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ABSTRACT

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Presented here is a preliminary analysis of the factors involved in designing a minimum weight baffle system, composed of rings and/or partitions, to prevent excessive fuel sloshing in the propellant tanks of large rocket vehicles. By specifying over a given frequency range a maximum permissible force response due to liquid sloshing, a set of permitted combinations among the ring and partition baffle structures is determined, each of which sufficiently suppresses the liquid motion. The overall minimum weight baffle system is then determined from a strength analysis of the permitted baffle structures. Results of a typical example indicate that for moderate damping a plain ring baffle system has minimum weight.

Author

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INTRODUCTION

General

If a cylindrical tank, partially filled with a liquid and subject to a finite axial acceleration, is abruptly displaced in a direction transverse to its axis, the tank will receive an oscillatory force response due to the "normal sloshing" of the contained liquid. If the tank experiences a periodic translation excitation at precisely one of the resonant frequencies of the contained liquid, the sloshing will become exceedingly violent, limited only through the damping provided by various suppression devices, by the natural wiping of the liquid against the tank walls, and by the internal viscosity of the liquid itself.

In the design of a liquid-fueled rocket vehicle, particular attention must be given to this phenomenon of liquid sloshing, as the resulting forces will interact with the vehicle's structure and control system, and may possibly cause disastrous failure of vital structural components within the vehicle or excessive deviation of the vehicle from its flight path. The effects of liquid sloshing are especially pronounced for large rockets, since for such rockets the weight of liquid propellant constitutes a very high percentage of the initial gross weight of the vehicle, and since the relative damping afforded by wiping against the tank walls decreases with an increase in tank size.*

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* Likewise, for usual liquid propellants the damping provided by internal viscosity is a negligibly small quantity.

The simplest and most effective known method of reducing the deleterious effects of propellant sloshing on the performance of a rocket vehicle consists of building into the fuel and oxidant tanks a system of mechanical baffles which suppresses the liquid by partially blocking its fluid motion. Of such baffle systems, two are of special interest: (1) a system of horizontal annular rings, both flat and conical, extending inward from the tank wall and spaced at regular intervals along its axis; and (2) a system of vertical rectangular flat partitions, both perforated and nonperforated, extending from the tank axis to the tank wall and separated by a uniform angle about the axis. For large rocket tanks, a baffle system composed of both rings and partitions, such as shown in Figure 1, may be considered.*

Ring baffles damp the liquid by transforming smooth axial flow along the tank walls into high speed turbulent flow around the baffles. A system of ring baffles is to be preferred over a single submerged ring because for multiple rings the minimum (or design) damping can be controlled by properly spacing the baffles, and because if one of the baffles should fail, say the one nearest the free surface, there will still remain other submerged baffles to damp the liquid. Nonperforated partition baffles provide very little damping, but rather derive their effectiveness from dividing the large sloshing mass associated with

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* See Figure 35 of Reference [1] for a proposed baffle system consisting of both rings and partitions.

liquid in an uncompartmented tank into several smaller sloshing masses having considerably higher first mode resonant frequencies. In addition to the partitioning effect of dividing the sloshing mass and raising the natural frequencies, perforated partition baffles provide damping and further alter the natural frequencies by allowing some of the liquid to flow between the various compartments.

The efficiency of a rocket increases with its thrust to weight ratio; it is therefore desirable that any fixed mass in addition to that of the necessary structure of the vehicle be kept as small as possible. Given a particular baffle configuration, its design is then determined by the minimum weight (mass) cross section necessary to withstand the maximum pressure loading of the sloshing liquid, where at certain points the baffle material is stressed to some characteristic ultimate strength. The two baffle systems mentioned above are of special interest because they provide a significant decrease in the effects of liquid sloshing with a minimum penalty of mass added to the vehicle.

Objective

The hydrodynamic (potential) theory of normal sloshing has been well developed for a variety of bare-wall tank configurations [1]* and a simple (mathematical) mechanical analogy involving masses, springs, and dashpot elements has been derived to represent the response of a

* Numbers in square brackets refer to the references listed at end of this report.

sloshing liquid when the effects of damping have been included [2]. Likewise, an analytic formulation has been developed to express the damping of normal sloshing obtained from a submerged ring baffle [3]. Finally, considerable experimental background has been provided on the normal sloshing characteristics of liquids in cylindrical tanks and on the damping characteristics of various liquid suppression devices [4, 5, 6, 7].*

The present study is a first attempt to integrate this existing knowledge of liquid sloshing with a weight analysis of ring and partition baffle structures, and has been conducted in order to provide a preliminary rational basis for designing a minimum weight baffle system and in order to serve as a guide for future investigations pertaining to the optimum design of antisloshing baffle systems. Many of the assumptions upon which this study is based are crude approximations to the operating phenomena in actual rocket vehicles, and the resulting weight equations and tentative design procedure are therefore inaccurate to the same extent. Presented at the end of this report is a discussion of the most apparent sources of inaccuracy, together with some recommendations for improving and sophisticating the present analysis.

Basic Assumptions

Although the design of an antisloshing baffle system for the liquid propellant tanks of a rocket vehicle must necessarily depend upon the

* See reference [8] for a bibliography and summaries of theoretical and experimental investigations of liquid sloshing.

overall design of the vehicle, the following assumptions are made in order to obtain applicable design information from the subsequent analysis.*

1. The liquid container is a circular cylindrical tank, considered to be perfectly rigid for all purposes, and subjected to a constant axial acceleration.

2. The most effective arrangement of antisloshing baffles is a system composed of either ring baffles or partition baffles, or a combination of both, as mentioned previously. Considering each of zero, four, six, and eight partitions is sufficient to indicate all the effects of varying tank compartmentation. Ring baffles are spaced uniformly.

3. The total liquid force, slosh height, ring pressure, and partition pressure, which are termed the sloshing effects,** are adequately described by their steady state amplitudes due to a given transverse sinusoidal tank excitation. The sloshing effects are calculated

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* Evidence of the complexity of designing minimum weight tank structures is provided by the many analytical approaches presented in [9].

** The total liquid force is the force exerted on the tank walls by the total liquid mass; the slosh height is the amplitude of the liquid free surface oscillations at the tank wall; the ring pressure is a representative maximum pressure exerted by the sloshing liquid on a ring baffle; the partition pressure is a representative maximum pressure exerted by the sloshing liquid on a partition baffle.

semiempirically from a mathematical analogy consisting of mass-, spring-, and dashpot-elements, referred to as an equivalent mechanical model [2], the constants of which are adjusted to agree with available experimental data. The effects of damping on liquid sloshing are adequately accounted for by the introduction of uniform linear dashpots into the equivalent mechanical model.

4. The tank excitation, being of random origin, is assumed to have components of equal amplitude for each of the first mode liquid resonant frequencies of the variously compartmented tanks. Furthermore, the sloshing effects at each of these first resonances are always the most severe, therefore the primary purpose of the proposed baffle system is to suppress first mode resonance sloshing effects, unless there is an additional requirement that liquid resonances be avoided throughout some designated critical range of frequencies.

5. Liquid damping is the result exclusively of axial flow past the ring baffles and of liquid exchange through perforations in the partition baffles. Natural damping from wiping against the tank wall and against the partitions, as well as that arising from the internal viscosity of the liquid are neglected. The minimum damping due to a system of ring baffles is obtained from Miles' formula [3], in its range of applicability, using the contribution of the first submerged baffle to represent the damping for a multiple ring baffle system The

damping due to perforated partitions must be obtained from experimental data [4, 5] as there exists no analytic formula to express this damping.

6. The minimum weight analysis is based upon the assumption that the baffles behave as perfectly elastic thin plates, either solid or composite, perforated or nonperforated, of uniform thickness, which derive their strength from resistance to bending. The baffles are assumed to be sufficiently rigid that their deflections do not contribute to the sloshing effects, and that a static stress analysis is valid for computing the responses of the baffle structures to the hydrodynamic pressure loadings. The pressure loadings and boundary conditions on the baffles are assumed to be as shown in Figures 16 and 17.

ANALYSIS OF LIQUID SLOSHING

Sloshing Analysis Approach

During the flight of a typical rocket vehicle, various conditions combine to produce transverse disturbances on the walls of its propellant tanks. These disturbances may be conceived of as random in direction of application and in frequency distribution; however, some maximum amplitude may usually be assigned to such disturbances. In addition to these disturbances, the level of liquid in the propellant tanks, and therefore the principal resonant frequencies of the vehicle are constantly changing with time. Presumably, for large tanks without liquid suppression devices, normal sloshing of the liquid propellant will become excessive at some point during the flight and the resulting forces applied to the vehicle will cause disastrous consequences to occur.

The two fundamental criteria for an antisloshing baffle system are therefore: (1) to prevent the total liquid force response from exceeding a certain prescribed maximum value under all possible combinations of liquid level, tank orientation and acceleration, and external tank excitation, and (2) to especially suppress the sloshing effects throughout certain designated frequency ranges in which the liquid oscillations might reinforce the fundamental vibration modes of the vehicle. Since knowledge of critical vibration frequencies depends upon the specific design of each particular vehicle, and since this report is intended to delineate a general design procedure, the following analysis is concerned primarily with the first of these criteria.

Sloshing Effects Derived from a Mechanical Model

In order to determine theoretically the effect of a sloshing liquid on a vibrating tank system, a rigorous hydrodynamic analysis is required which takes into account the presence of the free liquid surface. Such an analysis has been made for compartmented cylindrical tanks [1], as required for the present study; however, results from this analysis are only partly available. In the absence of such results, an equivalent mechanical (mathematical) model for the sloshing liquid similar to that presented in [2], was developed and made to conform semi-empirically with available experimental data.

Figure 2 shows horizontal cross sections of compartmented tanks having four, six, and eight partitions. The positions of the various compartments with respect to the direction of excitation are designated as orientation 1 if the direction of excitation lies in the plane of one of the partitions, and as orientation 2 is the direction of excitation lies in a plane which bisects one of the compartments.* Values of the nondimensional resonant frequency parameter, $\omega_1' = (\omega_1)^2 a/g$, are presented for the lowest resonances associated with each of the compartments in the particular orientations shown, as obtained experimentally from [4].

* This distinction exists only for an even number of compartments.

Figure 3 shows the higher harmonic frequencies corresponding to these first resonances and was obtained by passing smooth curves through a plot of the natural resonant frequencies presented in [5].

Figures 4, 5, and 6 present, respectively, the total force response, the slosh height response, and the maximum partition (or wall) pressure response, for a sloshing liquid in an uncompartmented tank. These figures show, respectively, the correlation of the theoretical results obtained from the equivalent mechanical model for various damping, with the total force response data obtained from [4], unpublished slosh height data, and wall pressure data obtained from [6]. Shown also in Figure 6 (and likewise in Figures 7, 8, 9) is the total force response of the liquid mass that would be obtained if the liquid were frozen solid.

Figures 7, 8, and 9 present the total undamped force responses for cylindrical tanks compartmented with four, six, and eight partitions, respectively, and excited in the directions designated in Figure 2 as orientation 1. The experimental data shown in Figure 7 for the four partition tank is that obtained from [4] for a half tank, and the experimental data shown in Figure 9 for the eight-partition tank was obtained from [5]. The resonant frequencies for the six-partition tank were obtained by interpolating between those for the four- and eight-partition tanks, since there is no experimental data available for the force response in a six-partition tank, as shown in Figure 8.

From this mechanical model was obtained the variation of total liquid force, slosh height, maximum ring pressure, and maximum partition pressure with damping for various resonant sloshing modes in variously compartmented tanks, as presented in Figures 10-13. The expressions used to obtain these results are given as follows:

$$\begin{aligned}
\zeta/X_o &= C_s (A'')^{1/2}, \\
(p_r')^2 &= p_r/\rho_L g_z a (X_o')^2 = C_r \omega' (\zeta/X_o)^2/2, \\
(p_p')^2 &= p_p/\rho_L g_z a X_o' = C_p \omega' (A')^{1/2}, \\
F_o' &= F_o/\rho_L g_z a^3 X_o' = \omega'(A' + 2 m_o' A + m_o'^2),
\end{aligned} \tag{1}$$

where ζ is the amplitude of free surface oscillation, or slosh height, X_o is the displacement amplitude of tank excitation, a is the tank radius, g_z is the maximum tank acceleration, p_r' is the nondimensional ring pressure parameter, p_p' is the nondimensional partition pressure parameter, and ω' is the nondimensional frequency parameter, and where:

$$\begin{aligned}
m_i' &= 2\pi C_m/\omega_i'(\omega_i'^2 - 1) \\
m_o' &= \pi h' - \sum_{i=1}^{\infty} (m_i') \\
a_i &= 1 - \omega'/\omega_i' \\
b_i &= 2\left[\omega'/\omega_i' (1 - \gamma^{-2})\right]^{1/2} \\
\alpha_i &= \left(\frac{1 + b_i^2}{a_i^2 + b_i^2} \right)^{1/2} \\
\beta_i &= \tan^{-1}(b_i) - \tan^{-1}(b_i/a_i); \quad \text{if } a_i < 0, \quad \beta_i = \beta_i + \pi \\
\alpha_i' &= \left[1 - 2\alpha_i \cos \beta_i + \alpha_i^2 \right]^{1/2}
\end{aligned} \tag{2}$$

$$\beta_i' = -\tan^{-1}\left(\frac{\alpha_i \sin\beta_i}{1 - \alpha_i \cos\beta_i}\right); \text{ if } a_i < 0, \ \beta_i' = \beta_i' + \pi$$

$$A_i = \alpha_i m_i', \quad A_i' = \alpha_i'/\omega_i'$$

$$A = \sum_1^{\infty} {}_i \, A_i \cos\beta_i$$

$$A' = \sum_1^{\infty} {}_i \sum_1^{\infty} {}_j \, A_i A_j \cos(\beta_i - \beta_j)$$

$$A'' = \sum_1^{\infty} {}_i \sum_1^{\infty} {}_j \, A_i' A_i' \cos(\beta_i' - \beta_j')$$

The fundamental resonant sloshing frequencies ω_1' (and sometimes ω_2') used in the above formulae are those obtained experimentally [4] as shown in Figure 2, and the higher harmonic frequencies corresponding to these first resonances were taken from Figure 3. A typical value of 2.00 was used throughout for h'. The appropriate values for C_s, C_r, C_p, and C_m were obtained by comparing the results of equations (1) and (2) with experimental data [4, 5, 6], as necessary to furnish the agreement shown in Figures 4-9.

For a given tank size, compartmentation, and excitation, Figures 10-13 provide the liquid sloshing effects from which the effectiveness of a particular baffle system may be determined. By prescribing a maximum permissible total liquid force, a value can be found (for each tank compartmentation) from the first mode resonance curves shown in Figure 10 for the minimum damping γ_{min} necessary to prevent the liquid force response from ever exceeding this prescribed maximum. Any baffle system, then, which will always provide damping greater than γ_{min} is considered sufficiently effective to be a permitted baffle system. The preferred antisloshing baffle system from among these permitted baffle systems is generally the one having minimum weight.

The pressure loadings by which the necessary strength (and therefore weight) of a permitted baffle system is to be calculated are given in Figures 12 and 13, plotted against the damping provided by the baffle system. The damping attributed to a given baffle system can be found by any means that is currently available. For perforated partition baffles the damping must be obtained from experimental data, as there exists no analytic expression for this damping. The damping provided by a system of ring baffles is found from Miles' formula, as presented in the next section.

Damping Due to a System of Ring Baffles

Miles' formula for the damping of liquid oscillations in a cylindrical tank due to the presence of a single submerged ring baffle is given by [3] to be

$$\gamma = C_r (\alpha')^{3/2} (\xi')^{1/2} \exp(-4.6d'), \tag{3}$$

where C_r is an appropriate drag coefficient, $\alpha' = w'(2 - w')$ is the fraction of the tank cross-sectional area blocked by the ring, ξ' is the nondimensional slosh height, and d' is the nondimensional depth of the ring below the liquid free surface. Figure 14 shows the variation of $\gamma/(\xi')^{1/2}$ with d' for various values of w'.

The effectiveness of a baffle system is governed by the minimum damping which can be obtained from such a system. Because of draining, the depth h of liquid propellant in a rocket tank is a slowly varying function of time. Therefore, the damping will likewise vary with time as the liquid surface encounters each of a series of rings. Since for a given slosh height the damping greatly increases at positions where the liquid surface intersects the plane of one of the baffles, and since the present concern is with relatively large slosh heights, the design damping value is defined for a liquid surface position exactly intermediate between two adjacent rings, and for the slosh height $\xi_d = D/2$, called the design slosh height, so that the liquid surface at its maximum excursion just touches each of these baffles (see Figure 15).

All submerged baffles are well below the liquid surface for the minimum damping positions described above; thus Miles' formula can be used to accurately predict the contribution of the i^{th} submerged baffle for such positions [7] as

$$\gamma_i = C_r (\alpha')^{3/2} (D'/2)^{1/2} \exp [- 2.3D'(2i - 1)]. \tag{4}$$

Assuming that the total damping from a system of N'' submerged ring baffles can be obtained simply by superimposing the damping contributions of the individual baffles, the design damping for such a system can be written as

$$\gamma_d/(\alpha')^{3/2} \simeq 2(D')^{1/2} \sum_{1}^{N''} \exp\left[-2.3D'(2i - 1)\right], \qquad (5)$$

where N'' is the integer which lies between $h'/D'-1$ and h'/D'. Figure 15 presents the variation of $\gamma_d/(\alpha')^{3/2}$ with D'; this figure shows also the damping contribution of the first submerged baffle versus D'. The latter has been shown generally to be the better assumption for expressing the damping due to a system of multiple ring baffles [7]. Here, as before, a typical value of 2.00 was used for h', and 2.83 was used for C_r.

ANALYSIS OF BAFFLE STRUCTURES

Bending Moments in Rings and Partitions

Throughout this analysis the baffles are assumed to be thin plates which derive their strength from resistance to bending. In particular, a composite sandwich plate will be considered which has a significantly lower specific weight (weight per unit area) γ_c than that of an ordinary solid plate for an equivalent bending stiffness. In order to establish limiting bounds on strength-weight characteristics, the baffles would have to be considered as large deflection membranes, which derive their strength from resistance to extension and shear deformations and exhibit negligible bending stiffness, and considered also from a plastic analysis approach whereby a general collapse load can be predicted. Inclusion of these latter effects into the strength analysis was considered beyond the scope of the present investigation.

A basic assumption of this preliminary study is that the walls of the cylindrical tank are perfectly rigid, or equivalently, that deflections of the internal baffles are independent of deflections of the tank, and conversely. This assumption provides two simplifications: First, since a given baffle does not contribute to the strength of the tank, the tank may be disregarded in computing and comparing the weights of various baffle systems. Second, since deflections of the tank and baffle are completely uncoupled, any convenient boundary conditions may be chosen for the baffles at the tank wall. The most realistic edge conditions for plates used as ring and partition baffle structures are as follows. The ring baffles are assumed clamped around their outer edges (welded to the tank wall) and either (1) free, or (2) simply supported (by stringers) around their inner edges. The partition baffles are assumed clamped along the edges in contact with the cylinder wall and coinciding with the cylinder axis, and simply supported along the other two edges.

The functions representing the transverse pressure distributions applied to the baffle structures (see Figures 16 and 17) were chosen so as to approximate as closely as possible those encountered in practice and yet provide analytical solutions to the thin plate problems. For a ring baffle, the maximum pressure distribution will vary in a way that is nearly sinusoidal around the ring circumference [3]. However, since the strength analysis is based only on the maximum bending moment incurred, and since this maximum bending moment is dependent largely upon the local (maximum) pressures, it was assumed that a uniform pressure distribution over the entire ring surface would be accurate enough for the present purposes.* The wall pressure distributions observed [6] for the first mode resonances in an uncompartmented tank vary from nearly zero at the tank bottom to

* This assumption is especially good for rings with small annular width w'.

a maximum value near the liquid free surface. Due to the obvious similarity, the pressure distributions on the partition baffles were therefore assumed to vary hydrostatically.

Ring baffle. Using the basic equations for the symmetrical bending of a circular plate having a centrally located hole, as given in section 17, pages 63-68 of [10], together with the simplified edge conditions prescribed above and the uniform transverse pressure distribution $P_p(r, \theta) = p_p$, as shown in Figure 16, the following expressions are obtained for the nondimensional (maximum) bending moment parameter m_r'.

(1) Free around $r = b$:

$$(m_r')^2 = \left[M_r(r,\theta)\right]_{max} / p_r a^2 = \frac{1}{8}\left[1 - \alpha + 2(2\beta - 1)b'^2\right], \qquad (6)$$

$$\alpha = \frac{(3+\nu)b'^2 - (1-\nu)}{(1-\nu)b'^{-2} + (1+\nu)}, \qquad \beta = \frac{1 + (1+\nu)\ln b'}{(1-\nu)b'^{-2} + (1+\nu)}.$$

(2) Simply supported around $r = b$:

$$(m_r')^2 = \frac{1}{8}\left[1 - \alpha + \left(\frac{2\beta - 1}{2}\right)\left(\frac{\alpha' - 2\alpha}{\beta' - 2\beta}\right)\right], \qquad (7)$$

$$\alpha' = \frac{(1 - b'^2)^2}{1 - b'^2 + 2\ln b'}, \qquad \beta' = \frac{1 - b'^2 + 2b'^2 \ln b'}{1 - b'^2 + 2\ln b'}.$$

Partition baffle. By a method similar to that used to obtain equation (ℓ), page 210 of [10], the following expression is obtained for the bending moment distribution along the edges $r = 0, a$, of a plate subject to a hydrostatic transverse pressure loading $P_p(r, z) = p_p z/h$, as shown in Figure 17, and given the edge conditions prescribed above

$$M_p[(0,a),z] = \left(\frac{2\rho_P a^2 \ell}{\pi^4 h}\right) \sum_1^\infty i \left(\frac{\sinh\alpha_i - \alpha_i}{\sinh\alpha_i + \alpha_i}\right)\left(\frac{\beta_i \cos\beta_i - \sin\beta_i}{i^4}\right) \sin\left(\frac{i\pi z}{\ell}\right),$$

$$\alpha_i = 2i\pi/\ell', \quad \beta_i = i\pi h/\ell. \tag{8}$$

The nondimensional (maximum) bending moment parameter m_p' is therefore given by

$$(m_p')^2 = \left[M_p\{(0,a),z\}\right]_{max} / \rho_P a^2. \tag{9}$$

Figures 16 and 17 show plots, respectively, of m_r' versus w', and m_p' versus ℓ' for $h/\ell \simeq 1$.

Plate Perforation Relationships

In seeking to determine more effective, light weight baffle structures, it is natural to consider perforation of the baffle plate material as a means of reducing overall baffle weight. Likewise, preliminary tests [5] on small models show that certain perforation improves the damping effectiveness of partition baffles.* Therefore, the effects of perforation on the strength-weight characteristics of a baffle structure will be considered as follows. Stress concentrations, due to the small radius holes are neglected. For holes of diameter d_p centered on a square lattice of spacing D_p, as shown in Figure 18, the following relationships apply.

* The damping provided by nonperforated ring baffles is always greater than that provided by equivalent perforated ring baffles.

$$\begin{aligned}
\epsilon_0 &= (\pi/4)(d_p/D_p)^2 , \\
k_0' &= 1 - \epsilon_0 , \\
k_0'' &= 1 - 2(\epsilon_0/\pi)^{1/2} , \\
k_0 &= \frac{k_0'}{(k_0'')^{1/2}} = \frac{1-\epsilon_0}{[1-2(\epsilon_0/\pi)^{1/2}]^{1/2}} ,
\end{aligned} \tag{10}$$

where k_0' is the perforation fraction, k_0'' is the minimum load bearing area fraction, and k_0 is the plate perforation factor which appears in all the subsequent weight equations. ϵ_0 is the percentage area removed.

For holes of diameter d_p centered on an equilateral-triangular lattice of spacing D_p, the following relationships apply:

$$\begin{aligned}
\epsilon_1 &= (\pi/2\sqrt{3})(d_p/D_p)^2 , \\
k_1' &= 1 - \epsilon_1 , \\
k_1'' &= 1 - (2\sqrt{3}\,\epsilon_1/\pi)^{1/2} , \\
k_1 &= \frac{k_1'}{(k_1'')^{1/2}} = \frac{1-\epsilon_1}{[1-(2\sqrt{3}\,\epsilon_1/\pi)^{1/2}]^{1/2}} ,
\end{aligned} \tag{11}$$

The variation of k_m with ϵ_m, where m is either 0 or 1 depending on the conventions established above, is shown in Figure 18; it is seen from this figure that the k_m are greater than 1 for all moderate values of ϵ_m. Therefore, contrary to widespread belief, there is no distinct weight saving to be gained by perforation, except for the case of very thin plates where the thickness is restricted by manufacturing tolerances rather than by optimum strength design. It may, nevertheless, be advantageous to use perforated partition baffles because of the hydrodynamic damping provided by plate perforation.

Including the effects of elastic stress concentrations would further degrade the strength-weight characteristics of perforated plates.

Minimum Specific Weights Based on Optimum Bending Strength

Expressions for the maximum bending moments encountered in ring and partition baffle structures due to the hydrodynamic pressure loading of a sloshing liquid have been presented previously. These bending moments are resisted by stresses induced in the baffle material, depending upon the particular plate cross section used for the baffle. By assigning a characteristic ultimate tensile strength S which the plate material can withstand before failing, and by designing the plate cross section such that the maximum induced tensile stresses are just equal to S, an optimum weight design will be established for the particular assumed plate cross section.

Solid plate. For a solid plate, the maximum stresses in terms of the bending moments are given by equations (44) of [10]. The maximum tensile stress condition due to bending, therefore becomes

$$\sigma_{max} = S = 6\, m_k / k_m'' t_s^2 = 6\,(p_k/k_m'')(m_k'/t_s')^2, \tag{12}$$

so that the optimum thickness is given by

$$t_s' = m_k'(6 p_k / k_m'' S)^{1/2}, \tag{13}$$

and the minimum specific weight (weight per unit area) of the solid plate can be written as

$$\gamma_s' = \gamma_s / \rho_B g_1 a = k_m' t_s' = k_m K_s m_k' (p_k/S)^{1/2}, \tag{14}$$

where $K_s = \sqrt{6} = 2.45$ is the cross section factor for a solid plate.

Simple composite sandwich plate. The weight of a baffle system can be significantly reduced by using a plate cross section with a greater moment of inertia (bending stiffness) per unit cross sectional area than that of a solid plate. Examples of such advantageous cross sections are the accordion, corrugated, and sandwich type constructions. To demonstrate the weight saving that can be gained in this manner, the strength-weight characteristics of a simple sandwich plate will be developed as follows.*

Consider a simple composite sandwich plate, consisting of two outer sheets of thickness t_o separated a distance τ_c by a vertical gridwork of square cells of sides d_c and thickness κt_c, as shown in Figure 19. The critical strength criteria for determining the dimensions of

* Sandwich type construction has found only limited use in missile shell designs primarily because of the inadequacy of welding techniques to make full use of the material strength and because the thermal stresses induced in the tank walls by cryogenic fuels tend to separate the outer sheets of a sandwich plate (see pages 3-4 of [9]). The use of sandwich type construction for baffles in large rocket tanks is attractive, however, since internal baffles are not exposed to thermal stresses, and since the recent development of high strength, low distortion aluminum welds [11] makes possible the fabrication of high quality sandwich plates.

this composite plate are obtained from the maximum bending condition, where one of the outer sheets is stressed in tension to the ultimate strength S of the material, and the other outer sheet, under uniform compression S, is neutrally stable. The gridwork is considered to contribute nothing to the bending strength of the composite plate, except to maintain the separation of the outer sheets, and to determine the positions of the simply supported buckling nodes.

Thus, the maximum tensile stress condition due to bending of this composite plate is given by

$$\sigma_{max} = S = m_k / k''_m t_c \tau_c = (m'_k)^2 p_k / k''_m t'_c \tau'_c , \tag{15}$$

or $$\tau'_c = (m'_k)^2 p_k / k''_m t'_c ,$$

and the local buckling condition, obtained from equation (j), page 310, of [10], is given by

$$\sigma_{max} = S \leq \pi^2 E t_c^2 / 3 k''_m d_c^2 (1 - \nu^2) , \tag{16}$$

or $$d'_c = \pi t'_c \left[E / 3 k''_m S (1 - \nu^2) \right]^{1/2} .$$

The minimum specific weight is obtained by substituting equations (15) and (16) into

$$d \Upsilon'_c / d t'_c = 0 , \tag{17}$$

where $\Upsilon'_c = \Upsilon_c / \rho_s g_1 a = 2 k'_m t'_c (1 + \kappa \tau'_c / d'_c) .$

From this minimum specific weight condition the following expressions are obtained,

$$t_c' = m_k' \left(\frac{\kappa p_k}{\pi}\right)^{1/2} \left(\frac{3(1-\nu^2)}{k_m'' S E}\right)^{1/4}, \tag{18}$$

$$\tau_c' = d_c/\kappa = m_k' \left(\frac{\pi p_k}{\kappa k_m' S}\right)^{1/2} \left(\frac{E}{3 k_m'' S (1-\nu^2)}\right)^{1/4},$$

and

$$\gamma_c' = 4 k_m' t_c' = k_m K_c m_k' (p_k/S)^{1/2}, \tag{19}$$

where $K_c = 4(\kappa/\pi)^{1/2} [3(1-\nu^2) k_m'' S/E]^{1/4}$

is the cross section factor for the simple composite sandwich plate. Equations (18) are expressions for the optimum dimensions of a simple composite sandwich plate, as shown in Figure 19, and equations (19) express its specific weight. Figure 19 shows the variation of K_c with $\kappa^2 k_m'' S/E$ for $\nu = 0.3$.

<u>Comparison of a solid plate with a sandwich plate</u>. Consider a solid plate and a simple composite sandwich plate, such as described above, having equivalent bending strengths, and constructed from the same material. Using the parameters $E = 10^7$ psi, $\nu = 0.3$, $S = 6 \times 10^4$ psi, and $\kappa = k_m'' = 1$, the following typical comparisons are obtained (see Figure 19).

$$\gamma_c/\gamma_s = 4 t_c/t_s = K_c/K_s \simeq \left(\frac{0.81}{2.45}\right) \simeq 0.33,$$

$$t_s/\tau_c = 6 t_c/t_s = (3/2)(K_c/K_s) \simeq 0.50, \tag{20}$$

$$t_c/\tau_c = 6(t_c/t_s)^2 = (3/8)(K_c/K_s)^2 \simeq 0.042.$$

It can be seen from the first of equations (20) that the theoretical specific weight of a simple sandwich plate is approximately one-third that for a solid plate of the same (typical) material. Of course, the use of sandwich plates is restricted to those applications where the thickness of the outer sheet material is large enough to be satisfactorily fabricated.

Total Weights of Various Baffle Systems

In order to determine the minimum weight baffle system for a particular application, it is necessary to have relationships expressing the total weights of baffle systems composed of rings and/or partitions. Given expressions (14, 19) for the specific weights γ_n of various plate structures, and knowing the geometry of the various baffle arrangements, the total weight of a baffle system is simply the product of γ_n, the area of each baffle, and the total number of baffles comprising the baffle system.

Weight of a ring baffle system.

$$
\begin{aligned}
W_r' &= W_r/\rho_B g_1 a^3 = \pi\alpha' N' \gamma_n' \\
&= \pi\alpha'[n'\ell/D][k_m K_n m_r'(p_r/S)^{1/2}] \\
&= K'\pi n'\alpha' X_0'(D')^{-1} m_r' p_r' ,
\end{aligned} \tag{21}
$$

where $n' = N'D/\ell \simeq 1$, $K' = k_m K_n \ell' S'$, and $S' = (\rho_L g_2 a/S)^{1/2}$, and where values of m_r' and p_r' are found from Figures 16 and 12, respectively.

Weight of a partition baffle system.

$$\begin{aligned} W_p' &= W_p/\rho_B g_1 a^3 = N \ell' \gamma_n' \\ &= N \ell' [k_m K_n m_p' (p_p/S)^{1/2}] \\ &= K' N (X_0')^{1/2} m_p' p_p' , \end{aligned} \tag{22}$$

where values of m_p' and p_p' are obtained from Figures 17 and 13, respectively.

Weight of a baffle system composed of both rings and partitions.

$$\begin{aligned} W_t' &= W_r' + W_p \\ &= K' [\pi n' \alpha' X_0' (D')^{-1} m_r' p_r' + N (X_0')^{1/2} m_p' p_p'] . \end{aligned} \tag{23}$$

Comparison of the weight of a ring baffle system to that of a partition baffle system, constructed from identical materials.

$$\frac{W_r}{W_p} = \left(\frac{\pi n' \alpha'}{N D'}\right) \left(X_0 \right)^{1/2} \left(\frac{m_r p_r}{m_p p_p}\right) . \tag{24}$$

These equations can be used to calculate the weights of various baffle systems if their dimensions are known.

DESIGN OF A BAFFLE SYSTEM

Equations 21-24, together with the curves shown in Figures 10-19, are sufficient to determine the minimum weight baffle system for a given rocket vehicle application within the limitations imposed by the basic assumptions. The analysis may be simplified, however, by determining algebraic approximations to these curves, and by substituting these approximations into the appropriate weight equations. The following analysis determines the minimum weight of baffle systems composed of rings and/or 0, 4, 6, or 8 partitions.

Approximate Relationships

Liquid sloshing relationships. Considering only the first resonant sloshing modes for each of the curves shown in Figures 10-13, since they are usually the most severe, the following approximations are valid for small damping ($\gamma \lesssim 0.02$),

$$
\begin{aligned}
&F_0' \simeq C_{1N}/\gamma, \quad \therefore \gamma(N) \simeq C_{1N}/F_0', \\
&\xi(N)/X_0 \simeq C_{2N}/\gamma, \quad \xi'(N) \simeq (C_{2N}/C_{1N})\, X_0' F_0', \\
&p_r'(N) \simeq C_{3N}/\gamma \simeq (C_{3N}/C_{1N})\, F_0', \\
&p_p'(N) \simeq C_{4N}/(\gamma)^{1/2} \simeq \left[C_{4N}/(C_{1N})^{1/2}\right](F_0')^{1/2},
\end{aligned}
\tag{25}
$$

where the coefficients C_{MN} are given by the following table, and where C_{rN}, C_{pN}, and C_{TN} are factors, involving products of the C_{MN}, which arise in the derivation of the weight equations to be presented later.

N	C_{1N}	C_{2N}	C_{3N}	C_{4N}	C_{rN}	C_{pN}	C_{TN}
0	1.33	0.81	1.31	0.81	2.01	0	0
4	0.60	0.60	1.15	0.54	0.97	2.79	1.96
6	0.43	0.52	1.05	0.46	0.70	4.22	2.32
8	0.17	0.46	0.99	0.29	0.28	5.53	2.04

TABLE 1. Coefficients of the Approximate First Mode Sloshing Relationships

Ring baffle damping relationships. Unfortunately, the curve (obtained by superposition) for the design damping of a system of multiple ring baffles shown in Figure 15 approaches infinity as the ring spacing D' is decreased to zero, whereas in actuality the damping should always remain finite and should, in fact, be zero for D' = 0. A preliminary experimental investigation [7] of multiple ring baffle arrangements has shown that the damping for such baffle systems is approximately that given by Miles' equation considering only the contribution of the first submerged baffle. Using this assumption, where $\xi \simeq D'/2$, the following relationships are obtained.

$$\gamma \simeq C_r (\alpha')^{3/2} (\xi')^{1/2} \exp(-2.3 D'), \quad \alpha' << 1 \tag{26}$$

or

$$D' \simeq \ln \alpha' \beta' / 1.5, \quad \beta' \simeq 2 (C_{2N} X_0')^{1/3} F_0' / C_{1N}.$$

Total damping relationships. For a general baffle system composed of both rings and partitions, the total damping is given by

$$\gamma \simeq \gamma' + C_r(\alpha')^{3/2}(\xi')^{1/2}\exp(-2.3D') ,$$

or

$$D' \simeq \ln \alpha'\beta'/1.5 , \quad \beta' \simeq \left(\frac{2}{\gamma}\right)\left[\frac{C_{2N}X_0}{(1-\gamma'')^2}\right]^{1/3} , \tag{27}$$

where γ' is all liquid damping other than that provided by ring baffles, where the various contributions to the damping are assumed to be additive, and where $\gamma'' = \gamma'/\gamma$.

Maximum bending moment relationships. The curves shown in Figures 16 and 17, for the maximum bending moment parameters of ring and partition baffle structures, are closely approximated by the following expressions

$$m_r' \simeq 0.34\,\varphi w' ,$$
$$m_p' \simeq 0.283 - 0.09/\ell' , \tag{28}$$

where $\varphi = 1$ for ring baffles simply supported around their inside edges and $\varphi = 2$ for ring baffles whose inside edges are free, and where the baffle width can be approximated by

$$w' = 1 - (1-\alpha')^{1/2} \simeq (\alpha'/2)(1+\alpha'/4) \simeq \alpha'/2 , \quad \alpha \ll 1 . \tag{29}$$

Minimum Weight Equations

Weight of a partition baffle system. Combining equations (22), (25d), and (28b), the following expression is obtained for the total weight of a system of partition baffles.

$$W_p' = K'N(X_0')^{1/2} m_p' \rho_p'$$
$$\simeq K'C_{pN}(0.283 - 0.09/\ell')(X_0'F_0')^{1/2}, \tag{30}$$

where $C_{pN} = NC_{4N}/(C_{1N})^{1/2}$ can be obtained from Table 1.

For a baffle system composed of <u>perforated partition baffles</u> alone, which provide a maximum damping γ_p, the following formulas apply,

$$F'_{0(p)} \simeq C_{1N}/\gamma_p\,,$$
$$W'_{p(p)} \simeq 0.283\,K'NC_{4N}(1 - 0.317/\ell')(X_0'/\gamma_p)^{1/2}. \tag{31}$$

<u>Minimum weight of a ring baffle system.</u> Combining equations (21), (25c), (26b), (28a), and (29), the following expression is obtained for the total weight of a system of ring baffles,

$$W_r' = K'\pi n'\alpha' X_0'(D')^{-1} m_r' \rho_r'$$
$$\simeq K'\pi n'\alpha' X_0'(1.5/\ln\alpha'\beta')(0.17\,\varphi\alpha')(F_0'C_{3N}/C_{1N})$$
$$\simeq 0.80\,K'\varphi n'(C_{3N}/C_{1N})X_0'F_0'(\alpha')^2/\ln\alpha'\beta'.$$

The value of α' for which W_r' is a minimum is obtained by equating the first derivative to zero, as

$$dW_r'/d\alpha' = 0 = 0.80\,K'\varphi n'(C_{3N}/C_{1N})X_0'F_0'(\alpha'/\ln\alpha'\beta')(2 - 1/\ln\alpha'\beta'),$$

from which

$$D' \simeq \ln\alpha'\beta'/1.5 \simeq 0.33\,,$$
$$\alpha' \simeq e^{1/2}/\beta' = 0.83\,C_{1N}/F_0'(C_{2N}X_0')^{1/3} \tag{32}$$
$$W_r' \simeq 1.11\,K'\varphi n'C_{rN}(X_0')^{1/3}/F_0'\,,$$

where $C_{rN} = C_{1N}C_{3N}/(C_{2N})^{2/3}$ can be obtained from Table 1.

In the absence of specific restrictions on the total liquid force amplitude F_o, set by the vehicle's control system, the most consistent method for limiting F_o is by restricting the maximum slosh height in an uncompartmented tank to the design slosh height (see page 14),

$$\zeta'_d = D'/2 \simeq 0.165 \;{}^*$$

$$= (C_{20}/C_{10}) X'_0 F'_{o(d)} \simeq 0.61\, X'_0 F'_{o(d)} ,$$

so that $\quad F'_{o(d)} \simeq 0.27/X_o \qquad (33)$

is defined to be the design liquid force for a system of ring baffles. The weight of the ring baffle system necessary to withstand sloshing to the design slosh height is given by

$$W'_{r(d)} \simeq 8.18\, K'\varphi n' (X'_0)^{4/3}. \qquad (34)$$

<u>Minimum total weight of a baffle system composed of both rings and partitions</u>. The expression for the total weight of a baffle system composed of both rings and nonperforated partitions can be written as

$$W'_t = W'_r + W'_p \simeq C/F'_o + D(F'_o)^{1/2},$$

where $\quad C = 1.11\, K'\varphi n' C_{rN}(X'_0)^{1/3}$ and $D = (0.283 - 0.09/\ell')K'C_{pN}(X'_0)^{1/2}.$

---

* The constant values of the ring spacing and the design slosh height found above are dependent on the form of the ring damping relationship (26), and might be considerably altered if this relationship is revised to agree more closely with experimental data.

It is seen from the above expression that the weight contribution of the ring baffles decreases and that of the partition baffles increases as the total force amplitude is allowed to increase. For some value of F_o', therefore, a minimum value of W_T' exists. By setting the first derivative of W_T' with respect to F_o' to zero, as

$$d W_T'/d F_o' = 0 = -C/(F_o')^2 + (D/2)(F_o')^{-1/2}$$

the expression for $F_{o(\min)}'$ which makes W_T' a minimum is found to be

$$\begin{aligned} F_{o(\min)}' &= (2C/D)^{2/3} \\ &\simeq 3.95\,(X_o')^{-1/9}\left[\left(\frac{C_{rN}}{C_{pN}}\right)\left(\frac{\varphi n'}{1-0.317/\ell'}\right)\right]^{2/3}. \end{aligned} \tag{35}$$

Substituting (35) into the expression for W_T' yields the following relationship for the minimum weight of a baffle system composed of both rings and nonperforated partitions,

$$\begin{aligned} W_{T(\min)}' &\simeq 1.89\,(C D^2)^{1/3} \\ &\simeq 0.841\,K' C_{TN}(\varphi n')^{1/3}(1-0.317/\ell')^{2/3}(X_o')^{4/9}, \end{aligned} \tag{36}$$

where $C_{TN} = (C_{rN} C_{pN}^2)^{1/3}$ can be obtained from Table 1.

If the partition baffles are perforated, such that they provide a damping contribution γ', and assuming that damping contributions are additive, the total damping γ necessary to make W_T' a minimum is a root of the equation

$$\gamma^{1/6}(\gamma + \gamma'/3)(\gamma - \gamma')^{1/3} = (C_{IN})^{3/2}(D/2C)\ , \tag{37}$$

where C and D retain their above definitions. The minimum weight baffle system composed of both rings and perforated partitions is then given by

$$W_T' = (C/C_{IN})\gamma(1-\gamma'/\gamma)^{4/3} + D(C_{IN})^{1/2}/\gamma . \tag{38}$$

<u>Overall minimum weight baffle system</u>. Equations (31b), (34), and (36) express, respectively: the weight of a system of perforated partition baffles which affords a normal slosh damping of γ_p ; the weight of a system of ring baffles necessary to withstand sloshing to the design slosh height $\zeta'_{(d)}$ in an uncompartmented tank; and the minimum weight of a baffle system composed of both rings and nonperforated partition baffles. Equations (31a), (33), and (35), express, respectively, the maximum liquid force amplitude provided by each of these baffle systems. Baffle systems composed of both rings and perforated partitions are omitted from this discussion as the resulting minimum weight equations are too complicated to be solved exactly.

Given a set of tank dimensions, and a value for the maximum transverse displacement amplitude at the frequencies of the first sloshing resonances, these equations can be used to calculate the weight W_k' and the force amplitude F_o' for each of the seven baffle systems which are obtained by letting N = 0, 4, 6, and 8. The overall minimum weight permissible baffle system is then the one having the minimum W_k' from among the set of baffle systems for which F_o' is less than some limiting value.

Dimensions of Baffle Structures

The following expressions for the plate dimensions of the various ring and partition baffles are provided to assist in designing a given baffle system.

For a ring baffle

$$\begin{aligned} D' &= D/a \simeq 0.33\,, \\ w' &= w/a \simeq \alpha'/2 \simeq 0.415\, C_{1N}/F_0'(C_{2N}X_0')^{1/3}, \\ t_s' &= t_s/a \simeq 0.83\, \varphi\, w'(C_{3N}/C_{1N})\, S' X_0' F_0'\,. \end{aligned} \qquad (39)$$

For a partition baffle

$$t_s' \simeq 0.283\,(k_m/k_m')(1 - 0.317/\ell')\, C_{4N}\, (C_{1N})^{-1/2} S' (X_0' F_0')^{1/2}$$

or

$$t_s' \simeq 0.283\,(k_m/k_m')(1 - 0.317/\ell')\, C_{4N}\, S' (X_0'/Y_p)^{1/2} \qquad (40)$$

For a composite sandwich plate instead of a solid plate,

$$\begin{aligned} t_c &= (t_s/4)(K_c/K_s) \simeq t_s K_c/9.80\,, \\ \tau_c &= d_c/\kappa = (2t_s/3)(K_s/K_c) \simeq 1.63\, t_s/K_c\,. \end{aligned} \qquad (41)$$

where K_c can be obtained from Figure 19.

ILLUSTRATIVE EXAMPLE

Problem Specifications

Consider the problem of designing a minimum weight baffle system for the liquid propellant tanks shown in Figure 20.* The approximate dimensions of equivalent cylindrical tanks, as required for this analysis, are $\ell_{(1)} \simeq 420$ in for the fuel tank (1), $\ell_{(2)} \simeq 720$ in for the oxidant tank (2), and a = 200 in for both tanks. Likewise, considering N' = 4 for the fuel tank,

$$n'_{(1)} = N'D/\ell \simeq 4(66\,\text{in})/(420\,\text{in}) \simeq 0.628\ ,$$

and considering N' = 8 for the oxidant tank,

$$n'_{(2)} = 8(66\,\text{in})/(720\,\text{in}) \simeq 0.733\ ,$$

Assume that the vehicle will undergo a range of axial accelerations from $g_1 = 386$ ips^2 to $g_2 = 3860$ ips^2, that the range of excitation frequencies ($0.3 \leq f \leq 1.1$ cps) will include the first resonant sloshing modes for the equivalent cylindrical tanks containing from zero to eight partition baffles, and that the transverse displacement amplitudes will never exceed $X_o = 2$ in, which is equivalent to $X_o' = 0.01$.

Assume that the baffles are to be constructed from high-strength aluminum alloy, having the properties $E = 10^7$ psi, $\nu = 0.3$, $S = 6 \times 10^4$ psi,

---

* This figure shows typical dimensions of the proposed S-1C booster for the C-5 advanced Saturn space vehicle, as depicted on page 65 of [9].

and $\rho_B = 259 \times 10^{-6}$ lb-sec^2/in^4, and that the liquid propellants have the density of water, $\rho_L = 93.3 \times 10^{-6}$ lb-sec^2/in^4. Because of the considerable weight saving to be gained, assume that all baffles are constructed from the type of composite sandwich plate described in a previous section, for which

$$K_c = 4(\kappa/\pi)^{1/2}[3(1-\nu^2)k_m'' S/E]^{1/4} \simeq 0.81 \ .$$

Thus, for the fuel tank considered above,

$$\begin{aligned} K'_{(1)} &= k_m K_c \ell'_{(1)} [\rho_L g_2 a/S]^{1/2} \\ &\simeq k_m (0.81)(2.10)[(93.3 \times 10^{-6})(3860)(200)/(6 \times 10^4)]^{1/2} \\ &\simeq 0.0588 \ k_m \ , \end{aligned}$$

where $k_m = 1$ for nonperforated baffles, and where k_m is obtained from Figure 18 for perforated partition baffles having ϵ percentage area removed. Similarly for the oxidant tank,

$$K'_{(2)} \simeq 0.1009 \ k_m \ .$$

Assume, finally, that the ring baffles are simply supported around their inside edges, so that $\varphi = 1$, and that the most severe sloshing always occurs when h' = 2.00. The total weight of liquid propellant in either tank when h' = 2.00, is given by

$$\begin{aligned} W_L &= \pi h' [\rho_L g_1 a^3] \\ &\simeq (3.14)(2.00)[(93.3 \times 10^{-6})(386)(200)^3] \\ &\simeq 1.81 \times 10^6 \ \text{lb} \ . \end{aligned}$$

Weight Comparison of Various Baffle Systems

Basic weight equations. Substituting the values of the parameters ℓ', n', K', φ, and X_0', specified above, into equations (30) and (32c), the following expressions are obtained for the weights of the various baffle systems which limit the first mode total force to some maximum amplitude F_0'.

For the fuel tank,

$$W_{r(1)} \simeq 1.11(\rho_B g_1 a^3) K'_{(1)} C_{rN} \varphi n'_{(1)} (X_0')^{1/3}/F_0'$$
$$\simeq 1.11(8 \times 10^5)(0.0588) C_{rN}(1)(0.628)(0.01)^{1/3}/F_0'$$
$$\simeq 7000\, C_{rN}/F_0', \quad (\mathrm{lb}),$$

$$W_{p(1)} \simeq 0.283(\rho_B g_1 a^3) K'_{(1)} C_{pN}(1 - 0.317/\ell')(X_0' F_0')^{1/2}$$
$$\simeq 0.283(8 \times 10^5)(0.0588\, k_m) C_{pN}(0.849)(0.01 F_0')^{1/2}$$
$$\simeq 1130\, k_m C_{pN}(F_0')^{1/2}, \quad (\mathrm{lb}).$$

Similarly, for the oxidant tank

$$W_{r(2)} \simeq 14{,}000\, C_{rN}/F_0', \quad (\mathrm{lb}),$$
$$W_{p(2)} \simeq 2080\, k_m C_{pN}(F_0')^{1/2}, \quad (\mathrm{lb}).$$

Design weight of a ring baffle system. By limiting the total liquid force amplitude to its design value, as expressed by equation (33),

$$F'_{0(d)} \simeq 0.27/X_0' \simeq 0.27/0.01 \simeq 27,$$

or

$$F_{0(d)} \simeq (\rho_L g_2 a^3) X_0' F'_{0(d)} \simeq (2.88 \times 10^6)(0.27) \simeq 778{,}000 \ \mathrm{lb},$$

and by using $C_{r0} = 2.01$, obtained from Table 1, the design weights of plain (nonperforated) ring baffle systems for the two (uncompartmented) tanks specified above are found to be

$$W_{r(1)} \simeq 7000(2.01)/(27) \simeq 521 \quad \text{lb},$$

and $$W_{r(2)} \simeq 1042 \quad \text{lb}.$$

The design damping provided by these ring baffles is determined from equation (25a) to be

$$\gamma_{(d)} \simeq C_{10}/F_0' \simeq 1.33/27 \simeq 0.049,$$

and the dimensions of these ring baffles are given by equations (39) as

$$D = D'a \simeq 0.33(200) \simeq 66 \text{ in},$$

$$w \simeq \frac{0.415 a C_{10}}{F_0'(C_{20} X_0')^{1/3}} \simeq \frac{0.415(200)(1.33)}{(27)[(0.81)(0.01)]^{1/3}} \simeq 20.3 \text{ in},$$

$$t_s \simeq 0.83 \varphi w (C_{30}/C_{10})(\rho_L g_2 a/S)^{1/2} X_0' F_0' \simeq 0.162 \text{ in},^*$$

$$t_c \simeq t_s K_c/9.80 \simeq (0.162)(0.81)/9.80 \simeq 0.0134 \text{ in},^{**}$$

$$\tau_c \simeq d_c \simeq 1.63\, t_s/K_c \simeq 1.63(0.162)/(0.81) \simeq 0.326 \text{ in}.^{**}$$

Weights of perforated-partition baffle systems. Assume the maximum damping that can be obtained from baffle systems composed of 4, 6, and 8 perforated partitions when subjected to a maximum excitation amplitude of $X_0' = 0.01$, is $\gamma_p = 0.10$, which agrees roughly

---

* Thickness of an equivalent strength solid plate.

** Figure 19 shows the physical significance of these sandwich plate dimensions.

with data presented in [5] for N = 8 and ϵ = 0.23. Figure 18 gives k_1 = 1.09 corresponding to ϵ = 0.23 for perforation holes centered on an equilateral triangular lattice. Putting these values of γ_p and k_1 into equation (25a) for the total force amplitude, into the weight equations derived above, and into equations (40b) and (41) for the plate dimensions, the following table is obtained for each of N = 4, 6, and 8.

	LIQUID FORCES		WEIGHTS		PLATE DIMENSIONS		
N	F_o'	F_o	$W_{p(1)}$	$W_{p(2)}$	t_s	t_c	τ_c, d_c
4	6.0	173,000	8,420	15,500	0.976	0.081	1.96
6	4.3	124,000	10,760	19,800	0.830	0.069	1.67
8	1.7	49,000	9,050	16,700	0.524	0.043	1.05
0*	13.3	383,000	1,060	2,120	0.171	0.014	0.344

* Equivalent damping ring baffle system (D = 66 in, w = 41.3 in)

Table 2. Comparison of the liquid force amplitudes (lb), baffle weights (lb), and plate dimensions (in), of perforated-partition baffle systems constructed from aluminum alloy sandwich plates, necessary to withstand liquid sloshing corresponding to $\gamma_P = 0.10$ and $X_o' = 0.01$.

Weights of baffle systems composed of both rings and nonperforated partitions. Equation (36) expresses the minimum weight W_T' of a baffle system composed of both rings and nonperforated partitions, necessary to withstand liquid sloshing due to a maximum excitation amplitude of X_o'. This value of minimum weight can be determined also by solving equation (35) for the liquid force amplitude F_o' corresponding to W_T', and then substituting F_o' into the above weight equations. By so doing, and by using equations (39), (40), and (41) for the baffle dimensions, the following tables are obtained for each of N = 4, 6, and 8.

	TABLE 3: FUEL TANK (1)					
N	γ	F_o'	F_o	W_r	W_p	W_t
4	0.227	2.65	76,300	2560	5120	7680
6	0.266	1.615	46,500	3030	6060	9090
8	0.232	0.732	21,100	2680	5350	8030
0*	0.200	6.65	191,500	2120	0	2120

	TABLE 4: OXIDANT TANK (2)					
N	γ	F_o'	F_o	W_r	W_p	W_t
4	0.214	2.80	80,600	4850	9,700	14,550
6	0.252	1.706	49,100	5750	11,500	17,250
8	0.200	0.774	22,300	5060	10,120	15,180
0*	0.200	6.65	191,500	4240	0	4,240

* Equivalent damping ring baffle systems (D = 66 in).

Tables 3 and 4. Comparison of the liquid force amplitudes and the minimum weights (lb) of ring/nonperforated-partition baffle systems constructed from aluminum alloy sandwich plate for $X_o' = 0.01$.

	TABLE 5: FUEL TANK (1)						
	RING BAFFLES (N' = 4)				PARTITION BAFFLES		
N	w	t_s	t_c	τ_c	t_s	t_c	τ_c
4	103.3	0.199	0.0165	0.400	0.457	0.0378	0.919
6	127.5	0.209	0.0173	0.420	0.361	0.0299	0.725
8	116.0	0.204	0.0161	0.392	0.269	0.0206	0.413
0*	82.6	0.201	0.0166	0.404	---	---	---

	TABLE 6: OXIDANT TANK (2)						
	RING BAFFLES (N' = 8)				PARTITION BAFFLES		
N	w	t_s	t_c	τ_c	t_s	t_c	τ_c
4	98.0	0.196	0.0162	0.394	0.505	0.0417	1.015
6	121.0	0.204	0.0169	0.410	0.400	0.0331	0.804
8	110.0	0.190	0.0157	0.382	0.264	0.0218	0.530
0*	82.6	0.201	0.0166	0.404	---	---	---

* Equivalent damping ring baffle system (D = 66 in).

Tables 5 and 6. Dimensions (in) of the minimum weight ring/non-perforated-partition baffle structures.

The Overall Minimum Weight Baffle System

It is evident from the weight comparisons presented on the previous pages that for moderate suppression of the effects of liquid sloshing, and for the particular parameters chosen in the present example, a baffle system composed exclusively of ring baffles (N = 0) will be far lighter in weight than any of the systems which include partition baffles, and thus would ordinarily be preferred over any of the partition baffle systems. If only mild liquid suppression is required, the overall minimum weight baffle system is the plain "design" ring baffle system based on the "design damping" defined on page 14, which for the present example was found to be $\gamma_d \simeq 0.05$. It should be noted, however, that the liquid force amplitude, $F_o' = 27$, for this "design" ring baffle system, resulting from an expected maximum first mode excitation amplitude of $X_o' = 0.01$, may be excessive, pending on the overall design requirements of the rocket vehicle. If this is the case, the overall minimum weight baffle system must be selected from the remaining baffle systems as follows.

Both perforated-partition baffle systems and a plain ring baffle system can be designed to provide damping of approximately $\gamma = 0.10$, which corresponds to moderate liquid suppression. Such a ring baffle system was shown to be far lighter in weight $(W_r:W_p \sim 1:10)$ than the perforated-partition baffle systems (see Table 2). For the same damping, however, the partition baffle systems result in significantly less liquid force amplitude, particularly the eight-partition system $(F_r:F_p \sim 8:1)$, and furthermore, the alterations in natural frequencies provided by the partitions may compensate for the large differences in baffle systems weight.

If very strong suppression of liquid sloshing is required, a baffle system composed of both rings and (nonperforated) partitions should be considered. Tables 3 and 4 present comparisons of such baffle systems, each of which provides the damping, $\gamma \simeq 0.20$, necessary to render its total weight W_T a minimum. It is observed that while the liquid force amplitudes resulting from the use of these ring/nonperforated-partition baffle systems, are considerably less than for any of the other baffle systems, their total weights are not excessive, and are actually lighter than the perforated-partition baffles which provide less liquid suppression. The arguments supporting the use of perforated-partition baffles are therefore even more applicable for the ring/nonperforated-partition baffle systems.

Baffle systems having six partitions weigh more in each instance than those having four or eight partitions (due to the equivalent mechanical model parameters used to represent the sloshing liquid for the various baffle systems), and therefore are not to be recommended. Furthermore, since the eight-partition baffle systems weigh approximately the same as the four-partition baffle systems and yet provide considerably more liquid suppression and considerably more control over liquid resonant frequencies, it is recommended that eight partitions be used whenever partitioning is desirable, unless some other design requirement dictates the use of four or six partitions.

Since the dimensions of the sandwich plates are quite small, particularly the skin thickness t_c of the ring baffles, and therefore difficult to manufacture, the use of solid plate baffle construction might be preferred. For baffles constructed from solid plate aluminum alloy, the corresponding weights would be approximately three times those given in Tables 2, 3, and 4. It is interesting to note that the thicknesses of the ring baffles remain essentially constant with varying annular widths w, so that their weights are virtually in direct proportion to w.

CONCLUSIONS AND RECOMMENDATIONS

The foregoing analysis represents a preliminary investigation of the problem of determining the optimum (sufficient damping - minimum weight) baffle structure to be used in suppressing liquid sloshing in the propellant tanks of large rocket vehicles. For the typical propellant tanks depicted in Figure 20, and for moderate liquid suppression, it was shown that a plain ring baffle system will be far lighter in weight than the baffle systems which include partition baffles. Since minimum weight is the criterion for selecting a particular baffle system from among those that provide sufficient damping, the plain ring baffle system would ordinarily be selected for use in these tanks.

There are, however, three instances in which one of the perforated-partition baffle systems or one of the ring/nonperforated-partition baffle systems (as compared in Tables 2-6) may possibly be preferred over a plain ring baffle system. (1) Partition baffles may actually contribute to the axial buckling strength of the tank structure — an interaction neglected in the present analysis — and may therefore result in a tank-baffle structure having less overall weight than an uncompartmented tank with ring baffles. (2) The minimum weight ring-partition baffle systems provide much stronger suppression of liquid sloshing (smaller F_O') than is attainable with ring baffles alone, and will therefore be preferred when such relatively strong liquid suppression is required. (3) Partitioning of a cylindrical tank considerably alters the liquid natural frequencies. If the fundamental sloshing frequency in an uncompartmented tank coincides with a critical range of natural frequencies of the rocket structure or control system, it may be possible to avoid liquid resonances in this critical frequency range by the introduction of partition baffles.

Whenever the use of partition baffles is desired, a baffle system having eight partitions is recommended over those having four or six partitions, because the eight-partition system affords greater liquid suppression and greater control of liquid resonant frequencies with very little additional weight penalty. A baffle system composed exclusively of nonperforated partition baffles is not to be recommended for use in large rocket vehicles. Since the liquid damping is merely that provided by the viscous wiping of the liquid against the partitions and the tank walls, and since for large propellant tanks (such as shown in Figure 20) this natural damping due to wiping is very small, the liquid sloshing will necessarily become excessive for tank excitation at one of the liquid resonant frequencies. Likewise, the use of perforated ring baffles is not to be recommended, as the damping effectiveness of ring baffles decreases with increasing percentage perforation and as it was shown that generally there is no weight saving to be gained by perforation.*

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* See the discussion of strength-weight characteristics of perforated plates presented on page 20.

The analysis and results presented above are approximate to the extent that certain of the functional relationships are not accurately known, that some of the approximations used were extended beyond their applicable ranges, and that certain of the basic assumptions are not strictly valid. It is suggested that future investigations pertaining to the design of minimum weight antislosh baffle systems be concerned with eliminating the existing inaccuracies and with generalizing the present analysis to include a more liberal set of basic assumptions. The principal inaccuracies in the present analysis are as follows:

Liquid sloshing analysis. The dependence of the sloshing effects (F_o', ξ', p_r', p_p') upon damping for liquid resonances in compartmented tanks, as shown in Figures 10-13, was obtained in part from experimental data, and in part by extrapolating from the known theoretical results for an uncompartmented tank. Expressions for these relationships from the available theoretical hydrodynamic analysis [1] would be valuable. Likewise, more complete experimental data for compartmented tanks would be useful, especially investigations of the six-partition tank and of the slosh heights in compartmented tanks, for which there is no existing experimental data. The first order approximations (25) used to represent the sloshing effects in the weight analysis are valid only for small damping ($\gamma \lesssim 0.02$), and should be replaced by more accurate, higher order expressions, since the present concern is with relatively large damping.

Damping analysis. Further experimental studies are needed for determining a more accurate relationship between the damping due to a system of multiple ring baffles and the axial spacing of the rings. Throughout the present analysis, the damping provided by a ring baffle system is considered merely to be the contribution of the first submerged baffle, as calculated from Miles' formula. Damping calculated by the above method appears to be in better agreement with experimental data than that obtained by superimposing the damping contributions from each of the submerged ring baffles. Experimental results are needed for determining analytic expressions for the damping provided by a system of perforated partition baffles and by a baffle system composed of both rings and partitions. Likewise, tests are needed for determining the influence of flexibility (finite deflections) of the tank and baffle structures on the liquid damping. Finally, there is some question as to whether the uniform damping introduced into the simple mechanical model used in this analysis is the same damping for each of the dependent sloshing variables and for each of the various liquid resonances.

Bending moment analysis. The bending moment analysis could be extended to include conical ring baffles, and the present analysis could be improved by using better approximations for the hydrodynamic pressure loadings on the baffles at the various liquid resonances. In considering the strengths of the various baffle structures, the baffles were assumed to be completely independent of deformations of the tank wall, and vice versa. However, the baffles may actually tend to stiffen the tank structure, as well as serving to damp the liquid sloshing. To be more nearly correct, therefore, an analysis should be made which includes the strength-weight characteristics of the entire tank-baffle structures. A further interaction which should be investigated is the mutual strengthening between the rings and partitions comprising ring/partition baffle systems such as shown in Figure 1.

General. The present weight analysis is based on the assumption that the baffles behave according to elastic thin plate theory. A valuable contribution would be a weight analysis which assumes the (ring) baffles to behave as large deflection membranes, and a strength analysis based on plasticity theory. The present analysis could also be extended to include additional tank compartmentation, such as the inclusion of five, seven, nine, ten, and twelve partitions, and to include nonuniform spacing of the ring baffles. A suggested extension of the present analysis is to consider a honeycomb baffle construction consisting of perforated prismatic triangular or rectangular cells repeated in each direction throughout the tank volume.

ACKNOWLEDGEMENTS

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NOTATION

Principal Variables

a	Radius of the cylindrical tank
b	Inside radius of a ring baffle
C_r	Hydrodynamic drag coefficient for normal sloshing around a ring baffle
C_m	Coefficient used in calculating the equivalent sloshing masses from the resonant sloshing frequencies
C_{MN}	Coefficients of the approximate relationships for determining the sloshing effects
C_p	Coefficient for adjusting the scale of the net pressure across a partition baffle
C_s	Coefficient for adjusting the scale of the slosh height
d	Depth of a single ring baffle beneath the liquid surface
d_c	Spacing between the square gridwork of a sandwich plate
d_p	Diameter of perforation holes in the baffle material
D_p	Spacing between perforation holes in the baffle material
D	Uniform axial spacing between ring baffles
E	Young's elastic modulus for the baffle material
ϵ_m	Percentage area of baffle material removed through perforation
F_o	Total liquid force response due to transverse tank excitation
g_1	Gravitational acceleration
g_2	Maximum axial acceleration of the tank

h	Average depth of liquid below the free surface
$k_m = \frac{k'_m}{(k''_m)^{1/2}}$	Plate perforation factor
$k'_m = 1 - \epsilon_m$	Plate perforation fraction
k''_m	Plate load bearing area ratio (due to perforation)
K_n	Plate cross-section coefficient
	Length of the cylindrical tank
M_k	Bending moment distribution throughout a baffle structure due to the maximum pressure loading P_k
$m_k = (M_k)_{max}$	Overall maximum bending moment occurring in a baffle structure
$m_T = \pi \rho_L a^2 h$	Total mass of liquid contained in the cylindrical tank
m_i	Equivalent mass associated with the i^{th} sloshing mode
m_o	Equivalent rigid liquid mass
N	Number of tank compartments (partitions)
N'	Total number of ring baffles
N''	Number of submerged ring baffles for a given h and D
P_k	Maximum instantaneous net pressure distribution over a baffle system
$p_k = (P_k)_{max}$	Overall maximum net pressure acting across a baffle structure
r, z, θ	Cylindrical coordinates
S	Characteristic ultimate strength of the baffle material
t_c	Skin thickness of a simple composite sandwich plate
t_s	Thickness of a solid plate

W_k	Total weight of a baffle system
w	Annular width of a ring baffle
X_o	Displacement amplitude of the transverse tank excitation
α_i, β_i, etc.	General variables used in various formulas
α'	Fraction of the tank cross-sectional area blocked by a ring baffle
$\gamma = \delta/2\pi$	Total damping factor (δ = logarithmic decrement)
γ_d	Design damping provided by a system of ring baffles, where $\zeta_d = D/2$
γ_n	Specific weight (per unit area) of a plate material
ρ_L	Mass density of the contained liquid
ρ_B	Mass density of the baffle material
σ_{max}	Maximum stress induced in a baffle structure by the maximum bending moment m_k
τ_c	Overall thickness of a simple composite sandwich plate
ν	Poisson's ratio for the baffle material
ω	Angular frequency of the transverse tank excitation
ω_i	ith lowest natural sloshing (angular) frequency
κt_c	Thickness of sandwich plate gridwork material
ζ	Amplitude of the liquid free surface oscillations at the tank wall (slosh height)

Subscripts

i, j	General indices; used for various summations, etc.
k = r, p, or T	Subscript depending on whether the quantity refers to ring baffles, partition baffles, or a baffle system composed of both rings and partitions, respectively
m = 0 or 1	Subscript depending on whether the perforation holes are centered on a square or a triangular lattice, respectively
n = s or c	Subscript denoting whether the plate material is solid or a composite sandwich plate, respectively

Convenient Nondimensional Parameters

$b' = b/a$	$(S')^2 = \rho_L g_2 a/S$
$D' = D/a$	$t'_c = t_c/a$
$d' = d/a$	$t'_s = t_s/a$
$d'_c = d_c/a$	$W'_k = W_k/\rho_B g_1 a^3$
$F'_o = F_o/\rho_L g_2 a^3 X'_o$	$w' = w/a$
$h' = h/a$	$X'_o = X_o/a$
$\ell' = \ell/a$	$\alpha' = w'(2 - w')$
$(m'_k)^2 = m_k/p_k a^2$	$\gamma'_n \quad \gamma_n/\rho_L ga$
$m'_i = m_i/\rho_L a^2$	$\omega' = \omega^2 a/g$
$m'_o = \pi h' - \sum_{i=1}^{\infty} m'_i$	$\omega'_i = (\omega_i)^2 a/g$
$n' = N'D/\ell$	$\tau'_c = \tau_c/a$
$(p'_r)^2 = p_r/\rho_L g_2 a (X'_o)^2$	$\zeta' = \zeta/a$
$(p'_p)^2 = p_p/\rho_L g_2 a X'_o$	

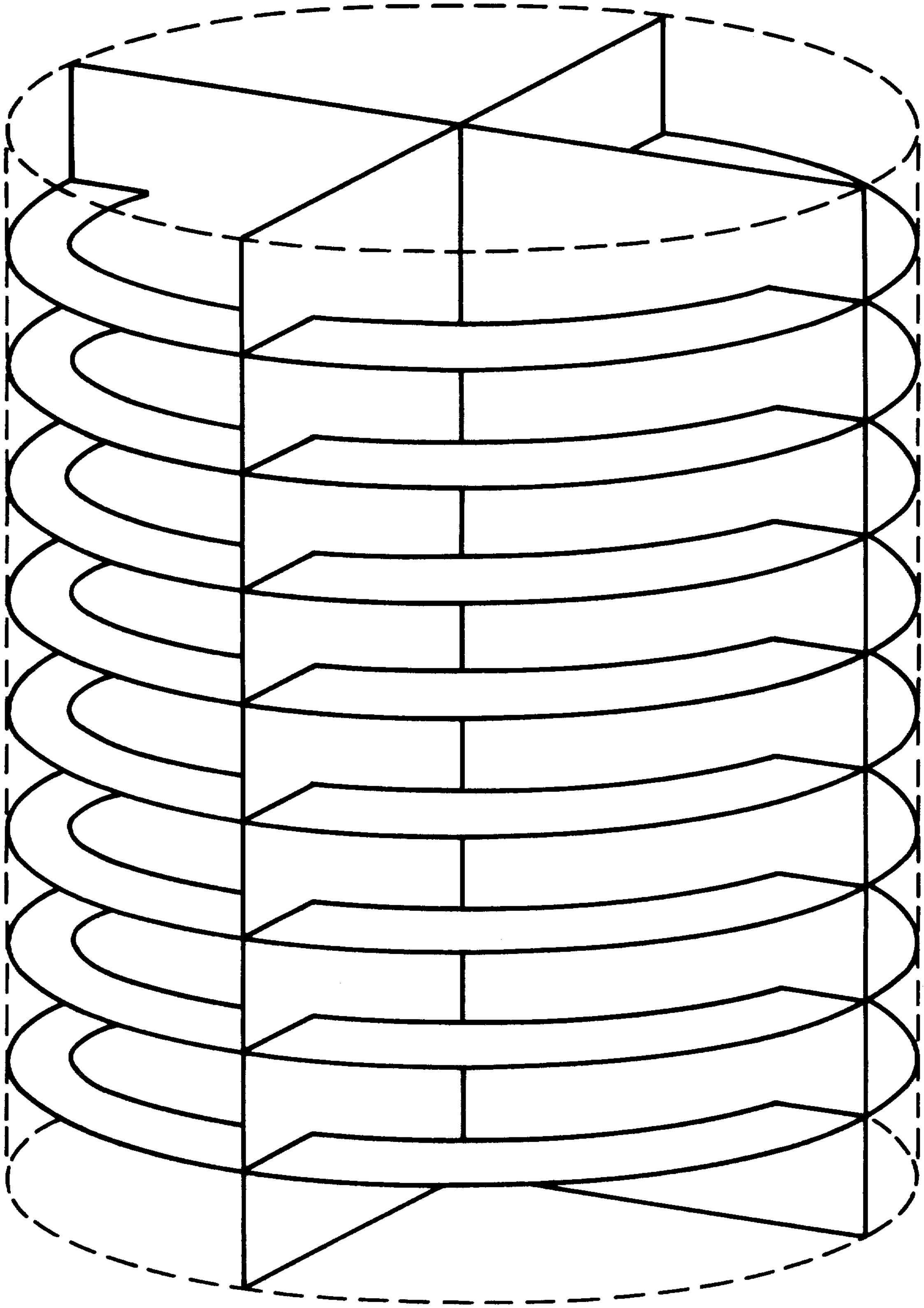

FIGURE 1. AN ANTISLOSHING BAFFLE SYSTEM COMPOSED OF FOUR PARTITIONS AND EIGHT RINGS

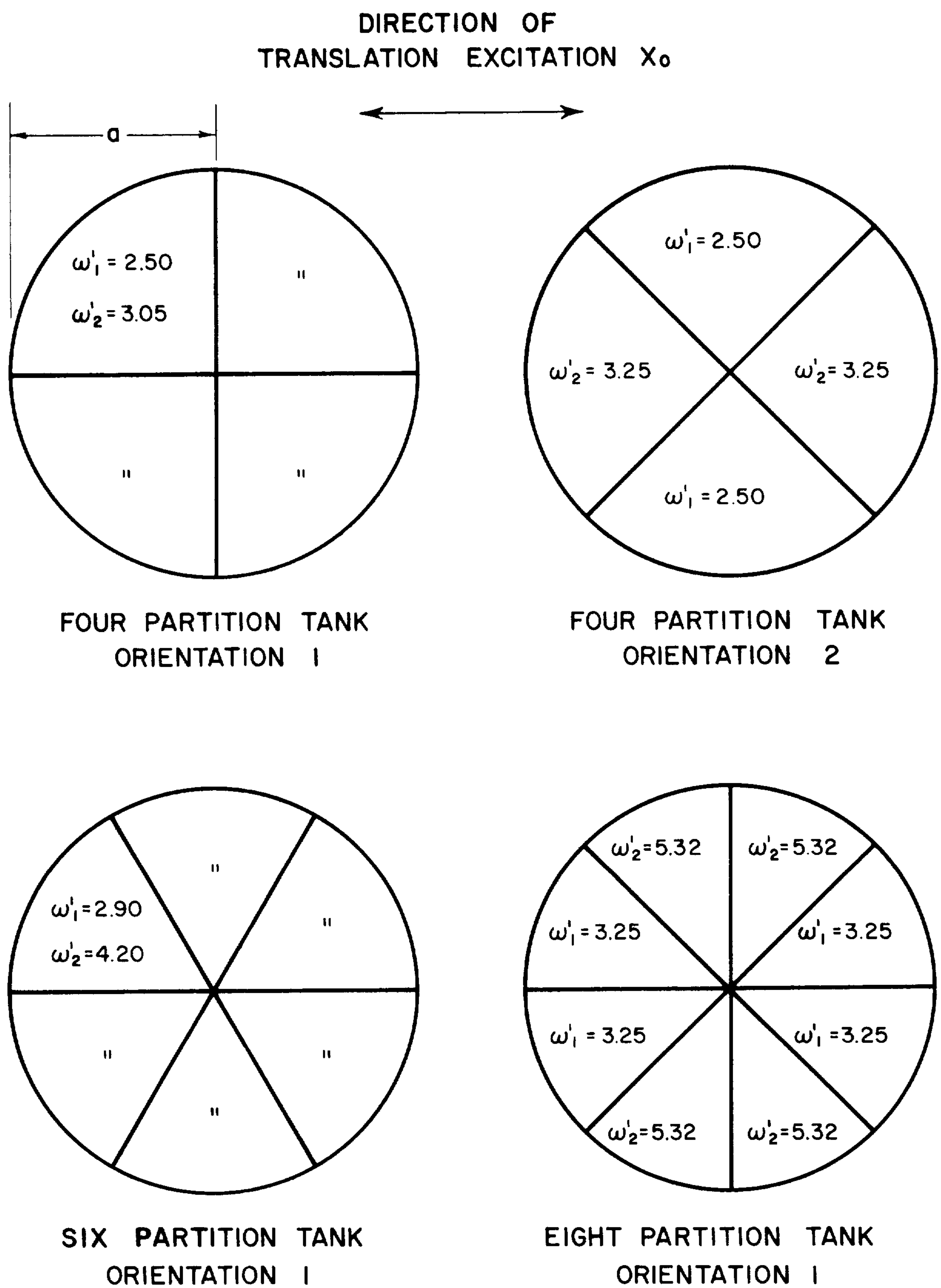

FIGURE 2. TANK COMPARTMENTATION, SHOWING VARIOUS FUNDAMENTAL RESONANT FREQUENCIES

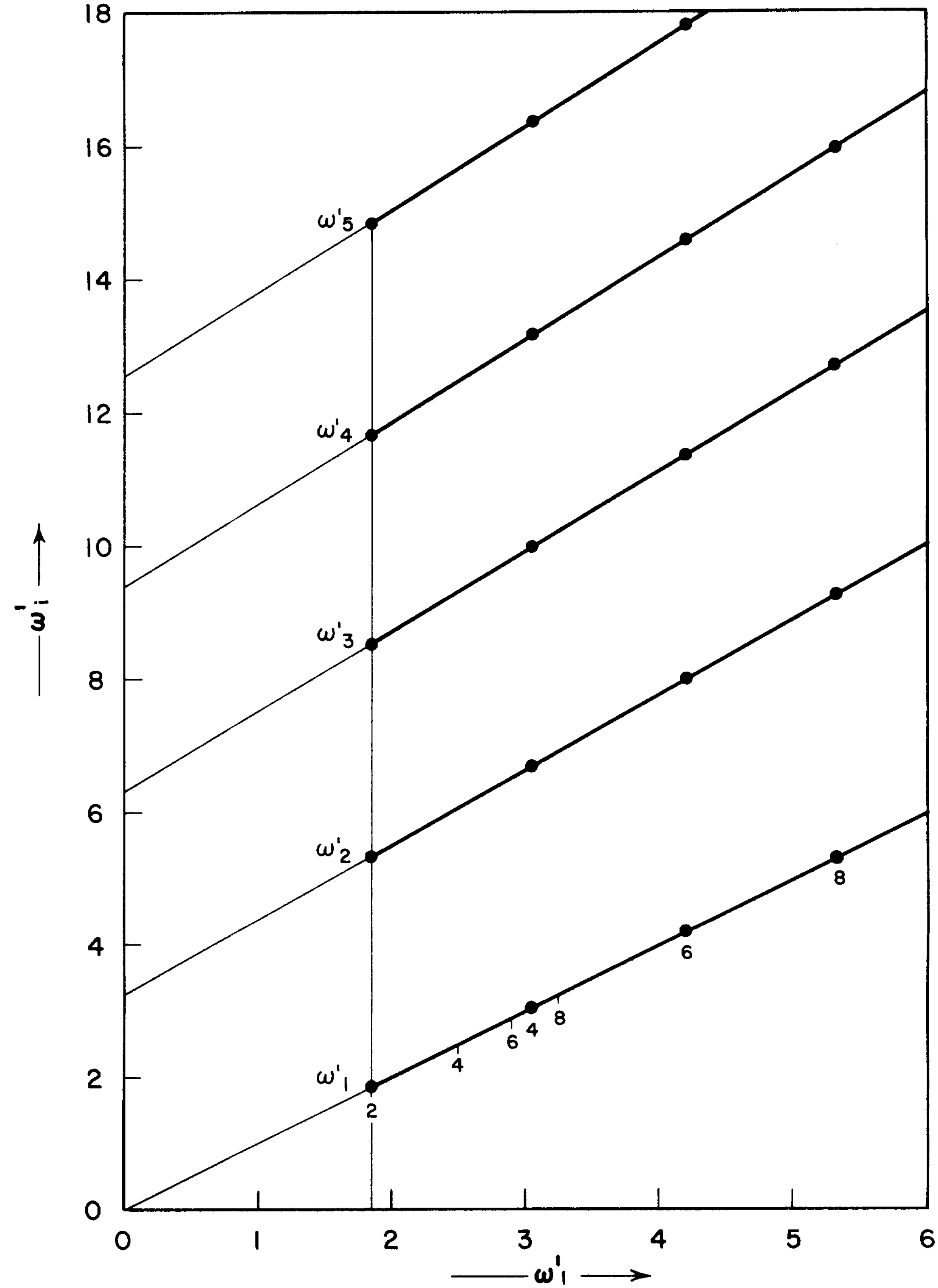

FIGURE 3. THE FIRST FIVE HARMONIC FREQUENCIES VERSUS THE FUNDAMENTAL RESONANT FREQUENCY FOR LIQUID SLOSHING IN A CYLINDRICAL TANK

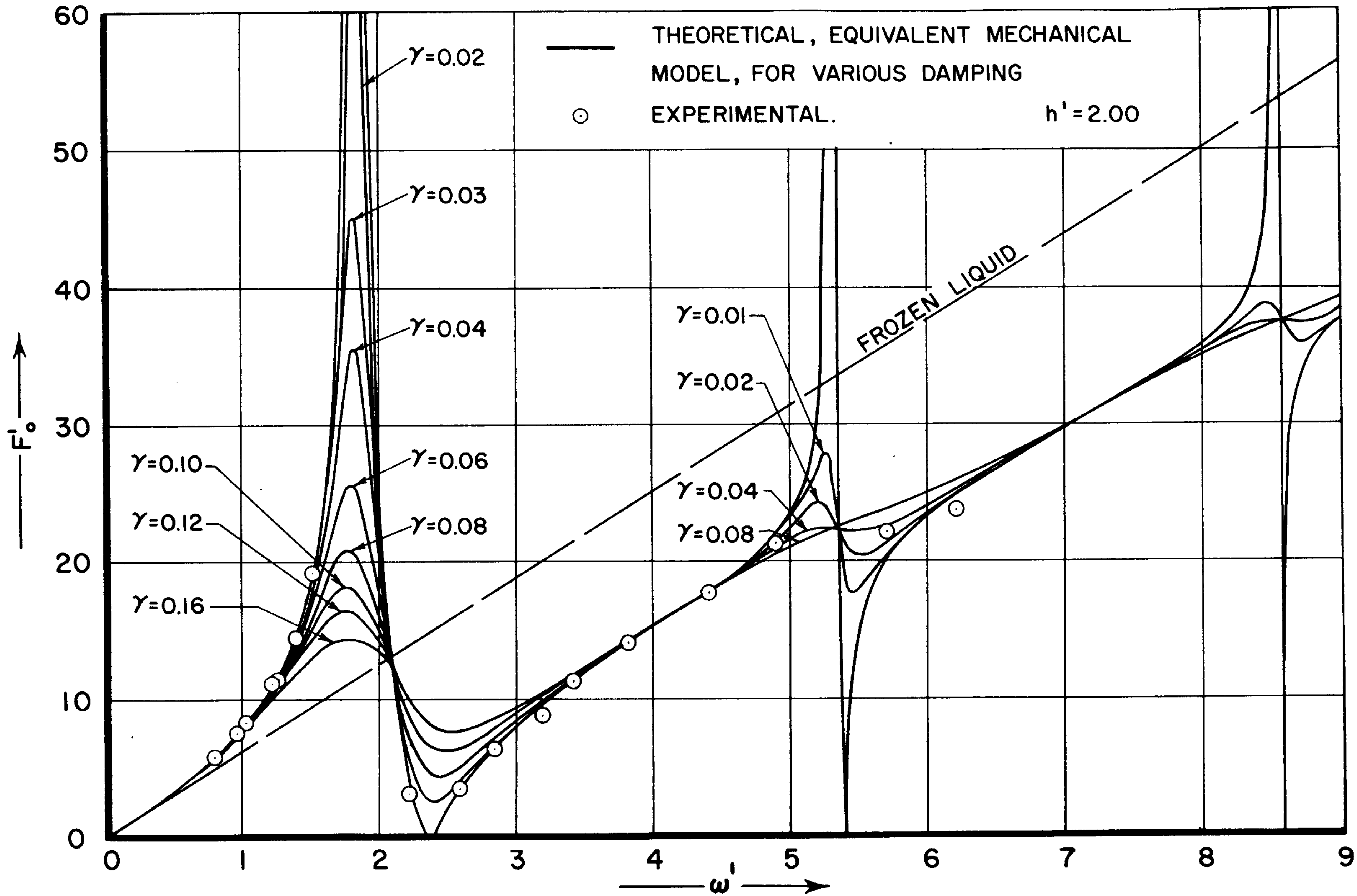

FIGURE 4. TOTAL FORCE RESPONSE OF A SLOSHING LIQUID IN AN UNCOMPARTMENTED TANK

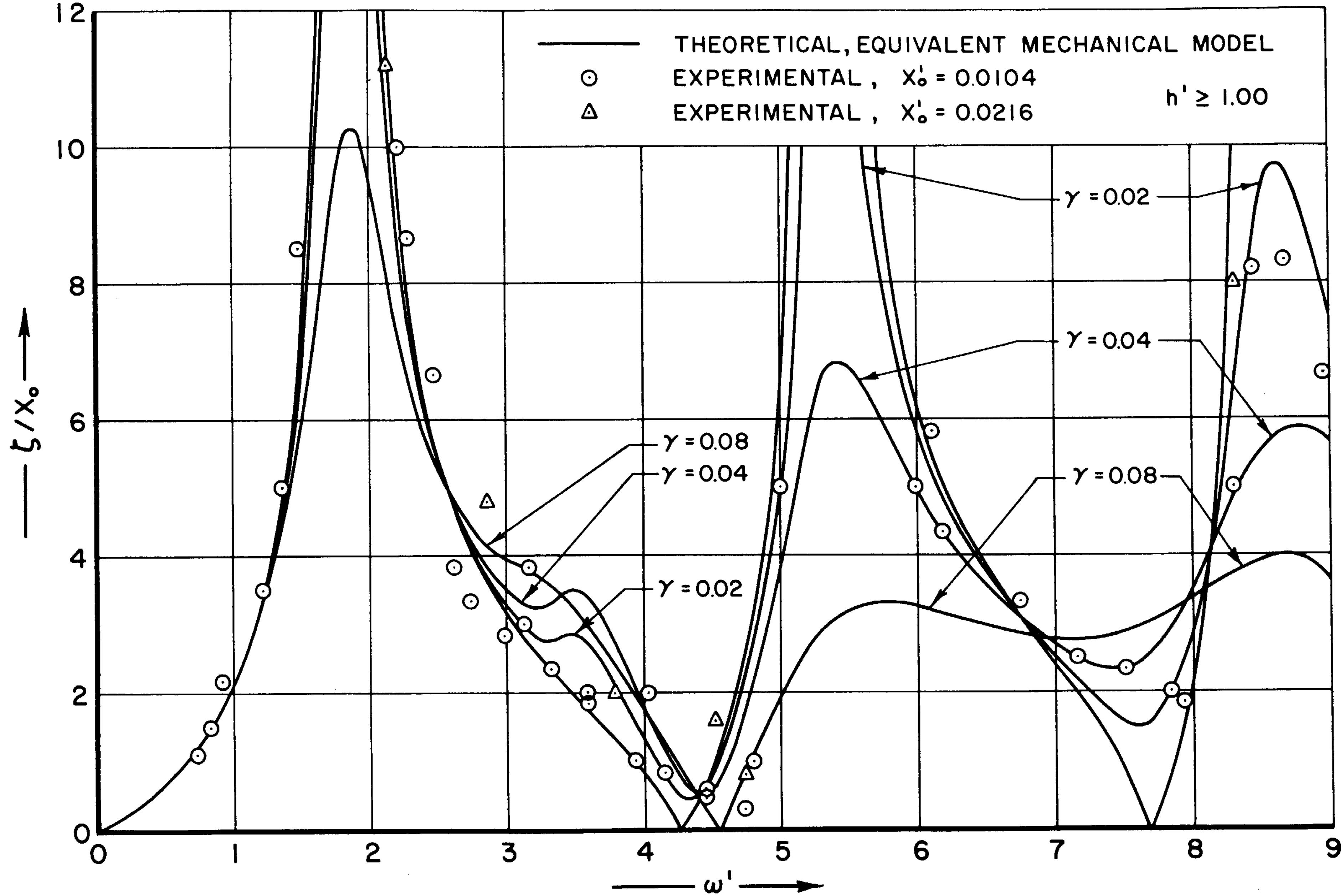

FIGURE 5. SLOSH HEIGHT FOR AN UNCOMPARTMENTED TANK

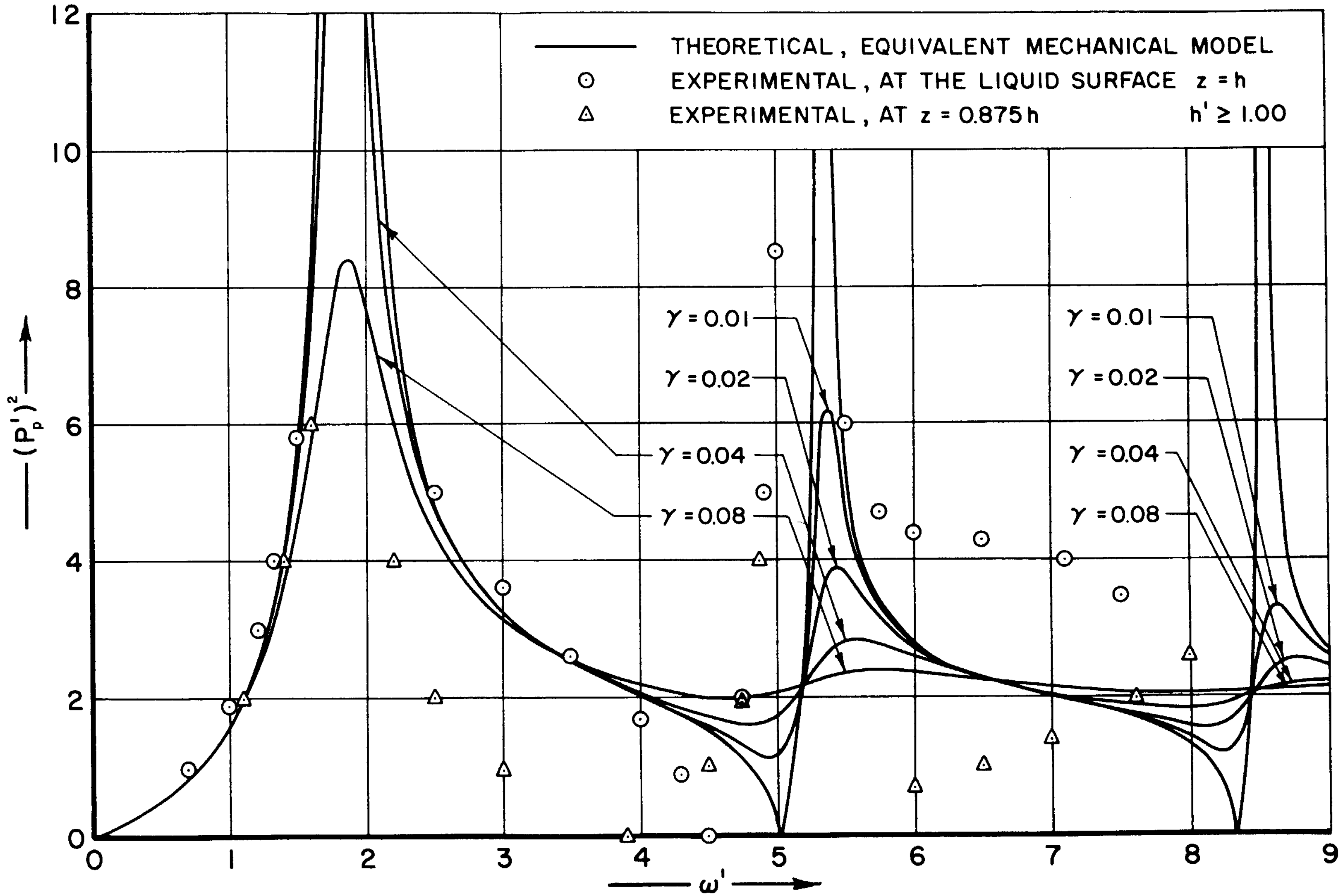

FIGURE 6. MAXIMUM PARTITION PRESSURE FOR AN UNCOMPARTMENTED TANK

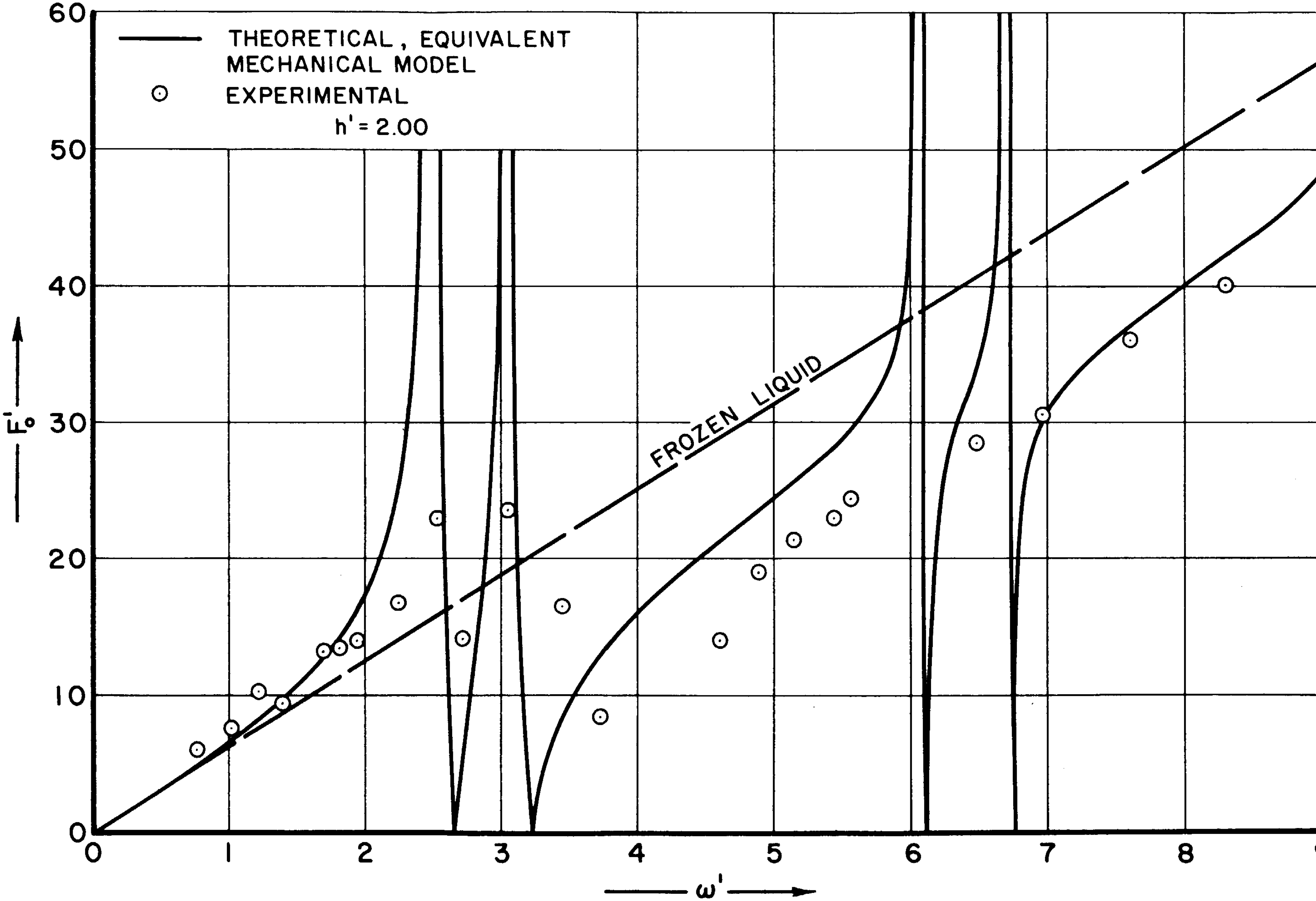

FIGURE 7. TOTAL FORCE RESPONSE FOR A FOUR PARTITION TANK

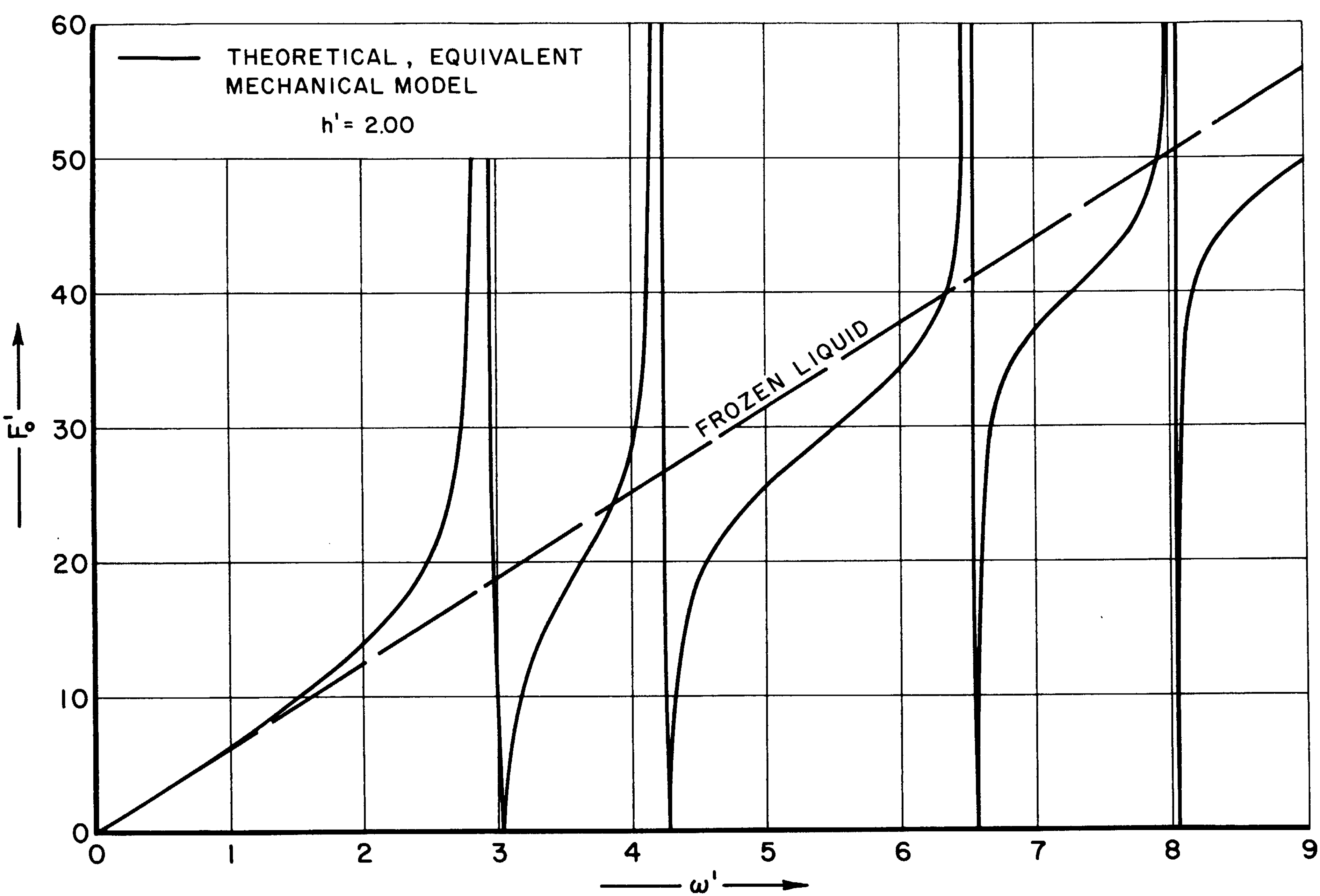

FIGURE 8. TOTAL FORCE RESPONSE FOR A SIX PARTITION TANK

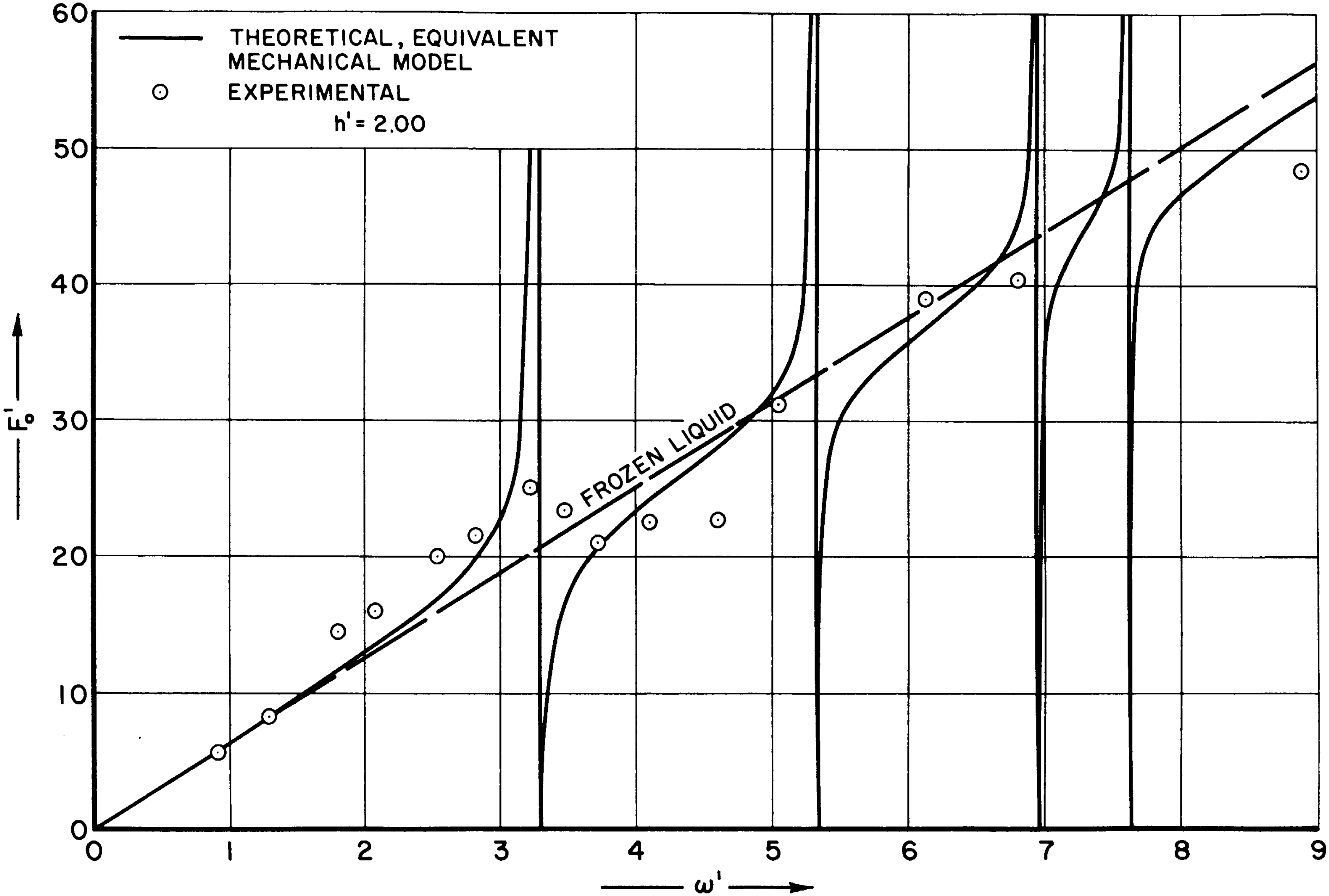

FIGURE 9. TOTAL FORCE RESPONSE FOR AN EIGHT PARTITION TANK

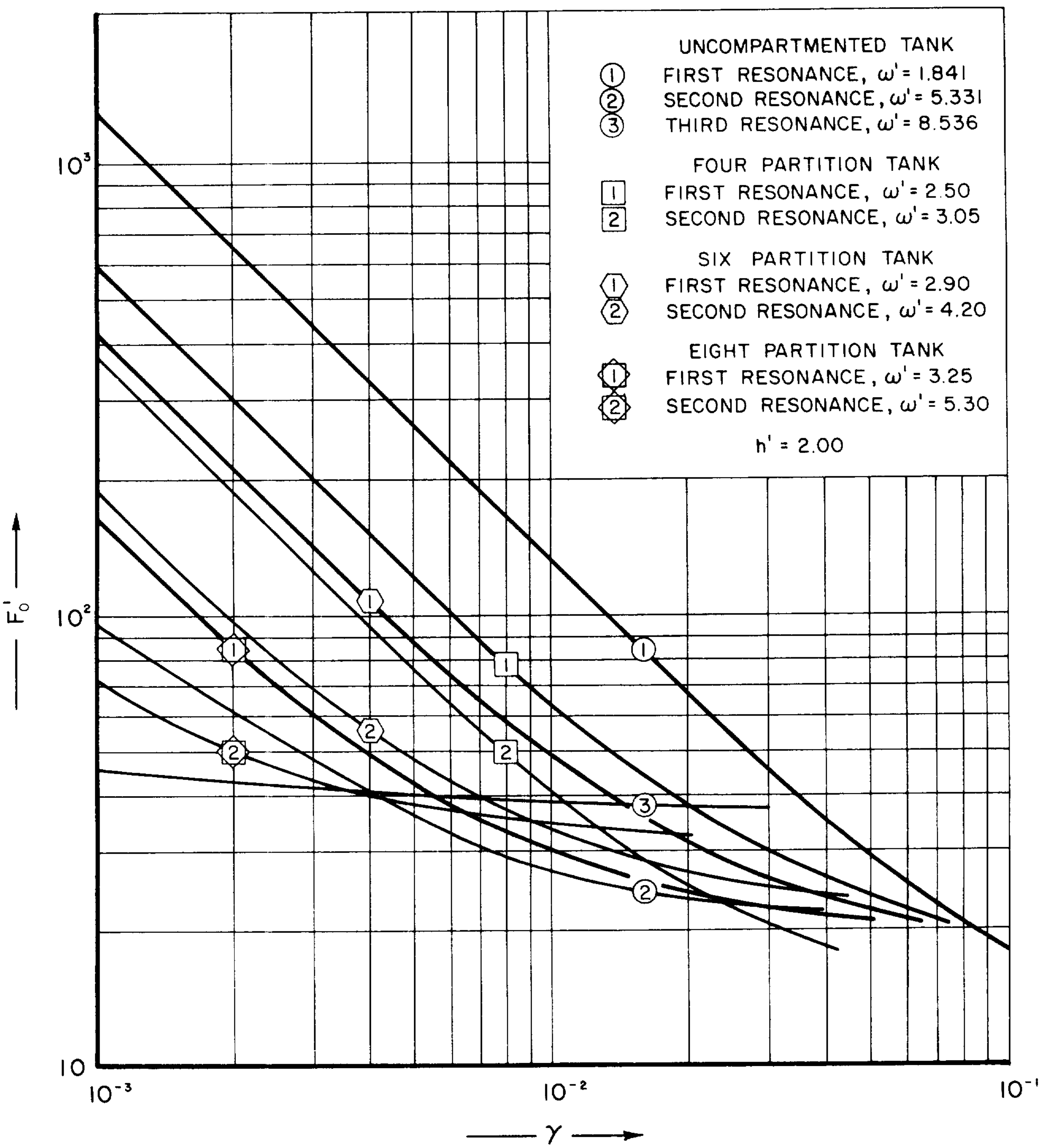

FIGURE 10. TOTAL FORCE DEPENDENCE UPON DAMPING FOR VARIOUS LIQUID RESONANCES

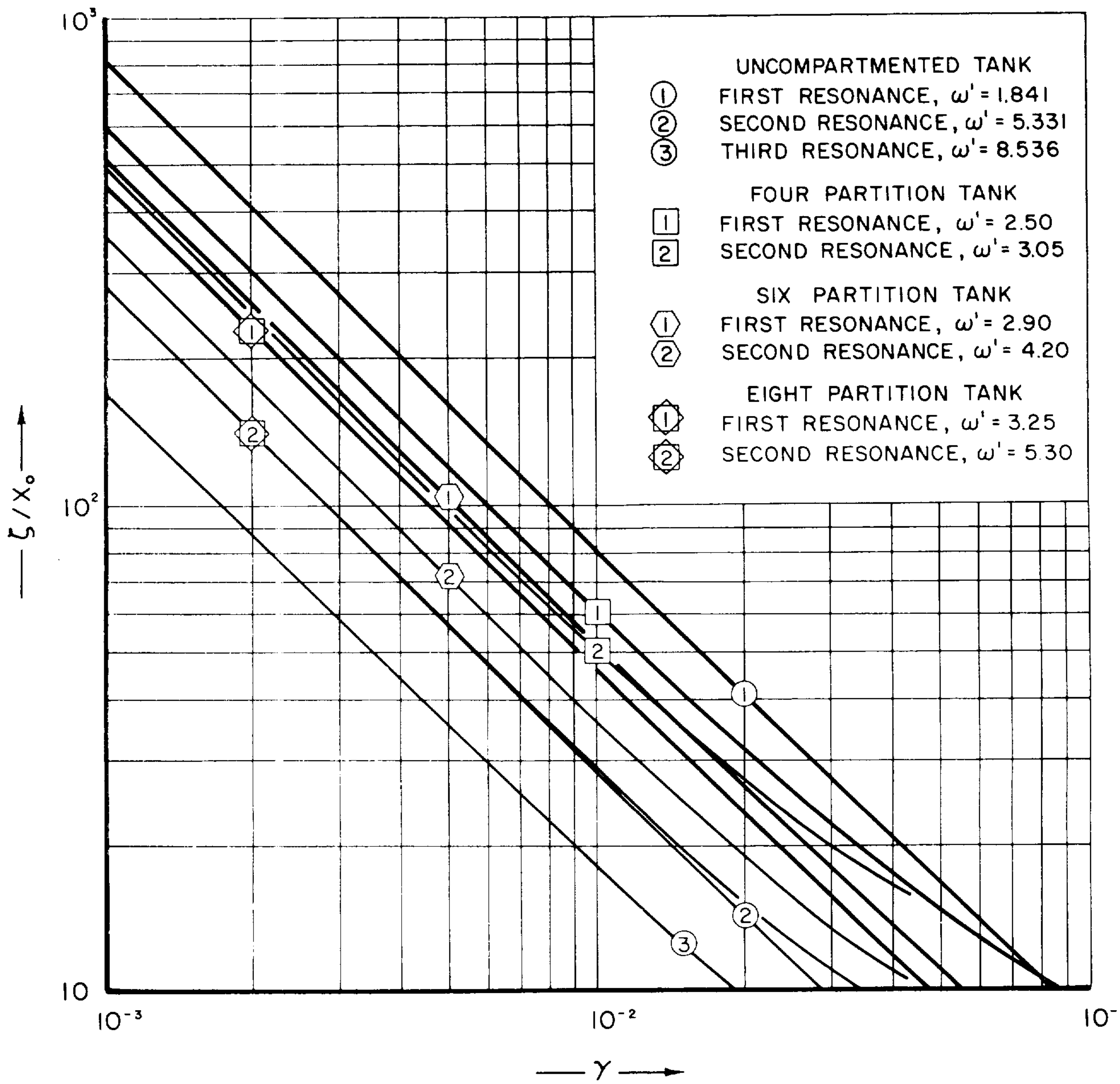

FIGURE 11. MAXIMUM SLOSH HEIGHT VERSUS DAMPING FOR VARIOUS LIQUID RESONANCES

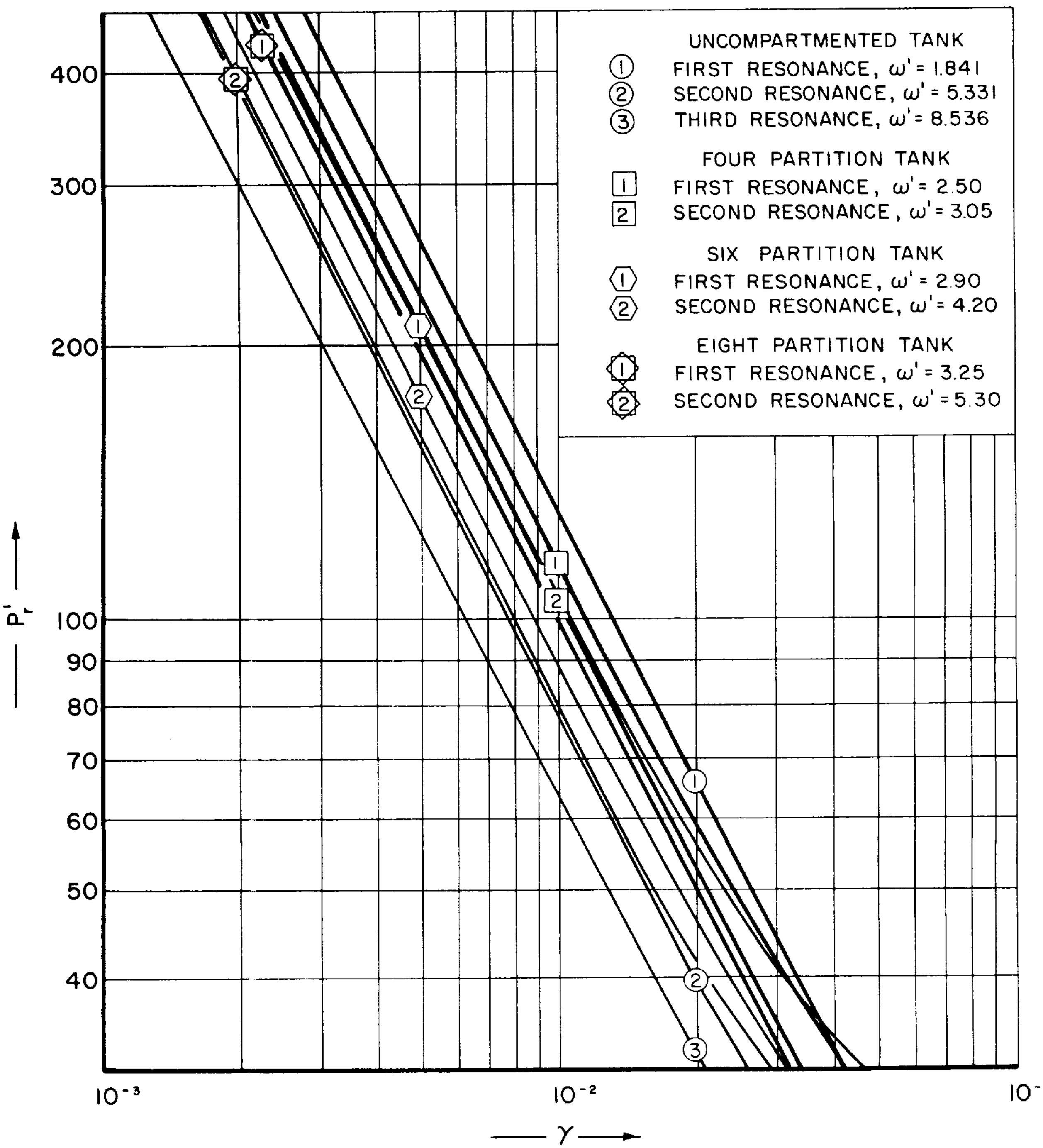

FIGURE 12. MAXIMUM RING PRESSURE VERSUS DAMPING FOR VARIOUS LIQUID RESONANCES

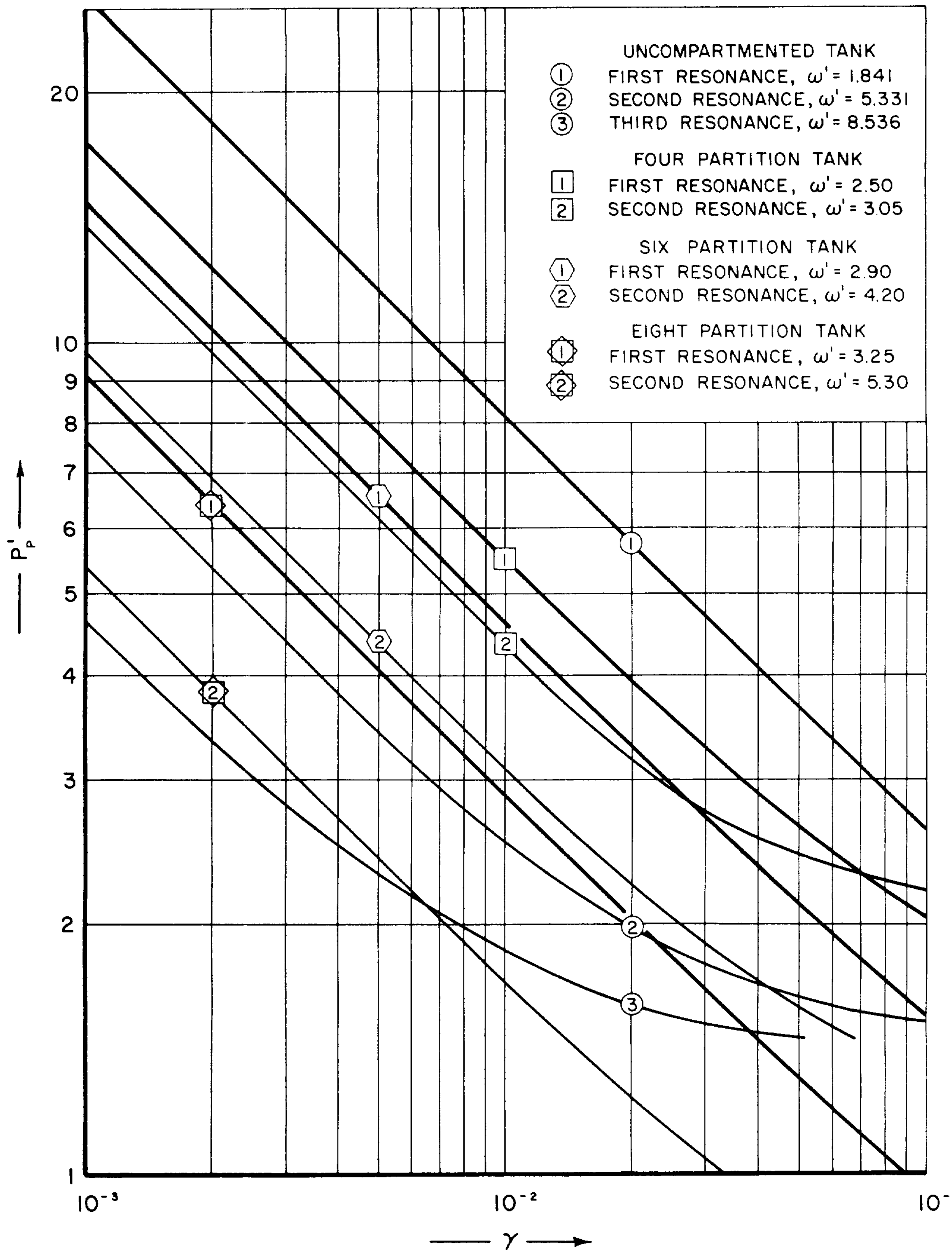

FIGURE 13. MAXIMUM PARTITION PRESSURE VERSUS DAMPING FOR VARIOUS LIQUID RESONANCES

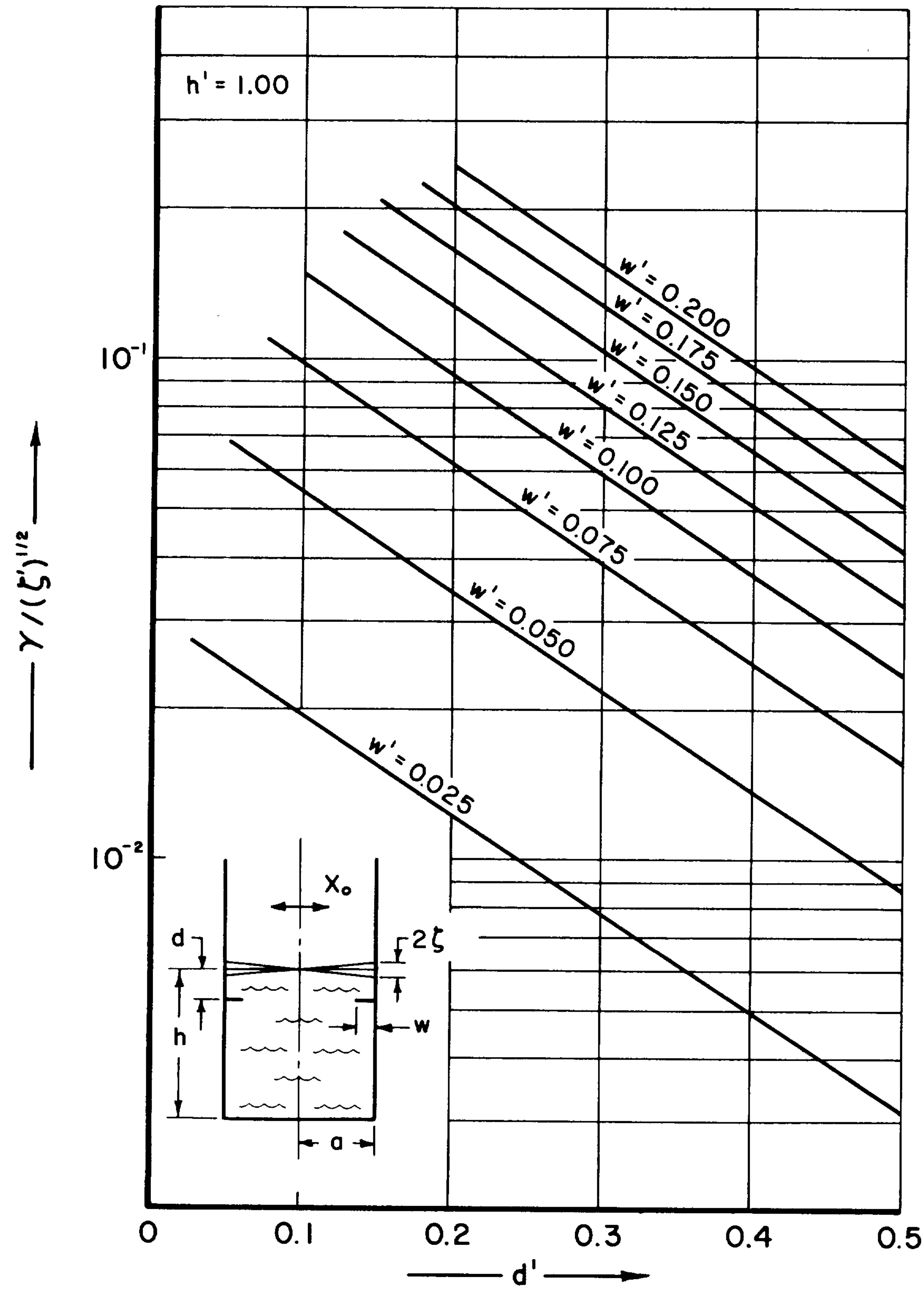

FIGURE 14. DAMPING OBTAINED FROM A SINGLE SUBMERGED RING BAFFLE

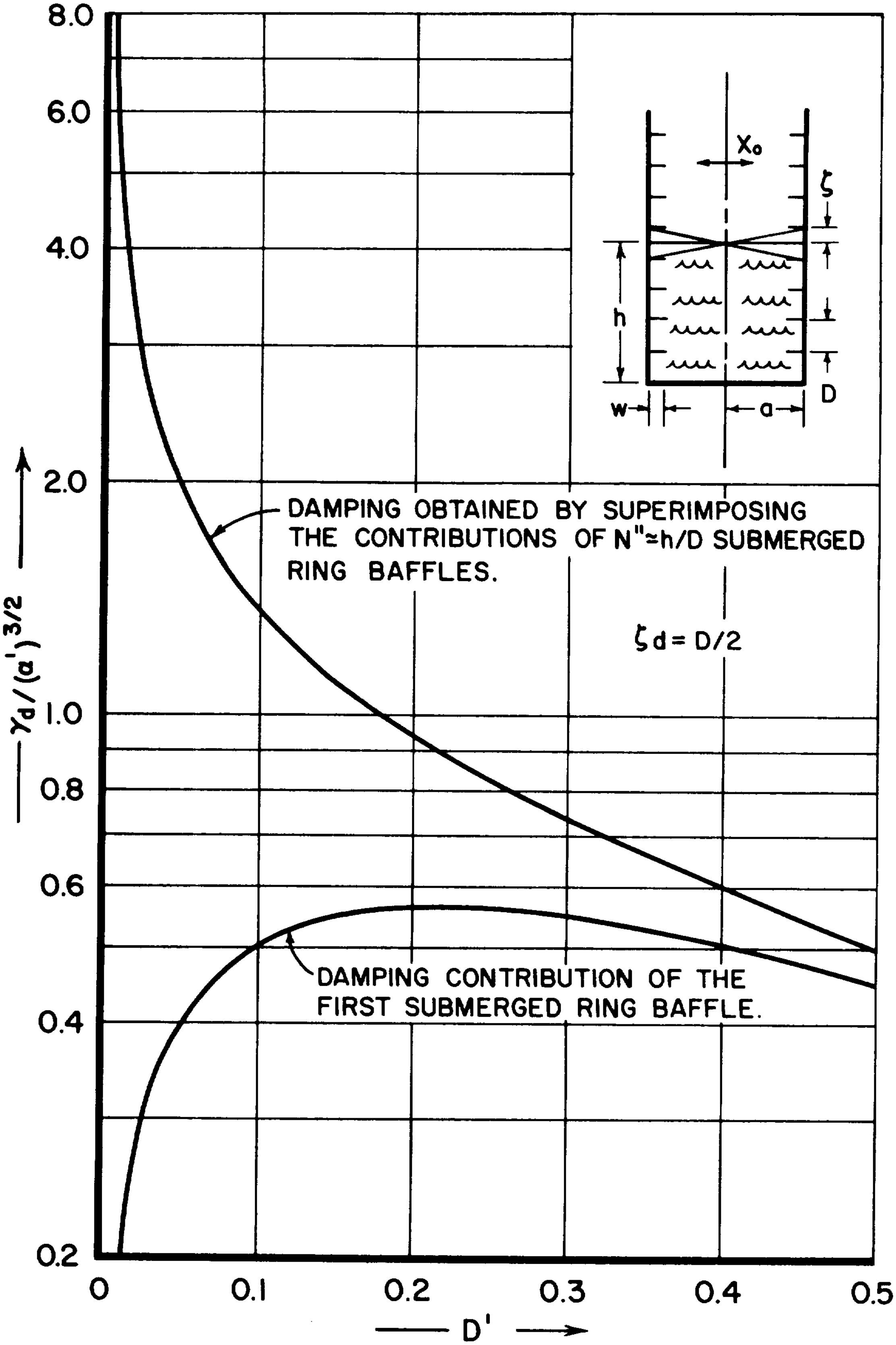

FIGURE 15. DESIGN DAMPING OBTAINED FROM A SYSTEM OF RING BAFFLES

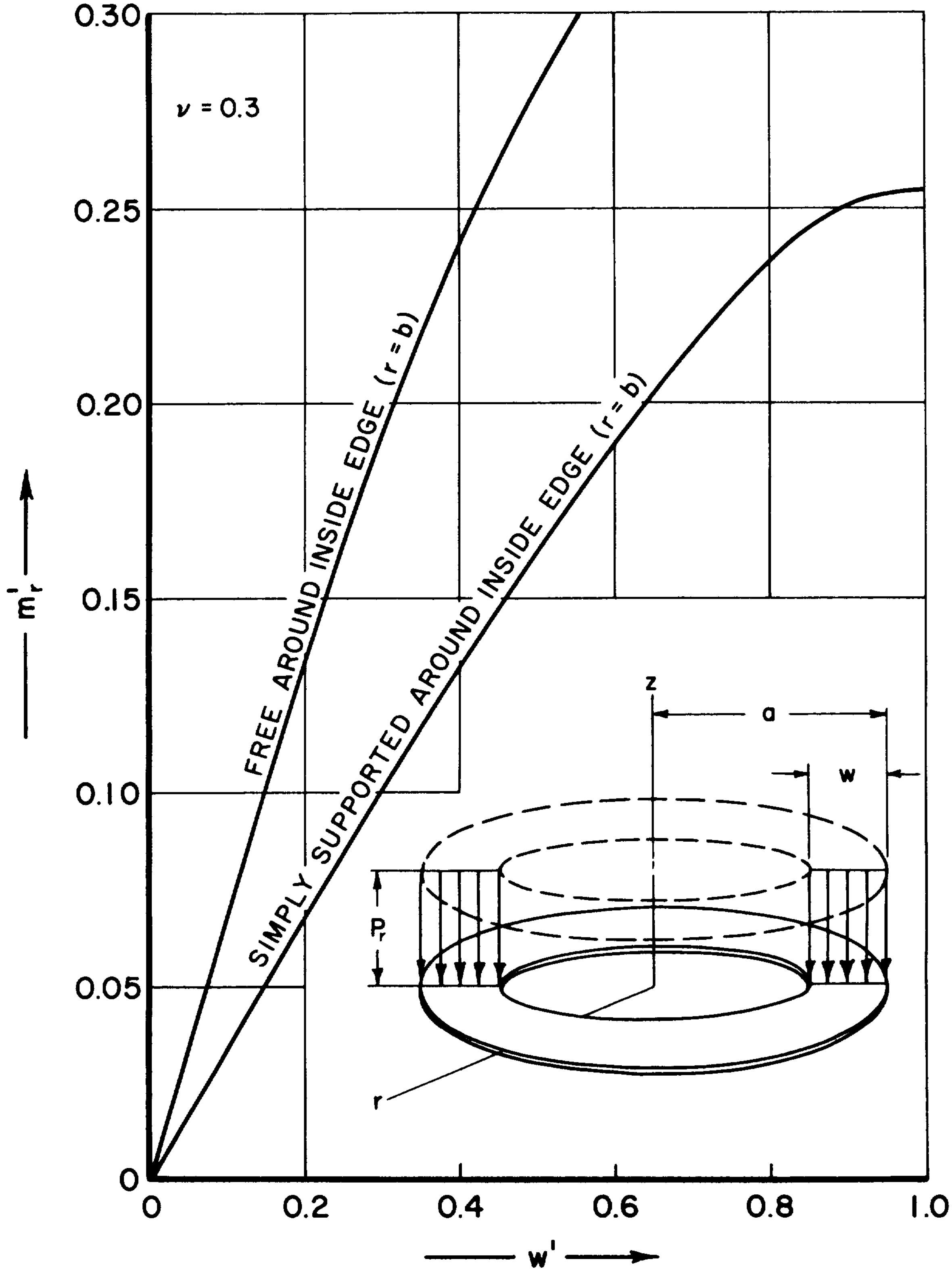

FIGURE 16. MAXIMUM BENDING MOMENT FOR A RING BAFFLE CLAMPED AROUND r=a, AND SUBJECT TO A UNIFORM TRANSVERSE PRESSURE LOADING

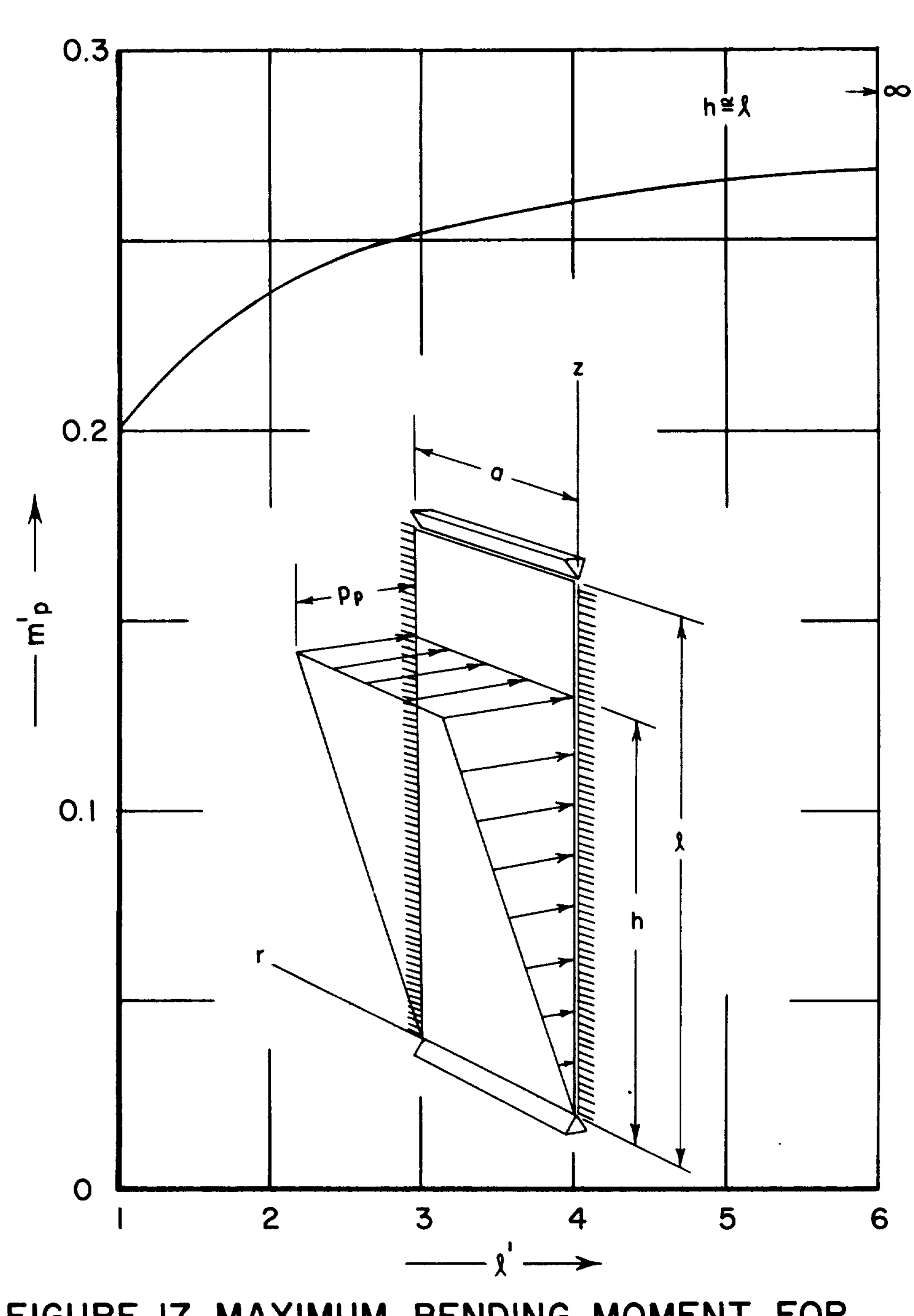

FIGURE 17. MAXIMUM BENDING MOMENT FOR A PARTITION BAFFLE, CLAMPED AT r = 0, a AND SIMPLY SUPPORTED AT z = 0, ℓ, AND SUBJECT TO A HYDROSTATIC TRANSVERSE PRESSURE LOADING.

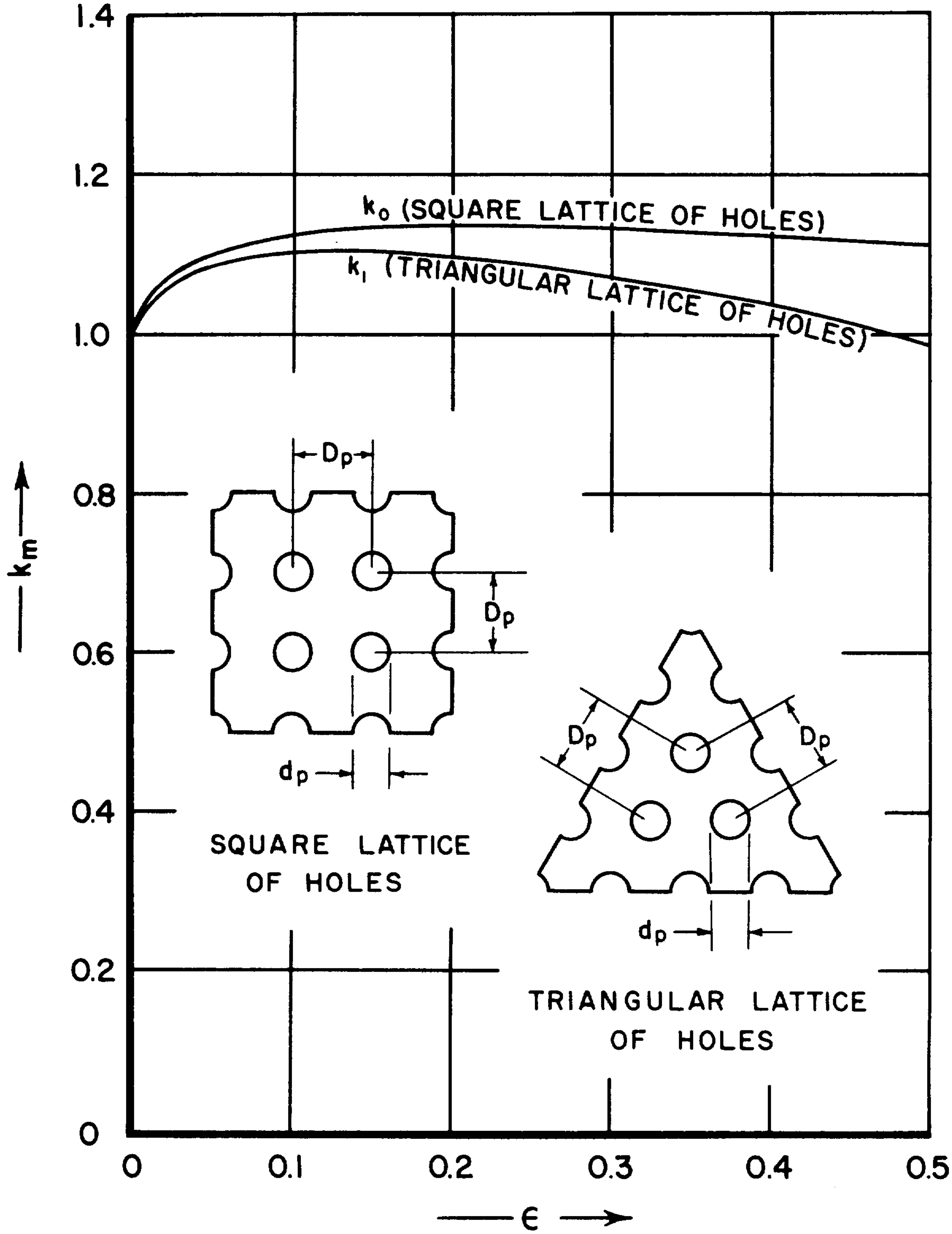

FIGURE 18. PLATE PERFORATION FACTORS VERSUS PERCENTAGE AREA REMOVED

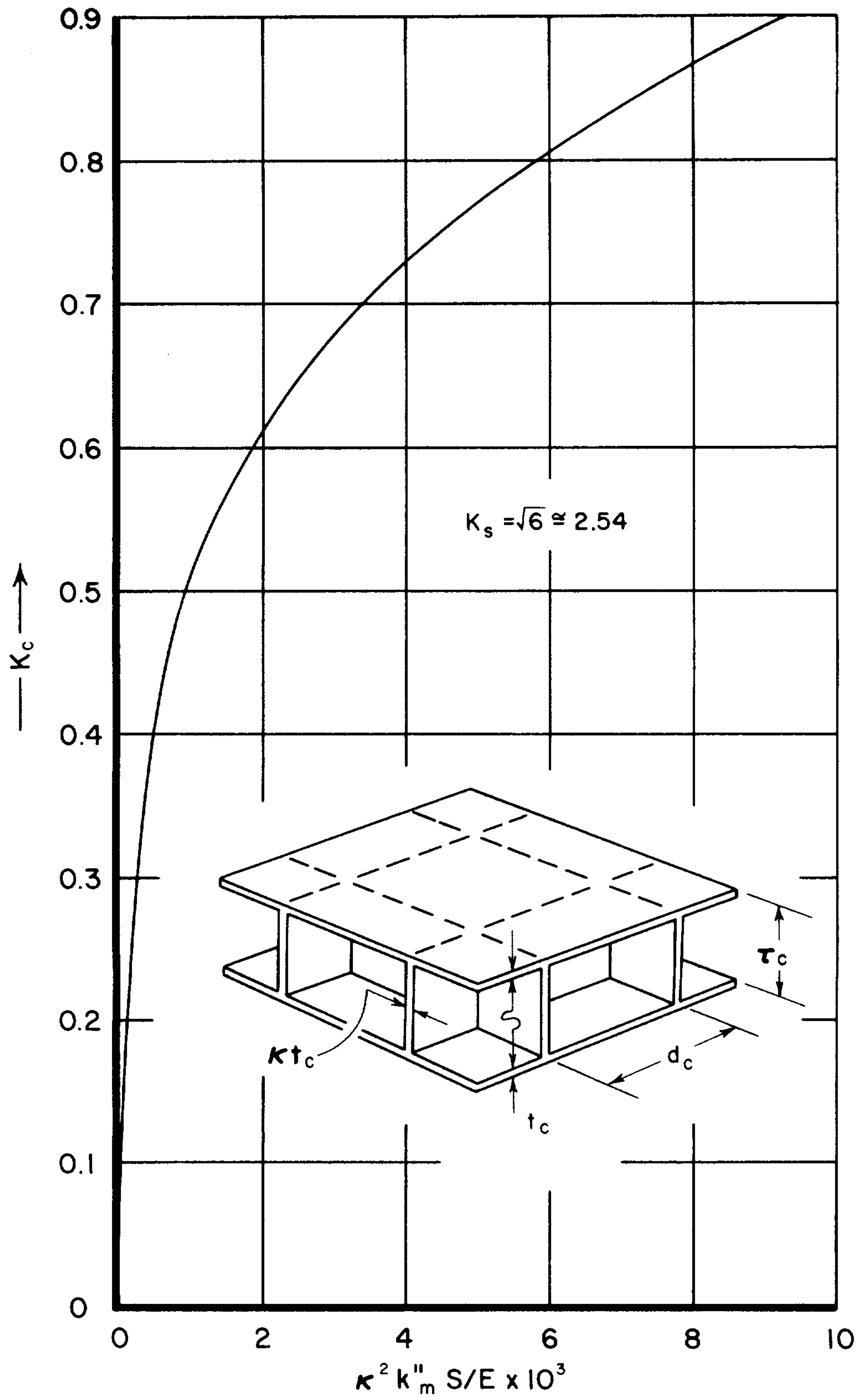

FIGURE 19. CROSS SECTION FACTOR FOR A SIMPLE COMPOSITE SANDWICH PLATE

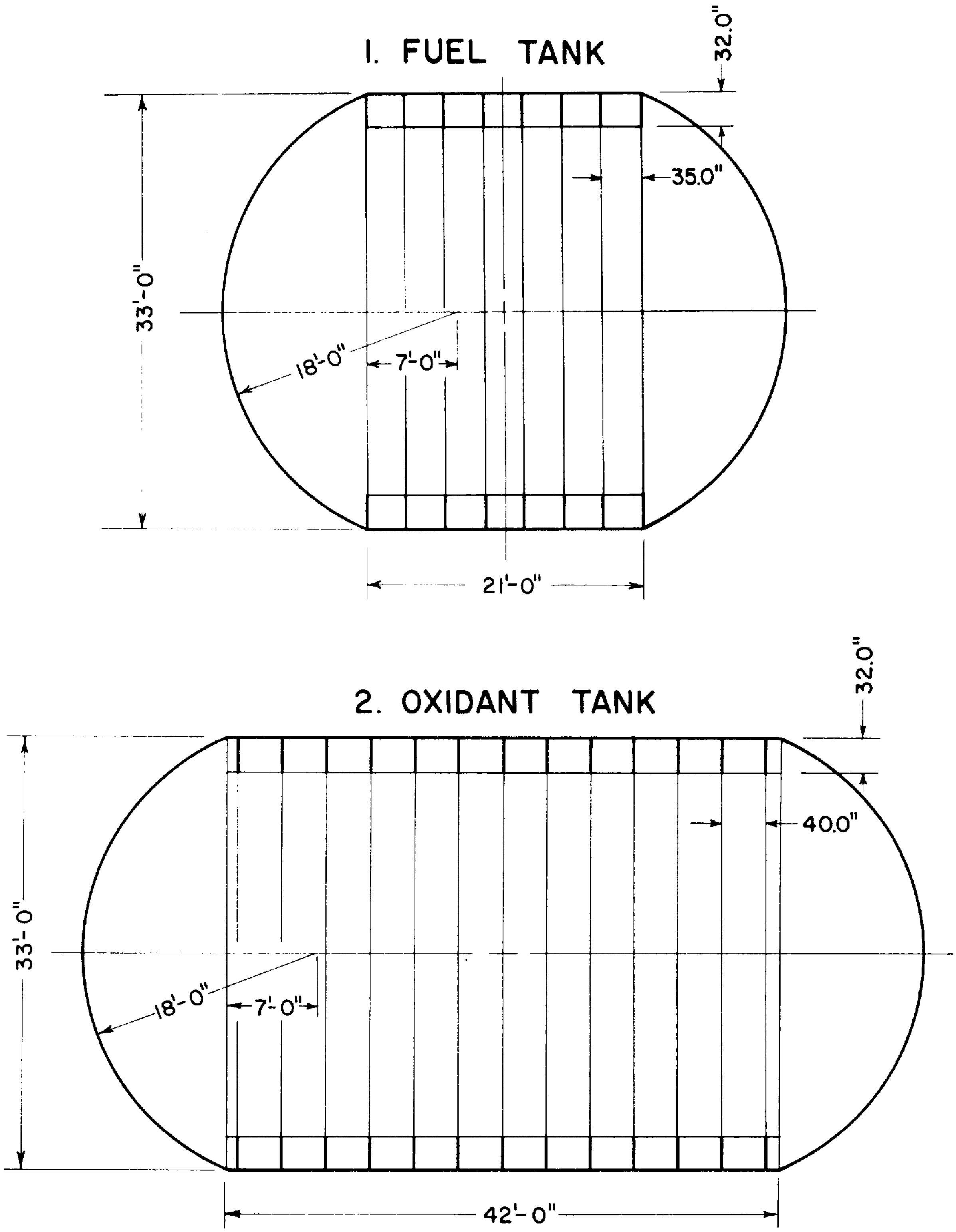

FIGURE 20. SCALE DRAWING OF THE LIQUID PROPELLANT TANKS FOR THE S-IC BOOSTER OF THE C-5 ADVANCED SATURN SPACE VEHICLE