SMITHSONIAN INSTITUTION
ASTROPHYSICAL OBSERVATORY

Research in Space Science

SPECIAL REPORT

Number 94

May 23, 1962

CAMBRIDGE 38, MASSACHUSETTS

SAO Special Report No. 94

ON THE MOTION OF EXPLORER XI AROUND ITS CENTER OF MASS

by

G. Colombo

Smithsonian Institution
Astrophysical Observatory

Cambridge 38, Massachusetts

TABLE OF CONTENTS

ON THE MOTION OF EXPLORER XI AROUND ITS CENTER OF MASS[1]

by

G. Colombo[2]

Summary.--In this paper we evaluate separately the magnitudes of the induced and the intrinsic magnetic dipoles of Explorer XI (Satellite 1961 Nu) needed to explain quantitatively and qualitatively the precessional motion of the tumbling axis as an effect of both the gravitational torque and the torque that the earth's magnetic field $\vec{H}$ exerts on the satellite. In addition, a good correlation between the first derivative of the frequency and the square of the component $H_\perp$ of $\vec{H}$ normal to the tumbling axis averaged over one orbital period has been found.

The rigid-body motion of Explorer XI around its center of mass can, after the complete damping of the spinning motion, be represented as a rotational motion around an axis defined by the vector $\vec{\Omega}$, normal to the figure axis Z of the satellite; the $\vec{\Omega}$ - axis is perturbed by several effects. The observational material (Naumann, 1961; Naumann, Fields and Holland, 1961) clearly indicates that the motion of $\vec{\Omega}$ is precessional (see fig. 2). In addition, within the limit of accuracy of the observations, the position of $\vec{\Omega}$ with respect to the body was observed to be

---

[1]This work was supported by a grant from National Aeronautics and Space Administration.

[2]Consultant, Smithsonian Astrophysical Observatory, Cambridge, Mass.; on leave of absence from the University of Padua, Italy.

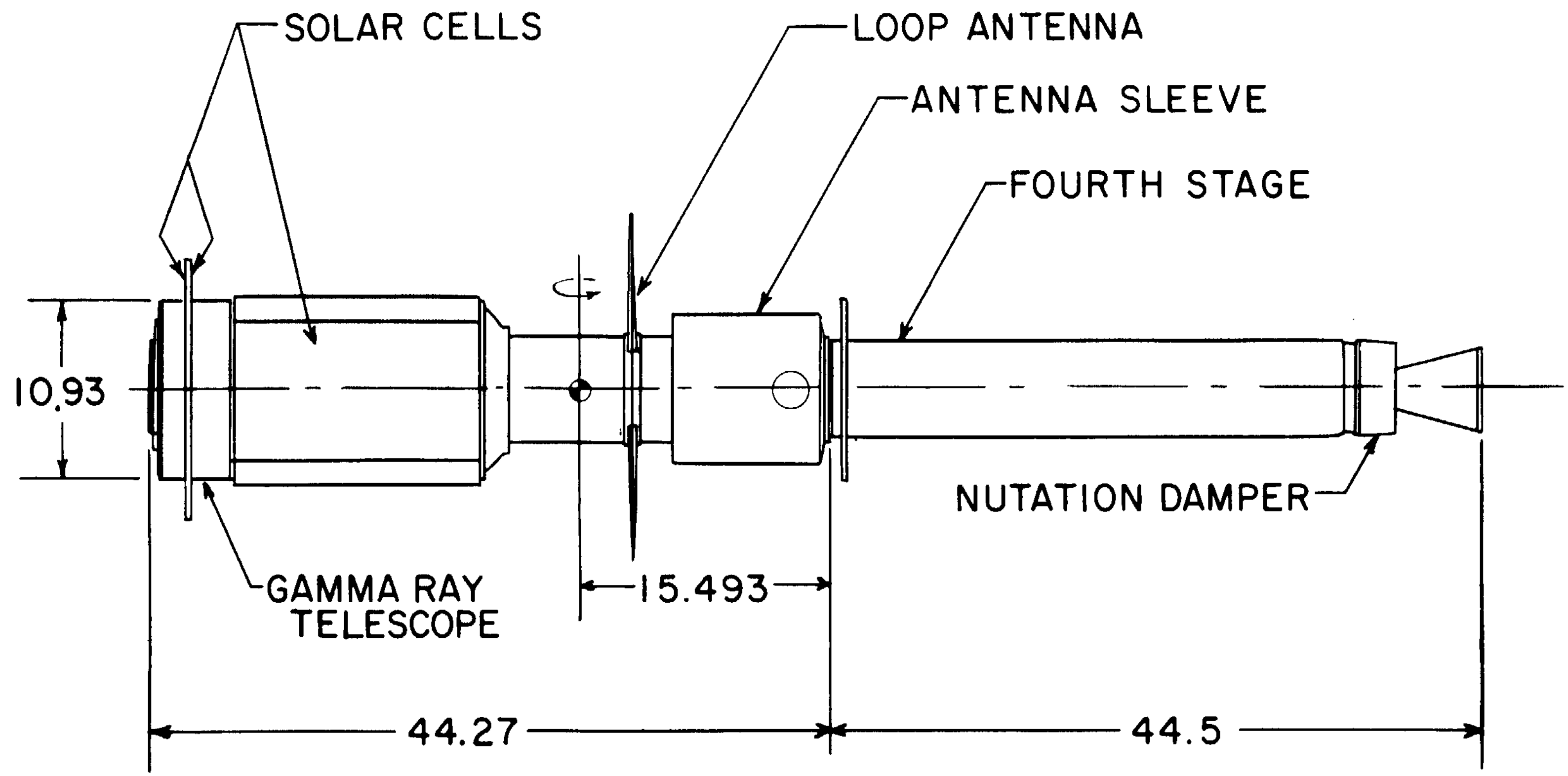

Figure 1.--Schematic drawing of Explorer XI. The total weight is 95.114 lbs. The moments of inertia are: $A_{max} = 143.792$ in.lb.sec^2; $A_{min} = 143.601$ in.lb.sec^2; and $C = 3.405$ in.lb.sec^2.

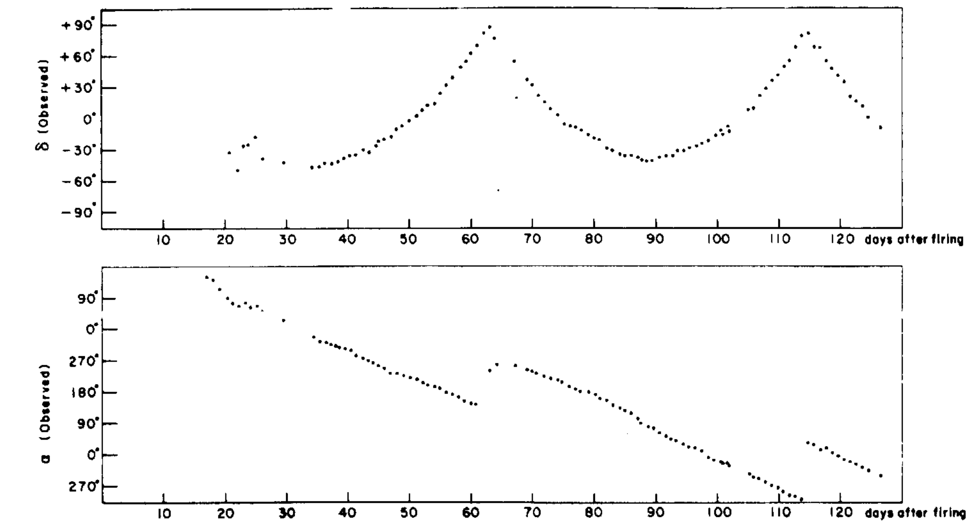

Figure 2.--Right ascension α and declination δ of $\vec{\Omega}$ versus time.

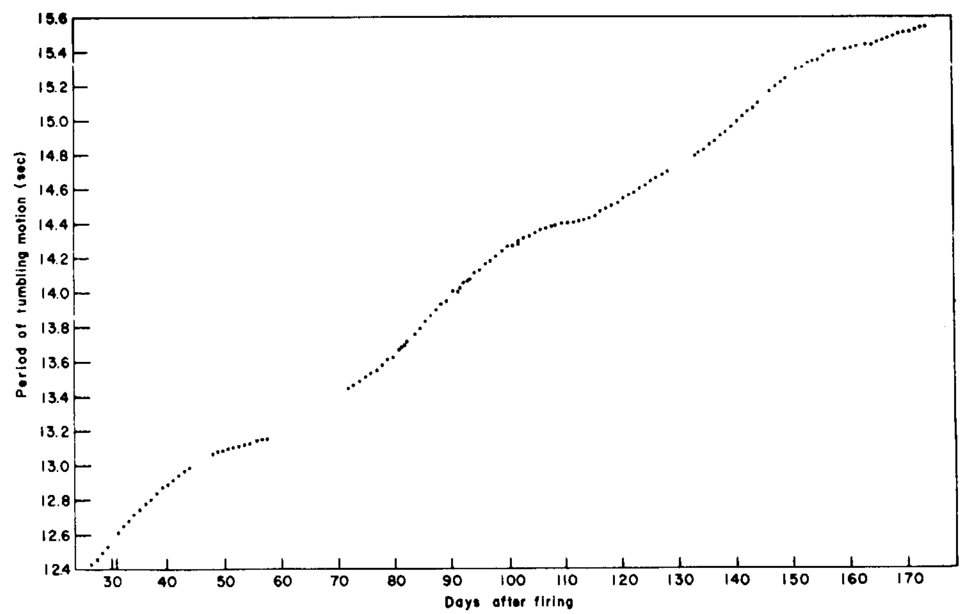

Figure 3.--Period of tumbling motion (sec) versus time.

fixed along the axis of maximum moment of inertia (private communication). Finally, a slow decay of the angular momentum L $\vec{\Omega}$, which is a second-order effect, has been precisely detected and studied, showing interesting features (see fig. 3). We will distinguish the first-order effects (motion of $\vec{\Omega}$) from the second-order effects (slow-down of the tumbling motion).

The precessional motion of $\vec{\Omega}$ can be explained by a torque whose components are quadratic functions of the components of $\vec{\Omega}$, that is, a torque with the same geometrical characteristics as the gravitational torque. However, the magnitude of the latter, which can be very well determined, is between one-half and one-third of the needed amount; and the orientation is quite different.

In this case of a satellite with a very large perigee distance and a complicated shape, but without any large amount of hard magnetic materials in its structure, the aerodynamic, eddy-current and hysteresis torques are two orders of magnitude less than the gravitational torque (Wilson, 1961). Only the magnetic torque coming from the interaction between the earth's magnetic field and the equivalent magnetic dipole of the satellite appears sufficient to explain the motion of $\vec{\Omega}$, as we have already observed in the case of Explorer IV (Colombo, 1961).

The torque produced by the earth's magnetic field is

$$\vec{M} = \vec{I} \times \vec{H}, \tag{1}$$

where $\vec{I}$ is the vector of the equivalent magnetic dipole of the satellite. Neglecting the second-order effects, we can write for the equivalent magnetic dipole of the satellite

$$\vec{I} = \vec{I}_0 + \vec{m}(\vec{H}), \tag{2}$$

where $\vec{I}_0$ is the intrinsic magnetization vector from the magnetized components and interior current loops; and $\vec{m}(\vec{H})$ is the magnetic dipole induced by the interaction of the earth's magnetic field with the ferromagnetic components. It seems reasonable to suppose that the permeable components of the satellite have, in the complex, the same property of symmetry with respect to the figure axis Z of the satellite, as the exterior shape has. Therefore, neglecting hysteresis effects, we may write equation (1) as

$$\vec{M} = \vec{I}_0 \times \vec{H} + \left\{\mu_1(\vec{H}\cdot\vec{K})\vec{K} + \mu_2(\vec{H}\cdot\vec{K} \times \vec{\Omega})\vec{K} \times \vec{\Omega} + \mu_3(\vec{H}\cdot\vec{\Omega})\vec{\Omega}\right\} \times \vec{H}. \qquad (1')$$

Here μ_1, μ_2, and μ_3 are coefficients related to the geometrical shape and the magnetic property of the permeable components.

When we average over a tumbling circle we will find that

$$\vec{M} = (\vec{I}_0\cdot\vec{\Omega})\vec{\Omega} \times \vec{H} + \mu^*(\vec{H}\cdot\vec{\Omega})\vec{\Omega} \times \vec{H}, \qquad (1'')$$

where $\mu^* = \mu_3 - \frac{1}{2}(\mu_1+\mu_2)$. It seems to me important to observe that for evaluating μ_1, μ_2, and μ_3, we must take into account that, while μ_1 and μ_2 are related to a component of the earth's magnetic field sinusoidally changing in a tumbling period (13 seconds), μ_3 is related to a very slowly changing component of the same field.

Before the launching no measurements of the intrinsic magnetic dipole of the payload were made; only a crude evaluation of the intrinsic magnetic dipole of the last stage of the rocket was carried out. Therefore, we do not know $\vec{I}_0$ with enough accuracy. All we know about the permeable structure of the satellite is that the material of the last stage of the rocket is 410 stainless steel, that small elongated cylinders of permalloy material were put into the payload for shielding purposes, and finally that there is an iron anular plate in the tail, lying in a plane normal to the Z-axis. We do not know anything about the permeable structure of the radio-transmitter complex. In order to obtain a possible explanation of the observed precessional motion of $\vec{\Omega}$, we must examine the values of I_Ω and μ^* that give the best agreement with the observations. For this purpose we need better observations of the orientation of $\vec{\Omega}$ and a precise numerical integration. Since the probable error in the orientation of $\vec{\Omega}$ is of the order of a few degrees, we studied separately, using the very quick averaging procedure, the cases where I_Ω or μ^* are negligible. In both we found the possibility of having good agreement with the observed motion of $\vec{\Omega}$, if $\mu^* = 4\pi \times 4.76 \times 10^5$ e.m.u., $I_\Omega = 0$ and also if $\mu^* = 0$, $I_\Omega = 0.63$ amp - m^2. While the needed value of μ^* seems too high, an estimate of the actual value is very difficult. The needed value of I_Ω in the second case is certainly not in good agreement with the measurement made on the last stage of the rocket before launching, although no measurements have been made for the payload. Our opinion is that perhaps the contribution of both magnetic dipoles, the induced and the intrinsic, are significant. In any case, our goal is to give a very easy method for a first-approximation study of the phenomenon. In a second stage this method may be improved by numerical integration on a high speed computer.

We assume that the following hypotheses are satisfied:

(a) the axis of rotation of $\vec{\Omega}$ is almost fixed in the body in the meaning we will state precisely later;

(b) the permeable structures of the satellite are such that we can write equation (1') neglecting second-order quantities;

(c) the aerodynamic, hysteresis, and eddy-current torques are negligible for the explanation of the main precessional motion of Ω.

We prefer to leave (a) as an hypothesis since we think a theoretical study of the mechanism of the stabilization of the $\vec{\Omega}$-axis in a region very close to the axis of maximum moment of inertia would require a good knowledge of the internal dissipation of energy (nutation damper). The $\vec{\Omega}$-axis cannot be precisely fixed in the body: if it were, it would also be fixed with respect to a fixed reference system and we would not have the observed precessional motion. However, the total angular-velocity vector of the satellite may be considered as the sum of the angular velocity vector $\omega \vec{\Omega}$ of the tumbling motion, fixed in the body, and the precessional velocity of the vector $\vec{\Omega}$, with respect to a fixed reference system. The magnitude of this second vector is of the order of 10^{-5} of the value of ω. This means that the total velocity vector may always be very close to the tumbling axis (less than one second of arc separation), which is consistent with the observed motion.

In hypothesis (b) we prefer to leave the parameters μ_i undetermined in view of our poor knowledge of the magnetic properties of the permeable components and the fact that these components are moving in the weak magnetic field of the earth. As we have already stated Dr. C. Lundquist made, before launch, a crude evaluation of the magnetization of the last stage of the rocket; his results gave a magnetization vector with a large component in the direction of the Z-axis and a small component (1/20 of the former) in a transversal direction that was not very well defined. For hypothesis (c) we can say first that the magnitudes of these torques are of two orders of magnitude less than the gravitational torque, and second that their orientation would not be in good agreement for the explanation of the observed variation in the direction of $\vec{\Omega}$. The aerodynamic torque can be fairly precisely determined using the hypothesis of neutral drag; the hysteresis and eddy-current torques cannot be determined so precisely. In any case, however, all these torques are dissipative, and the dissipation process involves torques of the order of 1 dyne-cm and not of one-hundred dyne-cm; the latter amount would be needed to explain the variation of the orientation of $\vec{\Omega}$ without a dissipation of the same order of magnitude.

The validity of the following procedures for the deduction of the equation of motion is postulated:

(d) to compute the torque acting on the satellite, we average over one period of rotation of the body around $\vec{\Omega}$ (tumbling period);

(e) we average the torque over one orbital period of the satellite;

(f) finally, we average over one day.

The procedures followed in (d) and (e) are the usual ones used in the perturbation method for determining the gravitational and aerodynamic torques (Beletsky, 1960). In one tumbling period, the center of mass of the satellite will move along a 100-km arc of the orbit. In the case of Explorer XI, this amount corresponds to a 1°-variation in the orientation of the radius vector from the earth's center E to the satellite's center of mass G; that means that the variation in the field is two orders of magnitude less than the intensity of the field. Furthermore, in one orbital period the variation of the orientation of $\vec{\Omega}$ is of the order of $0^\circ\!.5$. We prefer to use procedures (d) and (e) as working hypotheses since they also give good results for the gravitational torque. Dr. Leland Cunningham made for the Huntsville Center a step-by-step integration of the original equations to determine the effects of the gravitational torque on Explorer XI. His results were the same as those obtained by the averaging procedure (private communication from Dr. Lundquist). As for procedure (f), we prefer to use the same averaging procedure. Therefore, we are able to arrive quickly at the interesting results that follow.

At the end of this report we will make a preliminary analysis of the slow down of the spinning motion. This slow down is definitely a consequence of the eddy-current torque and hysteresis effects, since the aerodynamic torque is very small (less than 0.1 dyne-cm). Both effects are proportional to the square of the component $H_\perp$ of $\vec{H}$ normal to the tumbling axis. The correlation between the square of this component averaged over one orbital period of the satellite and the first derivative of the tumbling period states the nature of the breaking torque, even if it seems more complicated, but not hopeless, to distinguish between the two effects (Wilson,1961).

A more detailed analysis has been made at M.I.T. This analysis of the observations (a dozen per day) of the variation of the period seems to show definitely a term with the period of one day, which we think is correlated with the variation in one day of the position of the earth's magnetic dipole, which in 24 hours rotates around the earth's geographical axis.

The averaging procedure we used in our computation is related to the accuracy of the observations of $\vec{\Omega}$. In view of the good observations of ω, a more accurate knowledge of the value of this parameter may make a numerical integration of the general equation of motion worthwhile.

1. Magnetic torque from induced magnetic dipole

Let us compute the effective magnetic torque coming from the induced magnetic dipole. From equation (1"), assuming $\vec{I}_0 \cdot \vec{\Omega} = 0$, we have

$$\tilde{\vec{M}} = \mu^* (\vec{H} \cdot \vec{\Omega}) \, \vec{H} \times \vec{\Omega}. \tag{3}$$

In the usual notation, let

$$r \, \vec{r} = \frac{a(1-e^2)}{1+e \cos(\theta-\omega)} \left\{ \cos\theta \, \vec{i} + \sin\theta \cos i \, \vec{j} + \sin\theta \sin i \, \vec{k} \right\} \tag{4}$$

be the vector equation of the motion of the satellite's center of mass G. Here $\vec{r}$ is the unit vector of the direction EG from the earth's center;

$\vec{i}$ is the unit vector of the ascending node in the equatorial plane of the earth's equivalent (magnetic) dipole; and $\vec{k}$ is the direction of the axis of this dipole. We assume for the earth's magnetic field the usual first approximation (Chernosky and Maple, 1960).

$$\vec{H} = \frac{\mu_E}{r^3} \quad \vec{k} - 3(\vec{k} \cdot \vec{r})\vec{r} \quad . \tag{5}$$

The average value in one orbital period is given by the integral

$$\tilde{\vec{M}} = \frac{\mu^*}{T} \int_0^T (\vec{H} \cdot \vec{\Omega})\vec{\Omega} \times \vec{H} \, dt \ . \tag{6}$$

Since

$$\frac{d\theta}{dt} = \frac{2\pi a^2(1-e^2)^{\frac{1}{2}}}{Tr^2} \quad , \tag{7}$$

we obtain from equations (3), (4), (5), and (6) the following expression:

$$\tilde{\vec{M}} = \frac{\mu^*}{2\pi a^2(1-e^2)^{1/2}} \ \vec{\Omega} \times \int_0^{2\pi} r^2(\vec{H} \cdot \vec{\Omega})\vec{H} \, d\theta. \tag{8}$$

From equations (3) and (4) we have

$$\vec{H} = \mu_E \frac{[1+e \cos(\theta-\omega)]^3}{a^3(1-e^2)^3} \left\{ - 3 \sin \theta \cos \theta \sin i \ \vec{i} \right. \tag{9}$$

$$\left. - 3 \sin^2 \theta \sin i \cos i \ \vec{j} + (1 - 3 \sin^2 \theta \sin^2 i) \ \vec{k} \right\}$$

Let Ω_1, Ω_2, Ω_3 be the components of $\vec{\Omega}$ with respect to $(\vec{i},\vec{j},\vec{k})$; then from equation (7), neglecting some small terms in e^2, we finally obtain

$$\tilde{\vec{M}} = \frac{\mu^* \mu_E^2}{a^6(1-e^2)^{9/2}} \vec{\Omega} \times \left\{ \frac{9}{8} \sin^2 i \; \Omega_1 \vec{i} + \frac{27}{8} \sin^2 i \cos^2 i \; \Omega_2 \vec{j} \right.$$
$$+ (1 - 3 \sin^2 i + \frac{27}{8} \sin^4 i) \; \Omega_3 \vec{k} \tag{10}$$
$$\left. - \frac{3}{2} \sin i \cos i \; (1 - \frac{9}{4} \sin^2 i) \; (\Omega_2 \vec{k} + \Omega_3 \vec{j}) \right\} ,$$

2. Gravitational torque

We shall now compute the gravitational torque acting on the satellite. Let $(\vec{N},\vec{W},\vec{U})$ be an orthogonal reference frame centered at E, where $\vec{N}$ is the unit vector of the orbit's ascending node in the geographic equator, and $\vec{U}$ is the unit vector of the earth's axis. Also, let i_0 be the inclination of the orbit with respect to the geographic equator, and $\vec{n}$ be the unit vector normal to the orbital plane. Averaging the gravitational torque $\vec{G}$ over one tumbling period and then over one orbital revolution of G, we have, finally,

$$\tilde{\vec{G}} = \frac{3}{4} \tilde{\bar{\omega}}^2 \, (A-C)(\vec{\Omega} \cdot \vec{n}) \; \vec{\Omega} \times \vec{n} \; . \tag{11}$$

Here C is the moment of inertia of the satellite with respect to the $\vec{\Omega}$-axis, and A is the moment of inertia with respect to an axis normal to $\vec{\Omega}$ and

$$\tilde{\omega}^2 = \frac{h}{T} \int_0^T \frac{dt}{r^3} , \tag{12}$$

where h is the characteristic constant of the earth's gravitational attraction. For Explorer XI, we have

$$\begin{cases} i_0 = 28^\circ\!.8 \ , \\ \vec{n} = - \sin i_0 \vec{W} + \cos i_0 \vec{U} \ ; \end{cases} \tag{13}$$

and

$$\beta = \frac{3}{4} \tilde{\omega}^2 (A - C) = 1.2 \times 10^2 \text{ dyne-cm.} \tag{14}$$

The components of $\tilde{\vec{G}}$ with respect to $(\vec{N},\vec{W},\vec{U})$ are the following:

$$\begin{cases} G_x = \frac{1}{2} \beta \, \Omega_y \, \Omega_z + \frac{\sqrt{3}}{4} \beta \, (\Omega_z^2 - \Omega_y^2) \ , \\ G_y = - \frac{3}{4} \beta \, \Omega_x \, \Omega_z + \frac{\sqrt{3}}{4} \beta \, \Omega_x \, \Omega_y \ , \\ G_z = \frac{1}{4} \beta \, \Omega_x \, \Omega_y - \frac{\sqrt{3}}{4} \beta \, \Omega_x \, \Omega_z \ , \end{cases} \tag{15}$$

where Ω_x, Ω_y, Ω_z are the components of $\vec{\Omega}$ with respect to $(\vec{N},\vec{W},\vec{U})$.

3. Differential equation of perturbed motion

The equation of motion

$$\frac{L d\vec{\Omega}}{dt} = \tilde{\vec{M}} + \tilde{\vec{G}} \tag{16}$$

is now to be projected onto an inertial reference frame and then integrated with initial conditions corresponding to an observed orientation at the chosen initial time. We prefer to project equation (16) onto the moving reference system $(\vec{N},\vec{W},\vec{U})$. We need the equations for passing from the reference system $(\vec{i},\vec{j},\vec{k})$ to the reference system $(\vec{N},\vec{W},\vec{U})$. Let I be the angle $\widehat{\vec{k}\vec{U}}$; ψ the angle $\vec{N}$ makes with the intersection of the geomagnetic equator and the geographic equator; and α the angle $\widehat{\vec{i}\vec{N}}$. We have first

$$\begin{cases} \tan \alpha = - \dfrac{2 \tan \frac{\psi}{2} \sin I}{\sin (i_0 - I) + \tan^2 \frac{\psi}{2} \sin (I_0 + i_0)} , \\ \\ \cos i = \cos i_0 \cos I + \sin i_0 \sin I \cos \psi ; \end{cases} \tag{17}$$

and also

$$\begin{cases} \vec{i} = \cos \alpha \vec{N} + \sin \alpha \cos i_0 \vec{W} + \sin \alpha \sin i_0 \vec{U} , \\ \vec{j} = (- \sin I \sin i_0 \sin \alpha \cos \psi - \cos I \sin i_0 \sin \alpha)\vec{N} , \\ \qquad + (\cos I \cos \alpha - \sin \alpha \sin i_0 \sin I \sin \psi) \vec{W} , \\ \qquad + \sin I (\sin \psi \sin \alpha \cos i_0 + \cos \psi \cos \alpha) \vec{U} , \\ \vec{k} = \sin I \sin \psi \vec{N} - \sin I \cos \psi \vec{W} + \cos I \vec{U} . \end{cases} \tag{18}$$

Here $I = 11\overset{\circ}{.}5$; and $\psi = 2\pi t$ rad/day. In our approximation we will obtain from equations (17) and (18)

$$\begin{cases} \sin \alpha = -0.41 \sin \psi, \ \cos \alpha = 1 - 0.08 \cos \psi , \\ \sin i = 0.5 - 0.18 \cos \psi, \ \cos i = 0.86 + 0.1 \cos \psi ; \end{cases} \tag{17'}$$

and

$$
\begin{cases}
\vec{i} = (1 - 0.08 \cos \psi)\vec{N} - 0.36 \sin \psi \, \vec{W} - 0.2 \sin \psi \, \vec{U} \, , \\
\vec{j} = (0.35 \sin \psi + 0.02 \sin 2\psi)\vec{N} + (1 - 0.08 \cos \psi \\
\qquad - 0.02 \cos 2\psi)\vec{W} + (-0.04 + 0.2 \cos \psi)\vec{U}, \\
\vec{k} = 0.2 \sin \psi \, \vec{N} - 0.2 \cos \psi \, \vec{W} + 0.98 \, \vec{U}.
\end{cases}
\tag{18'}
$$

It is necessary now to note that for Explorer XI the reference system $(\vec{N},\vec{W},\vec{U})$ is rotating around the $\vec{U}$-axis in a uniform motion with an angular velocity (regression of the node) of - 0.087 rad/day. This means that equation (11) projected over the chosen reference system takes the form

$$
\begin{cases}
L \left(\dfrac{d\Omega_x}{dt} + 0.087 \, \Omega_y \right) = M_x(\Omega_x, \Omega_y, \Omega_z, t) + G_x \, , \\
L \left(\dfrac{d\Omega_y}{dt} - 0.087 \, \Omega_y \right) = M_y(\Omega_x, \Omega_y, \Omega_z, t) + G_y \, , \\
L \, \dfrac{d\Omega_z}{dt} = M_z(\Omega_x, \Omega_y, \Omega_z, t) + G_z .
\end{cases}
\tag{19}
$$

The functions G_x, G_y, G_z are quadratic functions of Ω_x, Ω_y, Ω_z with constant coefficients [equation (13)]; M_x, M_y, M_z are also quadratic functions of Ω_x, Ω_y, Ω_z but the coefficients are periodic functions of t through ψ, with a period of one day. It would be possible to perform a numerical integration corresponding to some initial condition similar to the observed conditions of the 35th day after firing. We choose this day since after this we have good observations of the precessional motion of $\vec{\Omega}$. The amount of work involved in this numerical computation, even if worthwhile, suggested to us, that we use a first approximation to compute the average values of the components M_x, M_y, M_z over one day.

The observed numerical variation of the orientation of $\vec{\Omega}$ is not more than 10° per day: in averaging, we consider the orientation of $\vec{\Omega}$ constant, using the mean orientation for the day. The displacement of $\vec{\Omega}$ from the mean value for the day is not greater than 5°. It is difficult to evaluate the error involved in this averaging procedure, but in any case we think that the approximation is quite good. We prefer to follow this method to confirm quickly our feelings about the nature of the torque needed to explain the precessional motion.

4. Evaluation of the induced dipole

We shall now discuss and integrate the differential system obtained by the procedure explained above. Let us put

$$\gamma = \frac{\mu^* \mu_E^2}{a^6(1-e^2)^{9/2}} \tag{20}$$

For our case , a quick evaluation gives

$$\gamma = 1.4 \times 4\pi\mu^* \times 10^{-3} \text{ dyne-cm.} \tag{21}$$

Averaging over one day we obtain

$$\vec{M} = \gamma \vec{\Omega} \times \left\{ 0.32\, \Omega_x \vec{N} + 0.62\, \Omega_y \vec{W} + 0.47\, \Omega_z \vec{U} - 0.25(\Omega_z \vec{W} + \Omega_y \vec{U}) \right\}. \tag{22}$$

It follows that

$$\left\{ \begin{aligned} M_x &= -0.15\gamma\, \Omega_y \Omega_z + 0.25\gamma\, (\Omega_z^2 - \Omega_y^2) \,, \\ M_y &= -0.15\gamma\, \Omega_x \Omega_z + 0.25\gamma\, \Omega_y \Omega_x \,, \\ M_z &= 0.30\gamma\, \Omega_x \Omega_y - 0.25\gamma\, \Omega_x \Omega_z \,. \end{aligned} \right. \tag{23}$$

To write equation (16) in the explicit form, we have to evaluate $L = A\,\omega$, where ω is the observed angular velocity. We find that

$$L = 1.616\,\frac{2\pi}{13}\cdot 10^{8}\ \text{gram-cm}^{2}\text{-sec.} \tag{24}$$

Choosing the day as the unit of time, now we can write, finally, equation (16) and we obtain

$$\begin{cases} 910\,\dfrac{d\Omega_x}{dt} = -\,(0.15\gamma + 60)\Omega_y\,\Omega_z + (0.25\gamma + 52)(\Omega_z^2 - \Omega_y^2) - 79\,\Omega_y , \\ 910\,\dfrac{d\Omega_y}{dt} = -\,(0.15\gamma + 90)\Omega_x\,\Omega_z + (0.25\gamma + 52)\Omega_y\,\Omega_x + 79\,\Omega_x , \\ 910\,\dfrac{d\Omega_z}{dt} = (0.30\gamma + 30)\Omega_x\,\Omega_y - (0.25\gamma + 52)\Omega_x\,\Omega_z\ . \end{cases} \tag{25}$$

The differential system in equation (25) has two first integrals; the obvious one,

$$\Omega_x^2 + \Omega_y^2 + \Omega_z^2 = 1\ , \tag{26}$$

and

$$\tfrac{1}{2}(0.15\gamma + 90)\Omega_z^2 + \tfrac{1}{2}(0.30\gamma + 30)\Omega_y^2 + (0.25\gamma + 52)\Omega_y\,\Omega_z - 79\,\Omega_z = E\,. \tag{27}$$

The intersection of the sphere in equation (26) with the cylinder in equation (27) gives the path of the vertex of the vector $\vec{\Omega}$ with respect to the rotating reference system $(\vec{N},\vec{W},\vec{U})$.

In figures 4a and 4b we plotted the projections on the $\overleftrightarrow{WU}$-plane and the $\overleftrightarrow{NU}$-plane of the observed position of the vertex of $\vec{\Omega}$. If we have

$$\mu^{*} = 4\pi \times 4.76\ 10^{5}\ \text{m.u.} \tag{28}$$

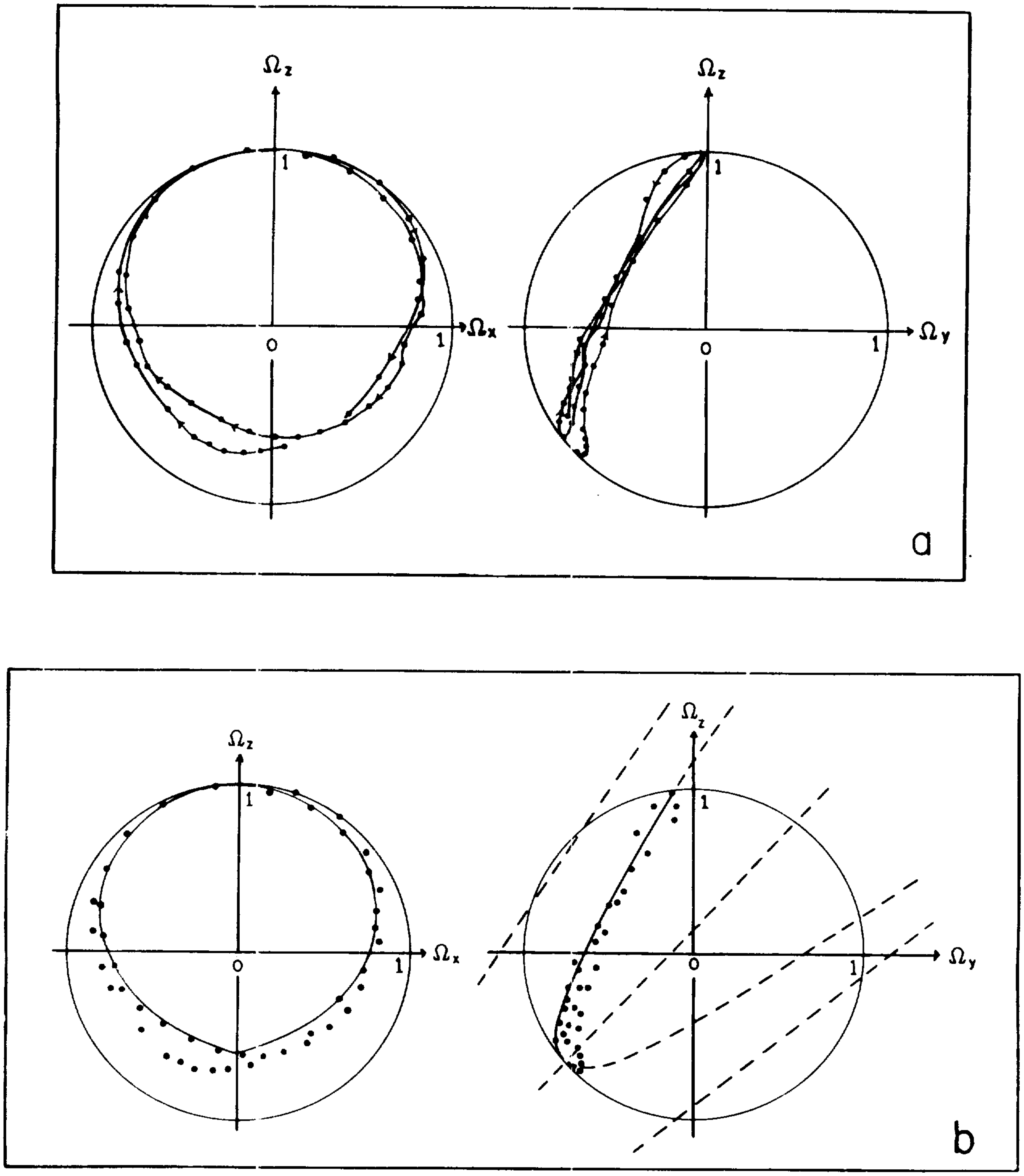

Figure 4, (a) and (b).--Projection of the observed path of the vertex of $\vec{\Omega}$ over the xz and yz planes of a rotating reference system (x, ascending node, z, earth's axis). Solid lines are computed path, and dots are observed position.

we will obtain

$$\gamma = 6.6 \times 10^2 \; ; \tag{29}$$

consequently equation (27) becomes

$$95\ \Omega_z^2 + 115\ \Omega_y^2 - 220\ \Omega_y\ \Omega_z - 79\ \Omega_z = E. \tag{30}$$

Equation (30) represents a family of hyperbolas. If we choose $E = 49.5$, we will obtain the hyperbola, $\overset{\frown}{J}$, shown in figure 4a. Figure 4b represents the projection on the $\Omega_x\ \Omega_z$-plane of the path Γ of $\vec{\Omega}$ corresponding to the arc AB of $\overset{\frown}{J}$. The motion of $\vec{\Omega}$ with respect to the reference system $(\vec{N},\vec{W},\vec{U})$ is periodic. The good agreement of the observed path emerges clearly (see fig. 4c)

We need also to compare the equation of motion along the path with the observations. The easiest way to do this is to compare the observed values of $\frac{d\Omega_z}{dt}$ as functions of Ω_z, with the value of the same derivative computed from the third part of equation (25), which when we take into account equation (29) becomes

$$\frac{d\Omega_z}{dt} = 0.25\ \Omega_x\ \Omega_y - 0.24\ \Omega_x\ \Omega_z\ . \tag{31}$$

In figure 4c we plotted the values of the second term of equation (31) as a function of Ω_z, evaluated using equations (26) and (27) with $E = 49.5$. The dots are the observed values of the same derivative $\frac{d\Omega_z}{dt}$ as a function Ω_z. The good agreement is evident.

5. Evaluation of the intrinsic dipole

We shall now show that a component of the permanent magnetization (evaluated as approximately 0.630 amp-m^2) normal to the Z-axis can explain the observed motion of $\vec{\Omega}$.

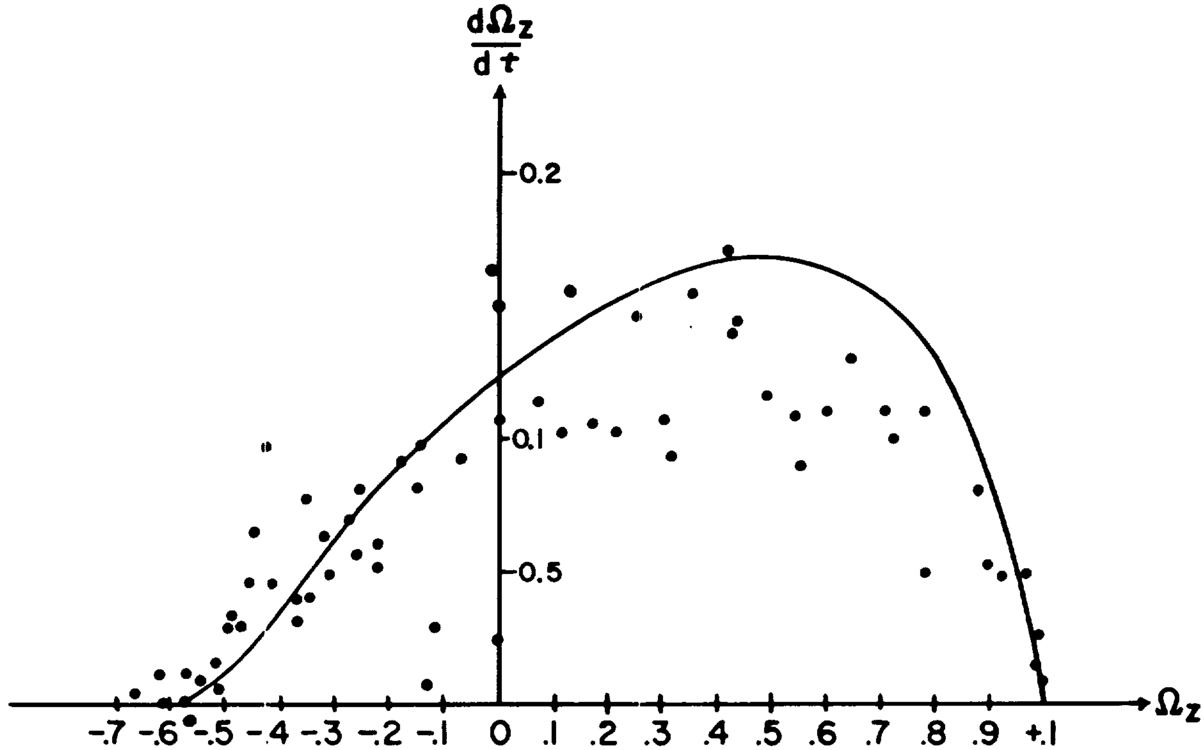

Figure 4(c).--Computed (solid lines) and observed (dots) value of $\frac{d\Omega_z}{dt}$ versus Ω_z.

Using the same procedure as above, we will find (Colombo, 1961)

$$L \frac{d\vec{\Omega}}{dt} = \lambda\vec{\Omega} \times \left(\frac{5}{8}\vec{U} - \frac{3\sqrt{3}}{8}\vec{W}\right) , \tag{32}$$

where

$$\lambda = \frac{\mu_E \ \mu^*}{a^3(1-e^2)^{3/2}} \tag{33}$$

If $\mu^* = -0.630$ amp-m^2, which corresponds to a value of λ of 126 dyne-cm, we will have for a first integral

$$45 \ \Omega_y^2 + 15 \ \Omega_y^2 - 52 \ \Omega_y \ \Omega_z - 82 \ \Omega_y = E \ . \tag{34}$$

Therefore, equations (30) and (31) become

$$910 \frac{d\Omega_z}{dt} = 30 \ \Omega_x \ \Omega_y - 82 \ \Omega_x - 52 \ \Omega_x \ \Omega_z \ , \tag{35}$$

and

$$45 \ \Omega_z^2 + 15 \ \Omega_y^2 - 52 \ \Omega_y \ \Omega_z - 82 \ \Omega_y = E \ . \tag{36}$$

We have again a family of hyperbolas. We choose the constant E in such a way that the curve given by equation (36) passes through the point $\Omega_y = -0.8$, $\Omega_z = -0.6$; that is, $E = 66.5$.

In figure 5a we have the projection of the path of $\vec{\Omega}$ on the $\Omega_x\Omega_z$, $\Omega_y\Omega_z$-planes, corresponding to $\mu^* = 0.630$ amp-m^2. The same results would be obtained if we assumed the satellite had a residual spin motion and a large axial component of the permanent magnetization.

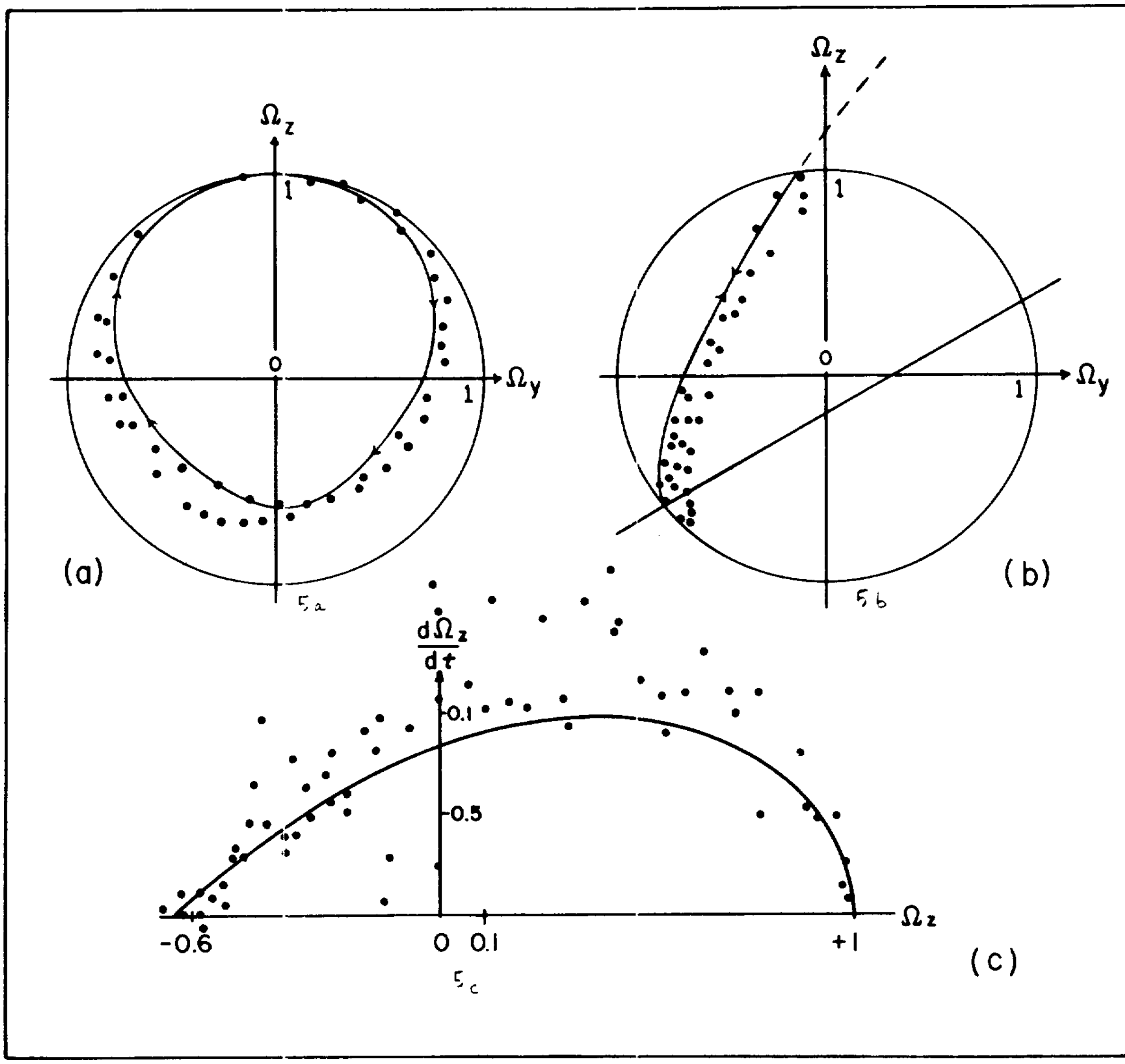

Figure 5.--Same as figure 4c, corresponding to the hypothesis of a constant component of the permanent magnetization in the direction of $\vec{\Omega}$.

While a residual spin velocity of the needed amount was not observed (private communication from Prof. Kraushaar of M.I.T.), the existence of the needed component of the residual magnetization has to be postulated.

6. Period of tumbling

In order to explain the variation in the period of the tumbling motion, we computed the value of the square of the component $H_{\perp}$ of $\vec{H}$ normal to $\vec{\Omega}$. For a first approximation, we averaged over one day, and then plotted the value $\frac{\widetilde{H}_{\perp}^2}{\widetilde{H}^2}$ versus time and the value of the first derivative of the period in sec/day in figure 6. The correlation looks good, at least for the positions of the maxima and minima. We have to take into account both the averaging procedure and the error of the observations of $\vec{\Omega}$, which also affects the computed value of $\widetilde{H}_{\perp}^2$.

If we assume the damping torque is the sum of the eddy-current torque (which is assumed proportional to the angular velocity ω) and the hysteresis torque (which is considered independent of ω), we can write

$$M_D = (\sigma\ \omega + \nu)\ H_{\perp}^2\ . \tag{37}$$

The magnitude of the torque needed to explain the breaking is of the order of 1 dyne-cm. The evaluation of the coefficients σ and ν is extremely difficult and requires good information about the physical properties of the conducting and ferromagnetic components of the satellite; we therefore plotted only the first derivative of the tumbling period versus time. Since the tumbling period varies from 12.4 to 14.6 seconds during the one-hundred day interval of observations of the orientation of $\vec{\Omega}$, the behavior of $\frac{d\omega}{dt}$ cannot be very different, and the position in time of the maxima and minima definitely cannot undergo appreciable changes.

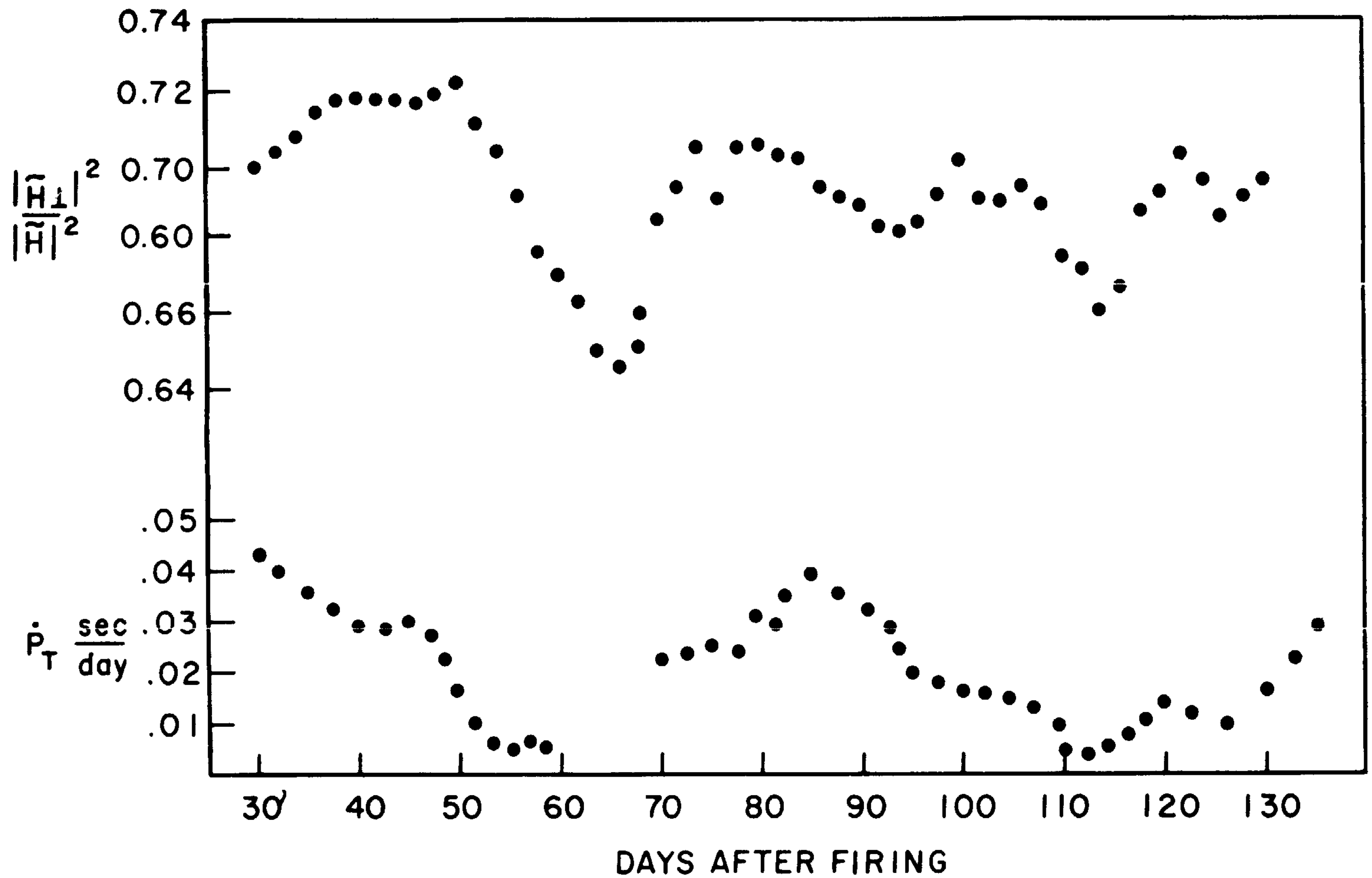

Figure 6.--Correlation between the first derivative of the period and the square of the component of $\tilde{\vec{H}}$ normal to $\vec{\Omega}$.

A deeper and more detailed analysis of the observed variation of ω, even during one day, is strongly suggested by the accuracy of the observations and by the interesting correlation of the diurnal periodic term with the variation of the position of the earth's magnetic dipole with respect to $\vec{\Omega}$. If we take into account that the earth's magnetic dipole makes an angle of $11\overset{\circ}{.}5$ with the earth's geographical axis, the maximum displacement of the earth's magnetic dipole will be of 23° in 12 hours, which is much larger than the maximum displacement of Ω (about 5°) in the same period. This fact makes the effect detectable.

I am indebted to Dr. C. Lundquist and R. Naumann of the Huntsville Center and to Prof. W. L. Kraushaar and Prof. G. Clark of M.I.T. for the observational material and for fruitful discussion. I am grateful to Miss Cara Munford for her help in the computation, and to Miss Joan Weingarten for her help in the writing of the paper.

References

BELETSKY, V. V.
1960. Motion of an artificial earth satellite about its center of mass. In Artificial earth satellites, L. V. Kurnosova, ed., Plenum Press, New York, vol. 1, pp. 30-54.

CHERNOSKY, E. J., and MAPLE, E.
1960. Geomagnetism. In Handbook of geophysics, MacMillan Co., New York, Chapter 10, pp. 10-1 - 10-68.

COLOMBO, G.
1961. The motion of Satellite 1958 Epsilon around its center of mass. Smithsonian Astrophys. Obs., Special Report No. 70, 25 pp.

NAUMANN, R. J.
1961. Recent information gained from satellite orientation measurement. In Ballistic Missiles and Space Technology, Pergamon Press, New York, vol. 3, pp. 445-453.

NAUMANN, R. J., FIELDS, S., and HOLLAND, R.
1961. Determination of angular momentum vector for the S-15 payload (Explorer XI). George C. Marshall Space Flight Center, Huntsville, Alabama, 16 pp.

SPITZER, L.
1960. Space telescopes and components. Astron. Journ., vol. 65, pp. 242-263.

WILSON, R. H., Jr.
1961. Rotational decay of Satellite 1960 η 2 due to the magnetic field of the earth. NASA Goddard Space Flight Center, Greenbelt, Md., 18 pp.