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Improved Geodetic Results from Camera Observations of Satellites

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Abstract. Analysis of Baker Nunn camera observations of satellites 1959α_1 over 1032 days; 1959η over 792 days; 1960ι_2 over 480 days; 1961δ_1 over 150 days; and 1961$\alpha\delta_1$ over 54 days yielded results greatly improved over those previously reported. This improvement is primarily due to the use of much more data and secondarily to various modifications in the method of analysis. As indicated by the discrepancies between results from appreciably different orbits, the datum shifts obtained have standard deviations of ±4 to ±23 meters. The tesseral harmonic of the gravity field most firmly determined appears to be J_{41}, followed by J_{22}, J_{31}, J_{42}, J_{43}, and J_{32}. The principal sources of error suggested are the influence of preassigned variances on separation of gravitational coefficients having the same periodic effects on an orbit (e.g., J_{22} and J_{42}) and the holding fixed with respect to each other stations on the same geodetic datum, but they do not seem adequate to explain all systematic discrepancies in the results. A comparison of gravitational and geometric geoid heights at station positions gave a mean equatorial radius of 6,378,196 ± 11 meters.

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Introduction. This paper describes appreciable improvement over the results obtained by *Kaula* [1963*a*], due both to additional observations and to revisions in the method of analysis. The discussion in this paper follows closely that of *Kaula* [1963*a*] and is limited to changes therefrom.

Observations. The precisely reduced Baker Nunn camera observations of 1959α_1, 1959η, and 1960ι_2 from launch until the end of 1961, of 1961δ_1 from launch until the middle of 1961, and of 1961$\alpha\delta_1$ in the spring of 1962 were analyzed. The observations through mid-1961 have been published in the catalogs compiled by *Veis et al.* [1961–1962].

No change was made in the methods of selection and conversion of observations. The number of observations of each satellite used is given in Table 1.

Geometry. The initial station positions used

TABLE 1. Satellite Orbit Specifications

	Satellite				
	1959α_1	1959η	1960ι_2	1961 δ_1	1961$\alpha\delta_1$
Epoch	1959 Feb. 28.5	1959 Sept. 28.5	1960 Sept. 22.0	1961 Feb. 20.0	1962 Mar. 8.5
Semimajor axis	1.304585	1.334500	1.250057	1.252779	1.568136
Eccentricity	0.16582	0.19008	0.01146	0.12135	0.01197
Inclination	0.57381	0.58212	0.82434	0.67835	1.67316
Argument of perigee	3.36062	3.20403	2.26377	2.02733	4.28853
Longitude of node	2.52442	3.48304	2.28139	2.76786	5.71336
Mean anomaly	6.00463	3.82408	2.72868	5.96587	1.51124
Perigee motion/day	+0.09181	+0.08501	+0.05186	+0.08315	−0.01733
Node motion/day	−0.06108	−0.05712	−0.05413	−0.06347	+0.00367
Max. A/m, cm²/g	0.21	0.27	0.27	15.9	0.08
Min. A/m, cm²/g	0.21	0.04	0.08	15.9	0.02
Perigee height, km	560	510	1500	640	3500
Number of days	1032	792	480	150	54
Number of observations	3513	3034	2502	1395	552

TABLE 2. Datum Shifts (in length units of 6.378165 meters)

Datum	Coordinate	$1959\alpha_1$	1959η	$1960\iota_2$	$1961\delta_1$	$1961\alpha\delta_1$	Weighted Mean
Americas	Δu_1	−02.5	−02.6	−03.7	−03.8	−06.4	−03.8 ± 1.0
	Δu_2	−04.7	−05.2	−09.6	−11.3	−04.2	−05.1 ± 0.8
	Δu_3	−00.9	−00.5	+00.6	−01.9	−00.3	−00.4 ± 0.2
Europe-Africa	Δu_1	+06.5	+07.3	+06.6	+11.6	+04.6	+05.8 ± 0.7
Siberia-India	Δu_2	−07.8	−07.8	−09.3	−04.1	−10.2	−08.9 ± 0.5
	Δu_3	+02.0	+01.3	+02.2	−01.4	+02.1	+01.9 ± 0.2
Australia	Δu_1	−16.3	−19.6	−19.6	−11.2	−26.6	−17.3 ± 1.5
	Δu_2	+09.6	+06.0	+06.7	+07.5	+03.0	+05.2 ± 1.7
	Δu_3	+10.8	+14.6	+10.0	+14.7	+09.4	+10.5 ± 0.4
Japan-Korea-	Δu_1	−08.9	−11.5	−08.3	−08.5	−06.5	−08.9 ± 0.5
Manchuria	Δu_2	+04.1	+05.2	+13.0	+08.7	+09.3	+09.4 ± 0.7
	Δu_3	+01.4	+00.1	+01.8	−00.1	+04.4	+01.5 ± 0.8
Argentina	Δu_1	+35.6	+37.9	+39.9	+50.7	+34.4	+38.3 ± 1.6
	Δu_2	−03.7	+03.8	−02.4	−02.1	+00.0	−02.3 ± 0.6
	Δu_3	+10.0	+07.5	+09.9	+04.2	−06.3	+05.7 ± 3.5
Hawaii	Δu_1	+03.3	+01.3	−05.2	+00.4	−00.3	−04.0 ± 1.6
	Δu_2	+06.1	+04.5	+16.0	+01.2	+15.2	+09.2 ± 2.8
	Δu_3	−45.4	−48.6	−47.7	−67.9	−25.0	−45.5 ± 3.6

were the solutions given in Table 1 of *Kaula* [1963*a*] with corrections for errors in the computed positions of three stations relative to the principal datums provided by I. G. Izsak of the Smithsonian Institution Astrophysical Observatory. Corrections to coordinates u_1, u_2, and u_3 in earth radii are listed in sequence (all values times 10^{-6}):

San Fernando	+5.6	−5.0	−8.5
Naini Tal	+2.7	−5.0	+4.9
Curaçao	−1.0	0.0	+2.8

The datum shifts listed in Table 2 of the present paper apply to the starting coordinates in column 4 of Table 1 of *Kaula* [1963*a*] with the local corrections given above.

Dynamics. The only change made in the dynamical aspects of the treatment was to omit entirely the long-period and secular perturbations that are due to lunisolar attraction, radiation pressure, and drag by a specified atmospheric model. For the orbital arc lengths of 10 to 20 days it was found that these effects were adequately absorbed by an arbitrary acceleration in the mean anomaly. Their inclusion made little difference in the solutions obtained for tesseral harmonics or station shifts—if anything, they may have distorted the results by shifting computed satellite directions farther from those observed.

Data analysis. Of the five techniques used on page 478 of *Kaula* [1963*a*] only three were employed: preassigning a covariance matrix **V** for the starting values of parameters, assigning higher weight to the across-track than to the along-track component of an observation, and using arbitrary polynomials.

The covariances and variances preassigned were identical with those in *Kaula* [1963*a*] with the exception of $\bar{C}_{21}$, $\bar{S}_{21}$, which were held fixed.

The observational variance employed was $(0.026 \text{ sec})^2$ time and $(9.2 \text{ sec})^2$ direction for 12-day arcs of $1959\alpha_1$ and 1959η; $(0.047 \text{ sec})^2$ time and $(13.4 \text{ sec})^2$ direction for 20-day arcs of $1960\iota_2$; $(0.146 \text{ sec})^2$ time and $(43.8 \text{ sec})^2$ direction for 10-day arcs of $1961\delta_1$; and $(0.047 \text{ sec})^2$ time and $(13.4 \text{ sec})^2$ direction for 18-day arcs of $1961\alpha\delta_1$. The principal criterion used in determining the observational variances was the χ^2 test; i.e., the quantity

$$s = (\mathbf{f}^T\mathbf{W}^{-1}\mathbf{f} - \mathbf{z}^T\mathbf{M}^T\mathbf{W}^{-1}\mathbf{f})/(n - p) \quad (1)$$

should average 1 for several orbital arcs, where **f** is the vector of observation equation residuals; **W** is the covariance matrix of observations; **z** is the vector of corrections to parameters; **M** is the matrix parameter coefficients in the observation equations; n is the number of observations; and p is the number of free parameters. In forming the covariance matrix **W**, observations in the same pass were treated as having the same timing error.

TABLE 3. Gravitational Coefficient Solutions
Multiply all numbers by a scaling factor of 10^{-6}.

Coefficient*	1959α_1	1959η	1960ι_2	1961δ_1	1961$\alpha\delta_1$	Weighted Mean
$\Delta\bar{C}_{00}$	4.96	−8.88	−0.75	−18.50	−9.85	−2.46 ± 2.36
$\Delta\bar{C}_{20}$	−0.06	−0.06	−0.05	−0.29	0.00	−0.03 ± 0.02
$\bar{C}_{22}$	1.30	1.36	1.99	1.80	2.52	1.88 ± 0.29
$\bar{S}_{22}$	−1.74	−0.76	−1.63	−0.32	−0.89	−1.38 ± 0.17
$\bar{C}_{30}$	0.97	0.96	0.98	1.01	0.97	0.97 ± 0.01
$\bar{C}_{31}$	1.30	1.62	1.53	−0.96	1.18	1.52 ± 0.03
$\bar{S}_{31}$	0.29	0.99	−0.10	−0.34	0.46	0.14 ± 0.16
$\bar{C}_{32}$	−0.14	−0.13	0.29	2.35	−0.84	−0.02 ± 0.26
$\bar{S}_{32}$	0.49	0.29	0.38	−0.16	0.98	0.42 ± 0.06
$\bar{C}_{33}$	0.36	1.11	0.42	2.36	1.70	0.70 ± 0.26
$\bar{S}_{33}$	0.83	1.11	0.89	0.43	−1.33	0.76 ± 0.29
$\bar{C}_{40}$	0.68	0.67	0.61	−0.35	0.62	0.67 ± 0.02
$\bar{C}_{41}$	−0.38	−0.38	−0.33	−0.48	−1.00	−0.33 ± 0.01
$\bar{S}_{41}$	0.43	0.53	0.45	0.39	−0.45	0.37 ± 0.15
$\bar{C}_{42}$	−0.10	−0.10	0.02	0.03	0.47	0.01 ± 0.02
$\bar{S}_{42}$	0.52	0.68	0.36	−0.43	0.06	0.35 ± 0.15
$\bar{C}_{43}$	0.18	0.35	0.50	0.44	0.17	0.17 ± 0.02
$\bar{S}_{43}$	0.29	0.11	−0.00	0.16	0.42	0.41 ± 0.03
$\bar{C}_{44}$	0.12	0.01	−0.20	0.20	−0.24	−0.01 ± 0.08
$\bar{S}_{44}$	0.11	0.22	0.36	0.29	0.32	0.18 ± 0.05
$\bar{C}_{50}$	0.02	0.03	0.03	0.01	0.02	0.02 ± 0.01
$\bar{C}_{51}$	−0.14	−0.02	−0.01	−0.63	†	−0.13 ± 0.02
$\bar{S}_{51}$	−0.06	−0.03	−0.01	0.23	†	−0.01 ± 0.01
$\bar{C}_{60}$	−0.09	−0.08	−0.04	1.10	−0.10	−0.09 ± 0.02
$\bar{C}_{61}$	−0.01	−0.03	0.00	−0.26	−0.09	−0.05 ± 0.03
$\bar{S}_{61}$	−0.09	−0.02	−0.07	−0.49	−0.06	−0.06 ± 0.01
$\bar{C}_{62}$	−0.04	0.05	−0.01	−0.07	0.05	0.01 ± 0.01
$\bar{S}_{62}$	−0.09	−0.18	−0.01	−0.07	0.01	−0.02 ± 0.03
$\bar{C}_{63}$	−0.02	−0.10	0.15	−0.02	†	0.15 ± 0.01
$\bar{S}_{63}$	−0.12	−0.01	−0.08	−0.06	†	−0.08 ± 0.01
$\bar{C}_{64}$	−0.00	0.06	0.06	−0.19	−0.01	−0.01 ± 0.01
$\bar{S}_{64}$	−0.06	−0.09	−0.42	−0.30	0.03	−0.03 ± 0.07
$\bar{C}_{70}$	0.12	0.12	0.07	0.09	0.12	0.12 ± 0.01

* $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are coefficients of spherical harmonic terms $kM/r(a/r)^n\, H_{nm}$ such that $\int H_{nm}{}^2\, d\sigma = 4\pi$ for integration over the sphere. $\Delta\bar{C}_{00}$ and $\Delta\bar{C}_{20}$ are corrections to $0.3986032 \times 10^{21}\,(1.0 - 0.00108236P_2)$ cgs.
† No determinations of $\bar{C}_{51}$, $\bar{S}_{51}$, $\bar{C}_{63}$, $\bar{S}_{63}$ were made from 1961$\alpha\delta_1$ because the partial derivatives of the orbit with respect to these coefficients were all smaller than the criterion $0.1n^{1.2}$ [*Kaula*, 1963*a*].

The arc lengths used were chosen after some experimentation as giving a reasonable compromise between magnitude of residuals and number of observations.

The use of arbitrary polynomials was held to a minimum; i.e., the only one used was a t^2 variation in the mean anomaly.

In determining the estimated mean value and its standard deviation from several orbital arcs of the same satellite, the weighting of a particular arc was considered to be proportionate to its degrees of freedom. The computer program limited to fifteen the number of arcs that could be combined at a time. In combining the results of several sets of fifteen (or fewer) arcs, the weight ascribed to the mean of each set was considered to be the inverse of its variance, or standard deviation squared.

In order that the final mean and standard deviation reflect as much as possible any systematic differences which were functions of orbital specifications, all sets were combined, with inverse-variance weighting, into four groups: 1959α_1 and 1959η, twelve sets; 1960ι_2, two sets; 1961δ_1, one set; and 1961$\alpha\delta_1$, one set. The final means and standard deviations given in Tables 2 and 3 are the result of an inverse-variance weighted combination of these four group solutions. However, for most of the variables, the standard deviations from combining the four

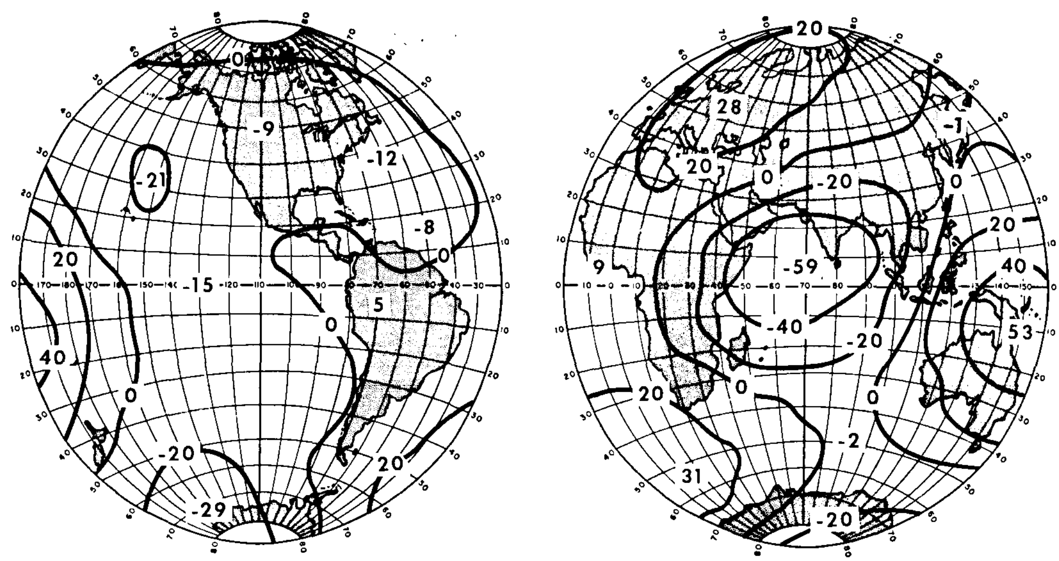

Fig. 1. Vanguard geoid. Geoid heights, in meters, referred to an ellipsoid of flattening 1/298.24, determined from observations of satellites, $1959\alpha_1$ and 1959η.

groups were smaller than the standard deviations combining all sixteen sets at once, primarily because the differences between the $1960\iota_2$ mean and the $1959\alpha_1$ and 1959η mean were smaller than the scatter of $1959\alpha_1$ and 1959η solutions about their own mean.

To avoid the tendency to prejudge the order of magnitude of the solution, which is the main defect of the preassigned-variance technique, some computer experimentation was tried in determining the amplitudes of specified periodic variations, in place of harmonic coefficients, in holding the reference orbit fixed, and in analyzing residuals. Applying these methods to one satellite at a time did not give as good results as the preassigned-variance method, to judge by the scatter of solutions. To apply them to data from more than one satellite simultaneously required considerable program revision which did not seem worth while because this method has been applied extensively by *Izsak* [1963]. Other changes tried and dropped as unnecessary were deleting orbital segments for which observations are scanty and holding fixed the station shifts obtained from the previous analysis of $1960\iota_2$ observations. Also dropped was the device of weighting observations inversely as their density with respect to the phase angle (node-GST).

Results. The analysis described above took much time to apply to the large quantity of $1959\alpha_1$ and 1959η data. The attempt to combine solutions from different sets of arcs was not made until this analysis had been completed. Consequently, the good agreement shown by Tables 2 and 3 between the results from $1960\iota_2$ on the one hand and from $1959\alpha_1$ and 1959η on the other came as a pleasant surprise. The combination of results is not as good, of course, as is suggested by the formal standard deviations given in the tables; in particular, the errors in difference of position between stations in North and South America—or between stations in Europe, Africa, and India—which were held fixed with respect to each other, are probably several times as great as some of the stated uncertainties. The good agreement is even more marked for the spatial representations given in Figures 1 and 2; e.g., for the seven most extreme maximums and minimums in the Vanguard geoid of Figure 1, there are maximums and minimums in the Echo rocket geoid of Figure 2 agreeing within 10° in location and within 11 meters in magnitude. The degree of independence in these solutions is fairly satisfying. The orbits differ by 0.23 in inclination, and 0.16 in eccentricity, the arc lengths used differed in a ratio of 5 to 3, and the observational weighting differed in a

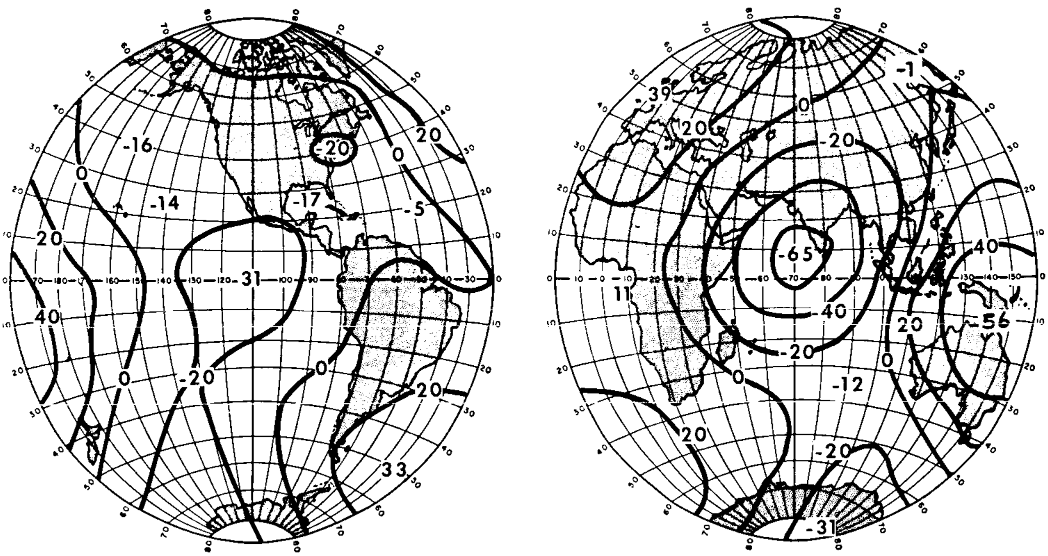

Fig. 2. Echo rocket geoid. Geoid heights, in meters, referred to an ellipsoid of flattening 1/298.24, determined from observations of satellite 1960ι_2.

ratio of 3 to 2. It would be very desirable, however, to obtain comparable series of observations of a satellite of much higher inclination.

The principal sources of systematic error likely to be common to satellites 1959α_1, 1959η, and 1960ι_2 seem to be (1) that the magnitudes of the results will be influenced by the preassigned variances and (2) that the relative positions of tracking stations on the same geodetic datum may be appreciably in error.

For a parameter whose effects are fairly distinct in periodicities, etc., from those of other parameters, it is implausible that its preassigned variance could cause a correction that is too large or of wrong sign, but it might cause a correction that is too small. However, the variance actually used in the analyses is not the estimated squared magnitude of the correction $\sigma^2(c)$, but rather $N\sigma^2(c)$, where N is the number of orbital arcs in a set. Since N was always between 10 and 15, this seems to be no more than a mild restraint preventing occasional ill-conditioned arcs from obtaining absurdly large corrections beyond the range of linearity.

Distortion caused by the preassigned variances seems most likely to occur in separating gravitational coefficients whose principal effects are of the same period; i.e., coefficients J_{nm} and J_{kl} such that $m = l$ and $n - k$ is even. The most prominent set of such coefficients is J_{22}, J_{42}, and J_{62}, all of which cause semidaily variations of argument $2(\Omega - \theta)$. A way of removing some (but not all) of the influence of the preassigned covariances would be to assume that what we have determined is not the coefficients themselves but the amplitudes of semidaily variations in the orbital elements; e.g., for the cos $2(\Omega - \theta)$ term in the variation of the inclination

$$\Delta i = \frac{\partial i}{\partial \bar{C}_{22}} \bar{C}_{22} + \frac{\partial i}{\partial \bar{C}_{42}} \bar{C}_{42} + \frac{\partial i}{\partial \bar{C}_{62}} \bar{C}_{62} \qquad (2)$$

The semimajor axis and the eccentricity have no semidaily variation. If we omit the 1961δ_1 and 1961$\alpha\delta_1$ results and assume that the similar 1959α_1 and 1959η orbits should be combined, we have two sets of eight equations for three unknowns. Using values $\bar{C}_{22} = 1.315 \times 10^{-6}$, $\bar{S}_{22} = -1.473 \times 10^{-6}$, $\bar{C}_{42} = -0.101 \times 10^{-6}$, $\bar{S}_{42} = 0.567 \times 10^{-6}$, $\bar{C}_{62} = -0.009 \times 10^{-6}$, $\bar{S}_{62} = -0.104 \times 10^{-6}$ for the combined 1959α_1 and 1959η solution (corresponding to Figure 1) and using values from Table 3 for 1960ι_2 we get the computed amplitudes of periodic perturbations in columns 6 and 10 of Table 4. Using these amplitudes as the observation equation constants and solving by the rule of minimizing $\Sigma(d\Delta El)^2$

TABLE 4. Semidaily Perturbations of Satellite Orbits

Satellite	Element (El)	$\frac{\partial El}{\partial \bar{C}_{22}}$	$\frac{\partial El}{\partial \bar{C}_{42}}$	$\frac{\partial El}{\partial \bar{C}_{62}}$	$10^6 \times$ Comp ΔEl_c	$\frac{\partial El}{\partial \bar{S}_{22}}$	$\frac{\partial El}{\partial \bar{S}_{42}}$	$\frac{\partial El}{\partial \bar{S}_{62}}$	$10^6 \times$ Comp ΔEl_s
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1959 α_1 and	M	−2.92	0.36	5.74	−3.93	2.92	−0.36	−5.74	−3.91
1959η	i	3.62	−5.89	3.83	5.33	3.62	−5.89	3.83	−9.12
combined	ω	1.68	12.21	−24.58	1.20	−1.68	−12.21	24.58	−7.09
	Ω	−5.54	4.22	4.88	−7.76	5.54	−4.22	−4.88	−11.10
1960ι_2	M	−6.04	0.00	−0.86	−12.05	6.04	0.00	0.86	−9.86
	i	5.48	−5.08	−0.34	10.81	5.48	−5.08	−0.34	−10.76
	ω	−2.59	20.86	−2.61	−4.76	2.59	−20.86	2.61	−11.70
	Ω	−5.07	−3.26	7.66	−10.12	5.07	3.26	−7.66	−7.07

yields

$$\bar{C}_{22} = 1.85 \times 10^{-6} \qquad \bar{S}_{22} = -1.75 \times 10^{-6}$$
$$\bar{C}_{42} = 0.05 \times 10^{-6} \qquad \bar{S}_{42} = 0.34 \times 10^{-6}$$
$$\bar{C}_{62} = 0.10 \times 10^{-6} \qquad \bar{S}_{62} = -0.22 \times 10^{-6}$$

All the coefficients are increased over the mean in Table 3 except $\bar{S}_{42}$, which hints of ill conditioning. However, it looks as though only $\bar{C}_{62}$ and $\bar{S}_{62}$ might have been significantly reduced by the preassigned-variance method.

The assumption made by *Kaula* [1963*a*] that the relative positions of tracking stations on the same datum should be known through the triangulation networks with a rms error of ±20 meters or less was based on standard methods of estimating triangulation accuracy [*Bomford,* 1962, pp. 143–159], as is confirmed by the misclosures of large loops of triangulation: (1) 15 meters in the 4000-km loop around the western Mediterranean [*Whitten,* 1952]; (2) less than 25 meters in the 10,000-km loop around the Caribbean [*Fischer,* 1959]; and (3) within 15 meters for the 10,000-km loop around the Black Sea and Caspian Sea, through Turkestan, and connecting in northwest India [*Fischer,* 1961].

The connections between the three northern hemisphere stations of the EASI system are closely associated with loops 1 and 3, and the connections between the three northern hemisphere stations of the Am. system are closely associated with loop 2. More in doubt are the positions of the stations in the southern hemisphere in Peru and South Africa, which depend on long single arcs of triangulation. A test run was therefore made on all the 1960ι_2 data, in which the stations Arequipa (in Peru) and Olifantsfontein (in South Africa) were assumed to be on separate datums. The results of this test corroborated the assumption as to triangulation accuracy; the station in Peru moved 24 meters with respect to those in North America, while the station in South Africa moved 14 meters with respect to those in Eurasia. The changes in the gravitational coefficients were insignificant—$\bar{C}_{22}$, from 1.99 to 2.11 × 10^{-6}; $\bar{S}_{22}$, from −1.63 to −1.60 × 10^{-6}; $\bar{C}_{31}$, from 1.53 to 1.49 × 10^{-6}; $\bar{C}_{41}$, from −0.33 to −0.28 × 10^{-6}; etc.—and the maximum effect on any geoid height in Figure 2 was 4 meters.

There still exists the possibility of errors in the local connection of tracking stations to the triangulation systems, a matter in which better standardization of procedures is needed [*Kaula,* 1963*b*]. To check this type of error for stations on the major datums we calculate the geometric geoid heights corresponding to the final positions in rectangular coordinates and then compare these heights with the gravitational geoid heights in Figure 3. To estimate the size of discrepancies to be expected, we have the geoid height variance of 1076 m² from autocovariance analysis of gravimetry [*Kaula,* 1959] and a mean square height of the satellite geoid of 466 m², obtained from the sum of the squares of the coefficients in Table 3. If the station positions and the equatorial radius were correct, the rms expected discrepancy between the geometric and gravitational geoid heights due to the inability of the satellite orbits to pick up the shorter-wave variations would be $(1076 - 466)^{1/2} =$ ±25 meters.

The results of the comparison are shown in

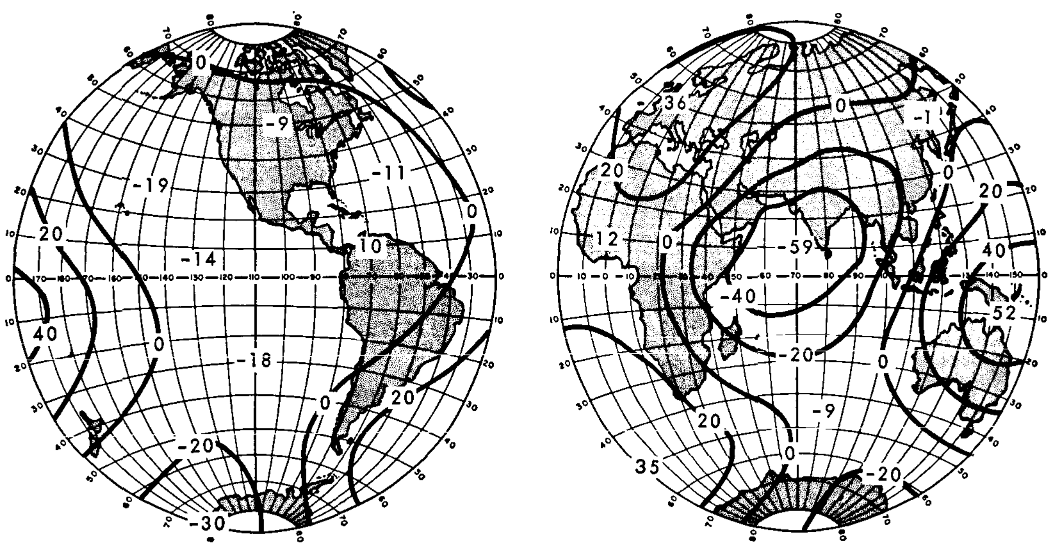

Fig. 3. Combined geoid. Geoid heights, in meters, referred to an ellipsoid of flattening 1/298.24, determined from observation of satellites $1959\alpha_1$, 1959η, $1960\iota_2$, $1961\delta_1$, and $1961\alpha\delta_1$.

Table 5. Applying the mean correction of +31 meters yields a mean equatorial radius of 6,378,196 ± 11 meters and a rms discrepancy of ±38 meters, which implies a rms radial position error of $(38^2 - 25^2)^{1/2} = \pm 29$ meters. Of the stations on the major datums, the 60-meter discrepancy for San Fernando causes suspicion of local connection error; however, there is also a 69-meter discrepancy for Villa Dolores, which was free to move to its correct position.

The agreement of the combined solution in Figure 3 with astrogeodetic [*Fischer*, 1961] and gravimetric [*Uotila*, 1962] solutions is an improvement over that in *Kaula* [1963*a*], particularly in showing a more pronounced negative in the western Atlantic. The discrepancies which exist may in part be ascribed to the method of analysis of the terrestrial data, since the agreement is appreciably better with the combination of astrogeodetic, gravimetric, and satellite zonal

TABLE 5. Comparison of Geometric and Gravitational Geoid Heights

Station	Datum	Geometric Geoid Height, m	Gravitational Geoid Height, m	Discrepancy for 6,378,165 *m* Radius, m	Discrepancy for 6,378,196 *m* Radius, m
Organ Pass	Am.	−4	−10	+6	−25
Arequipa		−16	−8	−8	−39
Curaçao		−25	−10	−15	−46
Jupiter		−18	−9	−9	−40
Olifantsfontein	EASI	+22	+5	+17	−14
San Fernando		+117	+26	+91	+60
Naini Tal		−17	−41	+24	−7
Shiraz		+14	−18	+32	+1
Woomera	Au.	+47	+24	+23	−8
Tokyo	JKM	+54	+4	+50	+19
Villa Dolores	Ar.	+104	+4	+100	+69
Maui	H	+54	−12	+66	+35

Note. Geoid heights referred to ellipsoid of equatorial radius 6,378,165 meters, flattening 1/298.24.

harmonic data of *Kaula* [1961], especially for western Europe.

The reference flattening of 1/298.24 is used in Figures 1, 2, and 3 to facilitate comparison with the results of *Kaula* [1961, 1963*a*]. The flattening equivalent to the solution obtained for $\Delta\bar{C}_{20}$ is 1/298.28. The J_2 equivalent is 1082.48×10^{-6}.

In conclusion, it can be said that better explanations are needed for the systematic discrepancies indicated by Tables 3 and 5. However, considering that the observations used herein depended on reflected sunlight; that they were all made more than 3 years before the minimum of solar activity; and that the orbital specifications are far from ideal, the prospects are bright for extracting more information on the gravitational field from more recent and anticipated satellites. It will be of particular interest to push the analysis to a good determination of some sixth- or eighth-degree harmonics to see whether or not they corroborate other indicators of a weak upper mantle.

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