NASA TECHNICAL NOTE

NASA TN D-2049

N64-10452

NASA TN D-2049

FEASIBILITY STUDY OF A BANG-BANG PATH CONTROL FOR A REENTRY VEHICLE

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Langley Research Center
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1963

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SUMMARY

A study of a bang-bang controller for range control of a direct descent reentry has been made. The path trajectory selected for the study was a dive into the atmosphere followed by a constant-altitude phase and a constant-lift-coefficient glide. This trajectory allows a closed-form solution of desired altitude as a function of desired range. During the dive phase the bang-bang controller is used to guide the vehicle to the desired altitude and flight path for the constant-altitude phase. An analytical equation which represents the switching curve for the bang-bang controller is developed and evaluated.

Results of this study show that the switching curve used for the bang-bang controller gives an excellent prediction of when a control maneuver should be made to achieve rapidly level flight conditions at the desired altitude. The results also show that the equation representing the switching curve was effective for both high-drag—low-lift reentry vehicles and high-lift reentry vehicles and for a wide range of reentry conditions.

INTRODUCTION

The problem of the single-pass reentry of a space vehicle into the earth's atmosphere is that of proceeding on a path that will place it at a selected point on the earth's surface at the end of the flight. This trajectory has a lower altitude limit that is dependent upon acceptable deceleration limits of the vehicle's passengers and an upper altitude limit that is dependent upon the aerodynamic forces available to prevent the vehicle from skipping out of the atmosphere. In order to follow such a trajectory the vehicle must be controlled. The design of a control system requires the selection of suitable parameters as references for control and the formulation of control laws in such a way as to minimize deviations from the desired trajectory.

In recent years numerous procedures for the control of a reentry vehicle into the earth's atmosphere have been presented and have been discussed in a comprehensive review given in reference 1. Workable systems using linear feedback have been formulated by experimental and analytic procedures. However, in any workable control system, the control laws are formulated such that overshoot is held to a minimum. This is difficult especially when the control forces vary with altitude and velocity. The price paid for overshoot is either an uncontrolled skip or over deceleration. There is one type of control system in which overshoot is eliminated entirely - time optimal. Rigorous application of this type of control to a reentry vehicle appears to be impracticable at present, however, because of complexities involved in the implementation of the control logic.

In view of the advantages inherent in the time-optimal control, a study was made of practical approximations thereof. This study, based on particle dynamics, indicated that an approximate-time-optimal (bang-bang) control could be used to advantage during the first part of the reentry. Satisfactory reentries may be achieved by a trajectory consisting of (1) an initial dive and flare-out to horizontal flight at a selected altitude under bang-bang control, (2) decelerating flight at this altitude under linear control until a given lift coefficient is reached, and (3) an uncontrolled glide at a constant lift coefficient to the terminal point.

A description of the development of the controls is presented herein together with results from a digital computer simulation showing the ability of the system to maintain the vehicle within the desired altitude-velocity limits and to achieve a number of desired flight distances for various values of reentry angles and velocities. Both low-lift and relatively high-lift vehicles are investigated.

SYMBOLS

a_D	deceleration, g units
$a_{D,max}$	maximum deceleration allowed for level flight conditions, g units
B	constant used in exponential approximation of atmospheric density, ft
C	constant used in switching logic, ft
C_D	drag coefficient
C_L	lift coefficient
$C_{L,max}$	maximum value of lift coefficient
C_L^*	arbitrary value of C_L selected for constant C_L phase
g	acceleration due to gravity, ft/sec^2
h	altitude of vehicle, ft
h_A	altitude for constant-altitude phase of desired mode of flight, determined from equation (20), ft
h_d	desired altitude for constant-altitude phase of desired mode of flight, ft

h_g deceleration limit on altitude for constant-altitude phase of desired mode of flight (eq. (22)), ft

h_s skip limit on altitude for constant-altitude phase of desired mode of flight (eq. (23)), ft

Δh $h_d - h$, ft

K_1 gain factor used with Δh in controller, radians/ft

K_2 gain factor used with $\dot{h}$ in controller, radians/ft/sec

k constant used with switching curve

m mass of vehicle, slugs

r radius of earth, ft

S surface area, sq ft

t time, sec

V velocity of vehicle, ft/sec

V_1 lower integration limit (present velocity of vehicle), ft/sec

V_2 upper integration limit (velocity of vehicle at beginning of constant C_L phase), ft/sec

W weight of vehicle, lb

γ flight-path angle, deg or radians

γ_d desired flight-path angle for a given altitude, deg or radians

γ_R magnitude of γ that can be arrested by using $|C_{L,max}|$ between present altitude and desired altitude, radians

ϵ_η $\eta_L - \eta_L'$, radians

η angular travel of vehicle about earth, deg or radians

η_A angular travel during constant-altitude phase of desired mode of flight, radians

η_d desired angular travel of vehicle, deg or radians

η_L angular travel during constant C_L phase of desired mode of flight, radians

η_L' approximation of angular travel during constant C_L phase of desired mode of flight, radians

ρ_c constant used for exponential approximation of density of atmosphere, slugs/cu ft

ϕ_c command roll angle, deg

ϕ_{tr} trim roll angle, radians

Subscript:

o initial conditions at beginning of dive phase

A dot above a quantity denotes rate of change of variable with respect to time.

ANALYTICAL DEVELOPMENT

Control Modes and Trajectory Phases

In recent years many studies have been made of time-optimal (bang-bang) controls, where the number of state variables is greater than two. In bang-bang theory a switching surface is required to tell when positive or negative control force should be used. (An adequate discussion of bang-bang theory is given in ref. 2.) Rigorous applications of this type of control to a reentry vehicle appear to be impracticable at present, however, because of complexities involved in the implementation of the control logic.

In the present study the complexities are avoided by reducing the switching surface to a curve. This is achieved by selecting the initial portion of the reentry trajectory as a maneuver at constant velocity. Thus, the number of state variables is reduced to two: altitude and flight-path angle. The switching curve is then used as a part of a bang-bang control for the initial portion of the reentry path.

In the present analysis it was assumed that the desired mode of flight was an initial dive into the atmosphere followed by a constant-altitude trajectory until equilibrium flight conditions are achieved and then a constant-lift-coefficient trajectory to the desired point on the earth's surface. This mode of flight allows a closed-form solution of desired altitude as a function of desired range.

Thus, the desired trajectory is divided into the following three phases (see fig. 1(a)): the dive phase of the reentry trajectory, which begins at the initial point of reentry and ends when desired level flight conditions are achieved; the constant-altitude phase, which covers the portion of the reentry trajectory where constant altitude is maintained; and, the constant C_L phase, which covers the remainder of the reentry trajectory.

The dive phase of the desired reentry trajectory is controlled by selecting the proper direction of maximum lift such that constant-altitude flight conditions ($\gamma = 0$) are achieved at the desired altitude. In general, at some time during the dive phase of the reentry trajectory, the proper direction of maximum lift will change from positive to negative or vice versa. For example, suppose the reentry flight-path angle is such that by using maximum positive lift the minimum altitude (where γ goes through zero) of the dive is below the desired altitude. Then, at some instant of time between the time of minimum altitude and the time when the desired altitude is obtained, the direction of lift changes from positive to negative in order to reduce γ to zero when the desired altitude is reached.

The constant-altitude phase of the trajectory is controlled by using the appropriate amount of lift to maintain level flight. In general, the vehicle will not be at the desired altitude with zero flight-path angle, and thus, a linear controller based on altitude error and rate of change of altitude is used to control the vehicle. The linear controller is used until equilibrium flight conditions for an assumed lift coefficient are achieved, and from this point, no control is used.

In the present study, motion is considered only in the vertical plane (see fig. 1(b)), and the vehicle is assumed to be trimmed at a constant angle of attack. Thus, lift control is obtained by rolling so that the drag coefficient is constant. Although the present study is limited to longitudinal range control, it is believed that lateral range control could be achieved by a simple control logic based upon rolling the vehicle in the proper direction during the constant-altitude and the constant C_L phases of the reentry trajectory.

Equations of Motion

The basic equations of motion will be used to derive an analytical expression for determining the required altitude for the constant-altitude phase of the reentry trajectory and the switching curve for the bang-bang controller. Also, the skip and deceleration limits upon the desired altitude will be defined in equation form.

The equations of motion of a reentry vehicle in two degrees of freedom, as given in reference 3, are as follows:

$$\dot{h} = V \sin \gamma \tag{1}$$

$$\dot{\eta} = \frac{V}{r} \cos \gamma \tag{2}$$

$$V\dot{\gamma} = \frac{\frac{1}{2}\rho_c C_L S V^2 e^{-h/B}}{m} - g \cos \gamma\left(1 - \frac{V^2}{gr}\right) \tag{3}$$

$$\dot{V} = -\frac{\frac{1}{2}\rho_c C_D S V^2 e^{-h/B}}{m} - g \sin \gamma \tag{4}$$

Derivation of Range Equation

In order to determine the range covered by the desired mode of flight, it was assumed that the range covered during the dive phase was equal to the range that would be covered if constant altitude were maintained during this phase. Thus, the following analysis (identical with that of ref. (4)) of the equations of motion is made.

Range of constant-altitude phase.- For constant-altitude flight, $\dot{\gamma}$ and γ must be zero and the equations of motion become:

$$\dot{h} = 0 \tag{5}$$

$$\dot{\eta} = \frac{V}{r} \tag{6}$$

$$V\dot{\gamma} = 0 = \frac{\frac{1}{2}\rho_c C_L S V^2 e^{-h/B}}{m} - g\left(1 - \frac{V^2}{gr}\right) \tag{7}$$

$$\dot{V} = -\frac{\frac{1}{2}\rho_c C_D S V^2 e^{-h/B}}{m} \tag{8}$$

Dividing equation (6) by equation (8) gives

$$\frac{d\eta/dt}{dV/dt} = \frac{d\eta}{dV} = -\frac{Vm}{r\frac{1}{2}\rho_c C_D S V^2 e^{-h/B}} = -\frac{2m}{\rho_c r S C_D} \frac{1}{Ve^{-h/B}}$$

or

$$\int d\eta = -\frac{2m}{\rho_c r S C_D e^{-h/B}} \int_{V_1}^{V_2} \frac{dV}{V} \tag{9}$$

Integrating equation (9) gives:

$$\eta_A = -\frac{2m}{\rho_c rSC_De^{-h/B}} \log V \Bigg|_{V_1}^{V_2} = \frac{2m}{\rho_c rSC_De^{-h/B}} \log \frac{V_1}{V_2} \tag{10}$$

where V_1 is present velocity and V_2 is the velocity at the beginning of the constant C_L phase.

Since constant altitude is maintained until C_L is equal to $C_L{}^*$ (the arbitrary value selected for the constant C_L phase), equation (7) may be solved for V_2 in terms of $C_L{}^*$. Hence,

$$V_2{}^2 = \frac{gr}{1 + \dfrac{\rho_c rSe^{-h/B}C_L{}^*}{2m}} \tag{11}$$

and

$$\eta_A = \frac{m}{\rho_c rSC_De^{-h/B}} \log\left[\frac{V_1{}^2}{gr}\left(1 + \frac{\rho_c C_L{}^*Sr}{2m}e^{-h/B}\right)\right] \tag{12}$$

which gives the range for the constant-altitude phase of the desired mode of flight.

Range of constant C_L phase.- An equation for the approximate range of the constant C_L phase is obtained in the following manner.

The term $g \sin \gamma$ in equation (4) is zero at the beginning of the constant C_L phase and is small in comparison with the first term on the right side of equation (4) during most of this phase. Therefore, this term is neglected in developing an approximate range equation. Thus, equation (4) becomes:

$$\dot{V} = -\frac{\frac{1}{2}\rho_c C_D SV^2e^{-h/B}}{m} \tag{13}$$

Dividing equation (2) by equation (13) gives:

$$\frac{d\eta}{dV} = -\frac{mV \cos \gamma}{r\frac{1}{2}\rho_c C_D SV^2e^{-h/B}} \tag{14}$$

This equation can be simplified by recalling that $\dot{\gamma} = 0$ at the beginning of the constant C_L phase and by rearranging equation (3) in the form

$$\frac{\frac{1}{2}\rho_c C_L^* S V^2 e^{-h/B}}{m} \frac{1}{g \cos \gamma\left(1 - \frac{V^2}{gr}\right)} = 1$$

Multiplying equation (14) by this equation yields

$$\frac{d\eta}{dV} = \frac{C_L^*}{C_D} \frac{V}{V^2 - gr} \tag{15}$$

Integrating equation (15) yields

$$\eta_L' = \frac{1}{2} \frac{C_L^*}{C_D} \log(V^2 - gr) \Big|_{V_2}^{0} = \frac{1}{2} \frac{C_L^*}{C_D} \log \frac{gr}{gr - V_2^2} \tag{16}$$

Substituting equation (11) into equation (16) and simplifying gives:

$$\eta_L' = \frac{1}{2} \frac{C_L^*}{C_D} \log\left(\frac{2m}{\rho_c r S C_L^*} e^{h/B} + 1\right) \tag{17}$$

Equation (17) gives the approximate range obtained for the constant C_L phase. Since this is an approximate equation an empirical correction term denoted by ϵ_η was obtained to supplement equation (17). The total range for the constant C_L phase is:

$$\eta_L = \eta_L' + \epsilon_\eta \tag{18}$$

In order to evaluate equation (17) several constant C_L trajectories were obtained by integrating equations (1) to (4) on a digital computer. Analysis of these data indicated that when $C_L^* = 0.2$, ϵ_η could be represented by $e^{0.491\frac{h}{B} - 7.525}$. By using this expression for ϵ_η, equation (18) becomes:

$$\eta_L = \eta_L' + e^{0.491\frac{h}{B} - 7.525} = \frac{1}{2} \frac{C_L^*}{C_D} \log\left(\frac{2m}{\rho_c r S C_L^*} e^{h/B} + 1\right) + e^{0.491\frac{h}{B} - 7.525} \tag{19}$$

Total range.- The range equation for the desired mode of flight is:

$$\eta_d = \eta_A + \eta_L$$

$$= \frac{m}{\rho_c S r C_D} \frac{1}{e^{-h_A/B}} \log\left[\frac{V_1^2}{gr}\left(1 + \frac{\rho_c C_L^* S r}{2m} e^{-h_A/B}\right)\right]$$

$$+ \frac{1}{2} \frac{C_L^*}{C_D} \log\left(\frac{2m}{\rho_c r S C_L^*} e^{h_A/B} + 1\right) + e^{0.491\frac{h_A}{B} - 7.525} \qquad (20)$$

The constant altitude of the desired mode of flight for a given range was determined from equation (20) by using an iterative procedure.

Altitude limits.- The altitude h_d for the constant-altitude phase of the desired mode of flight has a lower limit dependent upon the maximum allowable g's and an upper limit dependent upon negative $C_{L,max}$. These limits are determined in the following manner.

For level flight conditions ($\dot{\gamma} = \gamma = 0$), the deceleration, in g units, is

$$a_D = -\frac{\dot{V}}{g} = \frac{\frac{1}{2}\rho_c S C_D V^2}{mg} e^{-h/B} \qquad (21)$$

An expression for the lower altitude limit h_g is obtained by solving equation (21) for h when $a_D = a_{D,max}$. Thus,

$$h_g = B \log \frac{\frac{1}{2}\rho_c S C_D V^2}{m g a_{D,max}} \qquad (22)$$

The upper altitude limit h_s is the maximum altitude where aerodynamic forces are available to prevent the vehicle from skipping out of the atmosphere. An expression for h_s is obtained by solving equation (3) for h when $C_L = -C_{L,max}$ and $\dot{\gamma} = \gamma = 0$. Thus,

$$h_s = -B \log \frac{mg}{\frac{1}{2}\rho_c S C_L V^2}\left(1 - \frac{V^2}{gr}\right) \qquad (23)$$

In order to prevent either over deceleration or skipout, the vehicle must be controlled between these two limits. Thus, the following inequality relates the desired altitude h_d with the two limits:

$$h_g \leqq h_d \leqq h_s$$

The altitude h_A obtained from equation (20) is used as h_d if

$$h_g < h_A < h_s$$

Otherwise, the appropriate limit is selected as h_d.

Switching Curve

After the altitude for the constant-altitude phase has been determined, it is necessary to determine the point at which the vehicle should roll from positive to negative lift or vice versa. A bang-bang controller based on phase plane switching will force the vehicle to the desired conditions at the end of the dive phase of the trajectory. The switching point must be one so that γ and h will reach the desired values simultaneously. It is desirable to determine a function that can be generated and used in a practical control system. One such function would be the change in γ that can be effected by using $C_{L,max}$ between the present altitude and the desired altitude. Hence, a switching curve that lies in the flight-path-angle—altitude plane was developed as follows: With use of the small-angle approximations,

$$\sin \gamma = \gamma$$

$$\cos \gamma = 1$$

equations (1) and (3) become

$$\dot{h} = V\gamma \tag{24}$$

$$V\dot{\gamma} = \frac{\frac{1}{2}\rho_c C_L S}{m} V^2 e^{-h/B} - g\left(1 - \frac{V^2}{gr}\right) \tag{25}$$

Dividing equation (25) by equation (24) gives

$$\frac{d\gamma}{dh} = \frac{\frac{\frac{1}{2}\rho_c C_L S}{m} e^{-h/B} - \frac{g}{V^2} + \frac{1}{r}}{\gamma} \tag{26}$$

or

$$\gamma\ d\gamma = \left(\frac{\frac{1}{2}\rho_c S}{m} C_L e^{-h/B} - \frac{g}{V^2} + \frac{1}{r}\right)dh \tag{27}$$

If it is assumed that velocity is constant during the maneuver, then equation (27) can be integrated as

$$\frac{1}{2}\gamma^2 \Bigg|_{\gamma_R}^{\gamma_d} = \left[-\frac{\frac{1}{2}\rho_c S}{m} B C_L e^{-h/B} - \frac{g}{V^2}h + \frac{1}{r}h\right] \Bigg|_{h}^{h_d}$$

$$\frac{1}{2}\gamma_d^2 - \frac{1}{2}\gamma_R^2 = -\frac{\frac{1}{2}\rho_c S}{m} B C_L\left(e^{-h_d/B} - e^{-h/B}\right) + \left(\frac{1}{r} - \frac{g}{V^2}\right)\left(h_d - h\right) \tag{28}$$

Since $\Delta h = h_d - h$, then

$$-\frac{1}{2}\gamma_R^2 = -\frac{1}{2}\gamma_d^2 - \frac{\frac{1}{2}\rho_c S}{m} B C_L e^{-h_d/B}\left[1 - e^{\left(h_d/B - h/B\right)}\right] + \left(\frac{1}{r} - \frac{g}{V^2}\right)\Delta h$$

or

$$\gamma_R = \pm\left[\gamma_d^2 + \frac{\rho_c S}{m} B C_L e^{-h_d/B}\left(1 - e^{\Delta h/B}\right) - \frac{2\ \Delta h}{r}\left(1 - \frac{gr}{V^2}\right)\right]^{1/2} \tag{29}$$

where $\gamma_d = 0$, h_d is the desired constant altitude and h is the present altitude. In this equation, negative $C_{L,max}$ is used when Δh is positive, and positive $C_{L,max}$ is used when Δh is negative. The sign of Δh is the sign which γ_R takes in equation (29).

Control Logic

The control logic used with the bang-bang controller is dependent upon the vehicle altitude with respect to the desired altitude. The control logic for the switching function γ_R is stated: If $\gamma_R - \gamma > 0$, then $\phi_c = 0^{\circ}$, but if $\gamma_R - \gamma \leqq 0$, then $\phi_c = 180^{\circ}$.

This control logic applies for a constant velocity. The vehicle velocity, however, is not absolutely constant but decreases during the dive phase. The velocity decrease between the time the vehicle rolls over and the time level flight conditions are attained may cause the desired altitude to change several thousand feet. In order to prevent the vehicle from rolling excessively for a changing h_d, the control logic was modified in this manner: If $k\gamma_R - \gamma > 0$, then $\phi_c = 0^o$, but if $k\gamma_R - \gamma \leqq 0$, then $\phi_c = 180^o$ where k alternates from 1 to a value $0 \leqq k < 1$ each time the vehicle rolls. Unfortunately, in a practical control system, it is impossible for the system to reverse the control force at exactly the correct point and so the vehicle will not achieve the exact desired conditions at the end of the dive phase of the trajectory. In order to reduce this undesired effect, the system automatically shifts from the bang-bang control mode to the linear control mode associated with the constant-altitude phase when $\gamma = 0$ and the error in altitude is equal to or less than an assumed value of C.

The linear controller, which is used until $V \leqq V_2$, is represented by

$$\phi_c = \phi_{tr} - 57.3\left(K_1\Delta h + K_2\dot{h}\right) \tag{30}$$

where

$$\phi_{tr} = \cos^{-1} \frac{2m(gr - V^2)}{\rho_c C_L r S V^2 e^{-h/B}}$$

No control is used during the constant C_L phase of the trajectory.

DIGITAL COMPUTER STUDY

The dynamics of the reentry vehicle and control system as described by the preceding equations were simulated on a digital computer. A block diagram of the control system is presented in figure 2. This diagram indicates the equations which are used for certain functions of the control system. In the simulation, an 8g deceleration limit was assumed. The values of k used with the switching curve were 1 and 0.7. The value of C used in the control logic to switch from the bang-bang controller to the linear controller was 5,000 feet. The gains used in the linear controller during the constant-altitude phase of the trajectory were $K_1 = 0.0001$ radian/ft and $K_2 = -0.001$ radian/ft/sec.

A 6,000-pound high-drag—low-lift reentry vehicle having a lifting area of 100 square feet was assumed for all complete reentries simulated in the present study. It was assumed that the vehicle was trimmed at $C_L/C_D = 0.5$ which corresponds to $C_L = 0.5$ and $C_D = 1.0$. Lift control was obtained by rolling the vehicle between 0^o for $C_L/C_D = 0.5$ and 180^o for $C_L/C_D = -0.5$. Instantaneous roll maneuvers were assumed.

The study also included the simulation of a high-lift vehicle during the dive phase of the reentry trajectory. It was assumed that this vehicle was trimmed at $C_L/C_D = 1.5$ and had a W/S of 27.88 lb/sq ft. Lift control was obtained by rolling the vehicle between 0° for $C_L/C_D = 1.5$ and 90° for $C_L/C_D = 0$.

The initial altitude for all reentries was 300,000 feet. The reentries were terminated at $h = 100,000$ feet, below which the high-drag—low-lift vehicle assumed herein has little or no maneuver capability.

RESULTS AND DISCUSSION

Range Equations

Solutions of the range equation (eq. (20)) where $C_L^* = 0.2$, $C_D = 1.0$, and $W/S = 60$ lb/sq ft were obtained for $V = 45,000$ ft/sec, $V = 36,000$ ft/sec, and $V = 26,000$ ft/sec. The solutions that fall within the skip and deceleration altitude limits are presented in figure 3. These data indicate that for $V = 36,000$ ft/sec the desired altitude must fall between about 198,000 feet and 230,000 feet. It should be noted, however, that this figures does not indicate the maximum obtainable range for a given velocity. For example, if it is desired that a vehicle with a velocity of 36,000 ft/sec attain a range of 60°, this could be achieved by flying along the skip boundary until the range-to-go and present velocity indicate a desired altitude below the skip boundary.

Switching Curve

Figure 4 presents the switching curve for the dive phase of three typical trajectories. This figure shows the switching curve where the solid line represents the solution of equation (29) and the dashed line represents the switching curve when $k = 0.7$ is used in the control logic. The linear region around the equilibrium point, that is, the allowable error in Δh when γ goes to zero, is indicated in the figure.

High-Drag—Low-Lift Vehicle

Results of the simulation of complete reentries for the high-drag—low-lift vehicle are shown in figures 5 to 8 as time histories. The reentries were terminated at $h = 100,000$ feet and were within ±8 nautical miles of the desired range.

Reentry angle.- The reentries shown in figure 5 are for $\eta_d = 30^{\circ}$, $V_o = 36,000$ ft/sec, and reentry angles of -3°, -4°, -5° and -6°. This range of reentry angles represents initial flight-path angles that are steep enough for atmospheric capture but shallow enough not to exceed the acceleration limit. For the first three reentries $\left(\gamma_o = -3^{\circ}, -4^{\circ}, \text{ and } -5^{\circ}\right)$, one roll maneuver was required to obtain level flight conditions at the desired altitude. Several rolls were required for the -6° reentry trajectory. The reason for this is the fact that h_d increased because of the decrease in velocity resulting from the lower altitude obtained during the initial dive into the atmosphere. The bang-bang controller gave excellent control for the range of entry angles to the correct altitude for $\gamma = 0$.

Reentry velocity.- In order to determine the effect of reentry velocity on the control technique, V_o was varied from 28,000 ft/sec, 40,000 ft/sec, and 45,000 ft/sec for $\eta_d = 20^\circ$ as shown in figure 6. The vehicle was controlled equally well for any combination of initial velocity and acceptable initial flight-path angle.

Range.- Reentries for $\eta_d = 20^\circ$ and $\eta_d = 60^\circ$ were made for both $\gamma_o = -3^\circ$ and $\gamma_o = -5^\circ$ for $V_o = 36{,}000$ ft/sec and are shown in figures 7 and 8, respectively. Figure 7(a) shows that for the reentry where $\gamma_o = -3^\circ$, only one roll was required to obtain level flight conditions at the desired altitude. The time histories for $\gamma_o = -5^\circ$ (fig. 7(b)) show that three rolls were required to level off at the required altitude. Initially, negative lift was needed to push the vehicle down into the atmosphere. At t = 10 seconds, the switching curve indicated that positive lift was needed to level off at h_d. However, before the vehicle obtained this altitude, h_d had decreased such that the $0.7\gamma_R$ switching curve indicated that the vehicle would level off too high, so negative lift was needed to push the vehicle down to the new value of h_d. Entries for a desired range of 60° and $\gamma_o = -5^\circ$ (fig. 8(b)) show that several rolls were required before the level flight conditions were achieved. For entries for ranges of 40° or longer, h_d follows the skip boundary h_s for a portion of the trajectory. The number of control maneuvers made by the bang-bang controller is dependent upon the length of time h_d follows h_s.

Although not presented, both long- and short-range reentries were investigated during the present study. Several reentries where the desired range was 16° were made. The bang-bang controller was required during the complete reentry in each of these short-range reentries. This was due to the fact that h_d was decreasing at such a rate that the vehicle was unable to obtain $\gamma = 0$ within 5,000 feet of h_d. Reentries where the desired range was 90° were also obtained. For these long-range reentries the bang-bang controller was also required during the complete reentry since when h_d decreased below the skip boundary the vehicle overshot the desired altitude. The desired altitude was decreasing at such a rate that by the time the vehicle corrected the overshoot at this high altitude, h_d had decreased beyond the point where the vehicle could obtain $\gamma = 0$ within 5,000 feet of h_d.

Time histories for $\eta_d = 16^\circ$ and $\eta_d = 90^\circ$ are not shown because the large number of roll maneuvers required made the reentries impractical. However, it is of interest to note that both of these reentries were effected within 8 nautical miles of the desired range.

For reentries where the desired altitude is limited for a portion of the trajectory to prevent skipout or to restrict g, the bang-bang controller is attempting to bring γ to zero while the vehicle is flying an increasing or decreasing (depending upon the boundary followed) altitude. Thus, the vehicle makes numerous roll maneuvers while attempting to fly these long ranges as illustrated in figure 8(b). Although not investigated, one method that could eliminate the excessive control maneuvers and the overshoot which occurs when the vehicle leaves the boundary is to modify the logic based on altitude and flight-path angle for these restricted cases. The reentries shown in figure 8 indicate that the vehicle maintains a flight-path angle of approximately 0.5° while following the skip boundary. Thus, changing the switching line of the bang-bang controller so that $\Delta h = 0$ at $\gamma = 0.5^{\circ}$ (that is, use $\gamma_d = 0.5^{\circ}$ in eq. (29)) when the vehicle is following the skip boundary may prove to be the modification needed for the long-range reentries.

High-lift Vehicle

In order to determine the feasibility of using the same analytical expression to compute the switching curve for a bang-bang controller on a high-lift reentry vehicle, the following analysis was made. The vehicle was assumed to be trimmed at $C_L/C_D = 1.5$. The bang-bang controller was assumed to roll the vehicle such that the lift vector in the vertical plane would be either maximum $\left(\phi_c = 0^{\circ}\right)$ or zero $\left(\phi_c = 90^{\circ}\right)$. The initial velocity and altitude were 25,000 ft/sec and 300,000 feet, respectively. The task of the bang-bang controller was to achieve level flight conditions at 220,000 feet. Time histories for $\gamma_o = -2.5^{\circ}$ and $\gamma_o = -5^{\circ}$ are shown in figure 9. The controller achieved the task within a few hundred feet.

CONCLUDING REMARKS

A study of a bang-bang controller for range control of a direct descent entry from supercircular velocities has been made. The equation used as the switching line for the bang-bang controller has been developed. The results of this study show:

1. The equation used as the switching line for the bang-bang controller gave an excellent prediction when a control maneuver should be made to achieve rapidly level flight conditions at the desired altitude. The equation is effective for both high-drag—low-lift reentry vehicles and high-lift reentry vehicles and for a wide range of reentry conditions.

2. A single control logic prevented skipout and restricted g while controlling the vehicle to a desired range which could vary between 20° and 60°.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 11, 1963.

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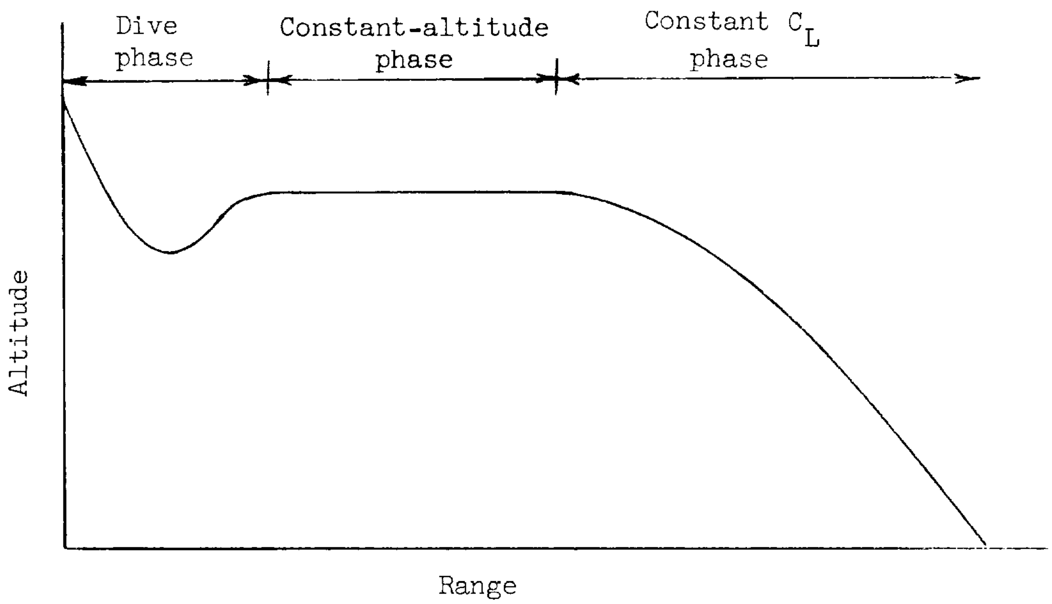
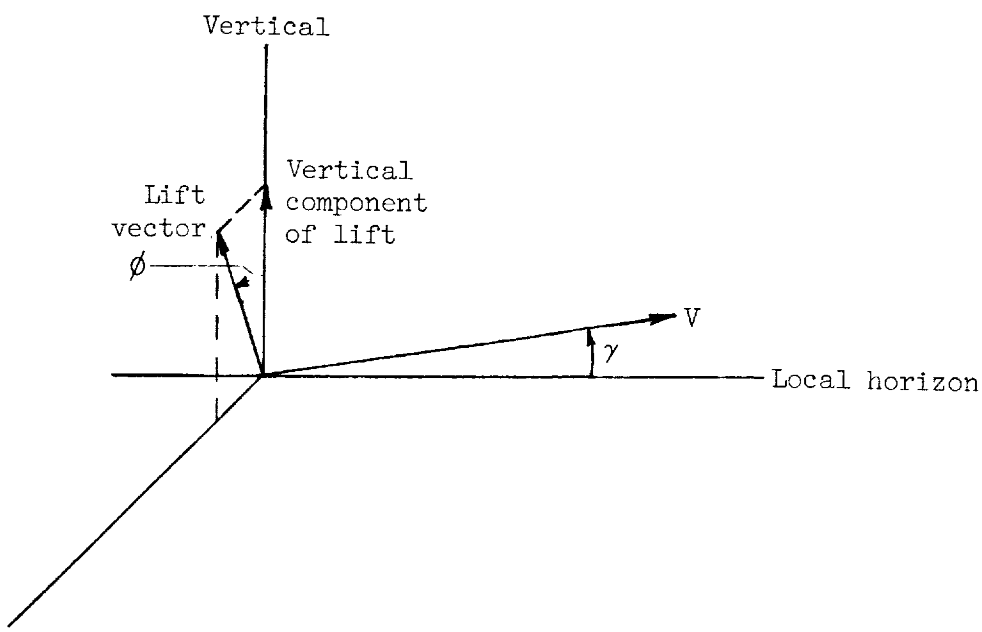


(a) Three phases of desired trajectory.



(b) Relation of vertical component of lift with roll angle.

Figure 1.- Schematics showing a typical reentry trajectory and the vertical component of lift.

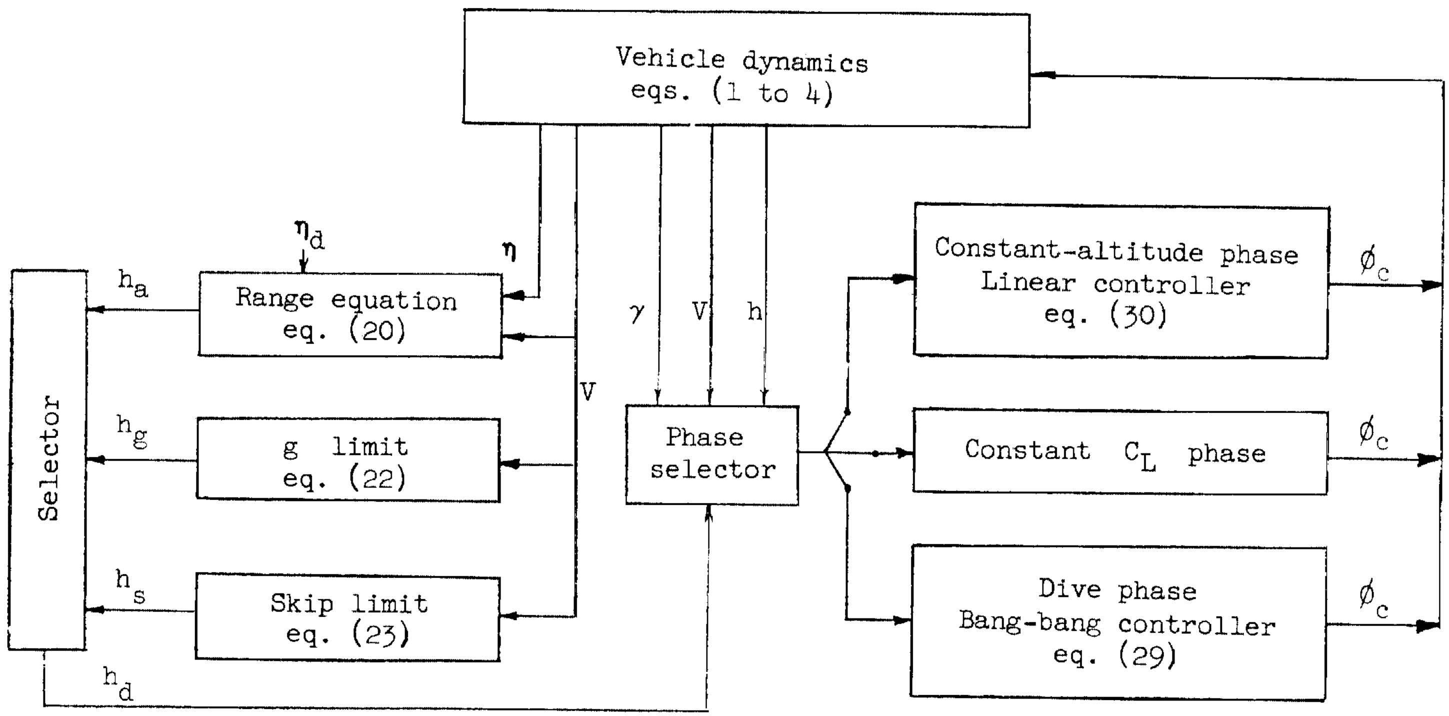


Figure 2.- Block diagram of control system.

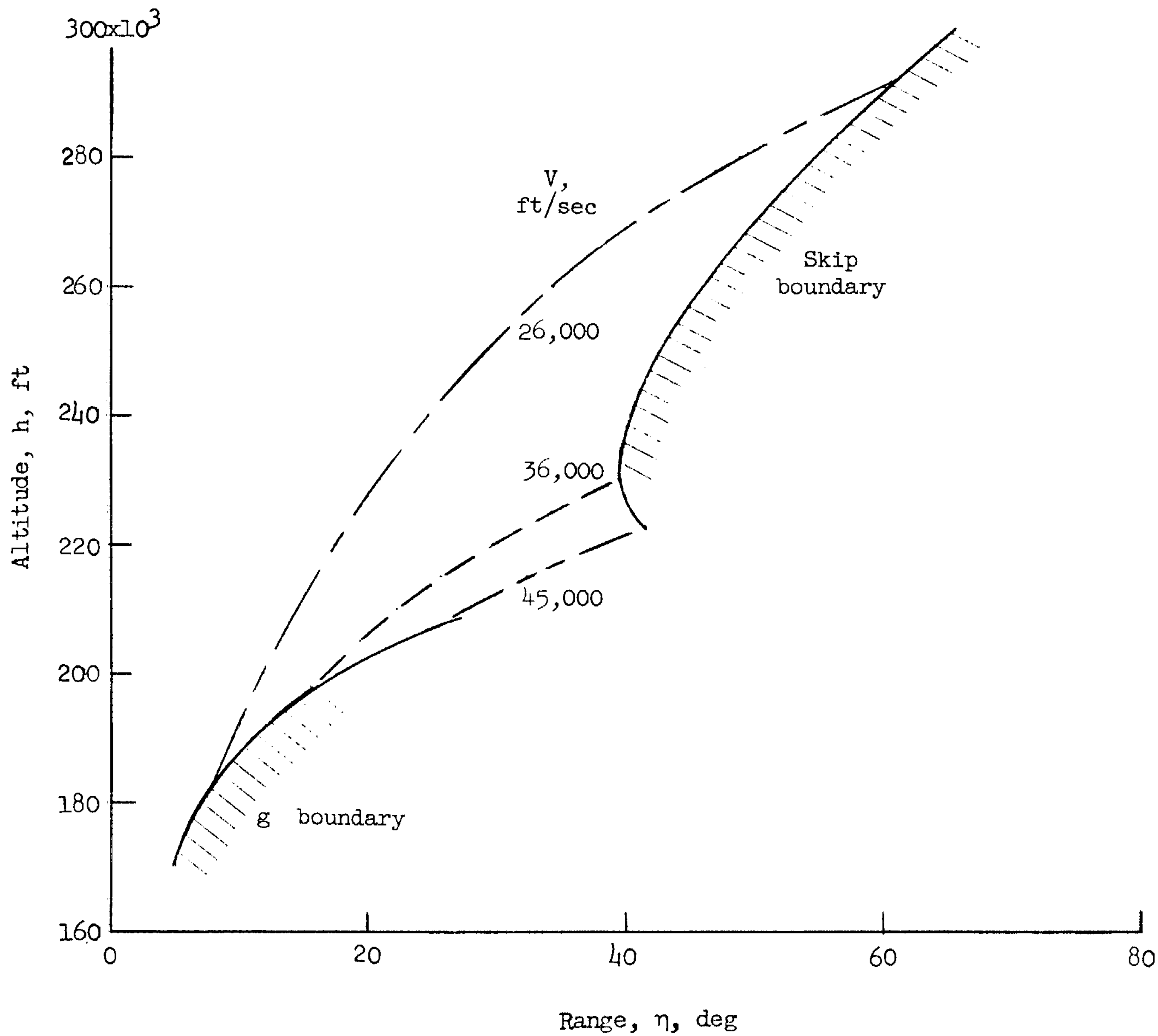
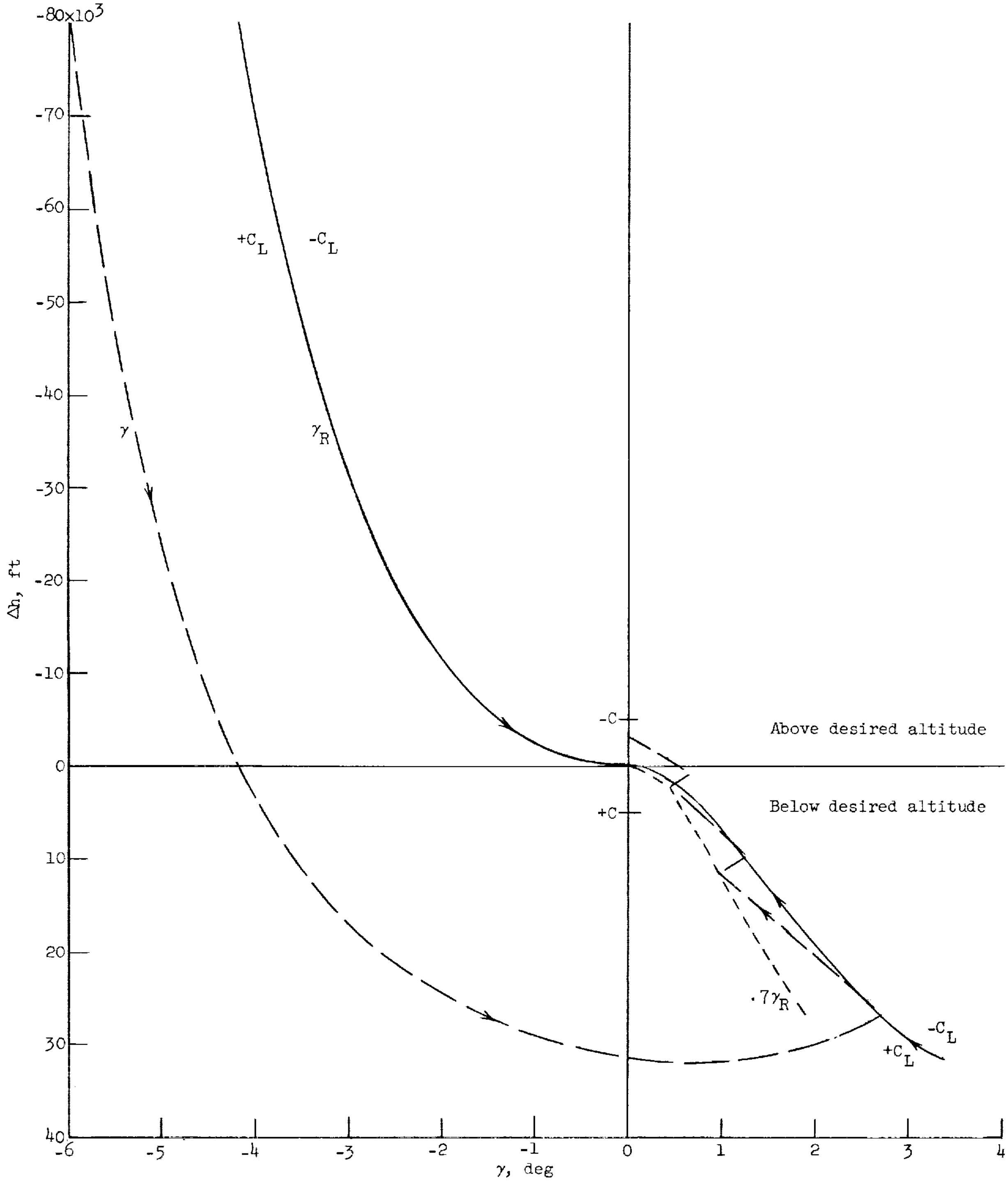
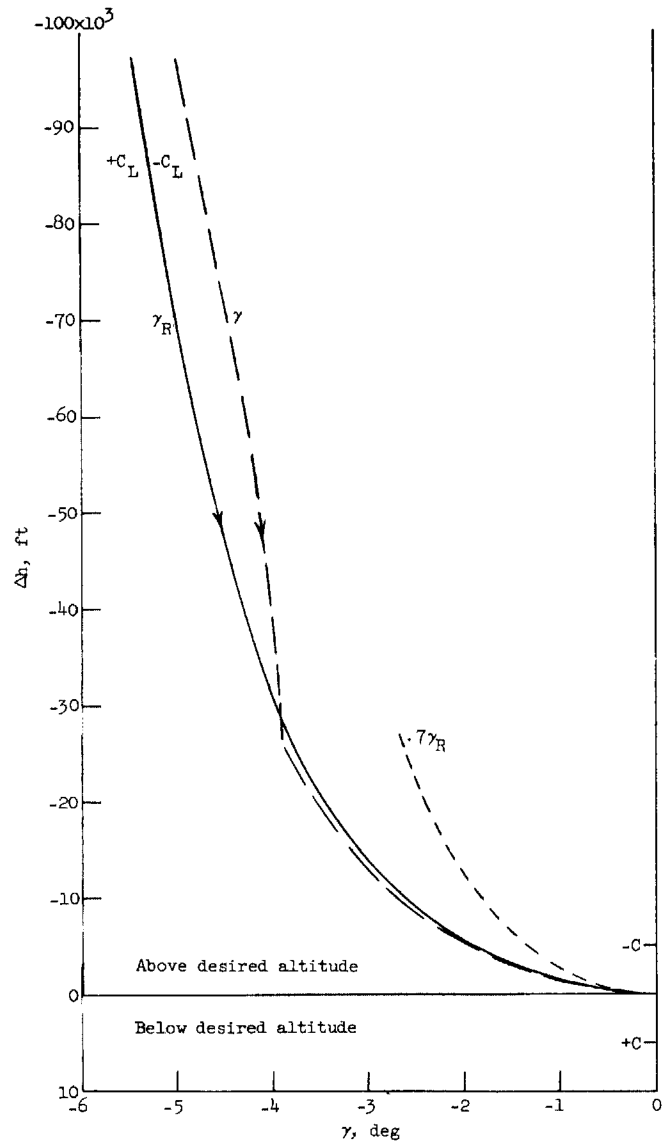


Figure 3.- Variation of angular range with altitude for specified velocities showing skip and deceleration limits on altitude.

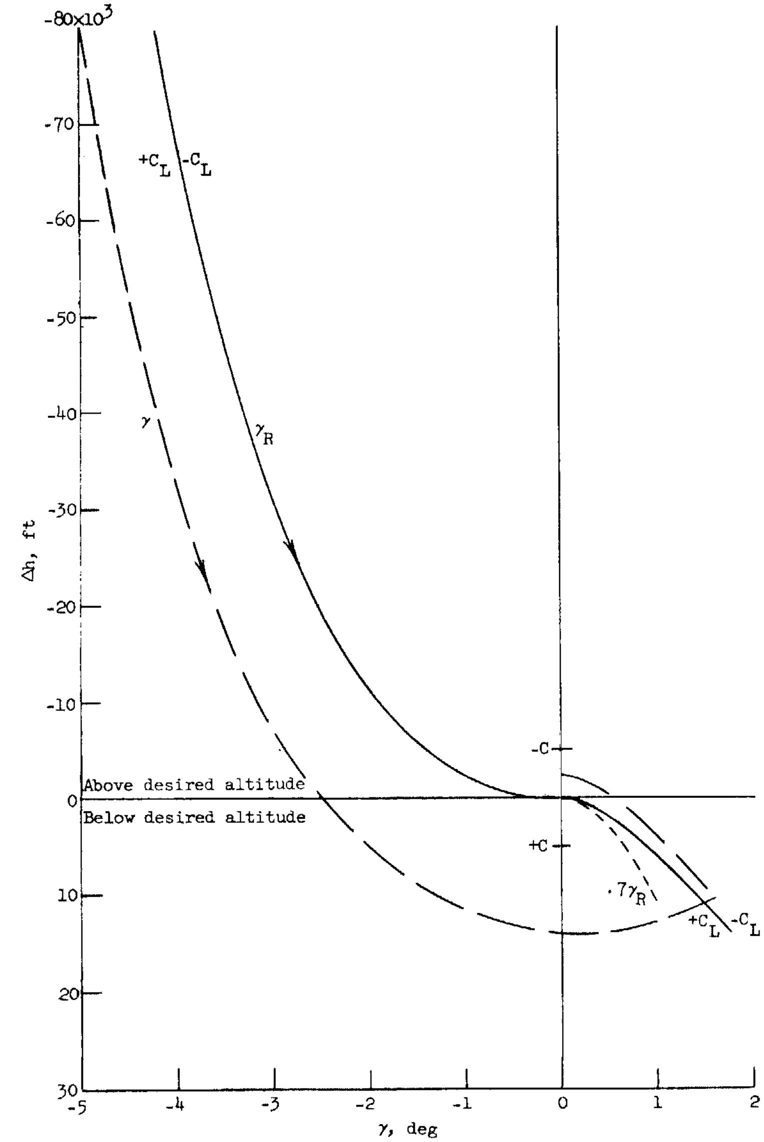


(a) $\gamma_o = -6^\circ$; $V_o = 36,000$ ft/sec; $\eta_d = 30^\circ$.

Figure 4.- Bang-bang switching curve for three typical trajectories.

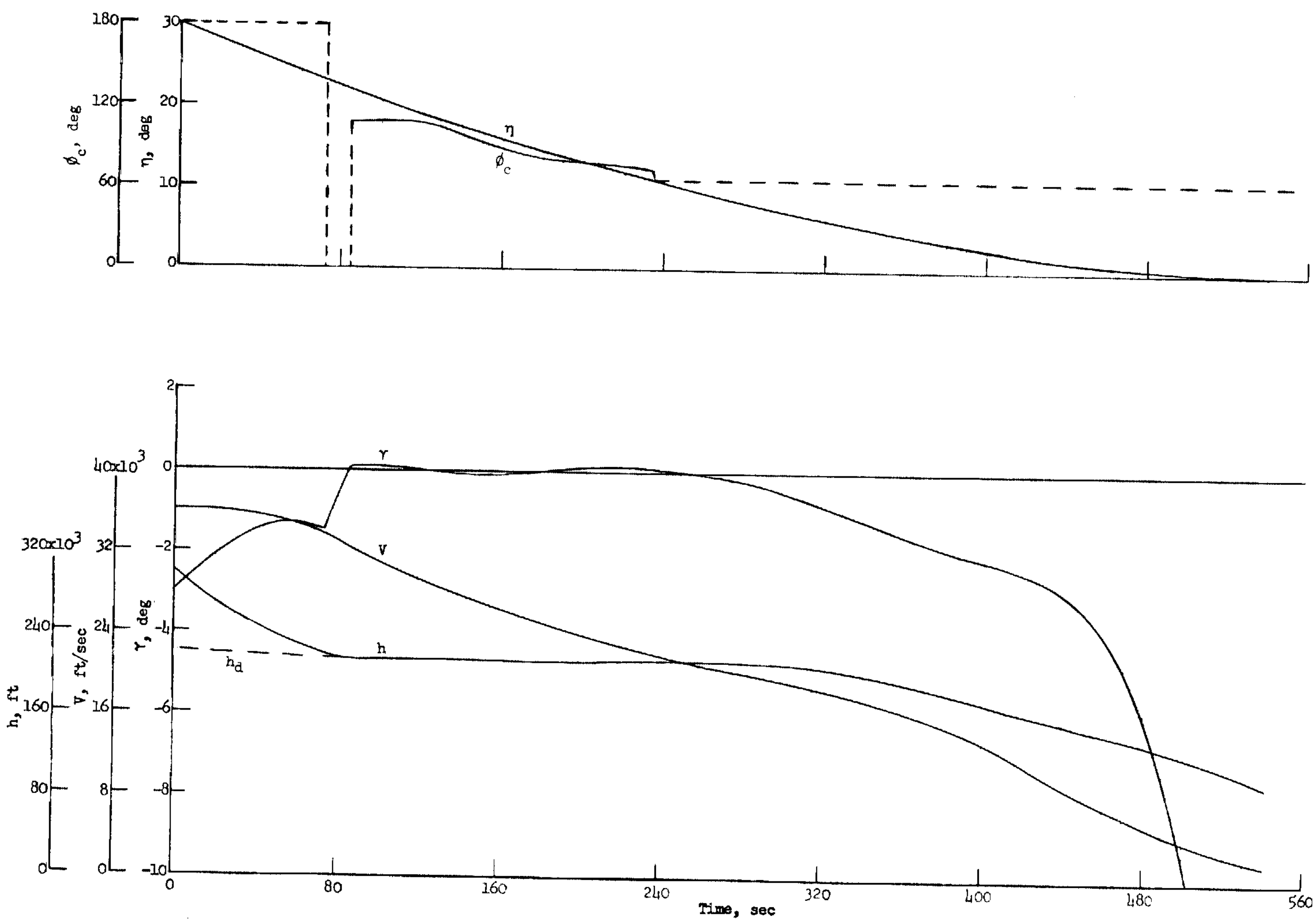


(b) $\gamma_o = -5^\circ$; $V_o = 40{,}000$ ft/sec; $\eta_d = 20^\circ$.



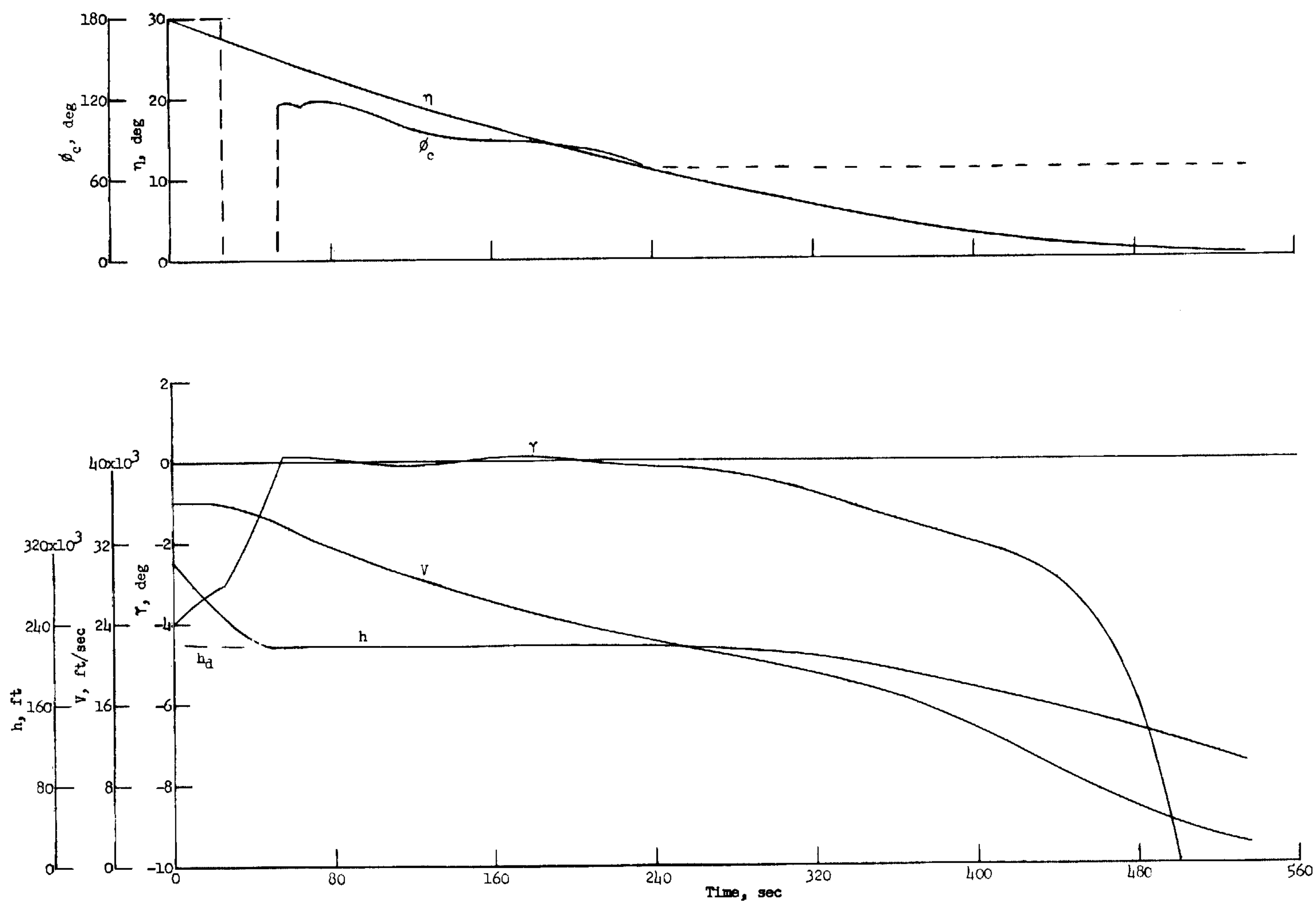
(c) $\gamma_o = -5^\circ$; $V_o = 36{,}000$ ft/sec; $\eta_d = 30^\circ$.

Figure 4.- Concluded.



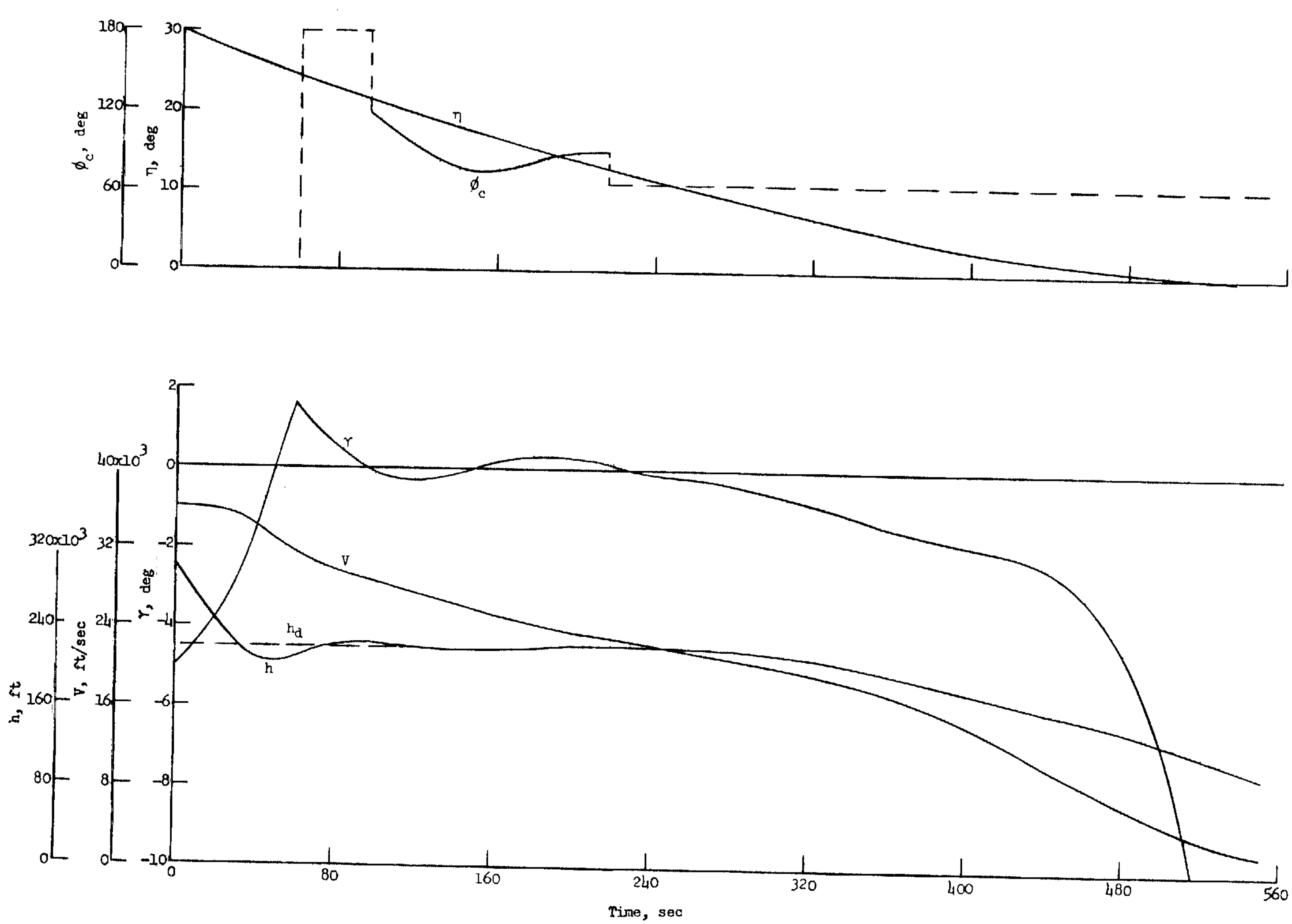
(a) $\gamma_o = -3^\circ$.

Figure 5.- Time histories of reentry trajectories for reentry angles of -3°, -4°, -5°, and -6° where $\eta_d = 30^\circ$, $V_o = 36{,}000$ ft/sec, $h_o = 300{,}000$ ft, and $C_L/C_D = 0.5$.



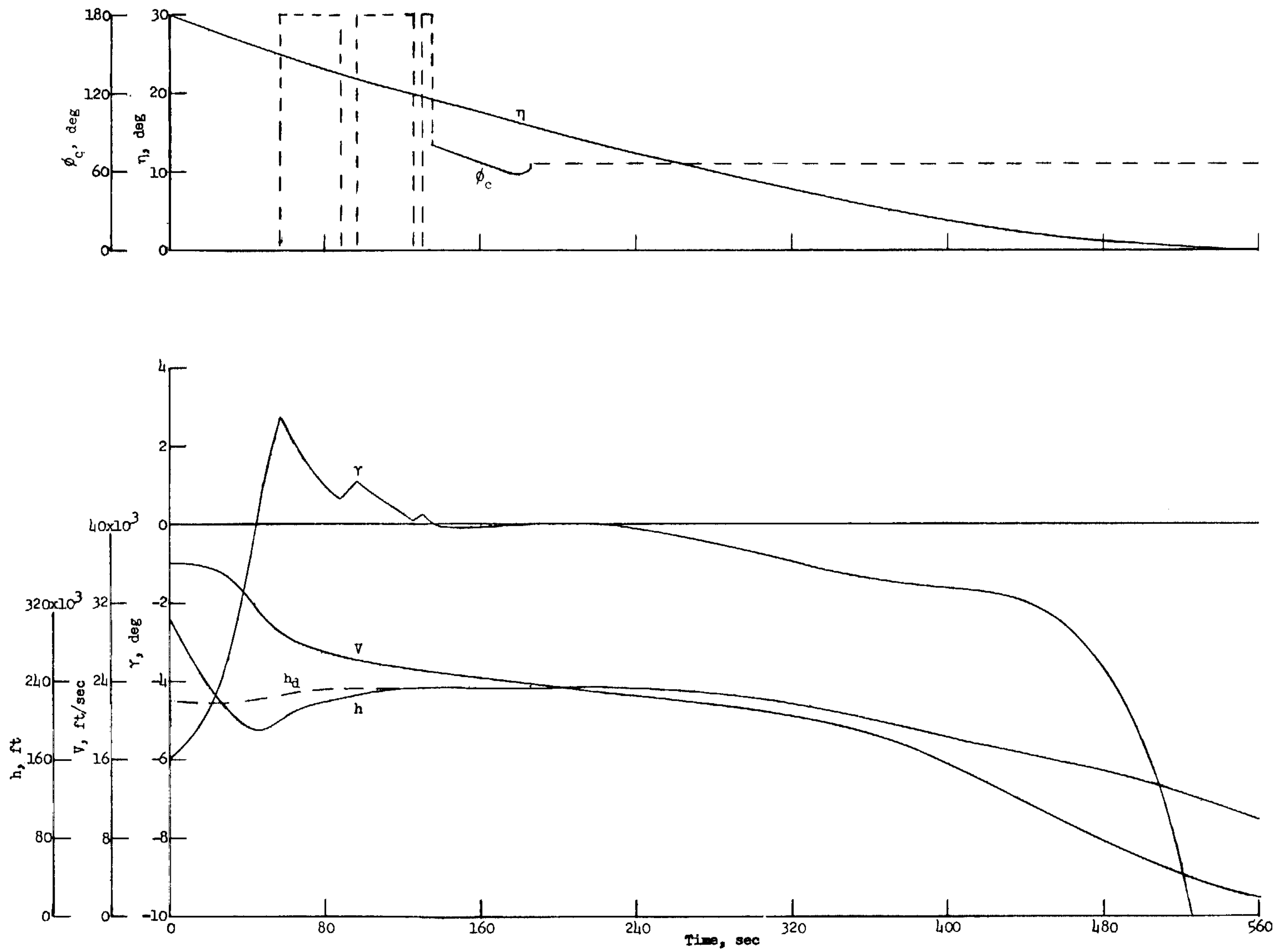
(b) $\gamma_o = -4^o$.

Figure 5.- Continued.



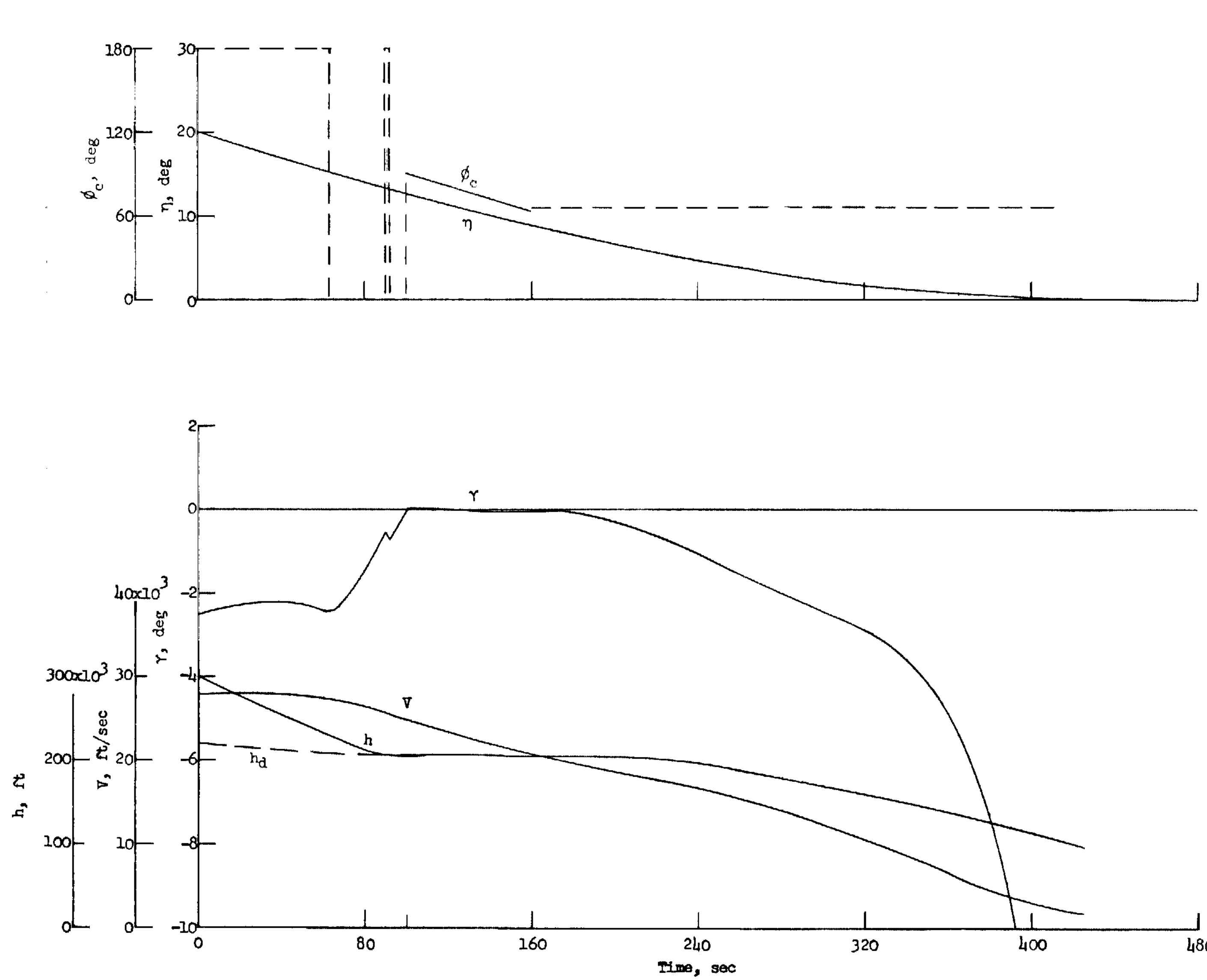
(c) $\gamma_o = -5^{\circ}$.

Figure 5.- Continued.



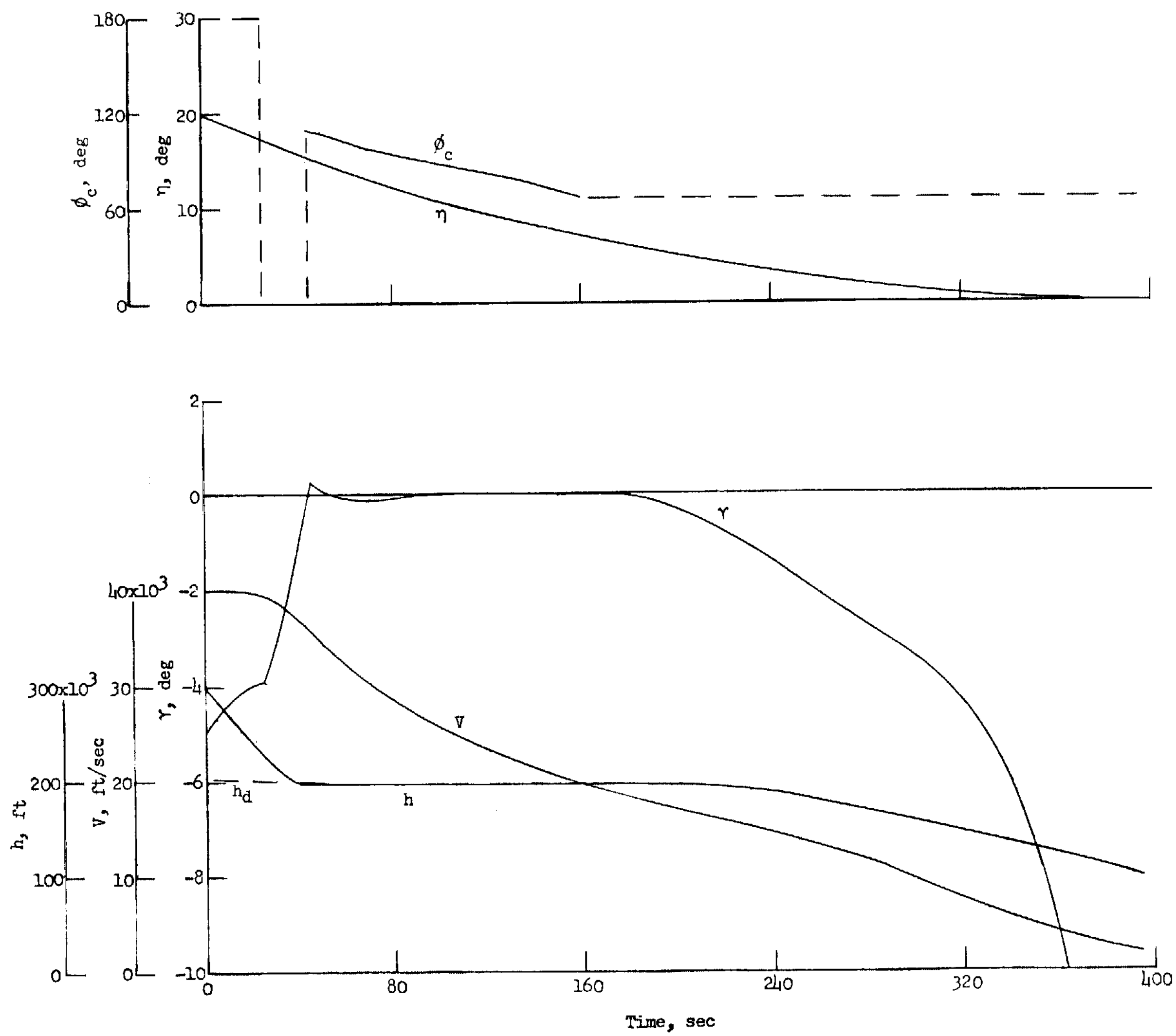
(d) $\gamma_o = -6^o$.

Figure 5.- Concluded.



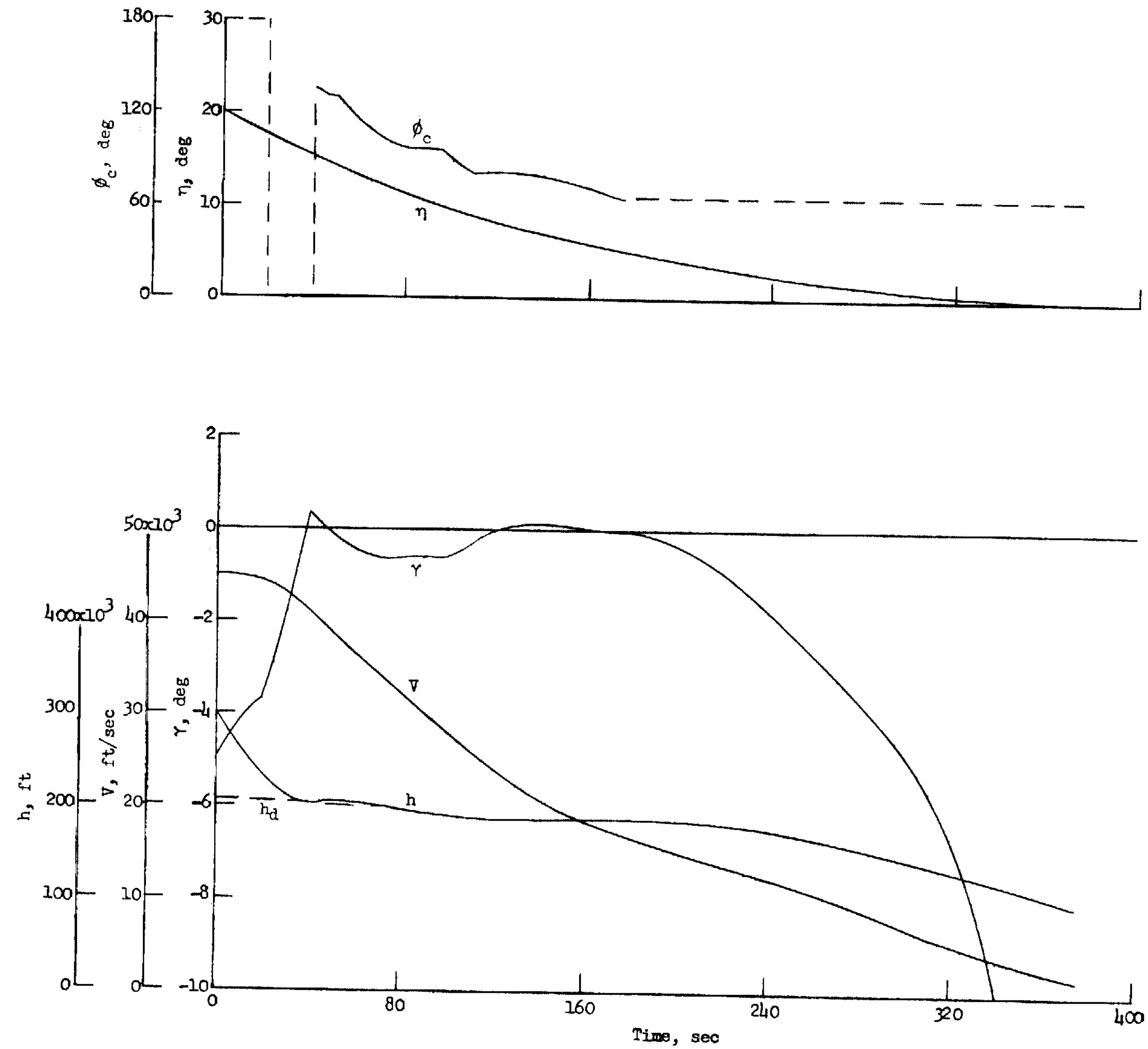
(a) V_o = 28,000 ft/sec; γ_o = -2.5.

Figure 6.- Time histories of reentry trajectories for reentry velocities of 28,000 ft/sec, 40,000 ft/sec, and 45,000 ft/sec where $\eta_d = 20^\circ$ and $C_L/C_D = 0.5$.



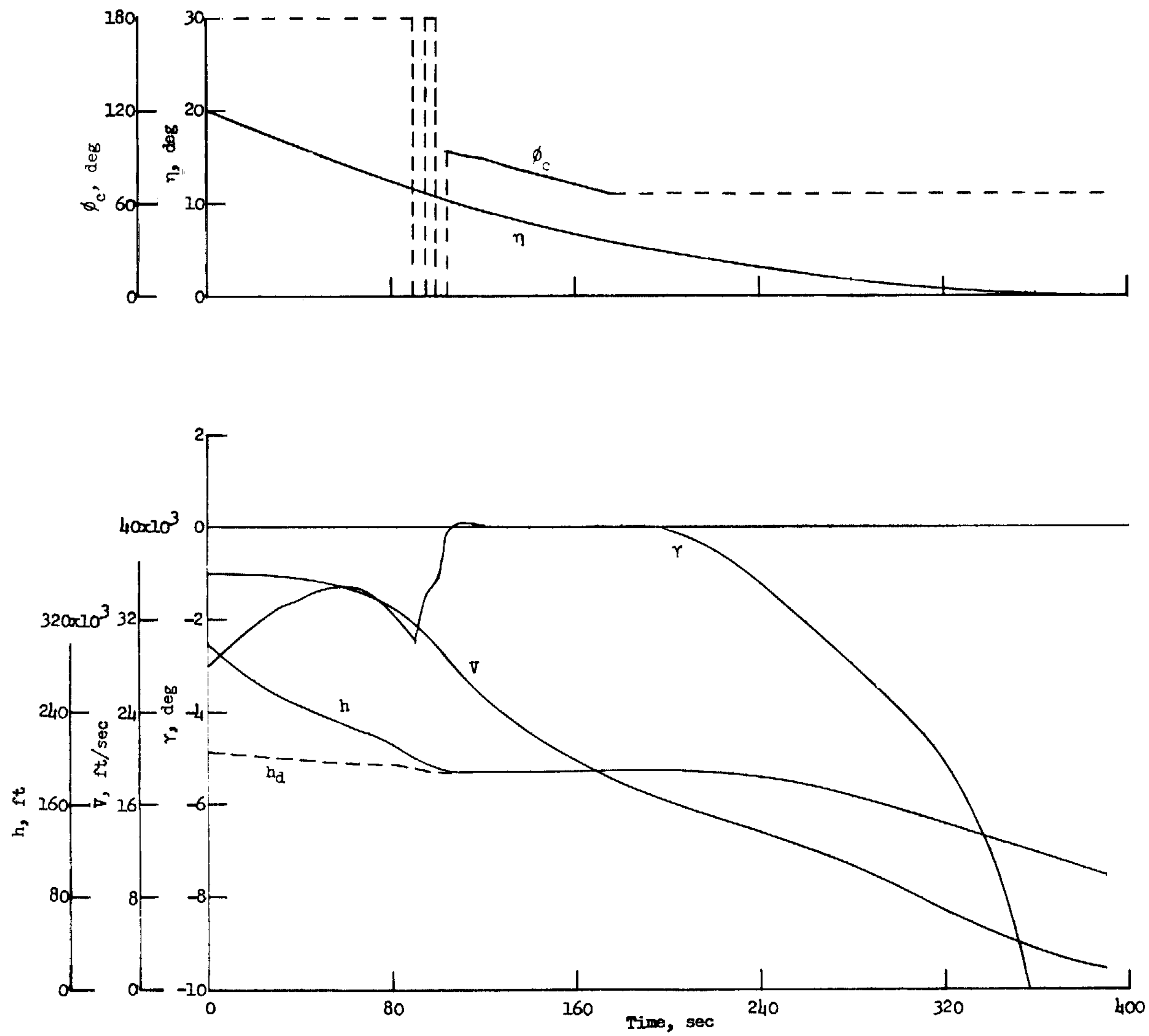
(b) $V_o = 40,000$ ft/sec; $\gamma_o = -5^\circ$.

Figure 6.- Continued.



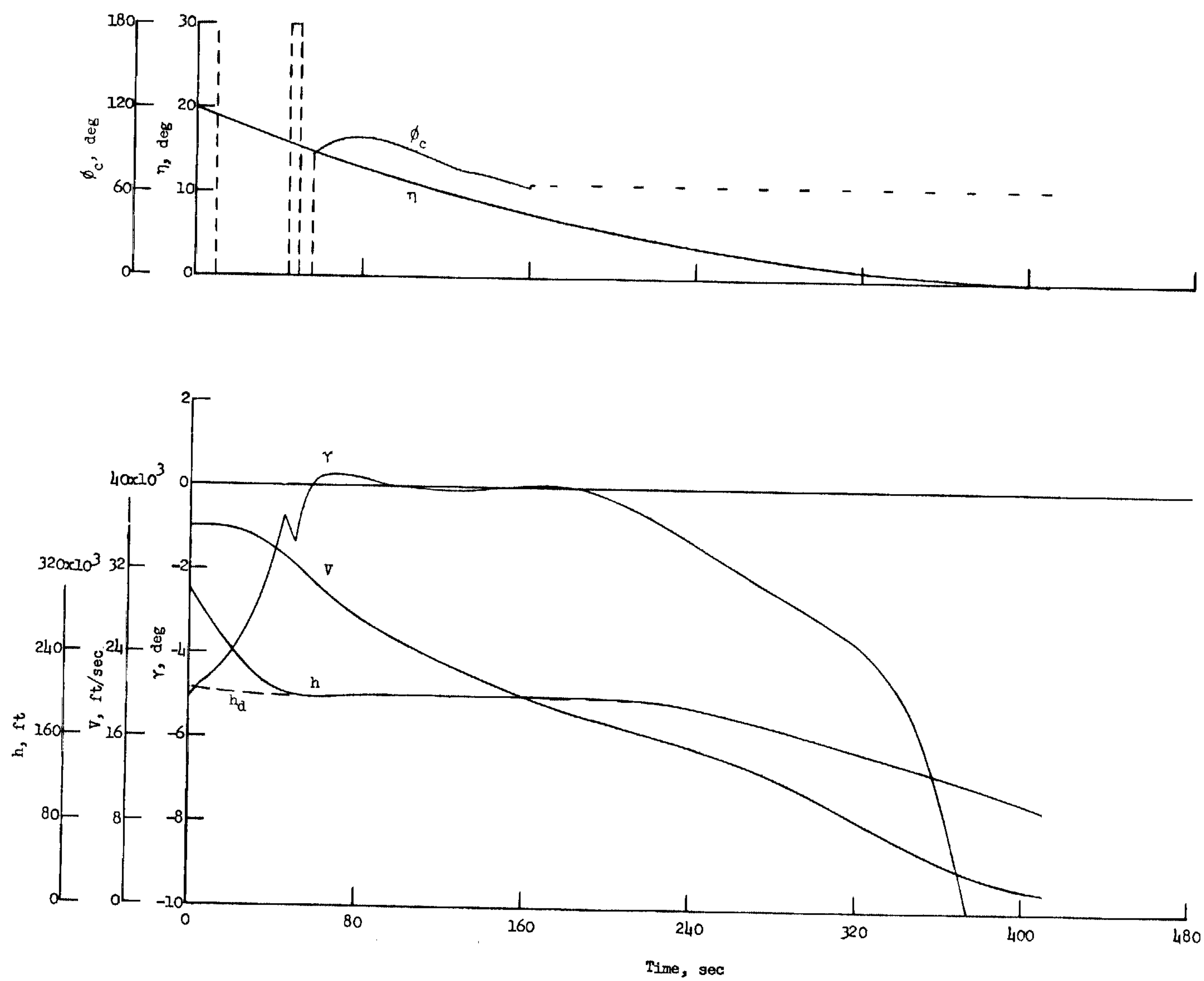
(c) $V_o = 45{,}000$ ft/sec; $\gamma_o = -5^{\circ}$.

Figure 6.- Concluded.



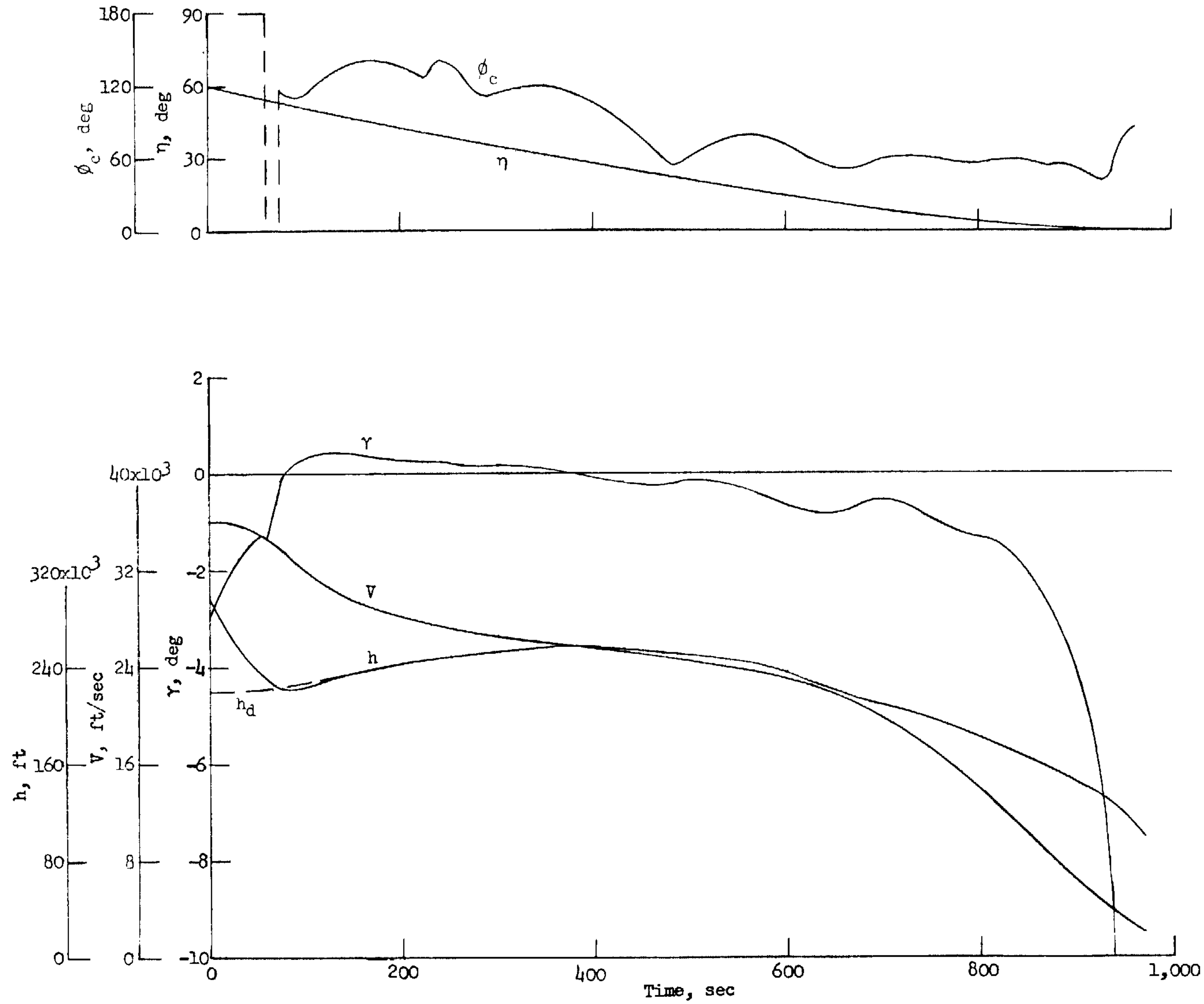
(a) $\gamma_o = -3^o$.

Figure 7.- Time histories of reentry trajectories for reentry angles of -3^o and -5^o where $V_o = 36,000$ ft/sec, $\eta_d = 20^o$, and $C_L/C_D = 0.5$.



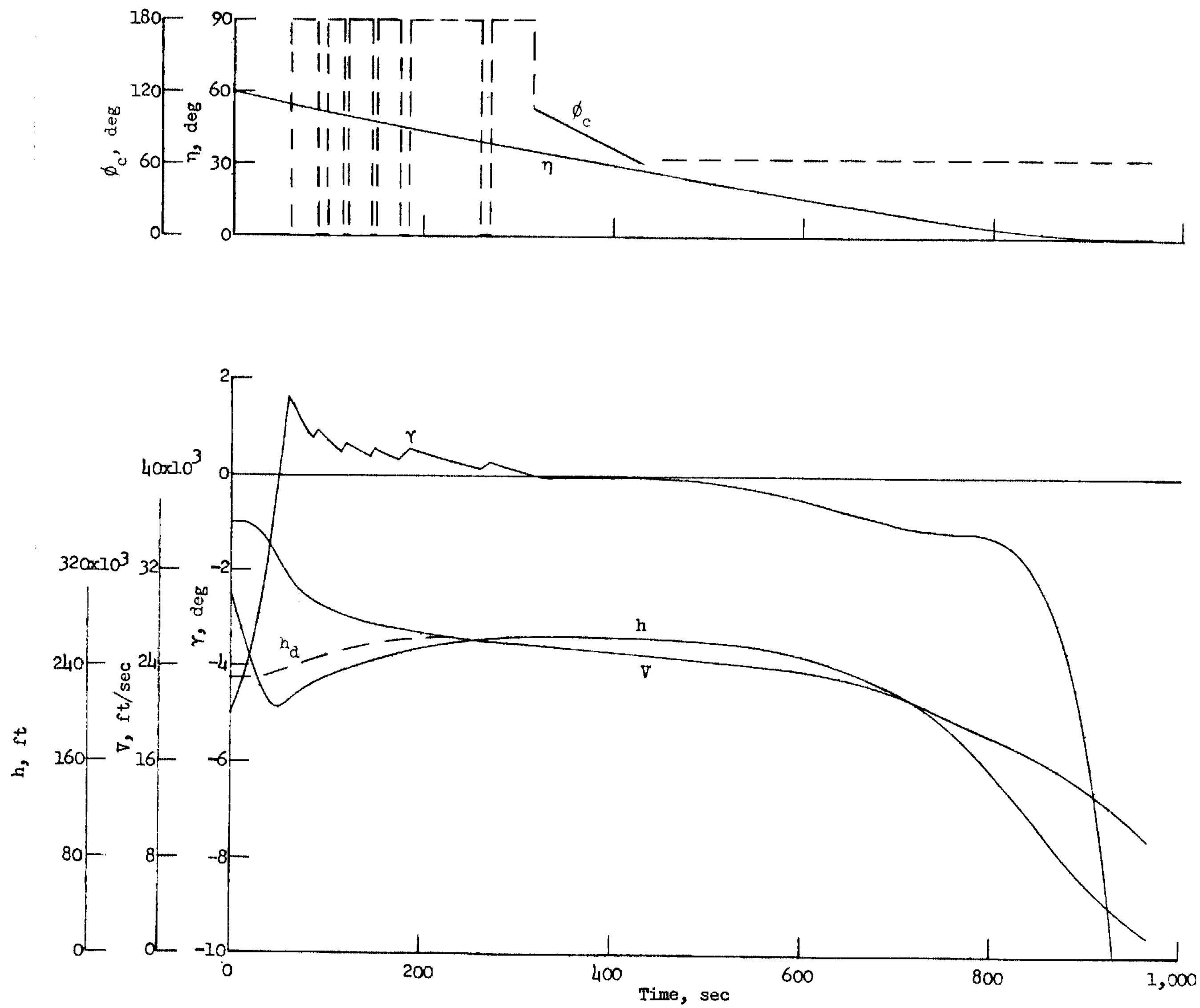
(b) $\gamma_o = -5^o$.

Figure 7.- Concluded.



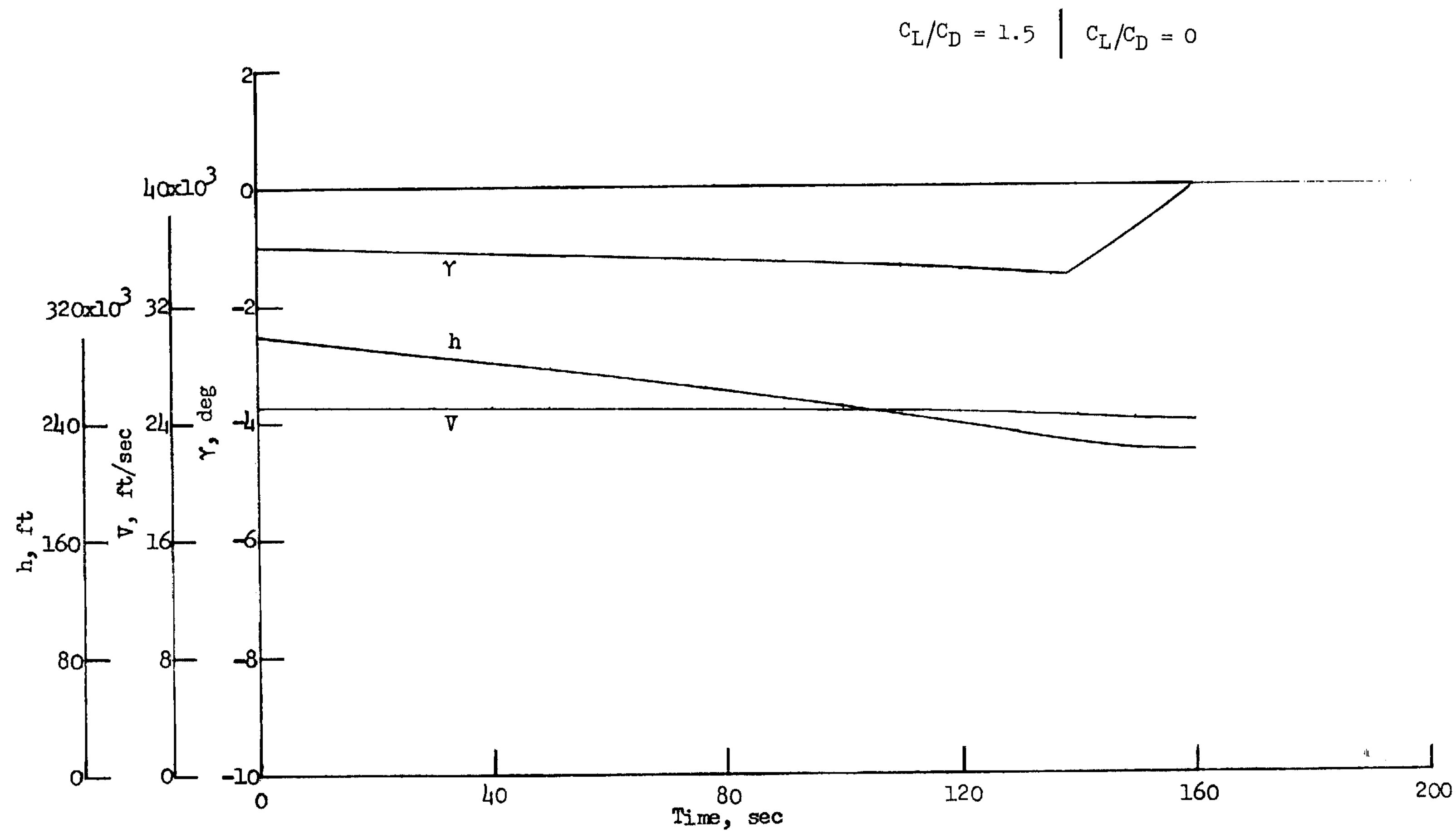
(a) $\gamma_o = -3^o$.

Figure 8.- Time histories of reentry trajectories for reentry angles of -3^o and -5^o where $V_o = 36,000$ ft/sec, $\eta_d = 60^o$, and $C_L/C_D = 0.5$.



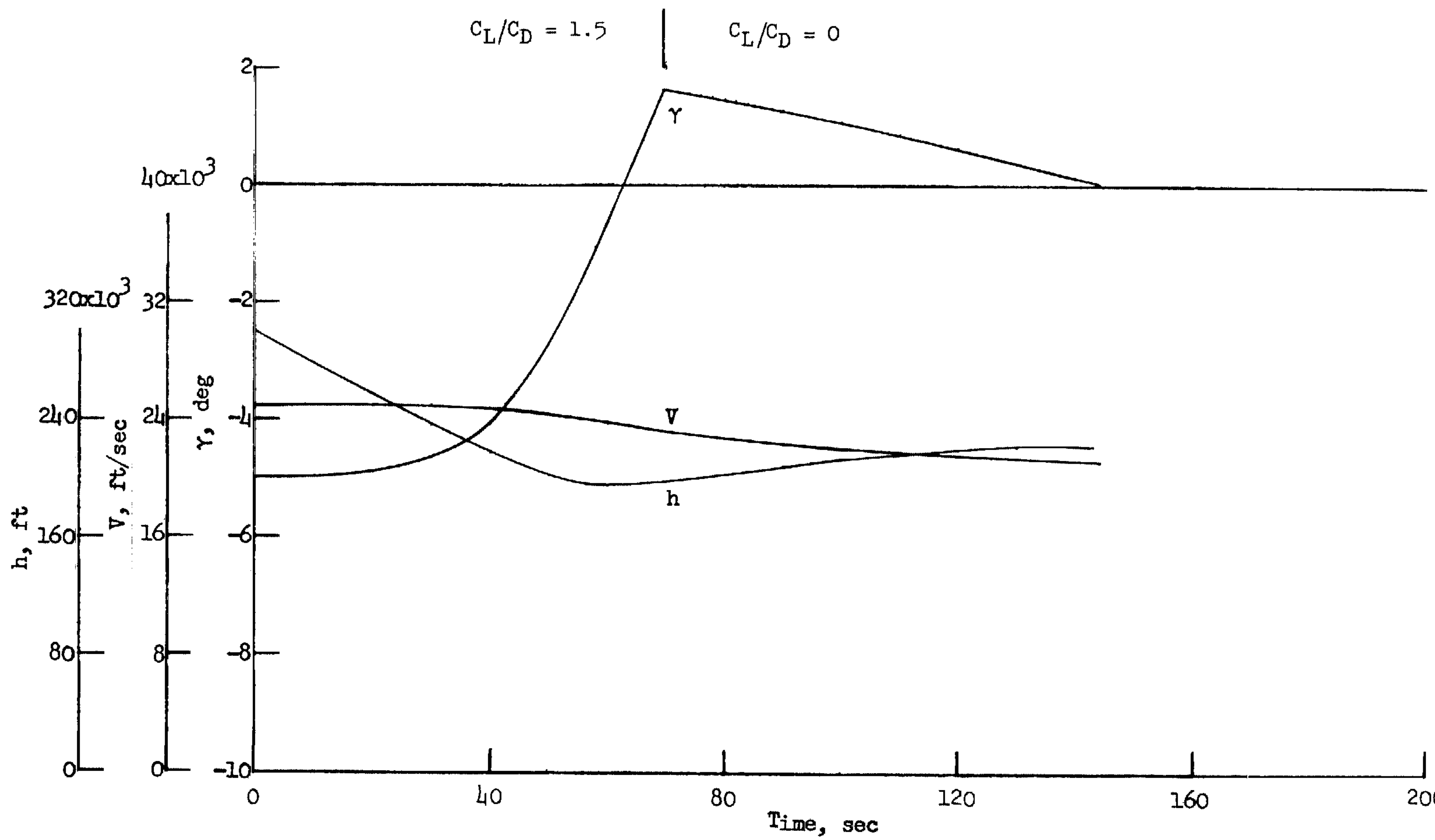
(b) $\gamma_a = -5^{\circ}$.

Figure 8.- Concluded.



(a) $\gamma_o = -2.5^o$.

Figure 9.- Time histories of dive phase of a reentry for a high-lift vehicle where the reentry angles were -2.5^o and -5^o. V_o = 25,000 ft/sec; h_o = 300,000 feet; h_d = 300,000 ft; $C_L/C_D = 1.5$.



(b) $\gamma_o = -5^o$.

Figure 9.- Concluded.

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