GEORGE C. MARSHALL SPACE FLIGHT CENTER

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A TECHNIQUE FOR CALCULATING SMOOTHING AND DIFFERENTIATION COEFFICIENTS

by

John P. Sheats and Jon B. Haussler

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FLIGHT EVALUATION BRANCH
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ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

d	The degree of the smoothing polynomial
h	The interval at which observations are available
t	The time of some observation
t_0	Time at which some smoothed or differentiated value is to be calculated
$C_0{}^4$, $C_1{}^4$, $C_2{}^4$	The smoothing, first derivative, and second derivative coefficients, assuming that the fourth derivative is constant
D	A column matrix composed of the computed first derivatives
E	The matrix composed of the time differences relative to t_0 where the desired smoothed or differentiated value is needed
F	Conditional equation used in the least square adjustment of data
N	Number of coefficients used in calculating a smoothed or differentiated value. Same as the number of observations
N_t	The smoothing interval over which a smoothed or differentiated value is calculated
R	A column matrix composed of the observed parameters
S	A column matrix composed of the computed parameters
U	The matrix composed of the unknowns $\overline{X}_0$, $\overline{\overline{X}}_0$, $\overline{\overline{\overline{X}}}_0$, ...
W	A diagonal weight matrix

v

LIST OF SYMBOLS (CONT.)

X_t	An observed parameter at time t
$\overline{X}_t$	A smoothed parameter at time t
$\dot{X}_t$	The first derivative of X (unsmoothed) at time t
$\overline{\dot{X}}_t$	The first derivative of a smoothed parameter at time t
$\ddot{X}_t$	The second derivative of X (unsmoothed) at time t
$\overline{\ddot{X}}_t$	The second derivative of a smoothed parameter at time t
σX_t	Standard deviation of X_t at time t
$\sigma \dot{X}_t$	Standard deviation of $\dot{X}_t$ at time t
$\sigma \ddot{X}_t$	Standard deviation of $\ddot{X}_t$ at time t
Δt	The time difference between the time (t) of some observed value $\dot{X}_t$ and t_0, $(t - t_0)$

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SUMMARY

The purpose of this report is to present a least squares procedure for calculating smoothing and differentiation coefficients and to present a procedure for calculating standard deviations of the data on which smoothing and differentiation formulas are used.

SECTION I. INTRODUCTION

External and onboard observations of various types are used in evaluating the performance of a flight test vehicle. All of these observations contain random errors as well as bias and systematic errors. Often first and second derivatives of the observations with respect to time are required. The presence of random error in the observations makes the determination of plausible derivatives considerably more difficult. Most observations fall into two categories: 1) essentially time varying functions (such as external tracking data, guidance output, pressures, flow rates, etc.) and 2) essentially oscillatory functions (such as normal accelerations, angular accelerations, angular rates, etc.). Appropriate smoothing and differentiation procedures can largely reduce the effect of random errors in the observations.

The derivation of coefficients that may be used for smoothing and differentiation of various observations is laborious, especially if a large number of points and a high degree approximating polynomial are

required. In addition, it is usually impractical to publish a sufficient variety of smoothing and differentiation coefficients to meet all needs since the major characteristics of the specific observation type should be considered. Most methods for computing smoothing and differentiation coefficients (Refs. 1, 2, and 3) derive them separately and with a lesser degree of flexibility than is desirable for evaluation purposes.

This report presents the development of a compact method for determining smoothing and differentiation coefficients that is easily adaptable to the characteristics of the observations and is readily usable on high speed electronic digital computers. The method presented is very effective with observations that can be approximated by a polynomial that is some function of an independent variable. The method can also be used with observations that are essentially periodic functions provided that only short arcs of a cycle occur during a given smoothing or differentiation span.

The methods were developed specifically for flight evaluation purposes but are equally valid for any type of observation or data for which smoothing or differentiation techniques are useful.

SECTION II. SMOOTHING AND DIFFERENTIATION COEFFICIENTS

A. PROCEDURE FOR CALCULATING THE COEFFICIENTS

One basic assumption is needed in the development of the proposed method of determining the smoothing and differentiation coefficients. The assumption is: The observation can be sufficiently well represented as a polynomial that is a function of an independent variable over a short span of the independent variable. Most observations used in flight evaluation would use the time of flight as the independent variable. The basic equation is obtained by expanding the function, X_t, in a Taylor's series about the point, t_0.

$$X_t = \overline{X}_0 + \Delta t\, \overline{\overline{X}}_0 + \frac{1}{2}\Delta t^2\, \overline{\overline{\overline{X}}}_0 + \frac{1}{6}\Delta t^3\, \overline{\overline{\overline{\overline{X}}}}_0 + \ldots + \frac{\Delta t^d}{d!}\, \overset{\overline{d}}{X}_0 \qquad (1)$$

where X_t is an observable parameter at time t; Δt is the time difference between the time of an observable parameter and the time at which the smoothed or differentiated value is to be calculated, i.e., $\Delta t = t - t_0$;

$\overline{X}_0$ is the smoothed parameter at t_0; $\overline{\dot{X}}_0$, $\overline{\ddot{X}}_0$, ... $\overset{\overline{d}}{X}_0$ are the successive derivatives of the smoothed parameter at t_0. Truncating equation (1) after the dth term determines the degree of the polynomial representing the observation. The point of truncation should be chosen such that bias induced by using a polynomial having its dth derivative constant is insignificant compared to random error remaining in the smoothed data.

The use of Eq. (1) will be illustrated in the following simple example which assumes the second derivative constant and which involves three observed values. The observed values can be expressed as:

$$X_{-1} = \overline{X}_0 + \Delta t_{-1} \overline{\dot{X}}_0 + \tfrac{1}{2} \Delta t_{-1}^2 \overline{\ddot{X}}_0$$

$$X_0 = \overline{X}_0 + (0) \overline{\dot{X}}_0 + (0) \overline{\ddot{X}}_0 \qquad (2)$$

$$X_1 = \overline{X}_0 + \Delta t_1 \overline{\dot{X}}_0 + \tfrac{1}{2} \Delta t_1^2 \overline{\ddot{X}}_0$$

This may be expressed in matrix form as:

$$
\begin{bmatrix} X_{-1} \\ X_0 \\ X_1 \end{bmatrix} =
\begin{bmatrix} 1 & \Delta t_{-1} & \tfrac{1}{2} \Delta t_{-1}^2 \\ 1 & 0 & 0 \\ 1 & \Delta t_1 & \tfrac{1}{2} \Delta t_1^2 \end{bmatrix}
\begin{bmatrix} \overline{X}_0 \\ \overline{\dot{X}}_0 \\ \overline{\ddot{X}}_0 \end{bmatrix}
\qquad (3)
$$

In this example, the smoothing polynomial will fit the observed points exactly. Consequently, there can be no removal of random error. By using more observations without increasing the degree of the smoothing polynomial, a least squares procedure results. With the additional observations Eq. (3) becomes:

$$R = EU \qquad (4)$$

where for example:

$$R = \begin{bmatrix} X_{-3} \\ X_{-2} \\ X_{-1} \\ X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & \Delta t_{-3} & \frac{1}{2}\Delta t_{-3}^2 \\ 1 & \Delta t_{-2} & \frac{1}{2}\Delta t_{-2}^2 \\ 1 & \Delta t_{-1} & \frac{1}{2}\Delta t_{-1}^2 \\ 1 & 0 & 0 \\ 1 & \Delta t_1 & \frac{1}{2}\Delta t_1^2 \\ 1 & \Delta t_2 & \frac{1}{2}\Delta t_2^2 \\ 1 & \Delta t_3 & \frac{1}{2}\Delta t_3^2 \end{bmatrix} \qquad U = \begin{bmatrix} \overline{X}_0 \\ \overline{\overline{X}}_0 \\ \overline{\overline{\overline{X}}}_0 \end{bmatrix}$$

The elements of the matrix E are the partial derivatives with respect to the unknowns $\overline{X}_0$, $\overline{\overline{X}}_0$, and $\overline{\overline{\overline{X}}}_0$ of the conditional equations:

$$F_i = \overline{X}_0 + \Delta t_i \overline{\overline{X}}_0 + \tfrac{1}{2}\Delta t_i^2 \overline{\overline{\overline{X}}}_0 - X_{t_i}$$

where $i = 0, 1, \ldots, N-1$.

The least square solution of Eq. (4) is now given by the expression:

$$\left[(E^T WE)^{-1} (E^T W) \right] R = U \tag{5}$$

where W is an arbitrary weight matrix.

The smoothing and differentiation coefficients are now the elements of the matrix $\left[(E^T WE)^{-1} (E^T W) \right]$ and are entirely independent of the observed parameters.

The matrix $\left[(E^T WE)^{-1} (E^T W) \right]$ will contain $d+1$ rows and N columns. The smoothing coefficients are the elements of the first row denoted $C_0{}^d$, the first derivative coefficients denoted $C_1{}^d$ are the elements of the second row, the dth derivative coefficients denoted $C_d{}^d$ are the elements of the $d+1$ row. Each set of coefficients forms a row matrix. The solutions become:

$$\overline{X}_0 = C_0^d \, R$$

$$\overline{\overline{X}}_0 = C_1^d \, R$$

$$\overline{\overline{\overline{X}}}_0 = C_2^d \, R \qquad\qquad (6)$$

$$\vdots$$

$$\overset{\overline{d}}{X}_0 = C_d^d \, R$$

This procedure for obtaining coefficients does not require that the observations be at equally spaced time intervals. There is an advantage, however, in having data available at equally spaced intervals since the coefficients need to be calculated only once. Thereafter, the data can be processed with a moving arc as long as the data remain available at the interval for which the coefficients were calculated.

If the data points are not equally spaced then the coefficients must be recalculated whenever a smoothed or differentiated value is desired.

A number of smoothing and differentiation coefficient ratios (the silhouettes or shapes of the plotted coefficients versus time) are shown on pages 12 to 15. The dashed lines in each figure indicate equal weights were used in the calculation of the coefficients, and the solid lines indicate the weights used in the calculations were of the form e^{-kt^2}, i.e., normally distributed (see Fig. 4).

Figure 1 shows the silhouettes of the smoothing coefficients C_0^1, C_0^3, and C_0^5. The smoothing polynomials for which these coefficients are applicable are of the degrees one, three, and five, respectively. Figure 2 shows the silhouettes of the first derivative coefficients C_1^2, C_1^4, and C_1^6. The smoothing polynomials for which these coefficients are applicable are of degrees two, four, and six, respectively. Figure 3 shows the silhouettes of the second derivative coefficients C_2^3 and C_2^5. The smoothing polynomials for which these coefficients are applicable are of degrees three and five, respectively.

B. INTERPOLATING AND EXTRAPOLATING

By slightly modifying the basic formulas, one may calculate the coefficients for interpolating and extrapolating. The following modification is made to Eq. (4) for obtaining interpolation coefficients.

$$
\begin{bmatrix} X_{-3} \\ X_{-2} \\ X_{-1} \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & \Delta t_{-3} & \frac{1}{2}\Delta t_{-3}^2 \\ 1 & \Delta t_{-2} & \frac{1}{2}\Delta t_{-2}^2 \\ 1 & \Delta t_{-1} & \frac{1}{2}\Delta t_{-1}^2 \\ 1 & \Delta t_1 & \frac{1}{2}\Delta t_1^2 \\ 1 & \Delta t_2 & \frac{1}{2}\Delta t_2^2 \\ 1 & \Delta t_3 & \frac{1}{2}\Delta t_3^2 \end{bmatrix} \begin{bmatrix} \overline{X}_0 \\ \overline{\dot{X}}_0 \\ \overline{\ddot{X}}_0 \end{bmatrix} \tag{7}
$$

This arrangement merely omits the t_0 observation since this is the time that an interpolated value is needed.

For extrapolation Eq. (4) becomes:

$$
\begin{bmatrix} X_{-7} \\ X_{-6} \\ X_{-5} \\ X_{-4} \\ X_{-3} \\ X_{-2} \\ X_{-1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t_{-7} & \frac{1}{2}\Delta t_{-7}^2 \\ 1 & \Delta t_{-6} & \frac{1}{2}\Delta t_{-6}^2 \\ 1 & \Delta t_{-5} & \frac{1}{2}\Delta t_{-5}^2 \\ 1 & \Delta t_{-4} & \frac{1}{2}\Delta t_{-4}^2 \\ 1 & \Delta t_{-3} & \frac{1}{2}\Delta t_{-3}^2 \\ 1 & \Delta t_{-2} & \frac{1}{2}\Delta t_{-2}^2 \\ 1 & \Delta t_{-1} & \frac{1}{2}\Delta t_{-1}^2 \end{bmatrix} \begin{bmatrix} \overline{X}_0 \\ \overline{\dot{X}}_0 \\ \overline{\ddot{X}}_0 \end{bmatrix} \tag{8}
$$

This extrapolation method will produce the coefficients necessary to obtain values ($\overline{X}_0$, $\overline{\dot{X}}_0$, $\overline{\ddot{X}}_0$) at some time after the observations are available. The same procedure could produce coefficients for extrapolating values prior to the time that observations are available by substituting observations X_{+1} through X_{+7} in place of X_{-1} through X_{-7}.

C. APPLICATION

The purpose of using smoothing and differentiation procedures is to eliminate random error from measured data and to obtain realistic derivatives. To obtain accurate results without inducing bias or

destroying real data, the degree (d) of the smoothing polynomial, the number (N) of observations, and the weights (W) to be used in the calculation of the coefficients must be properly chosen. To accomplish this the following suggestions are made:

1.   Since the smoothed values become more sensitive to random error as the degree of the smoothing polynomial increases, the polynomial of lowest degree which adequately represents the data should be used.

2.   Numerous factors must be considered in deciding the number of observations to be used in calculating the coefficients. Some of the more important are: (a) the degree of smoothing polynomial to be used; (b) the magnitude of the various derivatives of the smoothing polynomial; (c) the observation rate; (d) the amount of random error in the data. In other words a priori knowledge of the data is essential.

3.   All the observations in the smoothing interval should be used unless "stray" values have definitely been detected.

4.   If random error is unusually large, increase the number of coefficients or reduce the degree of the smoothing polynomial. The adverse effects of bias induced in this manner may be minor compared to the resulting effects of poor smoothing.

5.   There are no restrictions in formulating the weights to be assigned to the observed values. In most cases, however, the weights should be symmetrical about the midpoint of the smoothing interval and the weight of the value corresponding to the midpoint should be a maximum.

All of the above suggestions are applicable for observations that may be either approximated by a polynomial, or are periodic functions with only short arcs of a cycle occurring during a given smoothing or differentiation span.

SECTION III.   CALCULATION OF STANDARD DEVIATIONS

Standard deviations can be calculated easily by carrying the smoothing and differentiation coefficient calculations several steps

further than illustrated in Section II. These standard deviations will pertain to the raw data prior to any smoothing effects.

The coefficients $\left[(E^T WE)^{-1} (E^T W)\right]$ as obtained from Eq. (5) are used to calculate the unknowns (U) at some particular time, t_0. The standard deviations are then calculated at this particular time independent of the surrounding times except through the matrix (E).

The smoothed parameters (S) about t_0 are obtained from the expression:

$$S = EU.$$

The desired residuals are now given by:

$$V = R - S \tag{9}$$

where:

$$S = \begin{vmatrix} \overline{X}_{-3} \\ \overline{X}_{-2} \\ \vdots \\ \overline{X}_3 \end{vmatrix} \quad R = \begin{vmatrix} X_{-3} \\ X_{-2} \\ \vdots \\ X_3 \end{vmatrix} \quad V = \begin{vmatrix} X_{-3} - \overline{X}_{-3} \\ X_{-2} - \overline{X}_{-2} \\ \vdots \\ X_3 - \overline{X}_3 \end{vmatrix} \quad or \quad \begin{vmatrix} \Delta X_{-3} \\ \Delta X_{-2} \\ \vdots \\ \Delta X_3 \end{vmatrix}$$

The variance is then given by:

$$\sigma^2 X_0 = \frac{Y^T Y}{N} \tag{10}$$

where:

$$Y = \begin{vmatrix} \Delta X_{-3} - \overline{\Delta X} \\ \Delta X_{-2} - \overline{\Delta X} \\ \vdots \\ \Delta X_3 - \overline{\Delta X} \end{vmatrix} \quad and \quad \overline{\Delta X} = \frac{\sum\limits_{i=-3}^{3} \Delta X_i}{N}$$

The first derivatives of the smoothed values around t_0 are obtained in the following manner. The first derivative of Eq. (1) is written:

$$\overline{X}_t = \overline{X}_0 + \Delta t\,\overline{\dot{X}}_0 + \frac{1}{2}\Delta t^2\,\overline{\ddot{X}}_0 + \frac{1}{6}\Delta t^3\,\overline{\dddot{X}}_0 \ldots \tag{11}$$

and it is seen that the first derivatives of the smoothed values can easily be obtained with slight modifications to the matrix (E) and the matrix (U). Let $\overline{E}$ denote the matrix formed from the matrix (E) by dropping its last column and let $\overline{U}$ be the matrix formed from the matrix (U) by dropping its first element, then

$$D = \overline{EU} \tag{12}$$

The elements of the column matrix (D) are the desired derivatives. The residuals required for the calculation of the standard deviation of X at t_0 are the differences in the derivatives of the smoothed values and the derivatives obtained from the unsmoothed data. The second derivatives of the smoothed values are obtained in a similar manner.

The following simple example will illustrate the procedure. It is assumed in this example that the observed data can be represented by a third degree polynomial and that nine observed data points are available. The smoothed values around t_0 are given by:

$$
\begin{bmatrix}
\overline{X}_{-4} \\
\overline{X}_{-3} \\
\cdot \\
\cdot \\
\overline{X}_3 \\
\overline{X}_4
\end{bmatrix}
=
\begin{bmatrix}
1 & \Delta t_{-4} & \frac{1}{2}\Delta t_{-4}^{2} & \frac{1}{6}\Delta t_{-4}^{3} \\
1 & \Delta t_{-3} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
1 & \Delta t_3 & \cdot & \cdot \\
1 & \Delta t_4 & \cdot & \frac{1}{6}\Delta t_4^{3}
\end{bmatrix}
\begin{bmatrix}
\overline{X}_0 \\
\overline{\dot{X}}_0 \\
\overline{\ddot{X}}_0 \\
\overline{\dddot{X}}_0
\end{bmatrix}
\tag{13}
$$

The first derivatives of the smoothed values are given by:

$$
\begin{bmatrix} \bar{\ddot{X}}_{-4} \\ \bar{X}_{-3} \\ \cdot \\ \cdot \\ \bar{X}_{3} \\ \bar{X}_{4} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t_{-4} & \frac{1}{2}\Delta t_{-4}{}^2 \\ 1 & \Delta t_{-3} & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & \Delta t_{3} & \cdot \\ 1 & \Delta t_{4} & \frac{1}{2}\Delta t_{4}{}^2 \end{bmatrix} \begin{bmatrix} \bar{X}_{0} \\ \bar{\dot{X}}_{0} \\ \bar{\ddot{X}}_{0} \end{bmatrix}
\tag{14}
$$

and the second derivatives of the smoothed values are given by:

$$
\begin{bmatrix} \bar{\ddot{X}}_{-4} \\ \bar{\ddot{X}}_{-3} \\ \cdot \\ \cdot \\ \bar{\ddot{X}}_{3} \\ \bar{\ddot{X}}_{4} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t_{-4} \\ 1 & \Delta t_{-3} \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & \Delta t_{3} \\ 1 & \Delta t_{4} \end{bmatrix} \begin{bmatrix} \bar{\ddot{X}}_{0} \\ \bar{\dddot{X}}_{0} \end{bmatrix}
\tag{15}
$$

The appropriate variances are obtained in the same manner as above.

SECTION IV. ACCURACY OF METHOD

The matrix $\left[(E^T WE)^{-1}(E^T W)\right]$, which will be defined as B, is essentially the partial derivatives of the unknowns, U, with respect to the observed parameters, R. The variances of U can be determined from the usual equation if the variances of observed parameters are known.
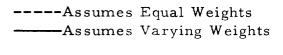
$$
\sigma U^2 = B \sigma_R{}^2 B^T
\tag{16}
$$

The variances of the smoothed positions, velocity, and acceleration were derived from the above equation assuming the raw position data were known within ±10 meters as an example of the accuracy obtained with this procedure. The standard deviations, which are the square roots of the variances, for the smoothed positions, velocities, and accelerations are shown in Figs. 5 through 7.

The standard deviations are plotted versus the number of points used where various degree polynomials were assumed to fit the data during the interval. Each of Figs. 5 through 7 indicates that the largest number of points with the lowest degree polynomial, commensurate with a good fit of the data, is required to produce the smallest standard deviation. The standard deviations shown are independent of the data sampling rate, the time interval chosen, or the absolute magnitude of the observations. The standard deviations from this method, are a function of the variances of the observed parameters, the number of points used, and the degree of the polynomial required to produce a fit to the data.

SECTION V. CONCLUDING REMARKS

A method for calculating smoothing and differentiation coefficients has been developed. It lends itself easily to various types of data with different characteristics. The method is particularly applicable to computer use since it eliminates the necessity for storing large numbers of coefficients in memory. Because of the cumulative effect of round-off error inherent in least squares techniques, the computations involved in the calculation of the coefficients should be done in double precision.
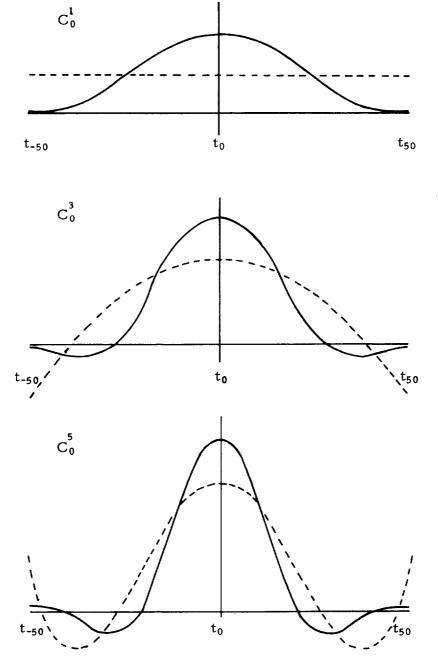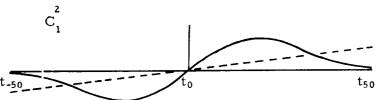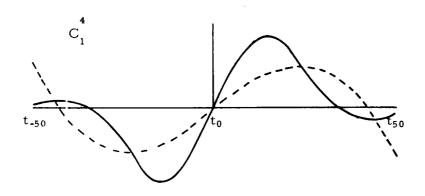
12

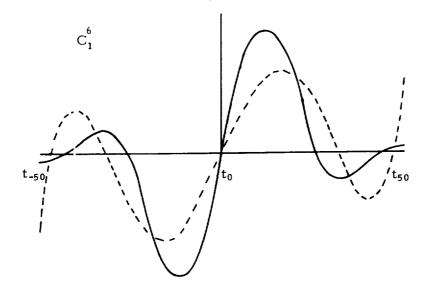Fig. 1. Smoothing Coefficients Assuming the First, Third, and Fifth Derivatives Constant

Fig. 2. First Derivative Coefficients Assuming
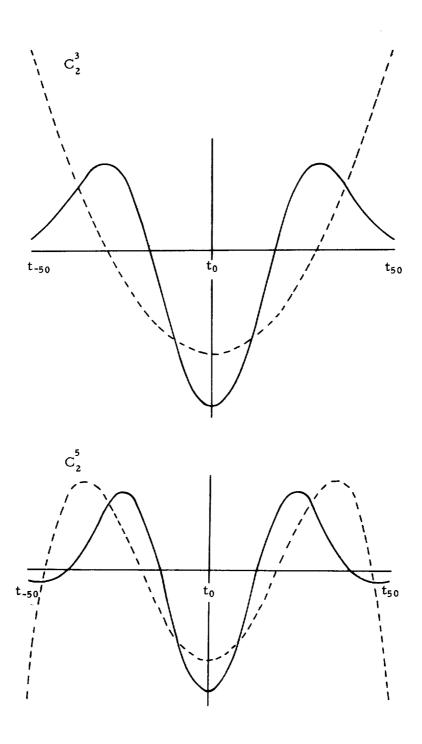the Second, Fourth, and Sixth Derivatives
Constant

Fig. 3. Second Derivative Coefficients Assuming
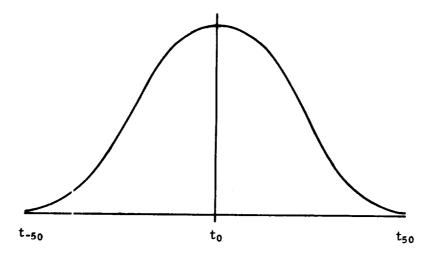the Third and Fifth Derivatives Constant

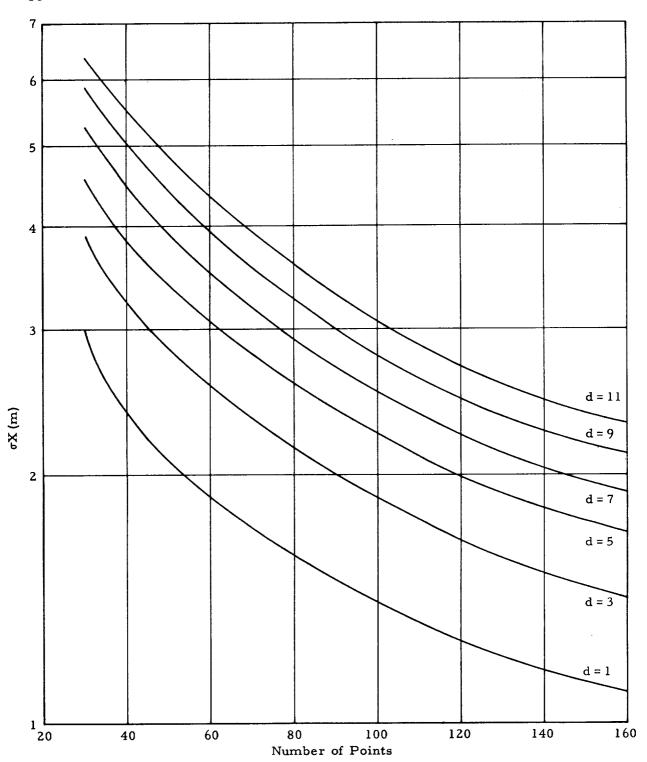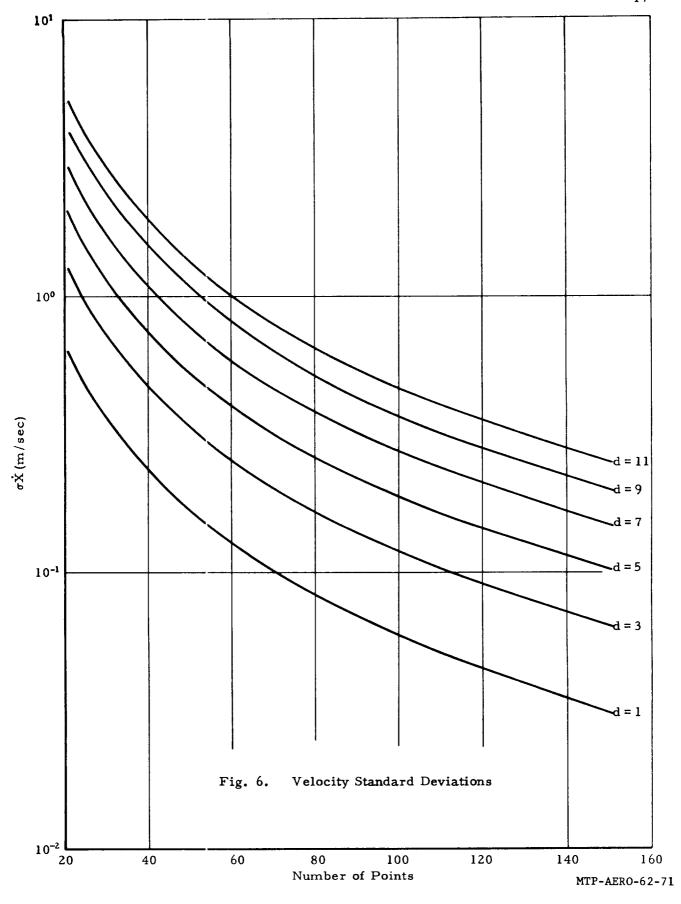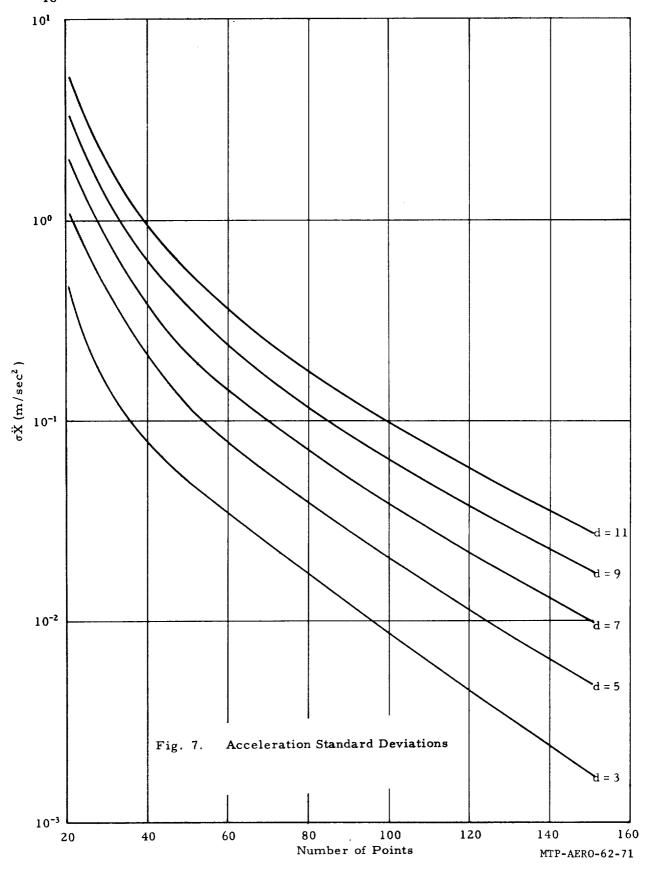Fig. 4.   Form of the Principle Diagonal
of the Weight Matrix

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Fig. 5.   Position Standard Deviations

Fig. 6. Velocity Standard Deviations

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Fig. 7.   Acceleration Standard Deviations

σẌ (m/sec²)

Number of Points

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3. Strand, Otto Neall, The Smoothing and Differentiation of an Empirical Function by the Use of a Least Squares Polynomial, U. S. Naval Ordnance Test Station, China Lake, California, 1952.

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