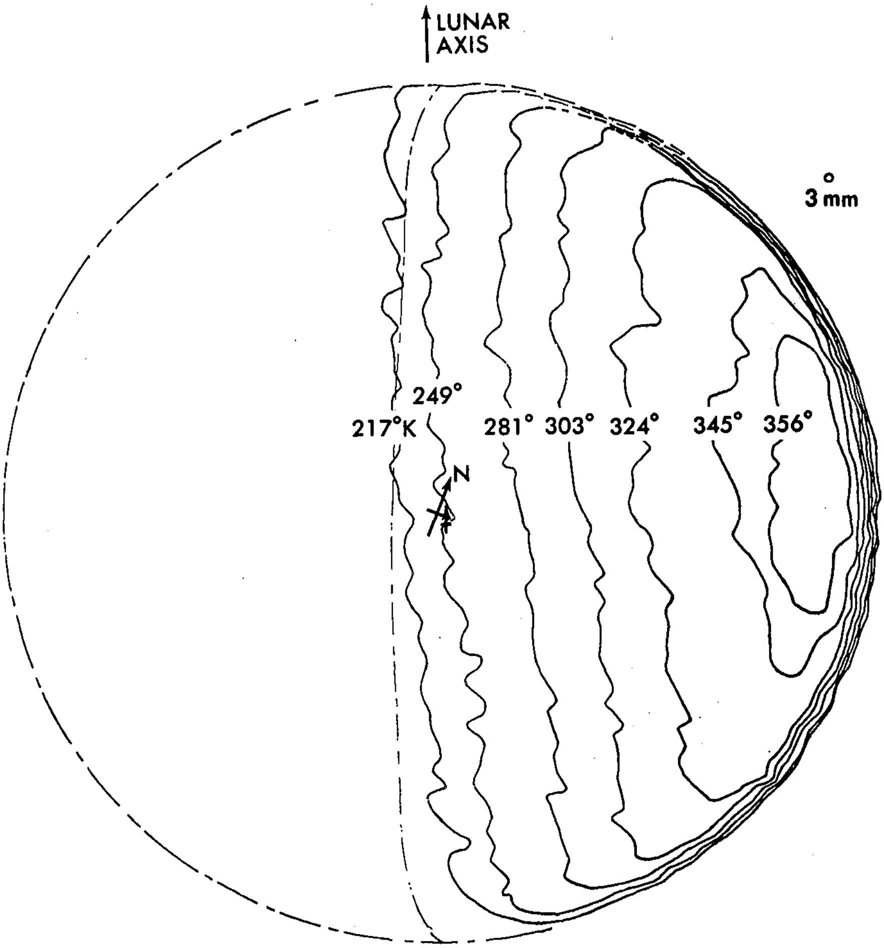
DISTRIBUTION OF PLANETARY HEAT ACROSS THE DISK OF THE FULL MOON.
AFTER PETTIT & NICHOLSON

Fig. 2



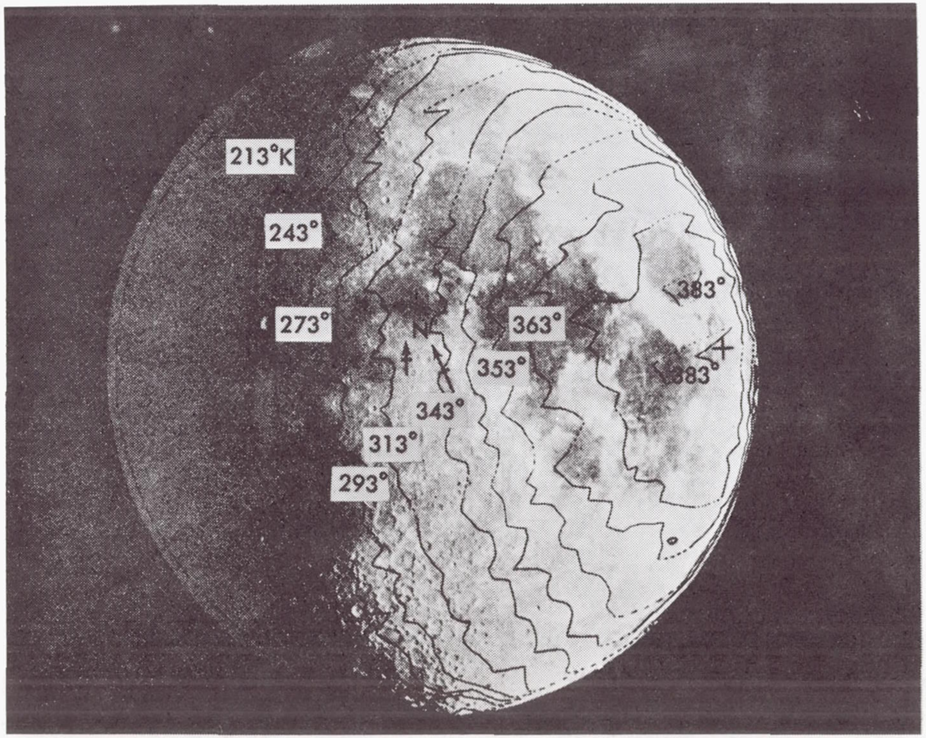
ROUGHNESS FACTOR EXPLANATION OF COS $^{2/3}$ θ VARIATION IN OBSERVED ENERGY OVER LUNAR DISK AT FULL MOON

Fig. 3



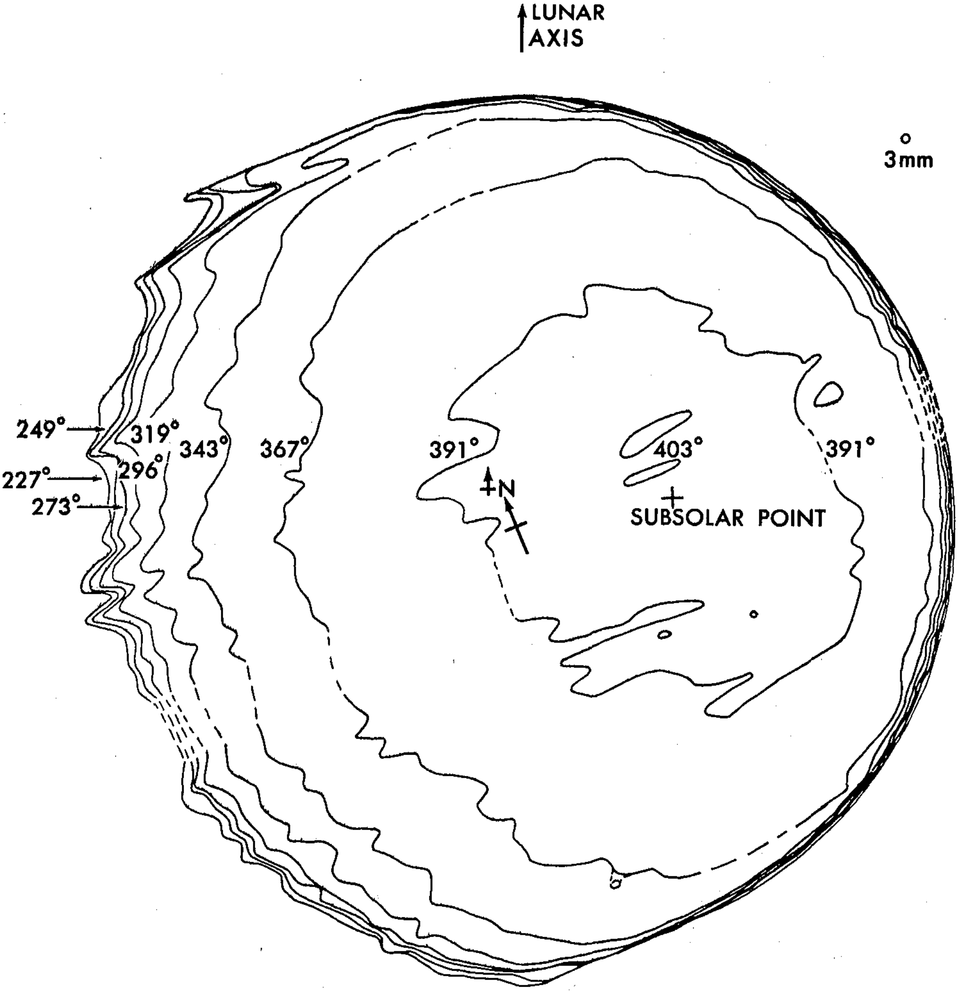
ISOTHERMAL CONTOURS OF MOON MAY 16, 1959 START 1:49, END 4:43 UT, 58 SCANS 0.53 ILLUMINATED (~8 DAY OLD MOON)

Fig. 4



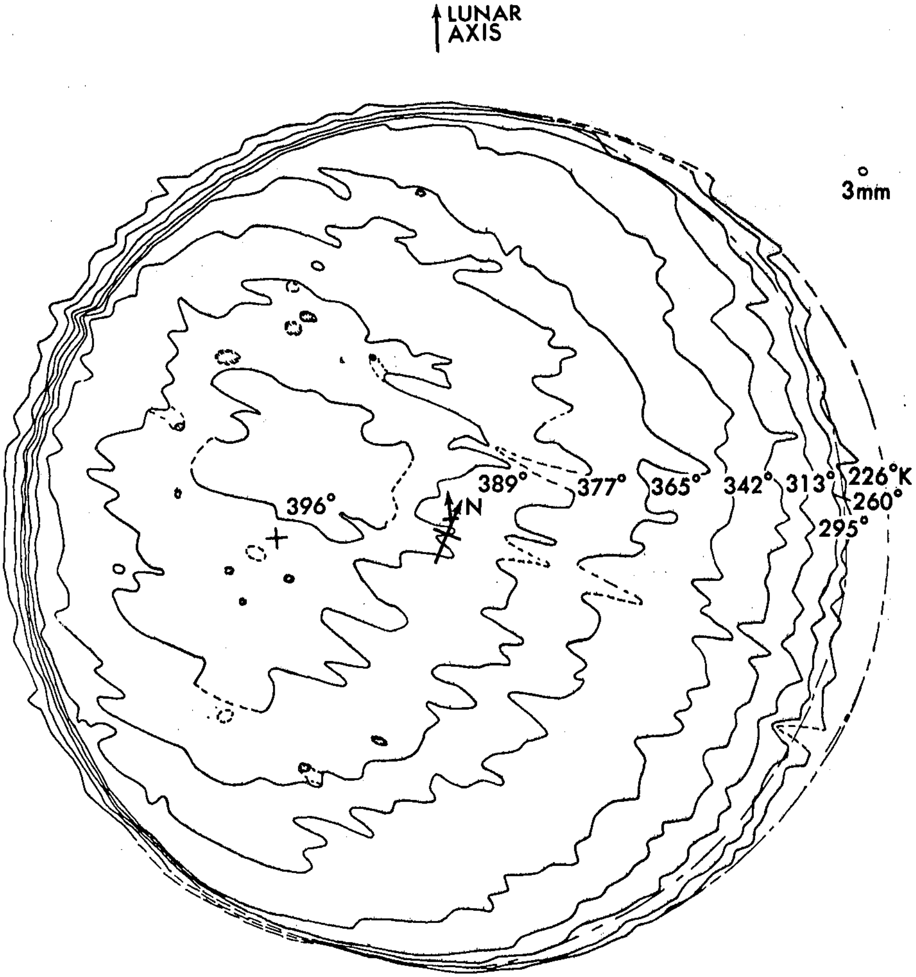
ISOTHERMAL CONTOURS OF NOVEMBER 10, 1959 START 2:17, END 5:23 UT, 63 SCANS 0.77 ILLUMINATED(~10 DAY OLD MOON) PLOTTED ON A SIMULTANEOUS PHOTOGRAPH.

Fig. 5



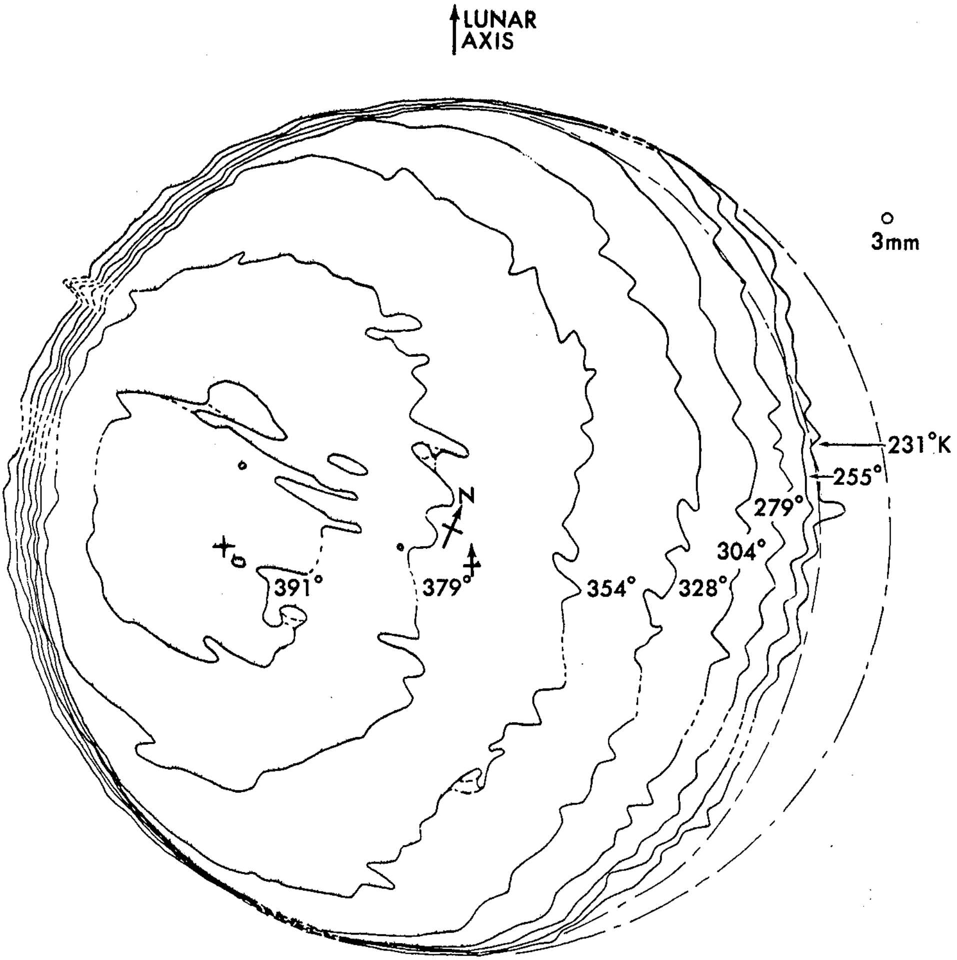
ISOTHERMAL CONTOURS OF SEPTEMBER 26, 1958, START 3:55, END 7:01UT
63 SCANS 0.98 ILLUMINATED (~14 DAY OLD MOON)

Fig. 6



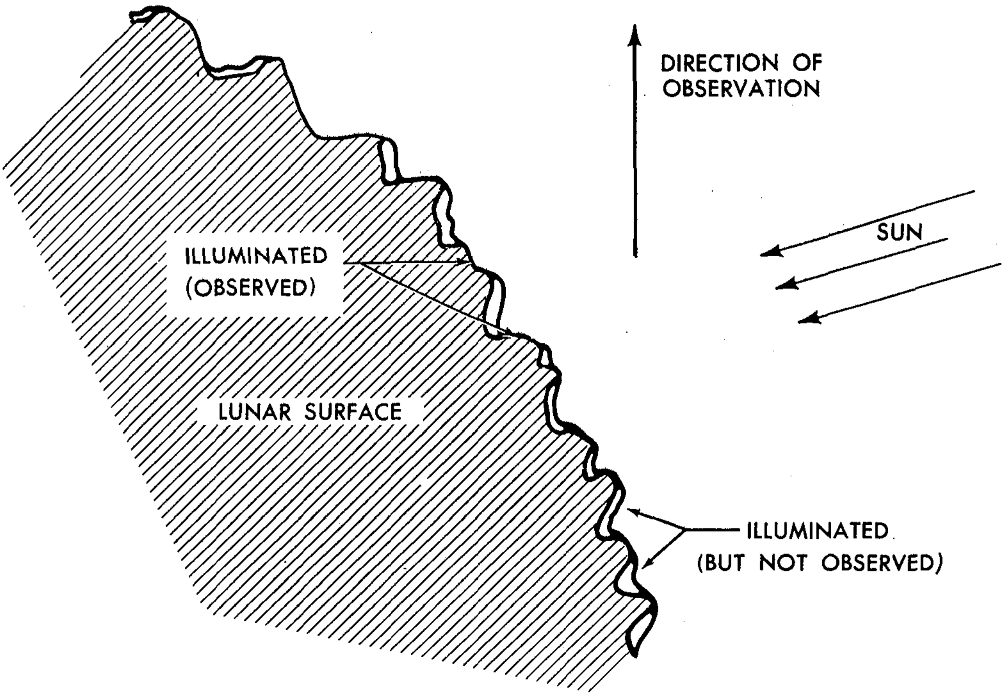
ISOTHERMAL CONTOURS OF MARCH 26, 1959, START 7:40, END 10:58 UT, 66 SCANS, 0.96 ILLUMINATED (~17 DAY OLD MOON)

Fig. 7



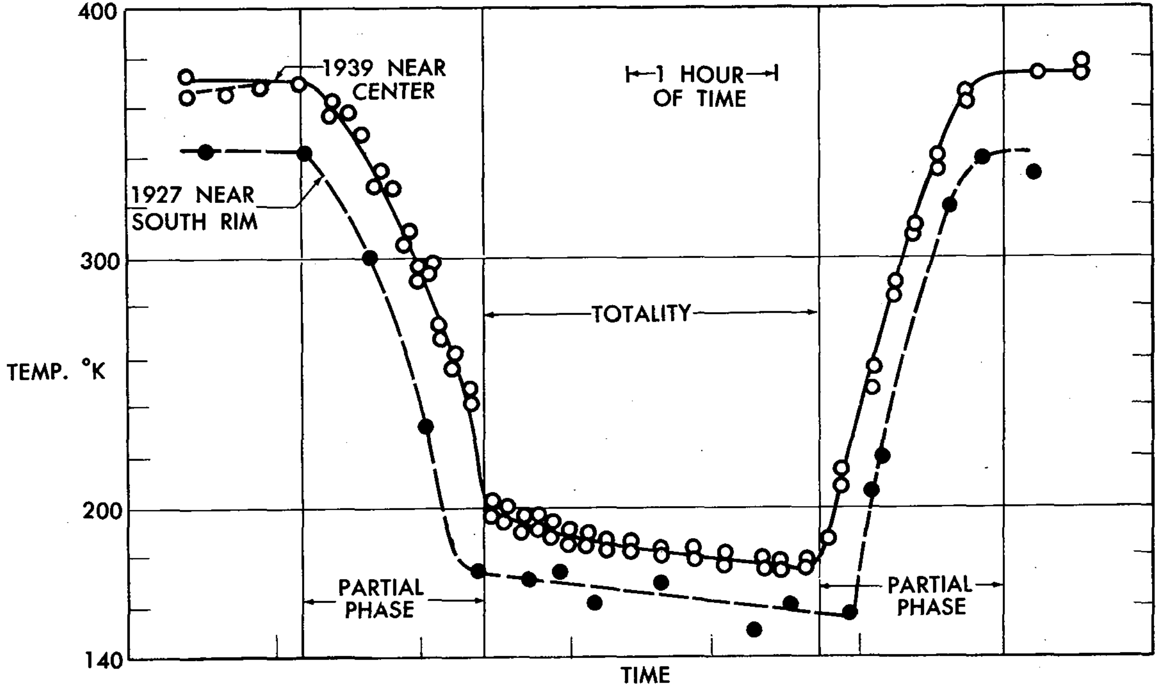
ISOTHERMAL CONTOURS OF JANUARY 27, 1959, START 7:42, END 10:39 UT, 60 SCANS, 0.92 ILLUMINATED (~18 DAY OLD MOON)

Fig. 8



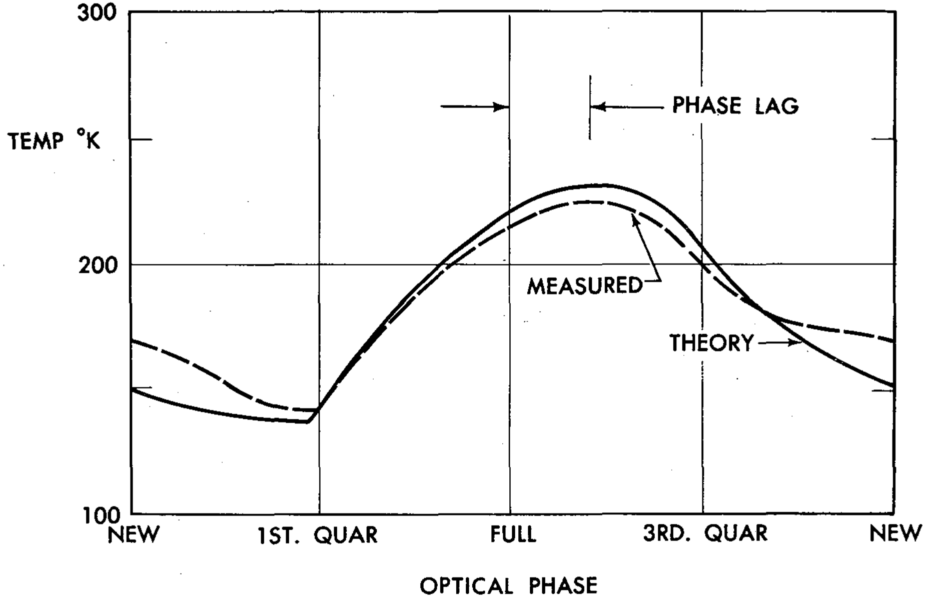
ROUGHNESS FACTOR EXPLANATION OF DECREASE IN OBSERVED SUBSOLAR ENERGY FOR PHASE ANGLES OTHER THAN AT FULL MOON

Fig. 9



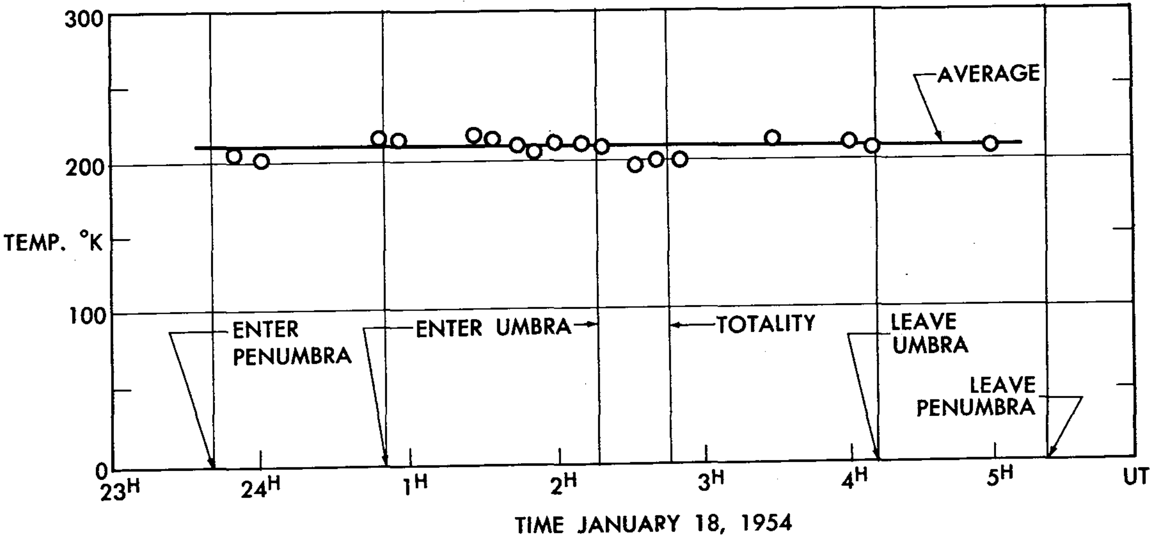
ECLIPSE COOLING CURVES FOR ONE POINT 1927 AND 1939 AFTER PETTIT & NICHOLSON

Fig. 10



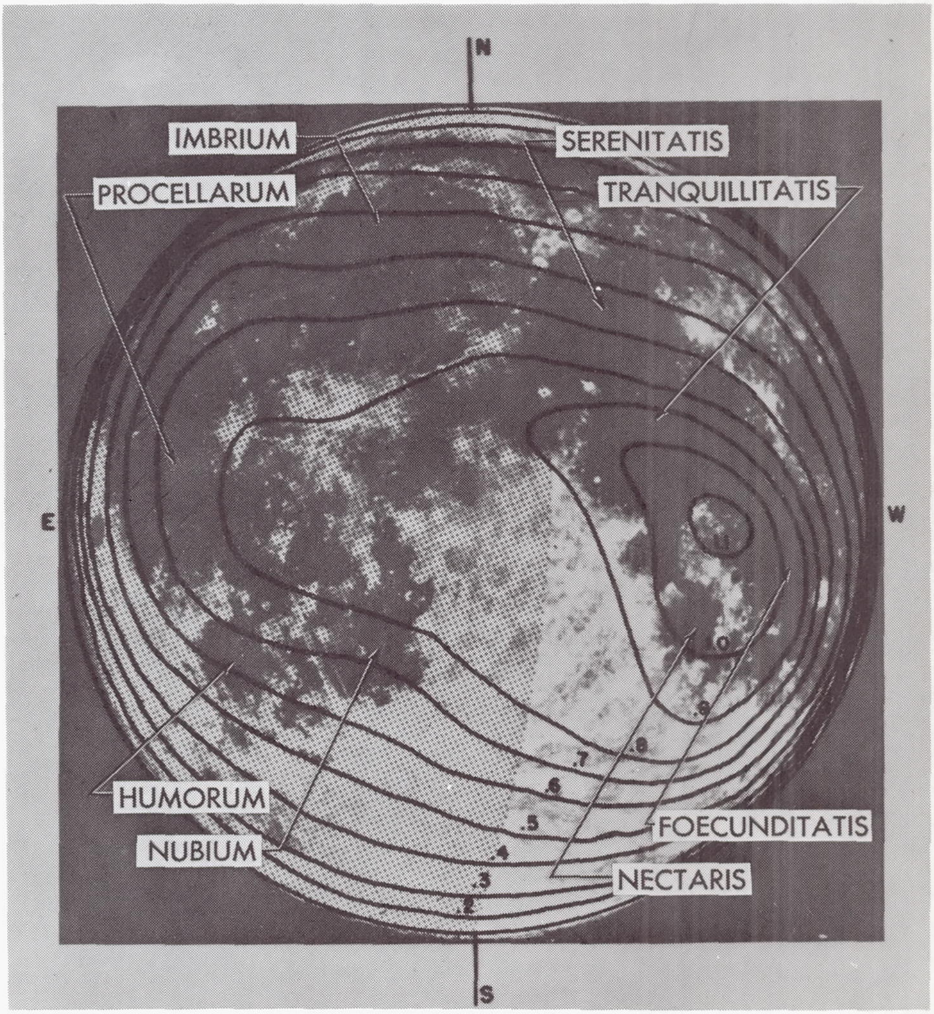
RESULT OF GIBSON'S RADIO MEASUREMENTS AT 8.6 MM
FOR CENTER OF LUNAR DISK.

Fig. 11



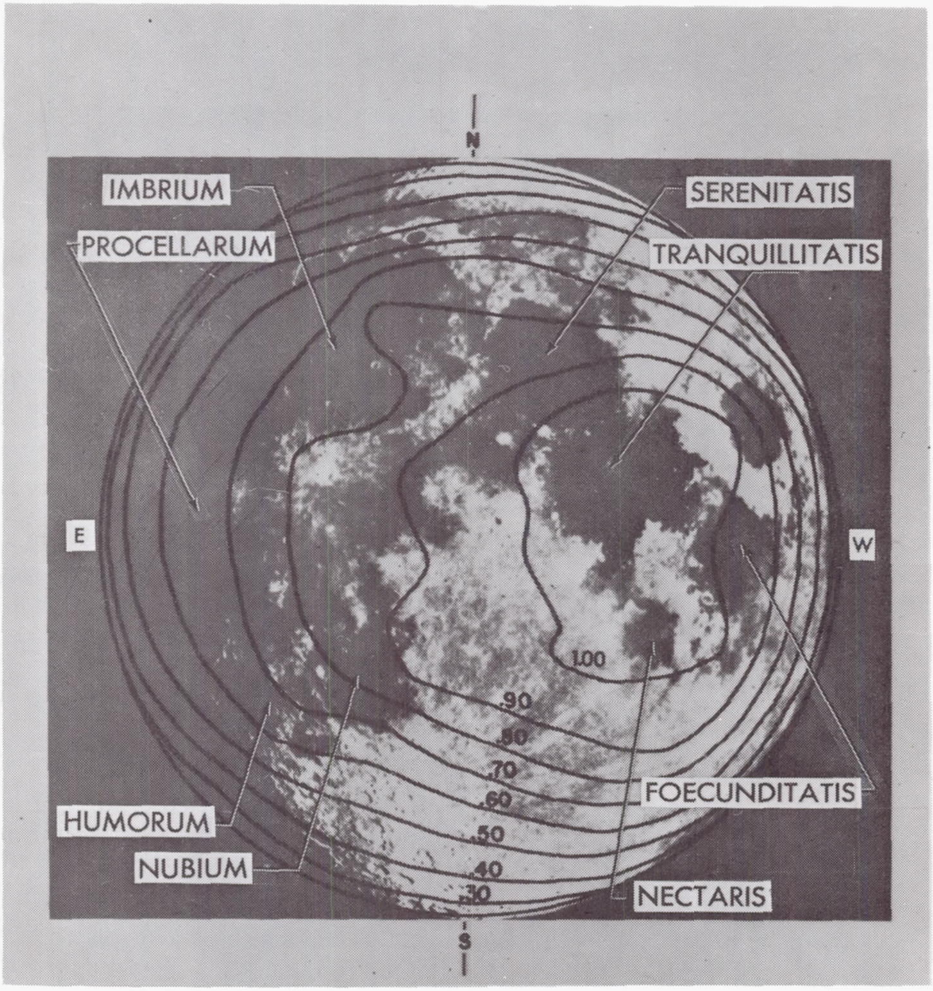
GIBSON'S OBSERVED TEMPERATURES DURING TOTAL ECLIPSE AT 8.6MM (ATLTITUDE 17° AT START)

Fig. 12



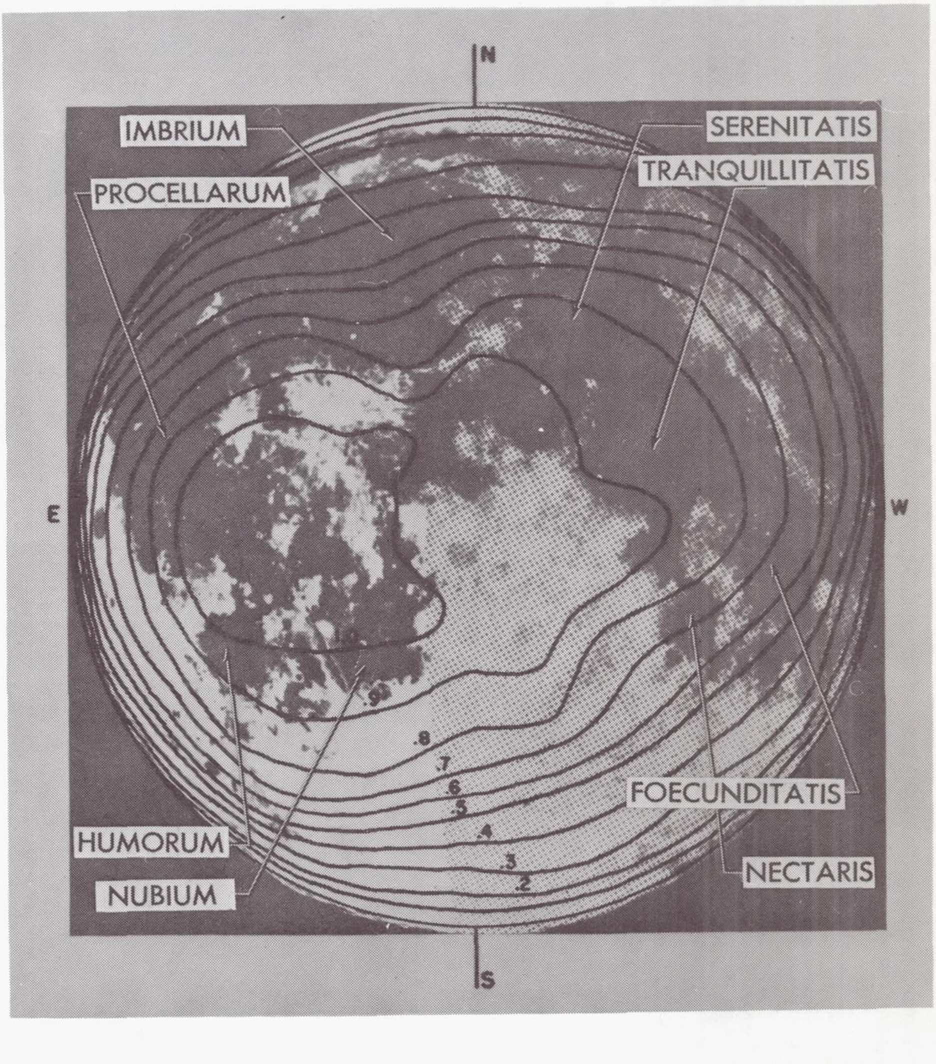
BRIGHTNESS CONTOURS AT 4.3mm WAVELENGTH
77° LUNAR PHASE

Fig. 13



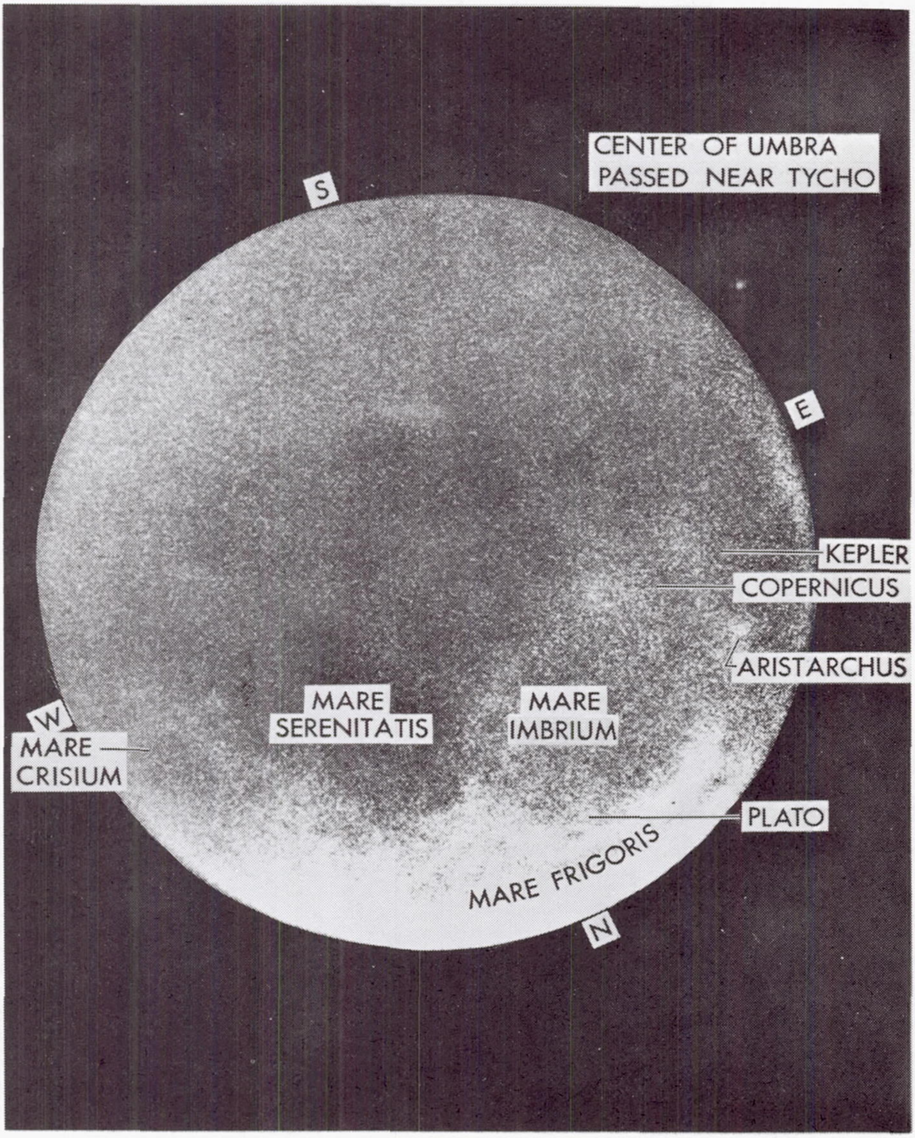
BRIGHTNESS CONTOURS AT 4.3mm WAVELENGTH
126° LUNAR PHASE

Fig. 14



BRIGHTNESS CONTOURS AT 4.3mm WAVELENGTH
280° LUNAR PHASE

Fig. 15

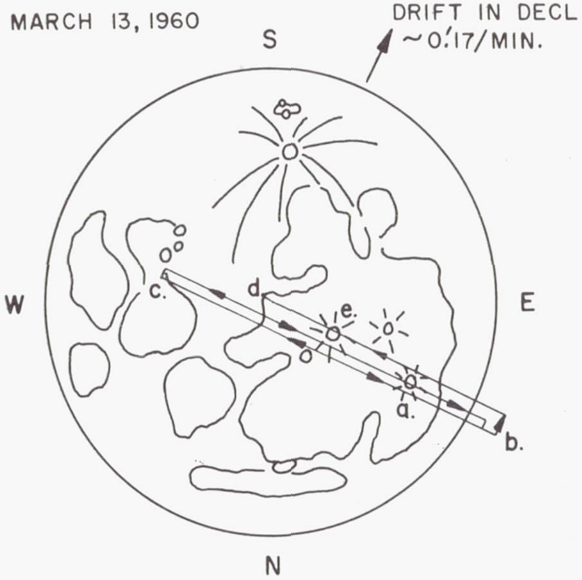


ECLIPSED MOON SEPTEMBER 5 1960 (DURING TOTALITY)

Fig. 16

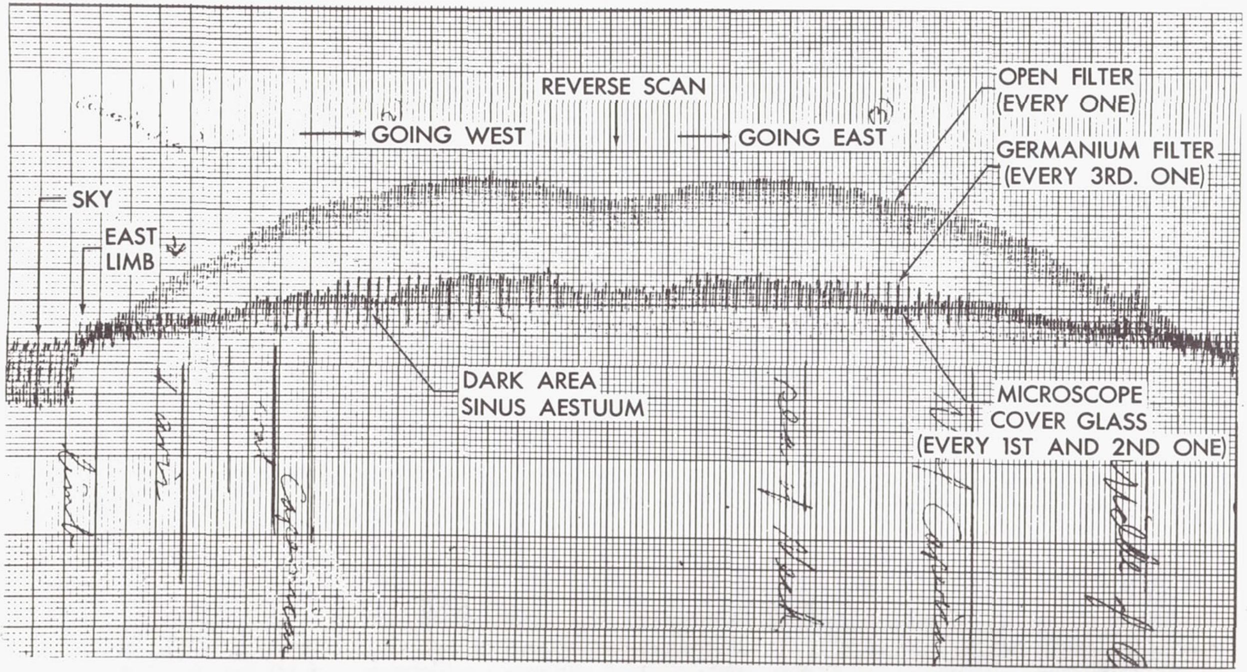
THE 73-INCH REFLECTING TELESCOPE OF THE DOMINION ASTROPHYSICAL OBSERVATORY, VICTORIA B.C. THE DETECTOR SUBTENDED 7'' OF ARC AT THE f/5 NEWTONIAN FOCUS.

Fig. 17



SCANNING CYCLE a. ARISTARCHUS b. END OF SCAN OFF EAST LIMB c. END OF SCAN NEAR THE SEA OF NECTAR d. CENTER OF DISK, END OF 14.5 MIN. SCAN CYCLE e. COPERNICUS

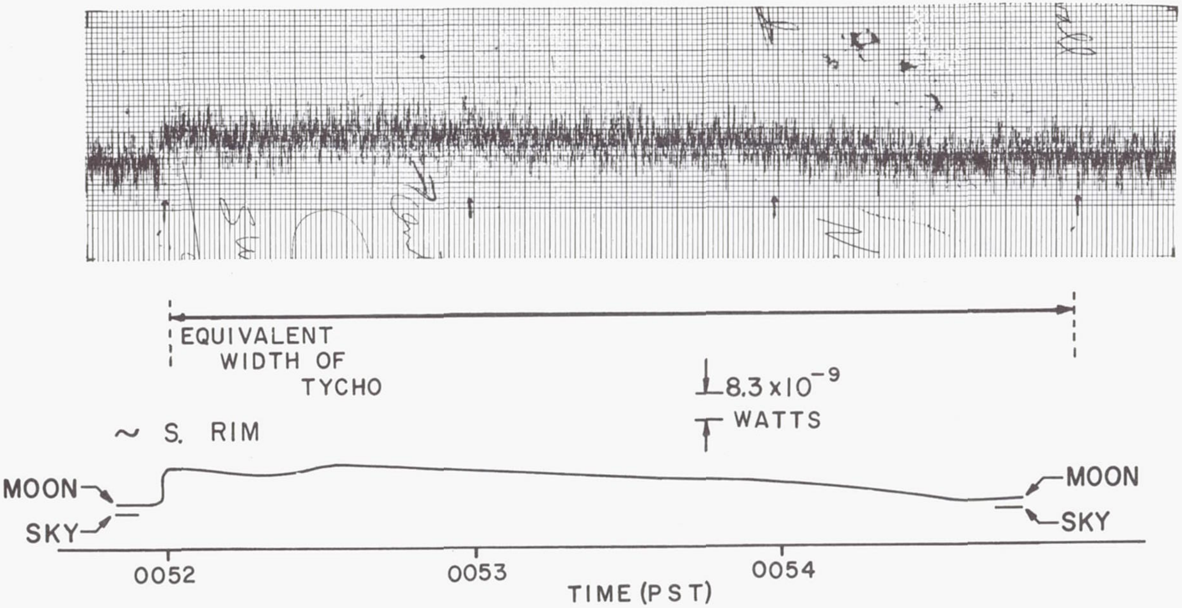
Fig. 18



SCAN OVER THE LUNAR SURFACE WITH ROTATING FILTER WHEEL DURING THE FULL MOON MARCH 12, 1960.

(THE OPEN FILTER SHOWS RADIATION FROM 0.5 TO 15 MICRONS. THE MICROSCOPE COVER GLASS SHOWS THE RADIATION 0.5 TO 5 MICRONS. THE GERMANIUM FILTER SHOWS THE RADIATION 2 TO 15 MICRONS.)

Fig. 19



ORIGINAL AND SMOOTHED DATA FROM A TYPICAL SCAN OF THE CRATER TYCHO

Fig. 20

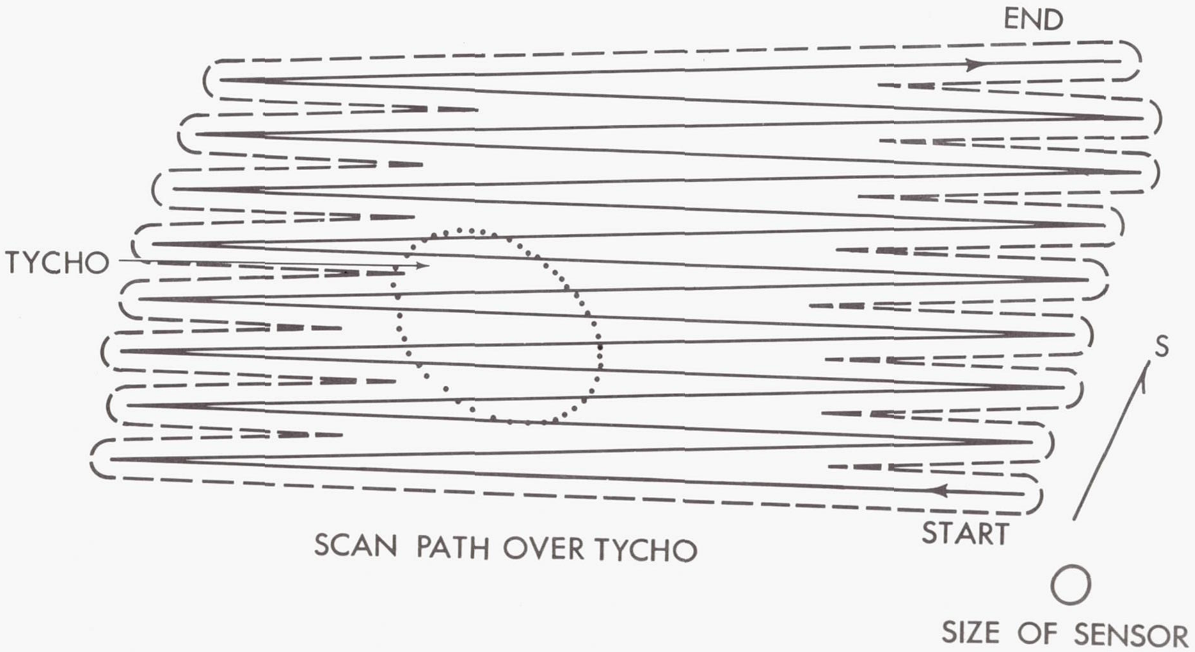
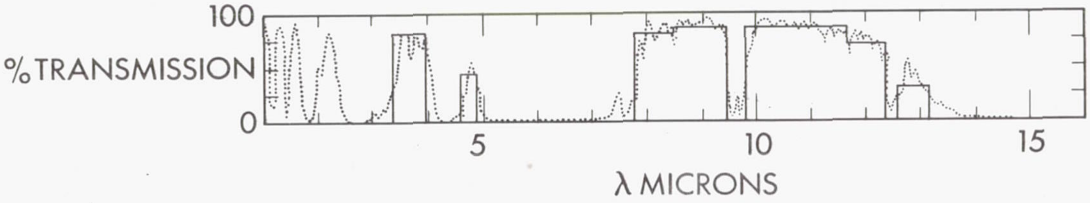


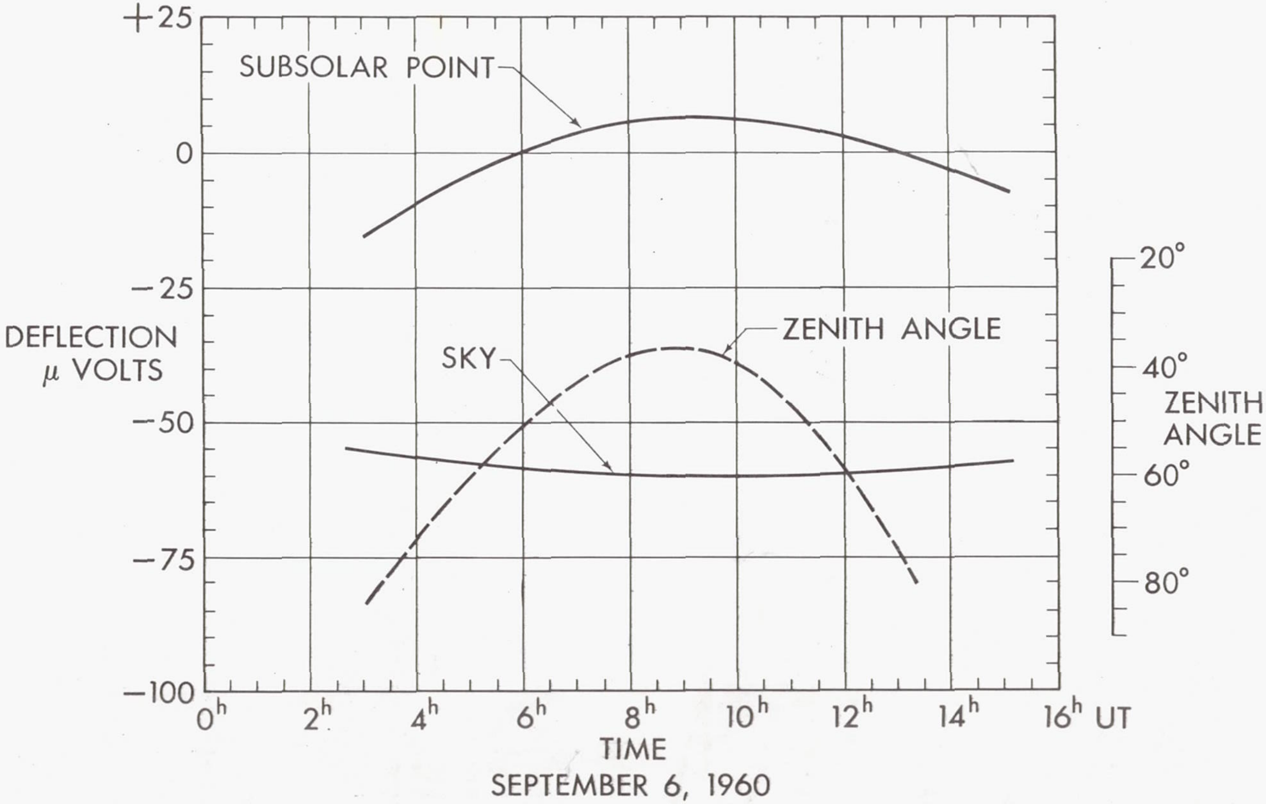
Fig. 21

ins 3-6



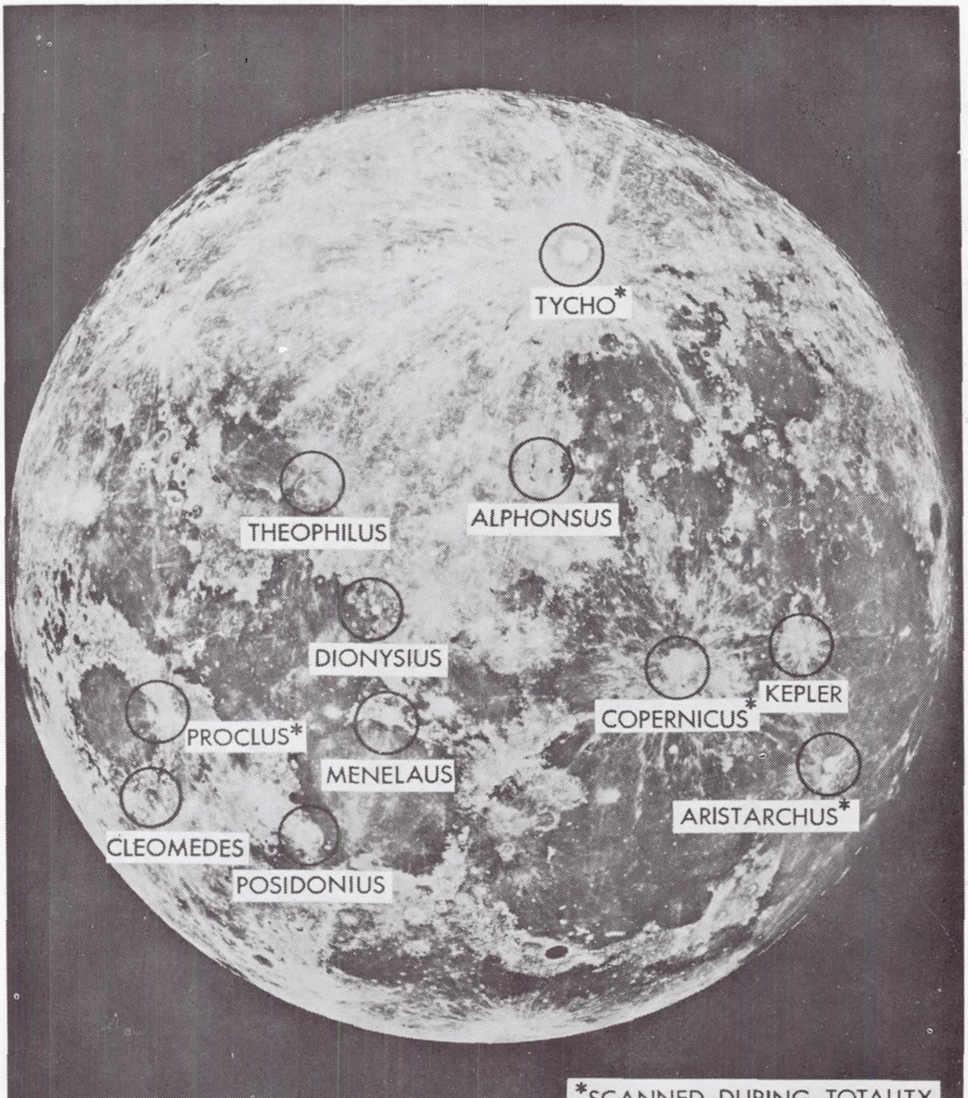
SIMULATED ATMOSPHERIC TRANSMISSION CURVE

Fig. 22



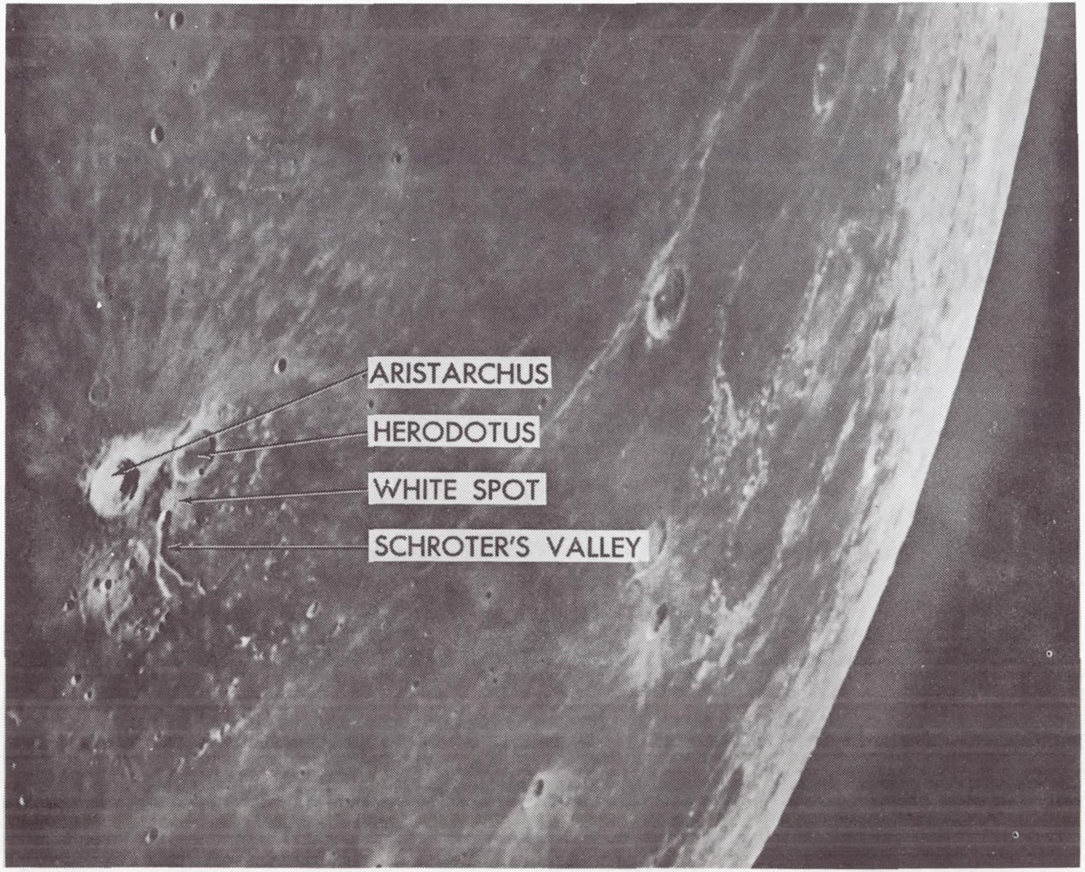
CALIBRATION CURVES FROM SKY AND SUBSOLAR POINT READINGS

Fig. 23



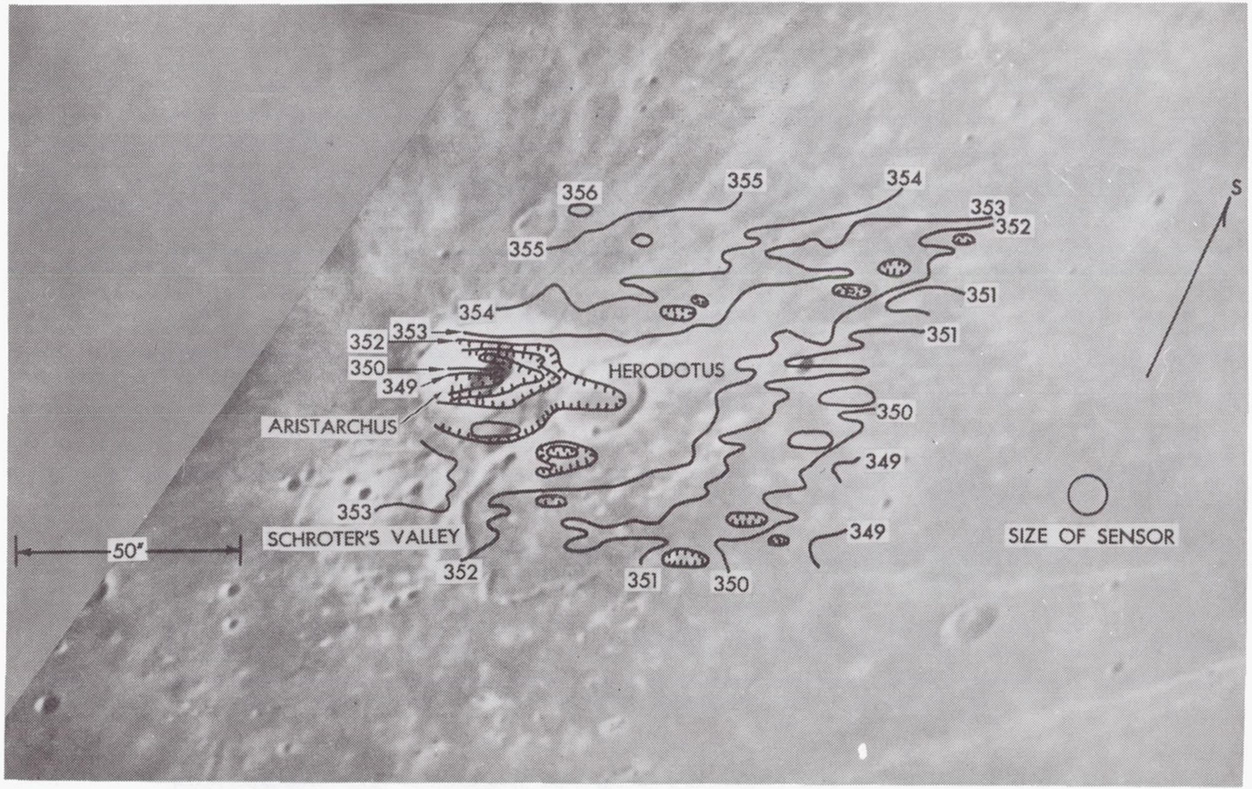
CRATER REGION SCANNED, SEPTEMBER 4, 5, 6, 1960

Fig. 24



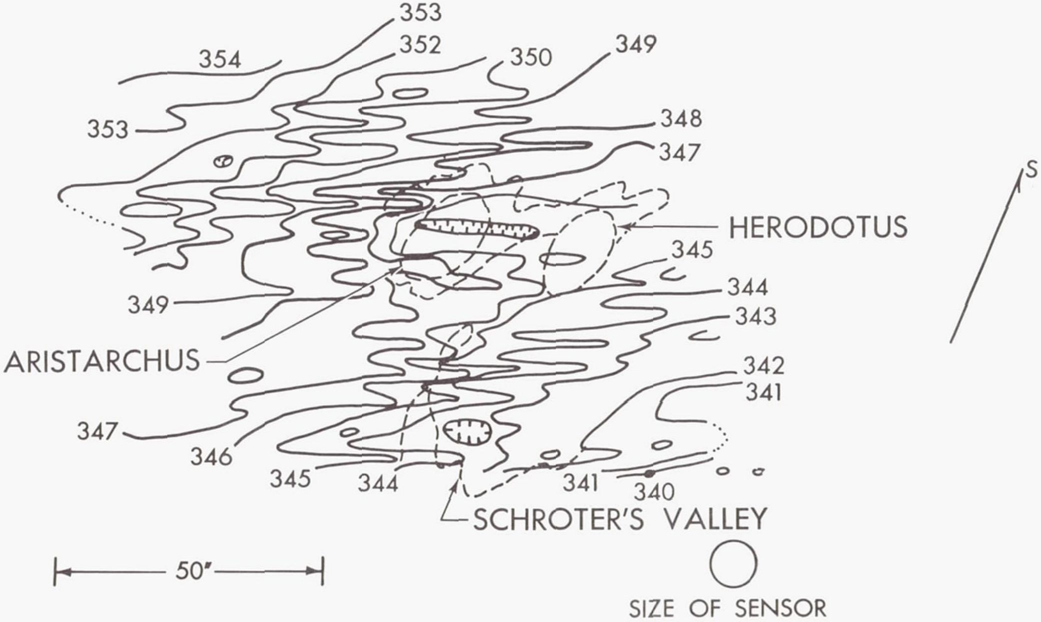
REGION OF ARISTARCHUS.

Fig. 25



ISOTHERMS IN THE REGION OF ARISTARCHUS SEPT. 6, 1960, 6:56 U.T.

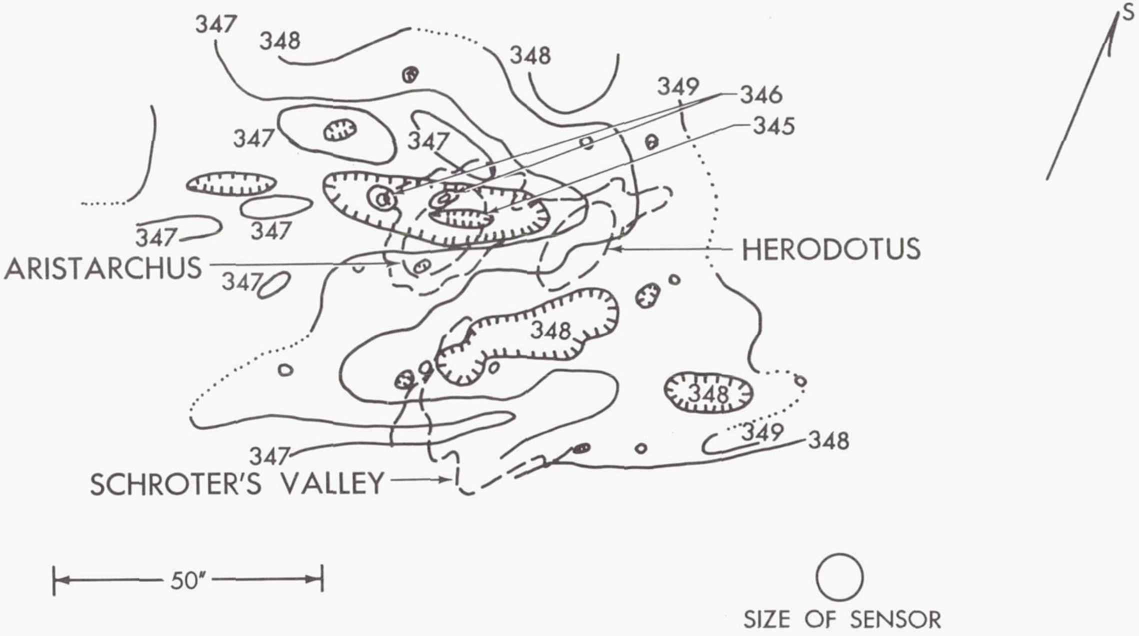
Fig. 26



ISOTHERMS IN THE REGION OF ARISTARCHUS

SEPT. 5, 1960 6:57 U.T.

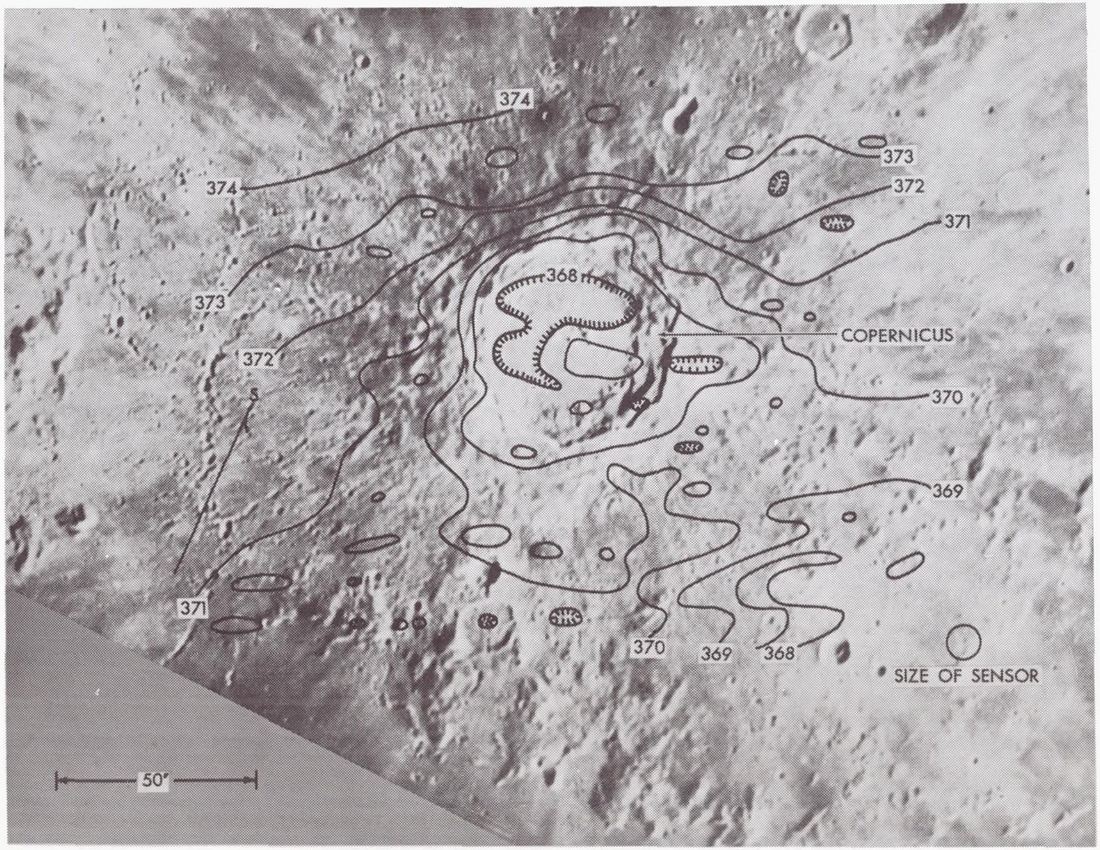
Fig. 27



ISOTHERMS IN THE REGION OF ARISTARCHUS RECTIFIED TO REMOVE THE EFFECT OF THE CURVATURE OF THE SURFACE

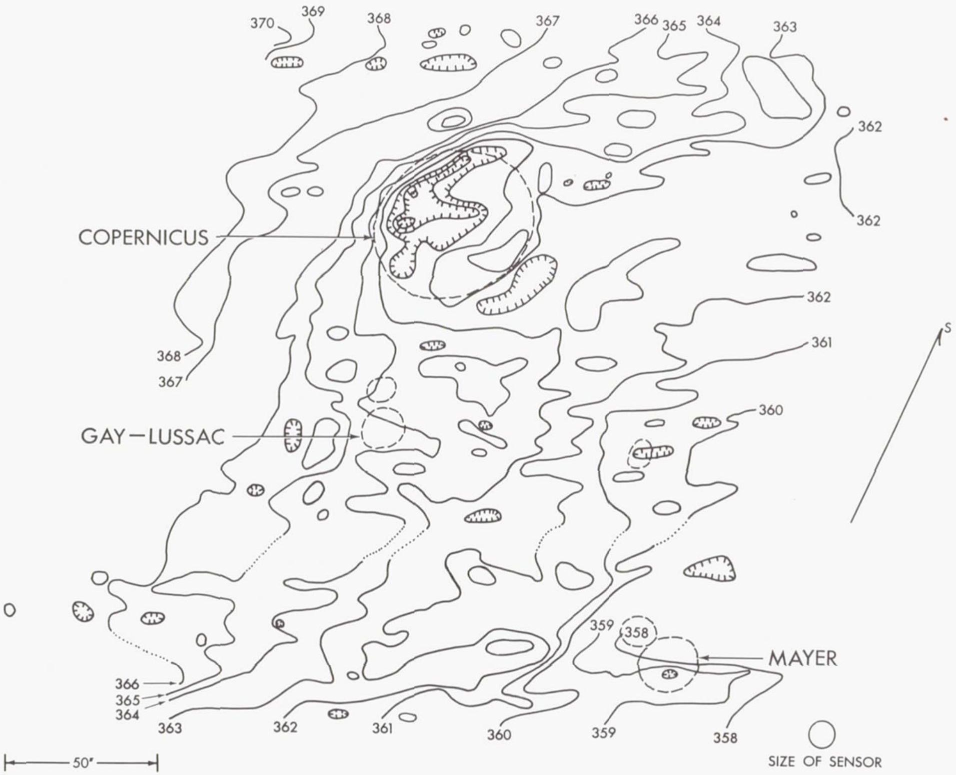
SEPT. 5, 1960, 6:57 U.T.

Fig. 28



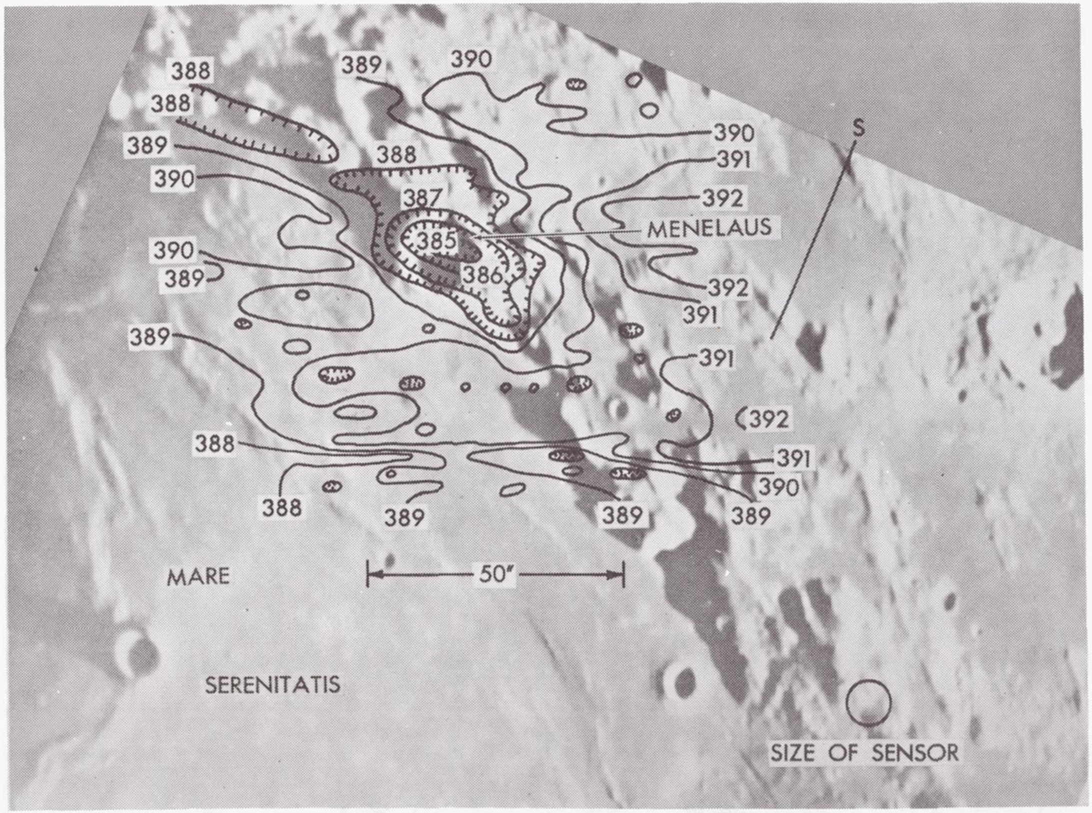
ISOTHERMS IN THE REGION OF COPERNICUS SEPTEMBER 6, 1960, 6:27 U.T.

Fig. 29



ISOTHERMS IN THE REGION OF COPERNICUS
SEPT. 5, 1960, 5:48 U.T.

Fig. 30

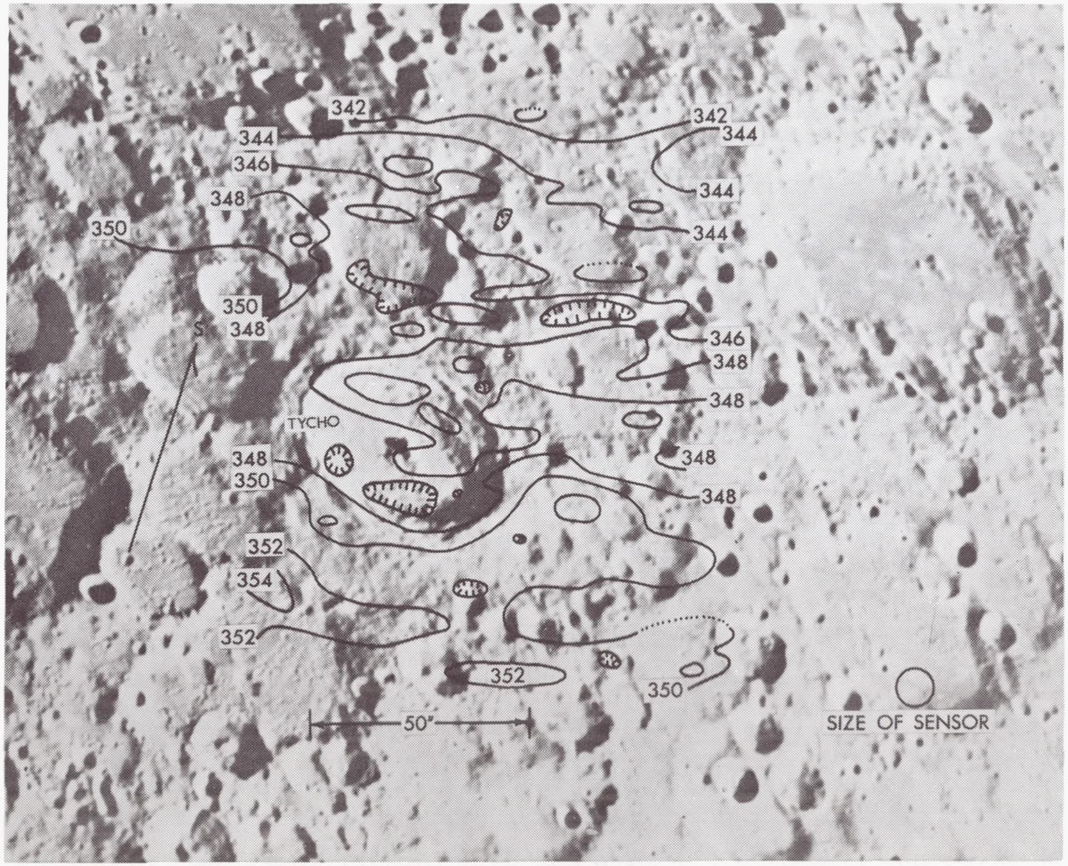


ISOTHERMS IN THE REGION OF MENELAUS SEPT. 5, 1960, 7:44 U.T.

Fig. 31

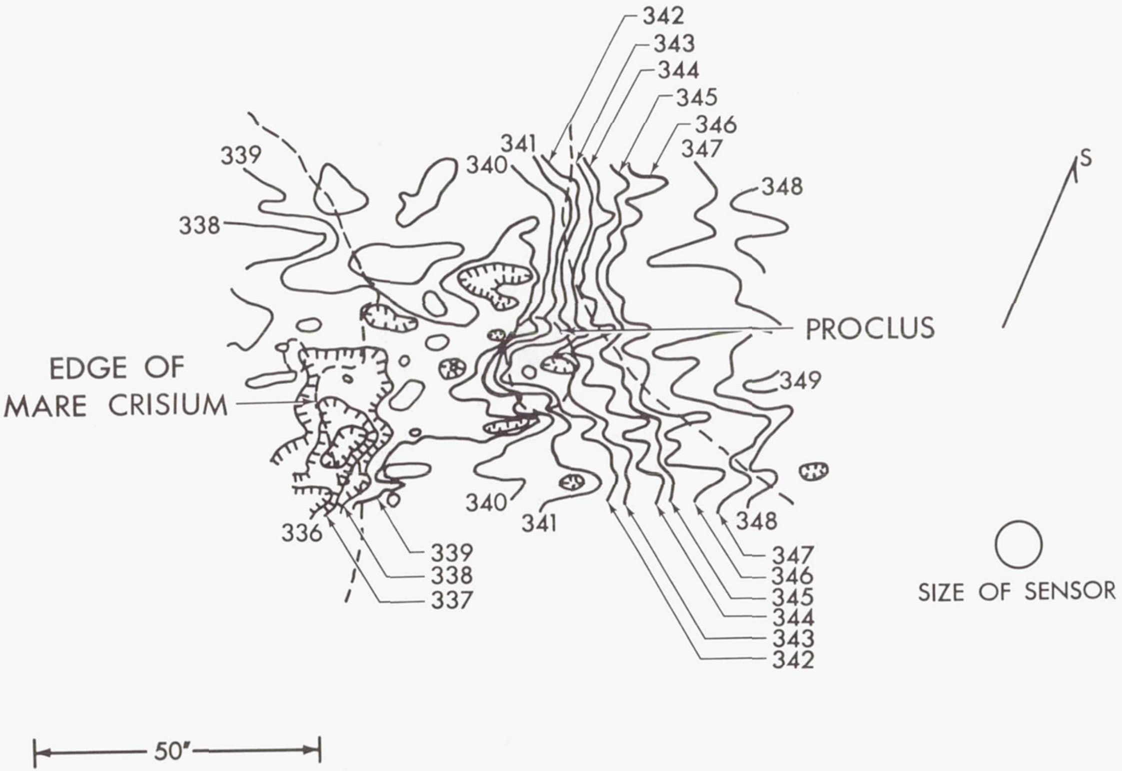
ISOTHERMS IN THE REGION OF TYCHO SEPTEMBER 5, 1960, 5:18 U. T.

Fig. 32



ISOTHERMS IN THE REGION OF TYCHO SEPTEMBER 6, 1960, 5:50 U.T.

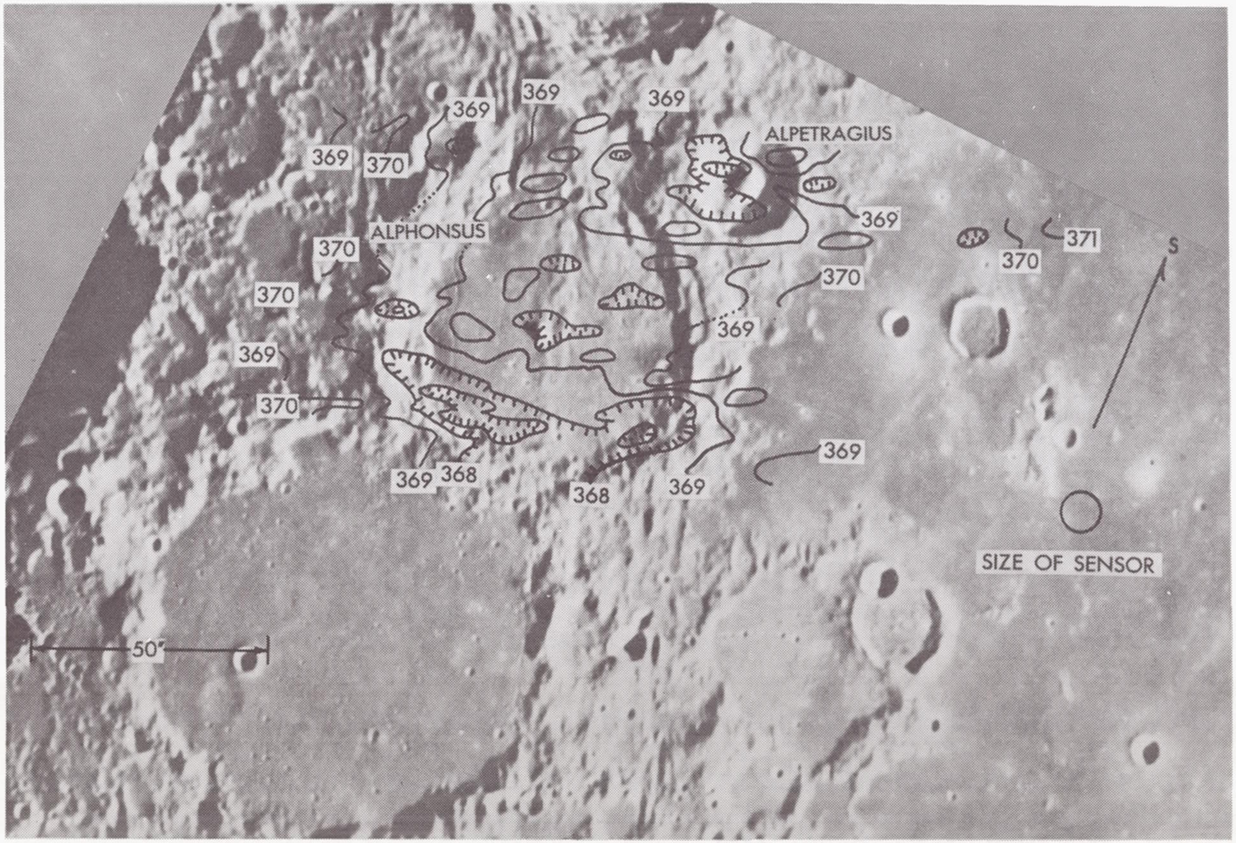
Fig. 33



ISOTHERMS IN THE REGION OF PROCLUS

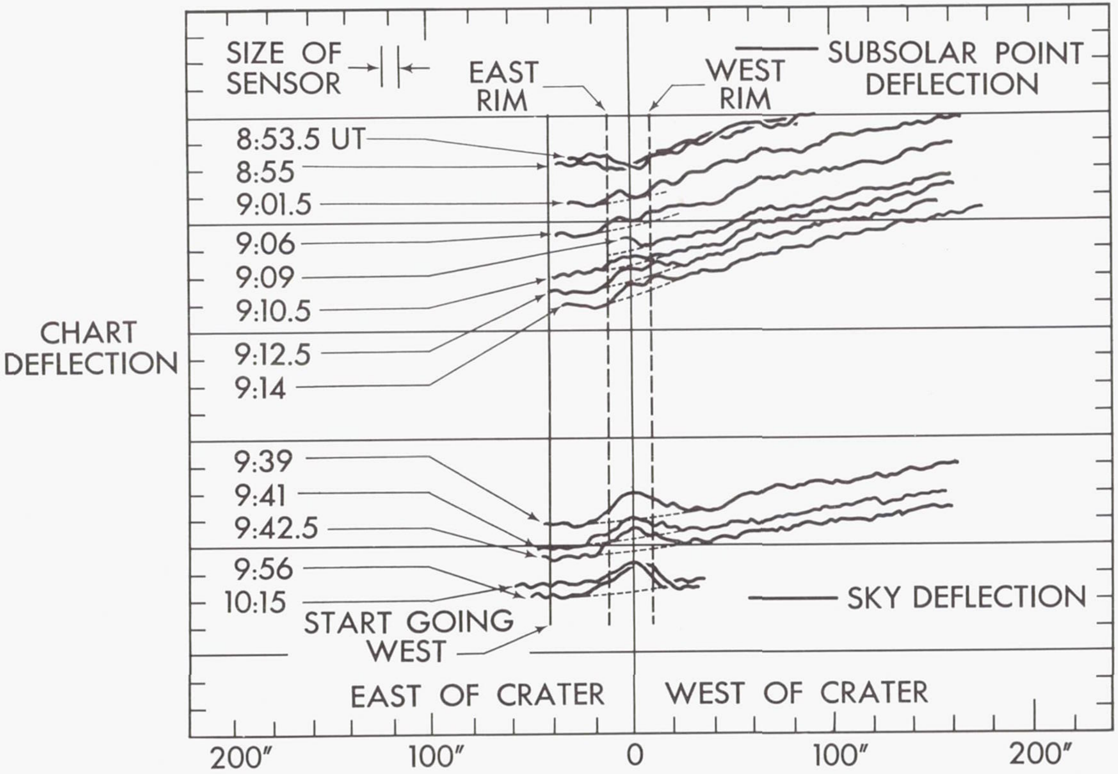
SEPT. 6, 1960, 7:54 U.T.

Fig. 34



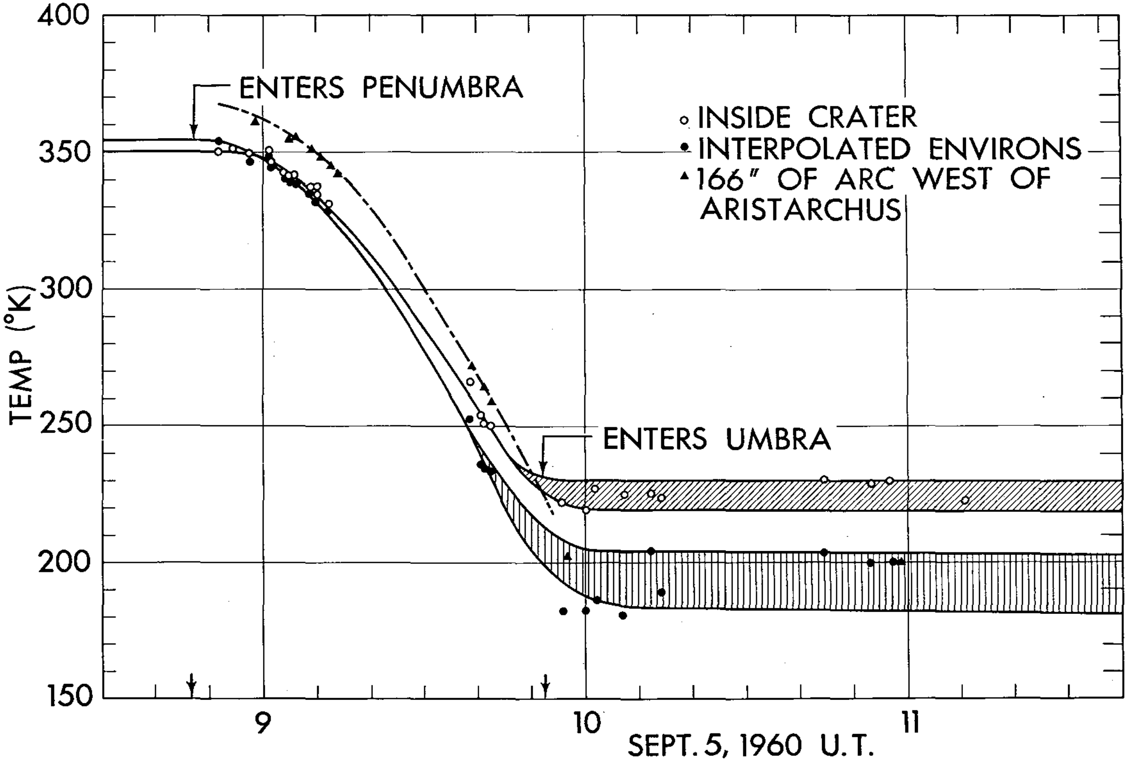
ISOTHERMS IN THE REGION OF ALPHONSUS SEPT. 5, 1960, 6:18 U.T.

Fig. 35



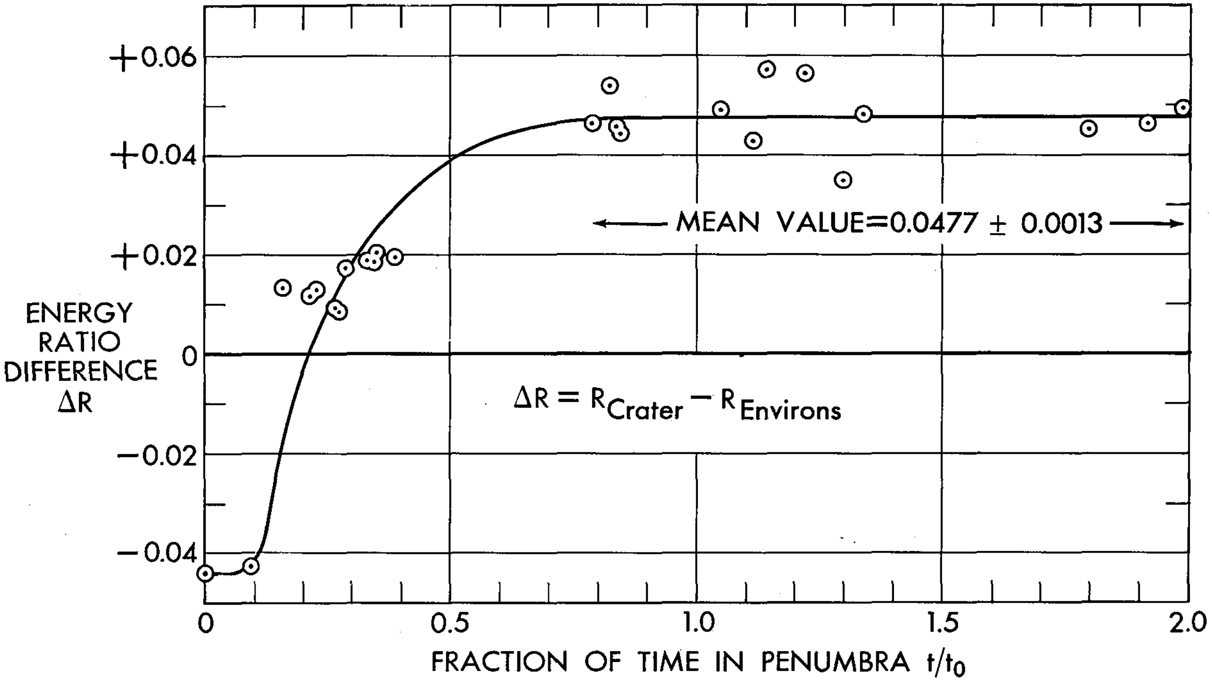
TEMPERATURE TRACES OVER ARISTARCHUS DURING ECLIPSE, SEPTEMBER 5, 1960

Fig. 36



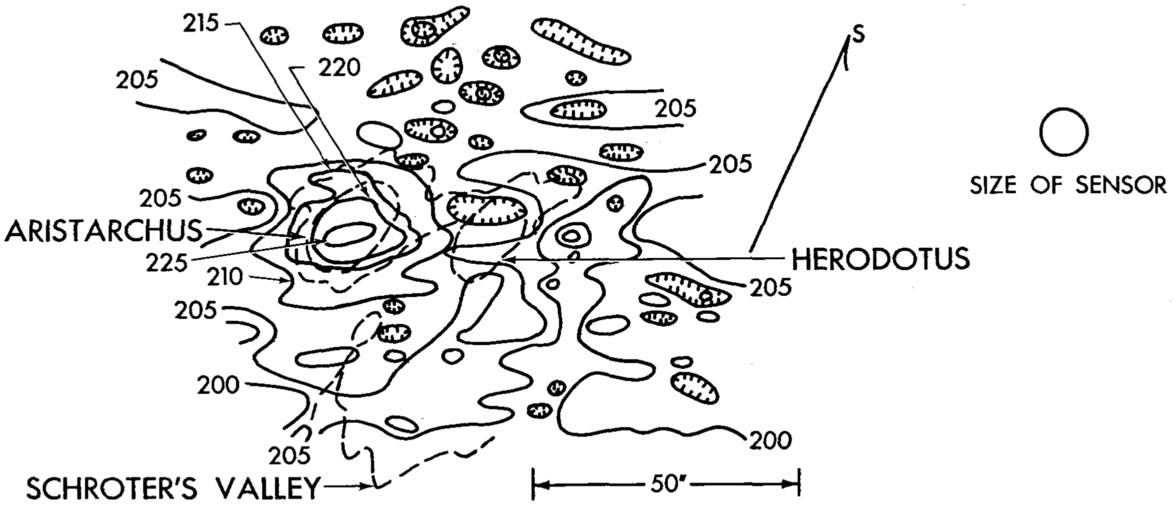
ECLIPSE COOLING CURVE FOR THE CRATER ARISTARCHUS AND ITS ENVIRONS.

Fig. 37



DIFFERENCE IN ENERGY RATIOS BETWEEN CRATER AND INTERPOLATED ENVIRONS
FOR ARISTARCHUS SEPTEMBER 5, 1960, ECLIPSE

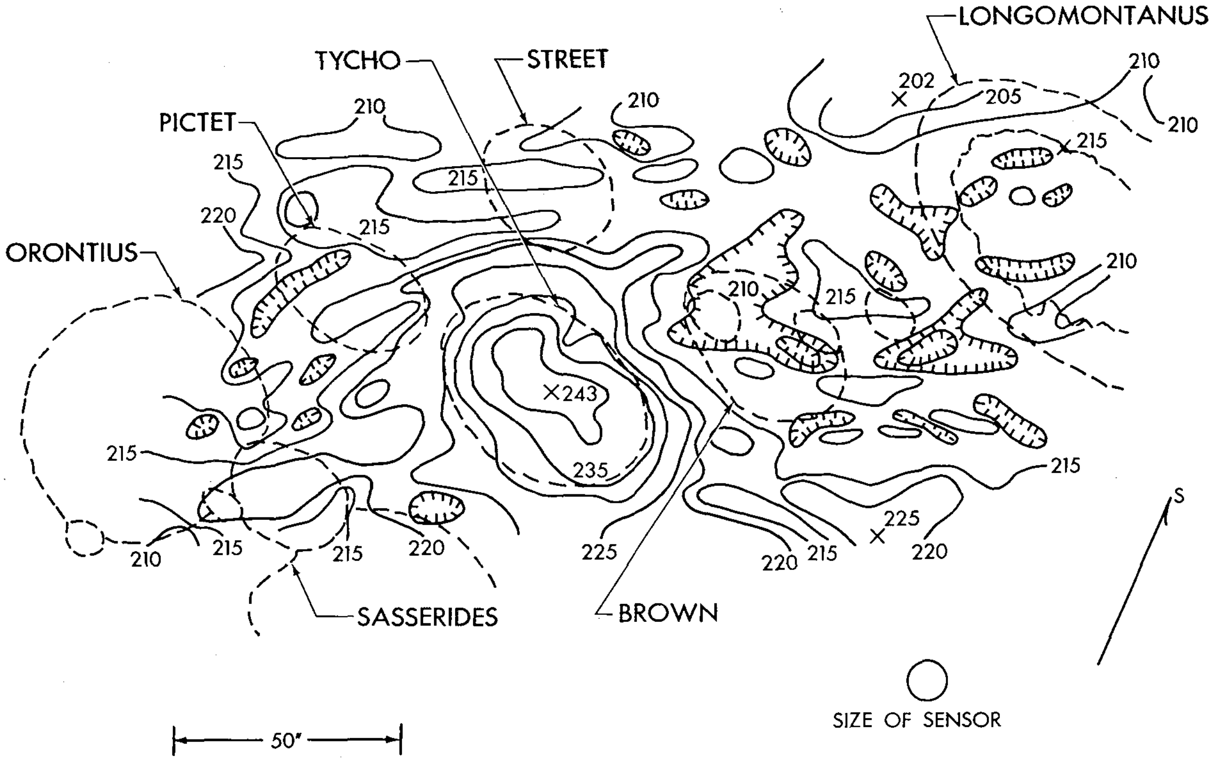
Fig. 38



ISOTHERMS IN THE REGION OF ARISTARCHUS DURING ECLIPSE

SEPT. 5, 1960 10:12 U.T.

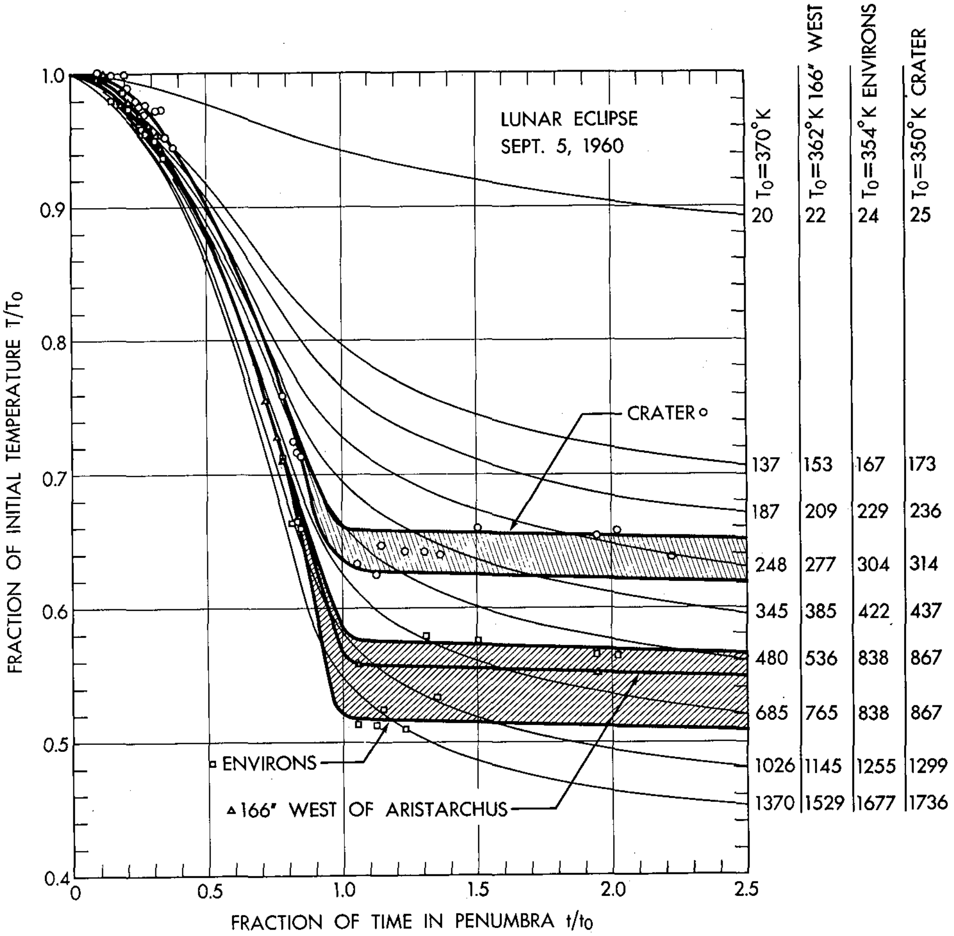
Fig. 39



ISOTHERMS IN THE REGION OF TYCHO DURING ECLIPSE

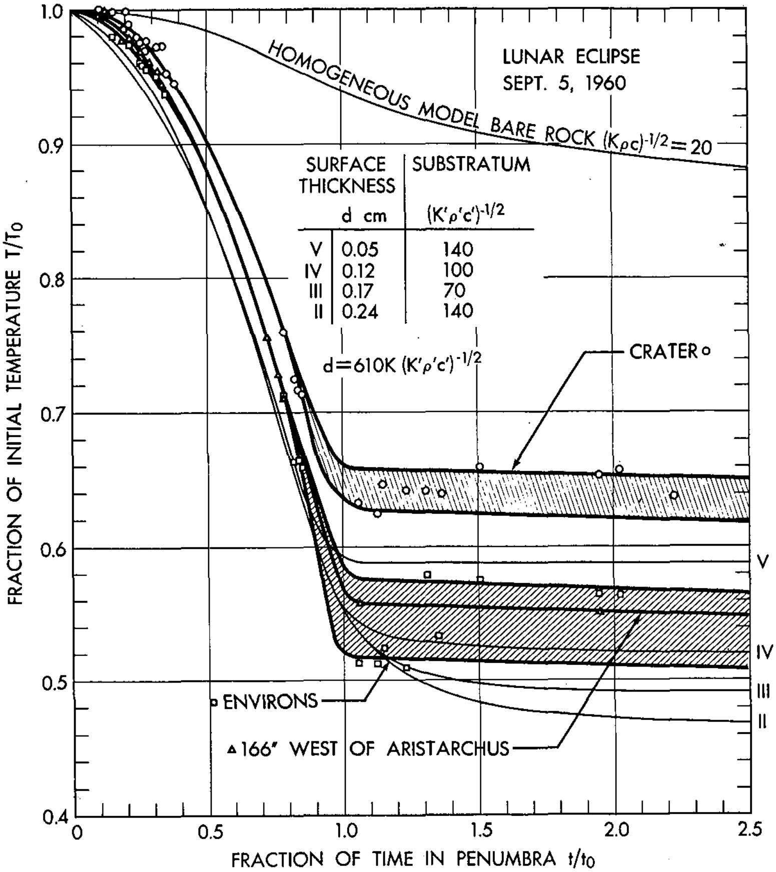
SEPT. 5, 1960, 10:34 U.T.

Fig. 40



NORMALIZED COOLING CURVES FOR ARISTARCHUS AND ITS ENVIRONS; EXPERIMENTAL VALUES AND THEORETICAL HOMOGENEOUS SURFACE WITH DIFFERENT $(K\rho c)^{-1/2}$ VALUES

Fig. 41



NORMALIZED COOLING CURVES FOR ARISTARCHUS AND ITS ENVIRONS; EXPERIMENTAL VALUES AND THEORETICAL TWO LAYER MODEL

Fig. 42

THE NATURE OF THE LUNAR SURFACE AND MAJOR STRUCTURAL FEATURES

BY

RALPH B. BALDWIN

OLIVER MACHINERY COMPANY

GRAND RAPIDS, MICHIGAN

THE NATURE OF THE LUNAR SURFACE AND MAJOR STRUCTURAL FEATURES

BY

RALPH B. BALDWIN

Much of the material to be presented in this paper is taken from my new book, The Measure of the Moon, to be published later this year and is given by courtesy of the University of Chicago Press.

Ever since the time of Galileo and his "optik tube," arguments have raged concerning the nature and origin of the craters of the moon and the great circular maria. At various times, diverse theories have been advanced, but at present most competent students of the subject are united in believing that it has been demonstrated that the circular lunar formations have been produced by a meteoritic impact mechanism. That is not to say that all details of the process are equally acceptable or that one cannot easily find an argument with some proponent of a form of volcanic genesis, but that the evidence now is almost overwhelmingly in favor of the meteoritic impact origin of many lunar structures.

The case is quite different when it comes to detailing the course of the history of the moon and its heat balance and its present conditions of internal heat. The nature of the lunar surface and the strengths of its materials are still considered to be subjects of major controversy.

It is my intent today to present certain observations, arguments, and conclusions which have a bearing on these matters. The solution of these problems could conceivably aid the space program of the United States in the sense that the range of conditions which will be met with on the moon may be more accurately defined.

Similarly, if certain theories of the moon can be rendered more probable by this analysis, it will be of aid to the "pure" scientists of many disciplines as contrasted with the "applied" scientists.

It is difficult to say when the moon was formed, for the process is continuing even today. As an arbitrary definition, the terminal phases of the moon's formation came when the moon reached approximately its present size. The latest evidence is that the crust of the earth is 4.5 to 4.6 billion years old and that meteorites and other objects in the solar system took their present forms about the same time in the past.

Several lines of evidence (1,2,3,4) have recently been advanced which suggest that the earth and moon were formed in a relatively short period after nucleogenesis, a period of the order of 3×10^8 years.

The terminal period of the formation of the moon's crust, as contrasted with the main building period was not extremely short. It must have stretched over a considerable period of time. Certain evidences of change in the processes occurring in crater formation and subsequent changes in crater shapes have been noted.

To put it rather briefly, the lunar craters may be divided into five classes. Those of Class 1, such as Tycho or Aristillus, are fresh and new appearing. The Class 2 craters, such as Clavius, have been considerably distorted by later craters which have appeared on the rim and basin. The Class 3 craters, as for example Maginus, are still more distorted by later appearing craters. Those of Class 4, such as the great unnamed crater east of Walter, are still more distorted. The Class 5 craters have been filled with a dark material.

Based on appearance only, the Class 1 craters are the newest, and the next three classes are progressively older.

The interesting thing is that the Class 2 craters are systematically shallower than the Class 1 craters, and the Class 3 and 4 craters are progressively even more shallow. Similarly, the rims become lower as we move from Class 1 to Class 4 craters.

However, the decrease in the crater volume is from 200 to 400 per cent greater at a minimum than the corresponding loss in volume of the rim. This effect is also progressive from Class 1 to Class 4.

Consequently, it is clear that erosion of the rim is not the cause of the decreased depth of craters with increasing age. This is emphasized when we realize that most of a crater rim slopes away from the pit, and hence only a portion of the eroded material could fall into the crater.

It is concluded that erosion cannot be the cause of the change of crater shape with increasing age, but that an isostatic adjustment which lowers the rim and raises the crater floor can explain the observed effects. The older the crater, the greater is the amount of isostatic adjustment which it has undergone. Conversely, the young Class 1 crater has been modified little, if any.

Urey (5) has given cogent reasons for believing that the moon and earth were built by accretion from cold planetesimals which had formed earlier from the primordial materials of the proto-earth.

Although there is considerable disagreement concerning the stages in the formation of the solar system before the cool planetesimals were formed, it is rather generally agreed that these small bodies did form and then were brought together to form the planets and the moon. There is a wide diversity of opinion concerning the history of the body of the moon after it reached its present size.

Yesterday Dr. Urey covered two sides of the cold moon - hot moon argument, but many of his publications (6-9) say that the moon was formed cold and remained cold, or at least solid, throughout all of its history. It has been gradually warming, due perhaps to radioactivity in its rocks, but never has become hot enough to liquify the lunar rocks. The dark areas he usually has felt to be lava, but at times he seems to lean toward Gold's deep dust hypothesis.

This paper was written before Dr. Ureys talk and will be presented as written, but I am glad to acknowledge his position that the moon could have been hot or even melted.

To explain the dark areas as lava flows, Urey has postulated that the impacts of large, low velocity planetesimals led to liquification of these objects rather than true explosions and this lava remained for the most part in the great impact craters, although in the case of the largest, Mare Imbrium, a great deal of overflow occured. The lava is thus of external origin and was deposited at the same time as the great circular maria were formed.

Kuiper's model (10) of the formation of the solar system also postulates that the moon was formed by planetesimals and was originally cold, but the radioactive heat, starting more than 5 billion years ago gradually warmed the moon until the impacts of the great planetesimals triggered a release of the subsurface lavas.

Gold (11) disinterred T. J. J. See's (12) old hypothesis that the dark areas were not lava but were thick layers of dust.

Baldwin (13) has pointed out that a considerable period of time elapsed between the impacts which formed Mare Imbrium and other similar structures, and the rise of the lava. The lava was postulated as coming from below.

It is certain that these four hypotheses are mutually exclusive.

Let us examine each hypothesis, starting with Gold's dust, for this is certainly the most unusual approach. Gold's solution to this problem rests on two assumptions.

> He says:
>
> "There are two reasons for departing from the lava hypothesis: firstly, because there are some severe obstacles to that interpretation connected with the distribution of subsequent impacts on the plains and the absence of a satisfactory time sequence interpretation; and secondly, because it may be unnecessary to invoke both processes if the impacts alone seem sufficient together with other effects which are known to be present or which must be inferred in any case." (11)

Gold's reasons for his hypothesis do not stand up under analysis.

It is quite apparent that the numbers of craters which were formed before the great dark areas developed are many times greater per unit area than the numbers of the post-mare craters. This observation simply places the maria as having been formed relatively, but not necessarily absolutely, late in the major sequence of events. It tells us that the crater-forming impacts had not ceased although the rate had markedly lessened. To test Gold's hypothesis, it is only necessary to show that the post-mare craters are, firstly, distributed at random on the maria, and secondly, that the numbers of these craters as functions of their sizes are similar for the various maria.

These two studies have already been made. Arthur (14) has shown beyond doubt that the craters within the borders of Mare Imbrium are distributed at random. This confirms their non-lunar origin as opposed to a crater-forming process operating from within. Shoemaker and Hackman (15) have made counts of post-mare craters on almost all of the major lunar dark areas and find that they are distributed with amazingly similar frequencies. With the possible exception of the dark matter in Mare Crisium, all other areas show the same frequency pattern. The great dark areas, judging from this test, were all formed at essentially the same time. Even Mare Crisium, which shows a slight deficit of craters under four miles in diameter, departs from the norm by less than a factor of two. The craters of Mare Crisium, coming so close to the limb as it does, are exceedingly difficult to see on many photographs. Close examination of several such pictures yielded counts similar to that of Shoemaker and Hackman, but a study under a six power glass of a contact glass positive from the original negative of a Lick Observatory photograph of June 2, 1938, when the moon was at age 4.6 days, disclosed 54 craters under the 4-mile-diameter limit. These pits in general were hard to see, but are considered real in every case. With this change, Shoemaker's and Hackman's value of 24.2 post-mare craters per 10^5 km^2 becomes 38.8 such craters. This value is close to the average of the other dark areas and indicates that the dark part of Mare Crisium was formed at about the same time as the others.

These observations eliminate Gold's first assumption. The observations lead to a consistent sequence of events on a relative time scale, and there are no difficulties concerning the distribution of the post-mare craters.

As for Gold's second reason, namely that impacts alone, or together with other effects known or inferred to be present, are sufficient to account for the dark areas, we can subject this to check also.

There is not the slightest question but that the moon is covered by a shallow layer of dust. This is not Gold's point. His suggestion is that various erosional forces have acted on the entire surface to produce dust. He then proposes a mechanism whereby the dust can be transported over the lunar surface, and finally he postulates that the dark areas are the places where the dust has accumulated and that the dust there has tremendous depth.

All authorities agree that craters are of widely differing ages. It has often been assumed that the older a crater is, the more chance there has been for another impact to deform the older pit. The more deformed pits are statistically more shallow and their rims average lower than do the Class 1, or new appearing craters.

Gold assumes that these variations in dimensions are due exclusively or dominantly to erosion and then looks for the eroded material, or dust, and finds it in the dark regions. He states:

> "If erosion is assumed to be the cause of the rounding of the features of old craters, as discussed earlier, then it is necessary to find the eroded material. This cannot reasonably be expected to have all left the Moon entirely. If an estimate is made of the amounts missing from the rims of all the many overlapping old craters, then this cannot come to less than the equivalent of a 300 foot depth if it were distributed over the entire surface, and probably a great deal more. Though on the lava flow hypothesis it could be supposed that the eroded material was so distributed that it was all drowned, the question would remain as to the whereabouts of the material eroded after the supposed lava flooding. This is still a substantial depth, at any rate deeper by a large factor than is necessary to make it optically opaque. Even if the erosion process were not to take the erosion product any further than is strictly required by the change of shapes, there would still have to be large areas of the flat plains covered by that material. This

> is clearest in the cases of flat-bottomed craters whose erosion on the inner crater walls must be assumed to have covered over at least part of the flat bottom, yet the colour there even close to the sharp edge is the same as that of the flat surfaces elsewhere. A more diffuse distribution of the eroded material would only strengthen this argument, so it must be concluded that the darker shade is that of eroded material even if it comes from lighter rock." (11)

Certain comments are pertinent. In essentially all rocky materials, including those which have been considered as possibly present on the moon, fragmented material is the same color as the original, but powdered rock such as would be produced by meteoritic impact is considerably lighter than the rock from which it was formed. The rays from Tycho in the bright uplands and Copernicus in the darker areas are all considerably lighter than the parent rock.

This is opposite to the Gold dust theory. There the dust is the dark material and the original rock is light.

Gold also does not distinguish between the colors of the bottoms of the great walled plains. Clavius does not show a different color on its interior plain from the walls, but Schickard, for example, shows two very dark interior areas. Most of the great craters in the upland regions do not possess dark centers. Some do, particularly the ones in the lower areas near the maria. The existence of the great flat floors in these old craters cannot be attributed to erosion and infilling from the walls.

This conclusion is buttressed by the earlier discussion where it was shown that the maximum possible loss of materials from the crater rims was from 200 to 400 per cent too small at a minimum to account for the flat bottoms of the old and large craters. From both points of view it must be concluded that the tremendous production of dust on the moon expected by Gold simply never occurred.

Some amount of dust must be expected on every part of the moon. With the known rates of infalls of meteorites, poorly defined though they are, this is unavoidable. All measures of the heat absorbed and emitted by the moon confirm it, but they merely show that a thin surface layer is present. Dust produced *in situ* by other than impacts might well be the color of the adjacent rock.

The fact that rays exist around many of the post-mare craters in not consistent with a large and continuing dust production. There are many hundreds of large and small ray craters scattered essentially at random over the moon's disk. There is no reason to assume that they are all very recent; they form too large a percentage of the post-mare craters. If a ray crater is produced on the dark material and the dark material is composed of dust, why are the rays lighter than the dust?

From satellite measurements, LaGow and Alexander (16), Dubin (17), and Nazarova (18,19) have found that the amount of meteoritic material entering the earth's atmosphere each day is of the order of 10^9 gm

($4x10^5$ tons). It is not yet clear how much of this material actually enters the atmosphere and how much is trapped in the higher satellite regions.

During major meteoritic showers, the rate of influx of particles increases by factors of from 10^1 to 10^4. The last named figure is unusual.

Whipple (20-22) showed that the number of particles falls off with increasing height above the earth by a factor of 10^2 to 10^4 toward the density of the cloud which produces the Zodiacal Light.

The above data correspond to a covering of dust on the earth of one centimeter thickness each 40,000,000 years if we assume an average density of such particles to be 3 and also that the above material does enter the earth's atmosphere as postulated.

The fall-off in particle density with increasing height above the earth leads to the conclusion that the moon would accumulate a layer one centimeter thick in 4,000,000,000 years or more.

Where the exact rate lies is still not certain, but the evidence of the ray craters suggests that the moon does not accumulate a great deal of dust from space.

The number of large ray craters, including Copernicus, Kepler, Aristarchus, Tycho and the numerous small craters with rays or haloes must have been accumulated over many hundreds of millions of years. All observations of meteoritic impacts as a function of crater diameter demand that the majority of ray craters cannot have been produced recently. The rays are often, but not always, present at craters where ages are measured in hundreds of millions of years and absent at craters whose ages are necessarily of the order of several billion years.

From all points of view then, erosion must occur on the moon, but is is of minor importance in explaining the present appearance of the ancient craters and the thickness of the maria.

Gold has reached an interesting conclusion from his premise that the maria are dust. The area of the bright highlands is greater than that of the dark lowlands. More than half of the dust must have been produced in the bright areas and then transported by some mechanism to the region of the maria. He then searched for a mechanism which would allow the eroded material, fine dust, to "flow" like a fluid over tiny gradients.

> "The requirement is for the average speed of flow not to diminish appreciably with the angle of the slope until a very small angle is reached. It should not be a flow restricted by the equivalent of viscosity in a liquid, for this would leave slight gradients persisting for long and remove steep gradients very quickly. The type of flow required would be one where at any instant a thin layer on the surface behaves like a non-viscous liquid whilst the remainder underneath remains stiff. This would then allow steep slopes to persist but would assure that the deposits of dust possess a flat surface. Such a 'fluidization' of a surface layer would have to be a process resulting from an external energy source, providing an agitation for the dust particles on or near the surface." (11)

Gold mentions several mechanisms which might "fluidize" the dust. Among these are the effects of an evaporation and condensation cycle of a suitable vapor, the effects of micrometeorites and the effect of electrostatic forces.

In most of his writings, he seems to lean most heavily on the last named. Such forces will arise either as a consequence of the photo-emission of electrons from the surface due to the ultra-violet light of the sun, or they could arise in some larger electrical process connected with solar events of the type that cause aurorae and magnetic storms on the earth.

The process of photo-emission in the absence of an atmosphere, and especially of moisture, will result in an erratic distribution of charge on the irradiated surface. The surface should be a good insulator. He states:

> "While the average positive charge will be indadequate to lift particles off, it is not clear that the chance distributions in small localities could not do it.--If particles were frequently dislodged by electrostatic forces, then again a net flow would result on the surface in accordance with the requirement." (11)

It is by some such process, a form of Goldian Movement, that the maria are supposed to have been formed.

If Gold is correct, then we have a choice. Either almost all of the dust was produced in the past and now has been transported onto the maria, or the dust changes color when it crosses the boundaries of the maria. Gold postulates a mechanism whereby the dust darkens with age.

Recently I published a new contour map of the moon (23), derived from 696 absolute height determinations.

This new contour map shows that at least half of the bright upland area drains not toward the maria, but toward the limb, and yet the limb is not dark.

The magnificent crater Theophilus is a post-mare crater. Under certain angles of illumination remnant rays may be seen spread on Mare Nectaris and Mare Tranquillitatis. Presumably the faintness of the rays means that Theophilus is quite old, approaching the maria in age.

The contour of this pit is accurately known from the measures made by McMath, Petrie and Sawyer (24) on McMath-Hulbert motion pictures. The bottom is everywhere curved and the inside of this crater shows a far greater slope than is usually to be found on the lunar surface. At the crater origin the materials of the walls must have been highly fragmented or pulverized so that the formation of dust should be greater than would be normal on the lunar surface outside of craters. Since Theophilus was formed there has been no appreciable production or transportation of dust in a place where conditions should have been highly favorable to Gold's mechanism if it existed as a major force.

The great rille systems in general mark the edges of the maria. Migrating dust, to reach the central regions of the maria, would have to cross these great trenches. Certainly the dust could not climb back out, once it has fallen in. The actual volume of the depressions of the rilles is negligible compared to the volume of the materials forming the maria. It must be very much less than one per cent. Because the rilles are not filled up, they must have been produced by a still active moon in the last few million years, or else the erosional forces have effectively ceased, or the Goldian Movement has stopped during the period since the rilles were formed.

Conversely, there is a great deal of evidence for a lava nature for the maria, only two bits of which will be mentioned here.

There is specific evidence of melting and erosion by very hot lavas. East of Alpetragius is a slightly smaller ring and only the highest portions of that ring project above the dark area. These projections are low and generally rounded. The following craters, to name but a few, show melting and erosive action on the seaward side: Fracastorius, Letronne, Doppelmayer and Posidonius. A great many other similar cases could be pointed out near the shores of any mare.

No distinction in principle can be made among Arzachel, where a moderate crater fill is evident, Ptolemaeus, where the crater is filled apparently to ground level, and Wargentin, which is filled to the brim. Lava forced from below can easily account for these examples. It would be unusual to find a mechanism which would allow humdreds of cubic miles of dust to climb a wall some thousands of feet high just to fill one crater.

Only one conclusion can be drawn. The dark areas are not dust and never have been dust. They are great lava flows and have a thin wash of dust over the surface. This dust has had three sources, micro-meteoritic material from space, ejected rock flour from distant craters, and dust generated from the local dark solidified lavas. The last is clearly the predominant type of dust.

Were it not so, we would not be able to distinguish the maria by color. The wide variations in color which appear on the maria in large and small areas give added evidence that no mass migration of dust has occured.

Gold's hypothesis does not satisfy the observations.

Urey's model for the moon apecifies that it was built by accretion from cold particles which have approximately the chemical composition of chondritic meteorites. The sequence of events before this is as follows:

> "(1) A disc nebula at temperatures so low that hydrogen is partially condensed to the solid state existed, though no process for its formation is discussed. (2) This broke up into masses of lunar size together with the quota of cosmic gases. (3) These objects collided later to produce very fine solid material, and part of this material and the gases were lost to space owing to particle radiation from the sun. (4) Subsequently the residue accumulated into the planets and the immediate parents of the meteorites." (25)

It is after this point that Urey's ideas and mine part company. He feels that the moon became melted throughout and that this conclusion is demanded by the theory of conduction of heat through a body as large as the moon. If the moon were ever liquid, the inner regions would still be liquid or essentially at the melting point, and therefore isostatic adjustments would quickly eliminate any major surface irregularities. My new contour map of the moon, the excellent work by Watts at the Naval Observatory on the variations in height of the limb, and the existence of a dynamical bulge all point out that the moon is not spherical and that surface irregularities within some hundreds of miles often amount to several miles. If this is so and the hot moon could not cool down, then such variations in height could not exist, but they do exist and therefore the moon is not hot. If it is not hot now, it never could have been hot. So runs Urey's argument.

Urey has not minimized the hot moon theory without thoroughly considering it. He has analyzed possible methods by which the moon could be heated. Among these are short-lived and long-lived radioactive elements, gravitational energy and chemical energies. He has argued against the hot moon idea by what seem to be three misinterpretations of the evidence. He has assumed that a solid crustal layer could not exist over a liquid and that the surface irregularities and the bulge could not be supported by a moon with a weak center. He has rejected the earlier views that the lavas came from below, and substituted the idea that the lavas were actually lavas but that they resulted from the melting of the low-velocity objects which struck the moon to form the circular maria.

Let us examine the last process first.

In the laboratory we have projected missiles of various compositions, including stone, against many targets (26-29). Heat is produced, and dust, and explosive energy, but never an appreciable amount of liquid. Poulter (30) has accelerated two slabs of anorthosite rock against each other at a relative velocity of about 2.4 km/sec and found that they were reduced to a moderately find power but were not melted. All of the material could not have been melted at this velocity of impact.

At terrestrial meteoritic craters, most of which were produced by low-velocity impacts of one to fifteen miles per second, there is little evidence of extensive melting. Even at the Wabar craters there is only a relatively little silica glass, a far cry from fluid lava.

Consequently, there is no reason to postulate that the impacts on the moon produced the observed hundreds of thousands of cubic miles of lava. The smaller, but still violent, impacts which formed the normal craters did not liquify rocks. The only possibility is that the very low velocity of impact and the large sizes of the objects which formed Mare Imbrium and others like it could have created conditions which would have melted the missiles and some of the lunar landscape rather than vaporizing them.

But, alas, even this possibility is denied. Without exception, it can be shown that the great circular maria were formed dry and that the lavas came considerably later.

In the Mare Imbrium structure there are three large craters which definitely were formed after the great crater and before the lavas came. They are Sinus Iridum, Archimedes and Plato. Sinus Iridum was formed after Imbrium, it developed at the wall of the larger object and eliminated the wall. The north half of Sinus Iridum is beautifully developed in the high rim area of Mare Imbrium. The southern half of the crater is buried in the impact area of the mare. The lost rainbow is not lost; it is merely misplaced, for it exists to this day under the lavas. Its outline is clearly shown for most of its length by wrinkles in the dark floor of Mare Imbrium.

Plato was also formed on the northern wall of an existing Mare Imbrium but it does not overlap the edge. It has formed a projecting rim into the sea.

Archimedes was formed on the shelf area outside of the impact region of the larger sea.

All three of these great craters are now flooded with frozen lava. There is no sign on the dark floor of Mare Imbrium that explosions produced them. These signs are buried. None of them could have been in its present position and come through the explosion that produced Imbrium and still escaped unscatched. They clearly are post Imbrium and pre-lava.

Five other similar but smaller craters can be detected. Wallace, on the shelf in the south, is a mere partial ring projecting above the lavas. Just north of Aristillus is a drowned ring, and Cassini is similarly placed in time. Sinus Gay Lussac is a flooded pit similar to the smaller Archimedes M.

It would be stretching the long arm of coincidence too far to postulate that these eight objects were all formed in the short period when the lava sheet was still liquid, a time to be measured in years at the most, when no others as large as four of the eight were formed on Mare Imbrium in the billions of years since.

The much smaller Mare Humorum also shows the same time sequence. First the great dry crater was formed. Then several smaller craters, including Doppelmayer, Lee and the larger crater it overlaps, Vitello, Hippalus,Gassendi, and some smaller craters on the east shore. Then the lavas came and drowned and melted and eroded the newer pits. All are now in a very sad state of repair.

Mare Serenitatis is adjacent to Mare Imbrium. It is older than Imbrium because the flying fragments from the larger explosion have nearly destroyed the Haemus Mountains bordering Serenitatis and forced an apparent alignment of them toward the impact area of Mare Imbrium. Some of the Haemus grooves disappear beneath the dark layers of Serenitatis, yet nowhere on the latter surface do we find any evidence of action from the Imbrium explosion.

Along the western shore the two prime examples of craters older than the lava but younger than the formation of Serenitatis are LeMonnier and Posidonius. At least three other nearby craters are similar.

Still farther to the west lies Mare Crisium. The craters Yerkes, Lick, Cooke, and several others on the floor, all postdate the Crisium impact and predate the lavas.

There is one, and there are suggestions of other such datable craters, in the limb sea, Mare Humboltianum.

Mare Nectaris completes the roster of major, easily visible circular maria. Fracastorius is the prime example of the craters of this special type, but Bohnenberger to the west and Beaumont to the east are of similar type.

All in all, there are 55 craters of this special type.

A histogram based on Young's (31) crater counts gives the numbers of craters in 3-km-diameter steps from 15 to 135 km. There are 1281 of these pits in the part of the disk lying within 70° of the center, and it includes named craters outside these limits. Cumulative totals of craters smaller than 135 km in diameter have been developed from this histogram. Similar cumulative totals were found for the 55 postcircular maria-prelava craters, omitting Sinus Iridum, which is outside of Young's limits. At each diameter interval, the ratio of the two cumulative totals was calculated. The data are consistent for craters larger than 42 km. The average ratio was found to be 19 to 1 in the sense that in all size ranges above 42 km, the Young list contained roughly 19 times as many craters as there are of our special type.

For craters smaller than 42 km, the numbers in the circular maria projecting above the lavas become steadily less relative to Young's data. This probably is due to the smaller craters' becoming completely covered by the lava in the thicker central parts of the flow and appearing only where the lava is relatively thin.

Fielder (32) states that Young's data should be multiplied by 3 to be representative of the entire lunar surface. Therefore, 19 x 3 = 57, or the special class craters are 57 times more rare than the total numbers of craters in these size ranges.

But the area covered by the circular maria totals approximately 1,620,000 km^2, while the area of the visible half of the moon is 19,000,000 km^2. The ratio is 11.7 to one. Therefore, the number of craters produced between the formation of the dry circular maria and the coming of the lava, relative to the number of all lunar craters of comparable sizes, is in the ratio of $11.7/57 \doteq 0.2$; or the observed numbers of all craters per unit area is 5 times higher than those within the circular maria. On the assumption that this rate was constant from the time of the earliest observable crater to the coming of the lava, the period of time between the formation of the dry circular maria and the coming of the lava was at least one-fifth as long as the total period on the average.

The lava flows divide the crater-forming period into two distinct times. In the earlier period, the rate of infall was heavy. After the lavas, the rate of crater formation was very slow. Because of this, the post-mare craters have been neglected in this computation.

Presumably the rate of infall declined continually from the origin of the moon to the present rate; and, therefore, the period of time during which the great circular maria were dry may have been substantially longer than was estimated. In face, the ratio must be considerably underestimated, for there are no datable craters observed in the central impact areas of any of the great circular maria, except Mare Nectaris and Mare Crisium. Several craters of this type project partially into the impact area of Mare Humorum.

We cannot give an absolute dating to the maria by this method, but all evidence suggests that the lavas came so late in the moon's history that there is no longer a need for us to postulate a sudden cessation of infalls at the time the dark areas were formed.

It is known from the crater counts of Shoemaker and Hackman that all of the dark maria are essentially of the same age. The great circular maria craters themselves were undoubtedly formed at rather widely differing times. Humorum and Nectaris seem to be the oldest. Imbrium definetely is the youngest. Based on the assumption of a continous decline in the rate of infalls to the moon, it is suggested that the lava flows may be hundreds of millions of years, or ever billions of years, younger than the earliest observable surface markings.

The rough dating of the lava flows is in better agreement with reasonable thermal histories of the moon such as have been developed by MacDonald. (33)

While the method is rough, it is conclusive. Urey's hypothesis that the lavas were formed by the impacts is not sustained.

The lavas came from the body of the moon considerably after the great circular maria craters were produced and came nearly at the end of the major period of impacts of meteorites with the moon's surface.

Inasmuch as Gold's dust hypothesis must be discarded and Urey's melting of the planetesimals on impact does not fit the observations, we must reach either of two conclusions. Either the dark areas are the result of some completely unknown or unrecognized process, or else the moon's body did produce the outpourings of lava. If the latter choice is made, and frankly I see no other realistic choice, then we must admit that the observations demand that the moon at least once became hot enough to melt at least a portion of its body.

All theories of the history and structure of the moon must then be in harmony with these observations. Those which do not permit the existence of liquid lavas in the moon well after the formation of the moon's crust must be discarded. The theories of the heat balance and the cooling of the moon need revision.

In 1954, Kuiper (10) presented his views on the early history of the moon and on its structure. The essence of his argument is that the moon had once been melted completely by long-lived radioactive heating and that the pre-mare craters were produced by left-over fragments from the formation of the earth and moon and that the dark areas were outpourings of lava triggered by the impacts which produced the great circular maria.

Urey (6) and Gilbert (34) have shown that the moon collected from cold objects. Urey has given theoretical reasons why the moon could never have melted completely. Kuiper has pushed the origin of the moon back to possibly 6×10^9 years in order to permit the melting to be produced from the increased amounts of thorium, potassium, and the two uraniums which could have been present then.

Both ideas cannot be right. Either the moon melted or it did not. Later in this paper, additional evidence will be cited to indicate that the moon did melt, but Kuiper's interpretation of the subsequent events is not completely in accord with the observations.

Both Urey and Kuiper have associated the great lava flows with the collisions which produced the circular maria. Urey found the lava to be from the body of the planetoid. Kuiper specified that the moon was liquid below a thin crust, perhaps 16 km thick, composed of uncompacted accreted material. Urey has correctly shown that this thin crust could not be stable over liquid rock of lesser density.

The same arguments just used against Urey's hypohesis are valid against Kuiper's. The crust of the moon was hard and thick when the giant circular maria were formed. These tremendous craters were formed dry. There was no extensive lava produced from the colliding body. There was no sudden release of liquid rock from below. The magmas did finally appear in these areas but they came much later.

If this is so, the lavas came from the body of the moon which must have been at least partially melted. If the body of the moon became partially melted, it was and is still hot at some unknown distance below the surface. If the moon was and is hot, we must account for the observed fact that there are real variations in height of the surface. A very real strength of the outer layers is indicated.

The bringing together of these apparently inconsistent observations and theories is the task to which we now set ourself.

We now have clear evidence from the changes in form of the ancient craters that the major period of crater formation on the moon covered an extended period of time, a period which is of unknown length but which was long enough to permit significant changes in the strengths of the lunar rocks to occur. The terminal phases of the moon's accretion marked the beginning of this era, and the lava flows mark its end or at least they are reasonably close to its end.

In this early phase of the moon's history, the surface layers were hard and fairly thick. Craters of Class 1 could be formed any time in this period. After they were formed, later impacts occured which allow us to date the age of a crater on a relative scale based on the number of later craters superimposed.

In this period, the older the crater, the more it has been distorted by an isostatic adjustment.

The conclusion is drawn that in the terminal phases of the building of the moon, the moon had developed a hard crust but that the interior was hot enough to weaken the regions of the moon corresponding to the mantle of the earth. Isostatic adjustment then occurred on the surface.

The amount of isostatic adjustment observed was greater at the largest craters and least at the small craters. It was greatest at the oldest craters and grew progressively less as time passed, so that the latest pre-mare craters, those of Class 2, show only moderate modification of form, and the post-mare Class 1 craters are distorted very little if at all. The crust was becoming thicker and more rigid during this period.

Inasmuch as the relief of load on the small scale of the craters could trigger an isostatic adjustment, the entire moon must have been able to adjust its form to exterior influences; and hence, during this period, the moon was close to the earth and was tidally distorted, for even to this day it shows a bulge.

As time went on, after the terminal period of the building of the moon, the depth to which the moon was hard and rigid became greater and greater. This implies that the outer layers of the crust and upper mantle were becoming cooler. At some indefinite period after the maria developed, the local isostatic adjustment ceased, and the later craters, few in number, remained in Class 1.

The new contour map is definitive in showing that the lava-covered areas are, without exception, low. They average about two miles deeper than the uplands. In all cases, the places where the lava are thickest are the lowest spots relative to a sphere. The lava flows are all of the same general age, and this is shown by crater counts.

Two possibilities exist. The surface of the moon could have had the observed contours and the lavas came from below and filled the lowlands, or the lavas came from below and filled whatever low spots then existed and formed much of the great basin system by isostatic adjustments due to the superimposed load of dense lava. The isostatic adjustments forced the shore areas of the maria to downwarp.

Inasmuch as the last pre-mare craters, those of Class 2, occurred when the moon could still adjust isostatically, the probabilities are very great that the lava flows did cause similar adjustments in height to occur. Since isostatic adjustments could still happen, the moon was still close to the earth at the time the maria appeared. Otherwise it would not now show a bulge. This is also apparent when we note that the maria are not distributed around the limb. We would expect them to be close to the limb, if the moon did not possess an equipotential surface when they were formed. The maria appear in widely separated regions of the moon and give no evidence of thicker lava on the sides away from the center of the disk.

When the maria developed, the moon had become considerably more rigid in its outer layers than earlier but was still incapable of maintaining a bulge such as we observe today. The earth was responsible for holding it up at that time. Therefore, the moon became more rigid after the development of the maria.

If we determine the shape of the moon from the available height measures on the bright parts of the face and of the limb which are distant from the mountains and the downwarping of the maria, we find that it is a triaxial ellipsoid. It is in good agreement with the theoretical bulge which would be produced by the tidal effects of the earth and the centrifugal forces of the moon's motions, provided that the earth and moon were only about 69,000 miles apart when the outer layers solidified to the extent that the moon could no longer adjust in shape as it receded from the earth. Of course, it is possible that the isostatic adjustments in the regions of the lava flows caused certain compensatory adjustments in the uplands, but this does not seem to be a major effect.

Inasmuch as the moon's outer layers solidified a very long time ago and the bulge still exists, it implies that the viscosity of the moon's upper mantle is higher than Haskell's theory would demand.

MacDonald and Urey have calculated from various assumptions what the heat balance of the moon might be.

It is unknown how much radioactivity there is in lunar rock. It is also unknown what the initial temperature of the moon was and how it varied in temperature throughout.

In spite of differing assumptions, the results are quite similar and informative. Unless the radioactivity were markedly less than that determined from the earth and from chondritic meteorites, the center of the moon is now hot and above the melting point of iron. Probably, then, the moon has a liquid metallic core which is considerably smaller in proportion than the earth's core.

If the original temperature were uniform and 600°C., then the center is now very hot, and the moon might be melted out to within 500 km of the surface, although probably it is now completely solid except for a liquid metallic core. It would be very difficult to devise a model of the moon which is cool now in the interior.

Various sources of energy could have been effective in raising the temperature in these early stages.

As the moon accreted, some of the energy of the infalling bodies could remain with the moon. The rate of accretion is important here.

Chemical energies could contribute some heat.

Adiabatic compression undoubtedly added its share of heat.

Short-lived radioactive nuclides, now extinct, may have been quite effective in raising the central temperature.

Long-lived radioactive nuclides added their portion, but their major effects would be slow in appearing.

The sun may have been very much brighter in this period.

When all of these effects are added together, it does not seem possible that the moon began as a cold body. It very probably was warm or hot throughout, and very early in the moon's history it became completely melted except perhaps for a thin sintered layer of less dense material which floated on the surface of the liquid.

It is also probable that the liquid interior was convective. If so, heat was rapidly translated from inside a core of reasonably uniform temperature to the outer layers where it became lost to space. This loss of heat could not have been replaced in a short period of time, as adiabatic heating, accretion energy, chemical action, and short-lived nuclides could not add much new heat in the period of rapid loss of heat.

As heat became lost, the average temperature of the liquid declined, and selective crystallization occurred. Some molten iron decended to the center, but the most ultrabasic silicates would crystallize out first and solidification would develop from the center outward. The great part of the moon, judging from the earth, would be ultrabasic rock with a thin wash of basalt on top of the olivine and possibly some of the more acid rocks above that. These rocks would presumably be beneath a layer of mixed composition sintered by the heat, and there probably was a thin layer of unconsolidated rubble above that. This is the layer which appears in the continental area. If the moon were ever completely melted, the uplands should be acidic in nature or possibly basaltic if there were not sufficient time to permit a more complete differentiation. If the outer layers did not become melted, they should be more like chondritic meteorites in composition. In either case, the light-colored surface materials are distinctly different in density and composition from the lower levels and from the lava flows. The moon has been chemically differentiated.

Under these conditions, the moon would have become a solid in a very few thousand years with the possible exception of a liquid iron core. At each point, the temperature would be just slightly below the freezing point of the particular silicates peculiar to those depths.

In these years of differentiation, it may be expected that the large molecules containing the long-lived radioactive nuclides of uranium and thorium, and--to a lesser extent--potassium, were selectively transported into the upper half of the moon's mantle.

On this model and for a long period of time, we would have a moon which was solid from the core to the surface. It was hot and capable of adjusting isostatically. The outer layers were continuously and fairly rapidly cooling and becoming more rigid.

The craters, which were formed when the outer crust became strong enough to show them soon were almost completely eliminated by isostatic adjustments. As time went on, the depth of cooling grew greater and the new craters became less and less distorted. Finally, the crust became deep enough and strong enough so that when the tremendous impacts of the bodies which produced the circular maria came, the crust was not shattered, but great dry craters appeared.

During the last stages before the lavas came, numerous craters were produced all over the moon's surface, even inside the dry circular maria.

In this period, small isostatic adjustments continued. The outer layers became cooler and more rigid, and the deep lower layers where the long-lived radioactives were concentrated were becoming warmer. It was only a little below the melting point anyway, and the entire region below about 200-300 miles probably was hotter than $1400^{\circ}K$.

Under these conditions, the moon must have begun to expand even though the crustal areas were still cooling.

Such expansion would lead to great cracks leading down to the hotter regions. Much degassing would be bound to occur.

Eventually the melting point would be reached deep below the crust, and the resulting increase of volume and lowering of density would force hot basic magmas toward the surface from reservoirs perhaps several hundred miles below.

This process was not simply the formation of isolated pools of lava in the body of the moon, although it may have started that way. It developed into almost a moon-wide spasm where boiling hot magmas, probably loaded with gases, were forced upward through great swarms of relatively narrow cracks.

Inevitably these cracks would open into the weak and brecciated zones beneath craters and circular maria. In these cases, individual craters would be filled from below. Close study indicates that most of the lava-filled craters are filled to less than normal ground level and that they are distributed primarily in the lowlands. These are the regions most accessible to the rising liquids.

Pressures would vary in different places. Wargentin was filled to the brim while nearby craters showed much less lava. The lavas which filled Wargentin must have come from an isolated pipe.

Hundreds and thousands of flows were thrust upward and out over the surface till vast areas were covered to depths counted in thousands of feet.

The lavas must have advanced and retreated many times.

Just as on the earth, the lavas at different places and different times were of different chemical compositions.

In general, the lavas were of basic character. This is concluded for two reasons. The depth from which these lavas had to come demands that they were basic, unless the moon contains far more acidic materials than does the earth.

Secondly, there are no evidences of terminal walls at the edges of what appears to be distinct flows.

A basic lava at the temperatures which would obtain at depth, particularly if it contained appreciable amounts of gas, would be very much more fluid than an acid lava, and hence the flows would fine out to low edges which could not be detected from the earth.

The viscosity of a silicate melted at a given sealed temperature is determined entirely by the ratio of the metal ions, Mg, Fe, Ca, etc., to the silicon atoms. Melts in which the ratio of metal ions to silicon atoms is small are highly viscous for a wide range of temperature above the melting temperature. A typical example is feldspar, in which the ratio of sodium to silicon plus aluminum, aluminum acting as silicon, is one-fourth; and it is almost impossible to form a homogeneous melt without repeated melting and grinding.

A melt of the composition of olivine is extremely fluid at a temperature just above the melting point and would fit the observations well, but it is extremely difficult to produce an ultrabasic magma. The lighter silicates would be melted first and extruded upward. If the lava were ultrabasic, the temperature would, of course, have had to be some two or three hundred degrees higher than in the case of the basalt. When any such high temperature lavas reached the surface of the moon, the radiational losses would be greater, but a thin frozen layer would be formed insulating the lava from the ground surface; and an outside crust would be formed insulating the lava from space. The material between would pour out the end, continuously forming new crusts. The lava would thus flow essentially in a two-dimensional tunnel relatively insulated from its surroundings. In such a case, with very fluid lavas, the front at the end of the flow might be so small that it would be below resolution from the earth.

At numerous places on the lava flows, particularly near Langrenus and Copernicus, there are tremendous evidences of degassing. Probably all over the moon, but particularly on the dark areas, there are thousands of relatively small craters formed by this process. Near Stadius they are often aligned along faults.

At this point of the moon's history, we find that tremendous amounts of heat were transported by liquid and by gas to the surface. The heat supply of the deep layers was seriously depleted, and they resolidified. This led to a compaction of the subsurface layers. Adjustments occurred over broad areas where the superimposed weight of layers of lava forced a sinking of the crust.

During this period, there never was a thick solid layer floating on a liquid; for as fast as liquids were produced, they were driven upward, melting and widening the cracks formed by the expansion and then solidifying, only to be split and penetrated by new lavas.

When the very hot lavas did finally reach the outside, they spread far and melted many parts of the earlier surface. Anywhere one looks at the edge of the maria, there may be seen craters, great and small, which have lost their seaward walls to the hot erosive lavas.

This tremendous paroxysm apparently is the only one like it that has occurred in the lunar history. In the case of the earth, the event on greater or lesser scale has occurred often.

If the inequalities in the figure of the moon were once in isostatic equilibrium,as appears probable, then the variation in level between the maria and uplands does not place a stress difference on the interior. In this event the stress difference is only that due to the bulge, which is far higher than can be accounted for by the present tidal pull of the earth.

Jeffreys says:

> "For the moon, which appears to be a very homogeneous body, the elastic theory..., indicates a stress difference at the centre of 2×10^7 dynes/cm^2. With the modified stress distribution this can be reduced to about 1.3×10^7 dynes/cm^2, say 13 atm. or the pressure of 130 m. of water, which would still need good masonry to hold it." (35)

The inequalities in the figure of the moon could be supported at the center by material with a strength over an order of magnitude less than granite. If the bulge is supported by a layer two hundred miles thick, it need have a strength capable of withstanding 130 bars, or about 1/6 that of granite. In all probability, the outer layers of the moon, above the region where the lavas of the moon developed, are amply strong enough to support the observed bulge and possible stress differences due to the maria on the assumption that the maria never becomes fully compensated. The inner parts of the moon may still be weak, even though solid.

Conversely, if the lunar rocks cannot support the observed bulge, then some mechanism such as Runcorn's convection process must be operating to produce an elongation toward the earth. In this event the interior of the moon must be hot still, and the moon is thus capable of adjusting its shape isostatically; but, then we run head on into the problem of why the Class 1 craters and in particular the older craters have not practically disappeared.

Based on the evidence only, and without regard to theory, it looks very much as though the moon's interior is now hot, but not as hot as in earlier times, and the outer layers have a finite strength sufficient to maintain a small bulge and to preserve craters for billions of years when similar objects on the earth would quickly vanish.

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